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Optimal Multi-Vehicle Type Transit Timetabling and Vehicle Scheduling

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Abstract

The public-transport (transit) operation planning process commonly includes four basic activities, usually performed in sequence: network design, timetable development, vehicle scheduling, and crew scheduling. This work addresses two activities: timetable development and vehicle-scheduling with different vehicles types. Alternative timetables are constructed with either even headways, but not necessarily even passenger loads or even average passenger loads, but not even headways. A method to construct timetables with the combination of both even-headway and even-load concepts is developed for multi-vehicle sizes. The vehicle-scheduling problem is based on given sets of trips and vehicle types arranged in decreasing order of vehicle cost. This problem can be formulated as a cost-flow network problem with an NP-hard complexity level. Thus, a heuristic algorithm is developed. A few examples are used as an expository device to illustrate the procedures developed.
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Keywords: Public-transit timetables; Even heaways; Even Loads; Multi-Vehicle type; Vehicle Scheduling

1. Introduction

The public-transport (transit) operation planning process commonly includes four basic activities, usually performed in sequence: (1) network route design, (2) timetable development, (3) vehicle scheduling, and (4) crew scheduling (Ceder 2007). Figure 1 shows the systematic decision sequence of these four planning activities. The output of each activity positioned higher in the sequence becomes an important input for lower-level decisions. Clearly the independence and orderliness of the separate activities exist only in the diagram; i.e., decisions made further down the sequence will have some effect on higher-level decisions. It is desirable, therefore, that all four activities be planned simultaneously in order to exploit the system's capability to the greatest extent and maximize the system's productivity and efficiency. Occasionally the sequence in Figure 1 is repeated; the required feedback is incorporated over time. However since this planning process, especially for medium to large fleet sizes, is extremely

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cumbersome and complex, it requires separate treatment for each activity, with the outcome of one fed as an input to the next.

This work focuses on two activities: timetable development and vehicle scheduling with the consideration different vehicle types. The aim of public timetables is to meet general public transportation demand. This demand varies during the hours of the day, the days of the week, from one season to another, and even from one year to another. It reflects the business, industrial, cultural, educational, social, and recreational transportation needs of the community. The purpose of this activity, then, is to set alternative timetables for each transit route in order to meet variations in public demand. Alternative timetables are determined on the basis of passenger counts, and they must comply with service-frequency constraints. The vehicle-scheduling activity in Figure 1 is aimed at creating chains of trips; each is referred to as a vehicle schedule according to given timetables. This chaining process is often called vehicle blocking (a block is a sequence of revenue and non-revenue activities for an individual vehicle). A transit trip can be planned either to transport passengers along its route or to make a deadheading trip in order to connect two service trips efficiently.

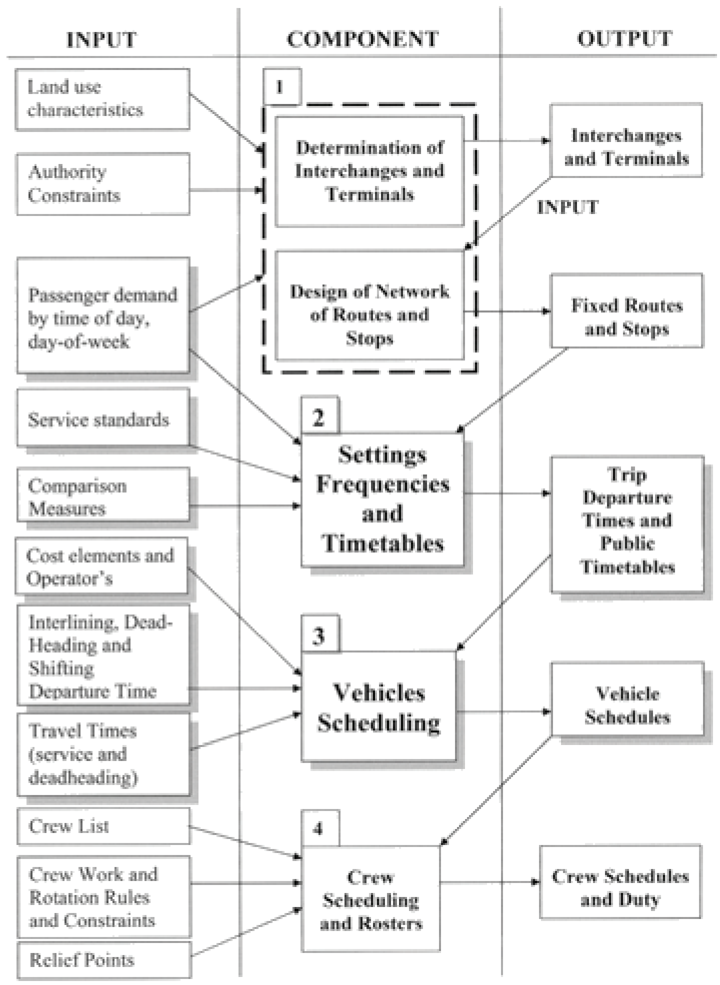

Figure 1. Functional diagram of a common transit-operation planning process

The vehicle-scheduling task described in Figure 1 usually considers only one type of transit vehicle. In practice, however, more than one type is used; e.g., a bus operation may employ minibuses, articulated and double-decker buses, and standard buses with varying degrees of comfort and different numbers of seats. Commonly the consideration of vehicle type in transit-operations planning involves two considerations: first, determining the suitable or optimal vehicle size; second, choosing vehicles with different comfort levels, depending on trip characteristics. Certainly a multi-criteria effort may treat both considerations simultaneously, but this is seldom done in practice. The issue of what vehicle type to consider arises when purchasing a vehicle or a fleet of vehicles, an undertaking that is not performed frequently. The purpose of this work is two-fold. First, to insert the variable of vehicle size and construct efficient timetables; that is, to approach both even headways and even loads. Second, to address a minimum-cost vehicle-scheduling problem, while taking into account the association between the characteristics of each trip (urban, peripheral, inter-city, etc.) and the vehicle type required for the particular trip. This means complying with a certain level of service for that trip: degree of comfort, seat availability, and other operational features.

2. Concise Literature Review

This section covers concisely the literature of the two activities: timetable development and vehicle scheduling.

2.1. Timetabling

The problem of finding the best dispatching policy for transit vehicles on fixed routes has a direct impact on constructing timetables. This dispatching-policy problem, which has been dealt with quite extensively in the literature, can be categorized into four groups: (1) models for an idealized transit system, (2) simulation models, (3) mathematical programming models, and (4) data-based models.

The first group, idealized transit systems, was investigated by, for example, Newell (1971), De Palma and Lindsey (2001), and Wirasinghe (2003). Newell (1971) assumed a given passenger-arrival rate as a smooth function of time, with the objective of minimizing total passenger waiting time. De Palma and Lindsey (2001) develop a method for designing an optimal timetable for a single line with only two stations. Wirasinghe (2003) considered the average value of a unit waiting time per passenger (C_1) and the cost of dispatching a vehicle (C_2) to show that the passenger-arrival rate in Newell's square root formula is multiplied by ($C_1/2C_2$).

In the second group, simulation models were studied by, for example, Adamski (1998), and Dessouky et al. (1999). Adamski (1998) employed a simulation model for real–time dispatching control of transit vehicles while attempting to increase the reliability of service in terms of on-time performance. Dessouky et al. (1999) used a simulation analysis to show that the benefit of knowing the location of the bus was most significant when the bus was experiencing a significant delay.

In the third group, mathematical programming methods have been proposed, for example by Furth and Wilson (1981), and Gallo and Di-Miele (2001). Furth and Wilson sought to maximize the net social benefit, consisting of ridership benefit and waiting-time saving, subject to constraints on total subsidy, fleet size, and passenger-load levels. Gallo and Di-Miele (2001) produced a model for the special case of dispatching buses from parking depots. Their model is based on the decomposition of generalized assignments and matching sub-problems. In the fourth and last group, the data-based models described in this work are based on Ceder (1986, 2007).

2.2. Vehicle Scheduling

Vehicle scheduling refers to the problem of determining the optimal allocation of vehicles to carry out all the trips in a given transit timetable. A chain of trips is assigned to each vehicle including possible deadheading (DH) or empty trips. The number of feasible solutions to this problem is extremely high, especially in the case in which the vehicles are based in multiple depots. Much of the focus of the literature is, therefore, on computational issues.

Löbel (1999) discussed the multiple-depot vehicle scheduling problem and its relaxation into a linear programming formulation that can be tackled using the branch-and-cut method. Freling et al. (2001) discussed the case of single-depot with identical vehicles, concentrating on quasi-assignment formulations and auction algorithms. Huisman et al. (2004) proposed a dynamic formulation of the multi-depot vehicle scheduling problem. The traditional, static vehicle scheduling problem assumes that travel times are a fixed input that enters the solution procedure only once; the dynamic formulation relaxes this assumption by solving a sequence of optimization problems for shorter periods.

Recent contributions noted are of Zak (2009) who developed a multi-criteria optimization method for bus scheduling using two criteria from the passenger perspective and two – from the operator perspective with satisfactory results. In addition studies of integrated multi-depot vehicle and crew scheduling can be found by Borndorfer et al. (2008), Gintner et al. (2008) and Mesquita et al. (2009) that use integer mathematical formulation, relaxation methods and heuristics to overcome the basic NP-Hard problem. Other related recent studies search for relief opportunities to approach optimal crew scheduling at transit stops where the drivers can be switched. Such studies are presented by Kwan and Kwan (2007) and Laplagne et al (2009).

3. Background on Even-Headway and Even-Load Timetables

Procedures to construct alternative timetables appear in Ceder (1986, 2007). The automated timetables are constructed with either even headways, but not necessarily even loads on board individual vehicles at the peak-load section, or even average passenger loads on board individual vehicles, but not even headways. Average even loads on individual vehicles can be approached by relaxing the evenly spaced headways pattern (through a rearrangement of departure times).

3.1. Even-Headway Timetables with Smooth Transitions

One characteristic of existing timetables is the repetition of the same headway in each time period. However, a problem facing the scheduler in creating these timetables is how to set departure times in the transition segments

between adjacent time periods. A common headway smoothing rule in the transition between time periods is to use an average headway. Many transit agencies employ this simple rule, but it may be shown that it can result in either undesirable overcrowding or underutilization. For example, consider two time periods, 06:00-07:00 and 07:00-08:00, in which the first vehicle is predetermined to depart at 06:00. In the first time period, the desired occupancy (desired load) is 50 passengers, and in the second 70 passengers. The observed maximum demand to be considered in these periods is 120 and 840 passengers, respectively. These observed loads at a single point are based on the uniform passenger-arrival-rate assumption. The determined frequencies are 120/50 = 2.4 vehicles and 840/70 = 12 vehicles for the two respective periods, and their associated headways are 25 and 5 minutes, respectively. If one uses the common average headway rule, the transition headway is (25 + 5)/2 = 15 minutes; hence, the timetable is set to 06:00, 06:25, 06:50, 07:05, 07:10, 07:15,…, 07:55, 08:00. By assuming a uniform passenger arrival rate, the first period contributes to the vehicle departing at 07:05 the average amount of (10/25)·50 = 20 passengers at the Max load point; the second period contributes (5/5)·70 = 70 passengers. Consequently, the expected load at the Max load point is 20 + 70 = 90, a figure representing average overcrowding over the desired 70 passengers after 7:00. Certainly, the uniform arrival-rate assumption does not hold in reality. However, in some real-life situation (e.g., after work and school dismissals), the observed demand in 5 minutes can be more than three times the observed demand during the previous 10 minutes, as is the case in this example. In order to overcome this undesirable situation, the following principle may be employed.

Principle 1: Establish a curve representing the cumulative (non-integer) frequency determined versus time. Move horizontally for each departure until intersecting the cumulative curve, and then vertically; this will result in the required departure time.

Proposition 1: Principle 1 provides the required evenly spaced headways with a transition load approaching the *average desired occupancies* of d_{oj} and $d_{o(j+1)}$ for two consecutive time periods, j and j+1.

Proof: Figure 2 illustrates Principle 1 using an example (Ceder, 2007) of determined frequencies of 2.68 and 3.60 vehicles/hour for 6:00-7:00 and 7:00-8:00 periods, respectively. Because the slopes of the lines are 2.68 and 3.60 for j = 1 and j = 2, respectively, the resultant headways are those required. The transition load is the load associated with the 7:05 departure, which consists of arriving passengers during 16 minutes for j = 1, and of arriving passengers during 5 minutes for j = 2. Therefore, (16/22)·50 + (5/17)·60 = 54 approximately. This transition load is not the exact average between d_{o1} = 50 and d_{o2} = 60, because departures are made in integer minutes. That is, the exact determined departure after 7:00 is (3-2.68)·60/3.60 = 5.33 minutes. Inserting this value, instead of the 5 minutes mentioned above, yields a value that is closer to the exact average. Basically, the proportions considered satisfy the proof-by-construction of Proposition 1.

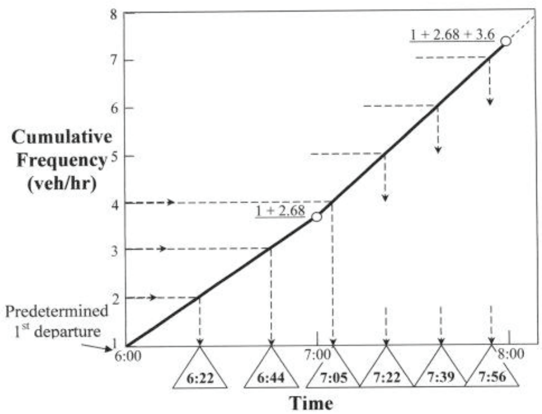

Figure 2. Determination of evenly spaced headways

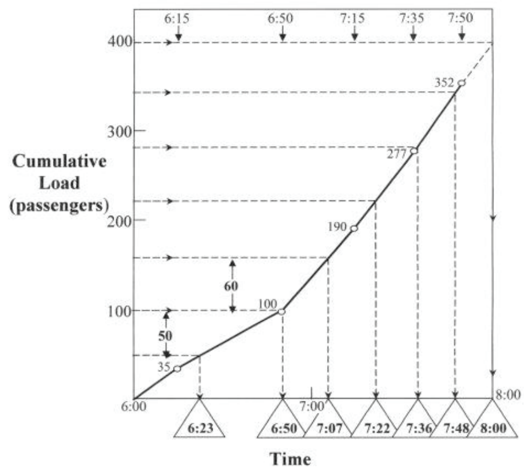

Figure 3 Determination of departure times with even load

3.2. Even-Load Timetables

A simple example is presented here to illustrate the underlying load-balancing problem. Consider an evenly spaced headway timetable in which vehicles depart every 20 minutes between 07:00 and 08:00; i.e., at 07:20, 07:40, and 08:00. The observed load data consistently show that the second vehicle, which departs at 07:40, has significantly more passengers than the third vehicle. The observed (average) Max load during this 60-minute period is 150 passengers, and the desired occupancy is 50 passengers. Hence, based on max-load consideration, three vehicles are required to serve the demand as in the case of the evenly spaced headways timetable. The average observed loads at the hourly Max load point on the three vehicles are 50, 70, and 30 passengers, respectively. Given that these average loads are consistent, then the transit agency can adjust the departure times so that each vehicle has a balanced load of 50 passengers on the average at the hourly Max load point. The assumption of a uniform passenger- arrival rate results in 70/20 = 3.5 passengers/minute between 07:20 and 07:40, and 30/20 = 1.5 passengers/minute between 07:40 and 08:00. If the departure time of the second vehicle is shifted by X minutes backward (i.e., an early departure), then the equation 3.5X = 70-50 yields the balanced schedule, with X = 5.7 ≈ 6 minutes, or departures at 07:20, 07:34, and 08:00. The third departure will add this difference of 20 passengers at the hourly Max load point. The even-headway setting assures enough vehicles to accommodate the hourly demand, but it cannot guarantee balanced loads for each vehicle at the peak point. In order to avoid this imbalanced situation, the following principle should be exploited.

Principle 2: Construct a curve representing the cumulative loads observed on individual vehicles at the hourly Max load points. Move horizontally per each d_{oj} for all j, until intersecting the cumulative-load curve, and then vertically; this results in the required departure times.

Proposition 2: Principle 2 results in departure times such that the average Max load on individual vehicles at the hourly j^{th} Max load point approaches the desired occupancy $d_{oj.}$

Proof: Figure 3 illustrates Principle 2 using loading data at the Max load point of individual vehicles observed. The derived departure times are unevenly spaced to obtain even loads at the Max load points for j = 1 and for j = 2. These even loads are constructed on the cumulative curve to approach d_{o1}=50 and d_{o2}=60. If we assume a uniform passenger-arrival rate between each two observed departures, it can be shown that the load of the first derived departure (6:23) consists of the arrival rate between 6:00 and 6:15 (35/15 = 2.33) and the rate between 6:15 and 6:50 (65/35 = 1.86). Thus, $2.33 \cdot 15 + 1.86 \cdot 8 \approx 50$. In the transition between j = 1 and j = 2, the value of $d_2 = 60$ is considered, because the resultant departure comes after 7:00. The load of the vehicle departing at 7:07 at its hourly

Max load point, is simply 17·(90/25) = 61.2 from rounding off the departure time to the nearest integer. That is, (10+y)·(90/25) = 60 results in y = 6.67 minutes. This completes the proof-by-construction of Proposition 2.

4. Combining Even-Load and Even-Headway Timetables Using Different Vehicle Types

4.1. Introduction and Background

As is mentioned above currently, in practice, bus timetables are commonly based on even-headway departures. The even-headway feature for a given time period reduces the flexibility of the scheduler to accommodate fluctuations in demand within this period. This lack of flexibility may result in undesirable operational scenarios such as overcrowding or vehicles running almost empty. Uneven loads lead to either passenger discomfort, in case of overcrowding, or uneconomical and energy inefficient operation of the vehicles in the latter case (Spicher 2004, Potter 2003). However, the even-load timetables can lead to long and exceedingly irregular headways and thus to increase the waiting times for passengers arrived randomly. To overcome the disadvantages of both approaches (even-headway and even-load) this section aims at making the transit service more attractive by creating timetables using different types and sizes of vehicles to achieve even headways with minimum uneven loads at the max-load point(s). The quality of the timetables will be based on two criteria: load discrepancy on the vehicles from a desirable load, and time discrepancy from a desirable headway.

The load discrepancy criterion serves as an indicator of how the actual max-point load on the buses deviates from a desired occupancy level (e.g., number of seats) including a buffer for demand fluctuations. The time discrepancy criterion provides information about how evenly the headways are spaced in the final timetable based on the calculation of average waiting time per passenger.

4.2. Methodology

As is shown in Ceder (2007) the expected waiting time for randomly arriving passengers, Wt, can be calculated as follows:

$$\text{Wt} = \frac{\text{average headway}}{2} * \left(1 + \frac{\text{Variance of headway}}{(\text{average headway})^2}\right)$$

This formula shows that the expected waiting time is minimal for even headways. For evenly distributed arrivals the best headway for a single type of vehicle can be easily found by dividing the total number of passengers observed at the Max load point by the desired passenger load. However in reality arrivals show fluctuations and are far from being evenly distributed. Consequently the proposed methodology uses a heuristic procedure to determine the optimal headway for a fluctuated demand using different vehicle sizes. For convenience the use of seat capacity determines the size of the vehicle; this seat capacity term is compared with the desired passenger load (desired occupancy or load factor).

In each step, of the heuristic procedure, buses are assigned departure times based on an even-headway timetable such that the Max load demand is satisfied. Having different vehicle sizes available, the choice of vehicle is sometimes ambiguous. Therefore, three main strategies are considered:

Strategy C1: Minimizing the size of the bus by assigning the largest bus size amongst all available buses such that its seat capacity is less than or equal to the average observed (hence expected) passenger load, That is, for a departure at time *t* with an expected load of *L(t)*, $S_{k-1} \leq L(t) < S_k$ where S_k and S_{k-1} are two following (w.r.t. to size) available bus sizes, the assigned bus is with the seat capacity of S_{k-1}. This may imply overcrowding on certain vehicles.

Strategy C2: Maximizing the size of bus by assigning the smallest bus size amongst all available buses such that its seat capacity is greater than or equal to the average observed (hence expected) passenger load. That is, for a departure at time *t* with an expected load of *L(t)*, $S_{k-1} < L(t) \leq S_k$ the assigned bus is with the seat capacity of S_k. This may imply empty seats on certain vehicles.

Strategy C3: Selecting the vehicle, whose size is closest to the average observed demand, per vehicle, at the Max load point. That is, for a departure at time *t* with an expected load of *L(t)*, select the bus sixe of seat capacity S_k such that $|s_k - L(t)|$ is minimal for all *k*. This can result in either overcrowding or running empty seats.

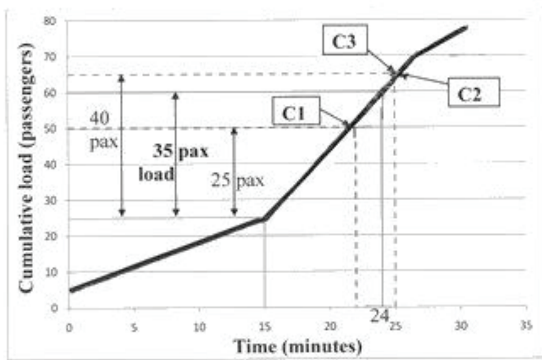

Figure 4. Strategies for selection of vehicle type

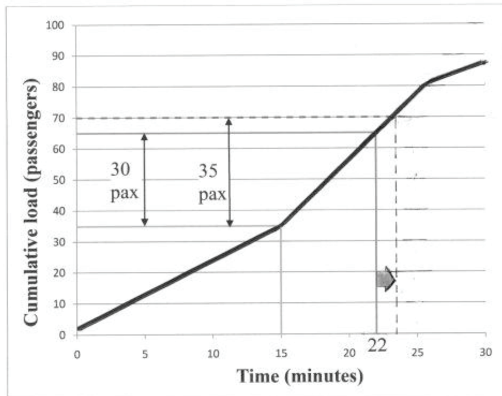

Figure 5. Shifting procedure

Figure 4 illustrates the three strategies on a cumulative observed load of individual buses at the Max load point. In this example, the examined departure is at $t = 24$ (after the beginning of the time period). The previous departure relates to $t = 15$ with 25 passengers on board. The load associated with the examined departure is: *L(24)*=60-25=35 passengers (pax). Vehicle size available are $S_1 = 25$ and $S_2 = 40$ seat capacity. Based on *Strategy C1* bus S_1 will be selected, and bus S_2 for *Strategy C2* and *Strategy C3* as is shown in Figure 4.

Although those strategies allow the creation of timetables with even headways, they might result in uneven loads even for the different bus sizes. Hence, there is a benefit of shifting departure times away from the even headway to approach a better balance of on-board passenger load. Figure 5 illustrates an example in which there is a departure at $t = 15$ and with even headway of 7 minutes the next departure is at $t = 22$. However *L(22)*=30 passengers and the assigned bus has a seat capacity of 35 passengers. The shifting to the right in Figure 5 (see arrow) will make the headway 10 minutes, but will provide a more efficient service without harming in average sense the quality of service.

The research examines different shifting policies with a total of 21 different combinations of strategies and shifting for constructing a set of feasible timetables. From this set the optimal timetable will be determined.

4.3. Results

The methodology developed has been applied to several sets of real data from Auckland, New Zealand. It refers to a city bus line which is currently running with even headways and with only one type of vehicle with 36 seats. The heuristic-based process provides several non-dominated sets of departure times; the Pareto frontier of these results exhibits significant improvement over the current set of departures. That is, the original passenger-load discrepancy from the desired load can be reduced from 38% to a discrepancy between 0% - 15% while preserving the time deviation from the determined even headway in the range of 0% - 7%. The work discusses the results in detail and also provides a sensitivity analysis

5. Vehicle Scheduling with Multi-Vehicle Types

In the vehicle scheduling activity in Figure 1 the scheduler's task is to list all daily chains of trips (some deadheading) for each vehicle so as to ensure the fulfillment of both timetable and operator requirements (refueling, maintenance, etc.). The major objective of this activity is to minimize the number of vehicles required in case of a single type, and minimum cost – for multi-type vehicles. The technique used is a step function termed deficit function, as it represents the deficit number of vehicles required at a particular terminal in a multi-terminal transit system (Ceder and Stern 1981, Ceder 2007). The value of embarking on such a technique is to achieve the greatest saving in number of vehicles while complying with passenger demand. This saving is attained through a procedure incorporating a man/computer interface allowing the inclusion of practical considerations that experienced transit schedulers may wish to introduce into the schedule.

5.1. Background on the deficit function

Following is a description of a step function approach described first by Ceder and Stern (1981) and Ceder (2007), for assigning the minimum number of vehicles to allocate for a given timetable. The step function is termed deficit function (DF), as it represents the deficit number of vehicles required at a particular terminal in a multi-terminal transit system. That is, DF is a step function that increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. To construct a set of deficit functions, the only information needed is a timetable of required trips. The main advantage of the DF is its visual nature. Let $d(k,t,S)$ denote the DF for the terminal k at the time t for the schedule S. The value of $d(k,t,S)$ represents the total number of departures minus the total number of trip arrivals at terminal k, up to and including time t. The maximal value of $d(k,t,S)$ over the schedule horizon $[T_1,T_2]$ is designated $D(k,S)$.

Let t_s^i and t_e^i denote the start and end times of trip i, $i \in S$. It is possible to partition the schedule horizon of $d(k,t,S)$ into sequence of alternating hollow and maximal intervals. The maximal intervals $[s_i^k, e_i^k], i = 1,...,n(k)$ define the interval of time over which $d(k,t)$ takes on its maximum value. Note that the S will be deleted when it is clear which underlying schedule is being considered. Index i represents the *i*th maximal intervals from the left and $n(k)$ represents the total number of maximal intervals in $d(k,t)$. A hollow interval H_l^k, l=0,1,2,…,$n(k)$ is defined as the interval between two maximal intervals including the first hollow from T_1 to the first maximal interval, and the last hollow-from the last interval to T_2. Hollows may consist of only one point, and if this case is not on the schedule horizon boundaries $(T_1$ or $T_2)$, the graphical representation of $d(k,t)$ is emphasized by clear dot.

If the set of all terminals is denoted as T, the sum of $D(k)$ for all $k \in T$ is equal to the minimum number of vehicles required to service the set T. This is known as the fleet size formula. Mathematically, for a given fixed schedule S:

$$D(S) = \sum_{k \in T} D(k) = \sum_{k \in T} \max_{t \in [T_1,T_2]} d(k,t) \qquad (1)$$

where $D(S)$ is the minimum number of buses to service the set T.

When deadheading (DH) trips are allowed, the fleet size may be reduced below the level described in Eq. 1. Ceder and Stern (1981) described a procedure based on the construction of a unit reduction DH chain (URDHC), which, when inserted into the schedule, allows a unit reduction in the fleet size. The procedure continues inserting URDHCs until no more can be included or a lower boundary on the minimum fleet is reached. The lower boundary $G(S)$ is determined from the overall deficit function defined as $g(t,S) = \sum_{k \in T} d(k,t,S)$ where $G(S) = \max_{t \in [T_1,T_2]} g(t,S)$. This function represents the number of trips simultaneously in operation. Initially, the lower bound was determined to be the maximum number of trips in a given timetable that are in simultaneous operation over the schedule horizon. Stern and Ceder (1983) improved this lower bound, to $G(S') > G(S)$ based on the construction of a temporary timetable, *S'*, in which each trips is extended to include potential linkages reflected by DH time consideration in *S*. This lower bound was even further improved by Ceder (2002) by looking into artificial extensions of certain trip-arrival points without violating the generalization of requiring all possible combinations for maintaining the fleet size at its lower bound.

In addition, it is worth mentioning the NT (Next Terminal) selection rule and the URDHC routines. The selection of the next terminal in attempting to reduce its maximal deficit function may rely on the basis of garage capacity violation, or otherwise on a terminal whose first hollow is the longest. The rationale here is to try to open up the greatest opportunity for the insertion of the DH trip. Once a terminal k is selected, the algorithm searches to reduce D(k) using the URDHC routines. Then all the d(k,t) are updated and the NT rule is again applied. In the URDHC routines there are four rules: R=O for inserting the DH trip manually in a conversational mode; R=1 for inserting the candidate DH trip which has the minimum travel time; R=2 for inserting a candidate DH trip whose hollow starts farthest to the right; and R=3 for inserting a candidate DH trip whose hollow ends farthest to the right. In the automatic mode (R=1,2,3) if a DH trip cannot be inserted and the completion of a URDHC is blocked, the algorithm backs up to a DH candidate list and selects the next DH candidate on that list.

5.2. Optimization framework of vehicle-type scheduling problem

The problem, entitled the vehicle-type scheduling problem (VTSP), is based on given set S of trips (schedule) and set M of vehicle types. The set M is arranged in decreasing order of vehicle cost so that if $u \in M$ is listed above $v \in M$, it means that $c_u > c_v$, where c_u, c_v are the costs involved in employing vehicle types u and v, respectively. Each trip $i \in S$ can be carried out by vehicle type $u \in M$ or by other types listed prior to u in the above-mentioned order of M.

The problem can be formulized as a cost-flow network problem, in which each trip is a node and an arc connects two trips if, and only if, it is possible to link them in a time sequence with and without DH connections. On each arc (i,j), there is a capacity of one unit and an assigned cost C_{ij}. If the cost of the lower-level vehicle type associated with trip i is higher than the cost of the vehicle type (even if of a lower level) required for trip j, then $C_{ij} = c_i$. That is, $C_{ij} = \max(c_i, c_j)$. The use of such a formulation was implemented by Costa et al. (1995), who employed three categories of solutions: (a) a multi-commodity network flow; (b) a single-depot vehicle- scheduling problem; and (c) a set-partitioning problem with side constraints. The mixed-integer programming of these problems is known to be NP-complete as may be seen, for example, in Bertossi et al. (1987). The math-formulation concepts for the third category are further explained by Ceder (2007, 2011).

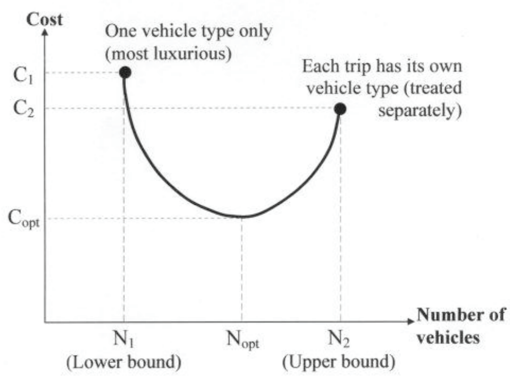

Figure 6. Optimization framework

Because of the complexity involved in reaching an optimal solution for a large number of trips in S, a heuristic method is considered a more practical approach. The heuristic procedure developed is called the VTSP algorithm. It begins by establishing lower and upper bounds on fleet size. The upper bound is attained by creating different DFs, each associated with a certain vehicle type $u \in M$, which includes only the trips whose lower-level required vehicle type is u. Certainly, this scheduling solution reflects a high cost, caused by the large number of vehicles demanded. The lower bound on the fleet size is attained by using only one vehicle type: the most luxurious one with the highest cost that can clearly carry out any trip in the timetable.

Between these bounds on fleet size, the procedure searches for the best solution, based on the properties and characteristics of DF theory. The algorithm VTSP developed is heuristic in nature while incorporating all DF components. It is detailed in Ceder (2007, 2011). Because of the graphical features associated with DF theory, the algorithm can be applied in an interactive manner or in an automatic mode, along with the possibility of examining its intermediate steps. The following is a general description of algorithm VTSP in a stepwise manner:

5.3. Example

An example is provided for the general demonstration of the VTSP algorithm. The example is illustrated in Figure 7, and consists of 8 trips, three terminals (*a,b,c*), and three types of vehicles, with the cost of 12, 5, and 3 cost-units, respectively. Figure 7(a) presents the simple network of the routes, in which the DH travel time between each two terminals is 20 minutes. The timetable and trip travel times are shown in Figure 7(b) according to vehicle type. The DFs of algorithm VTSP for the example are depicted in Figure 7(c); all trips are served initially by the same vehicle type (Type 1).

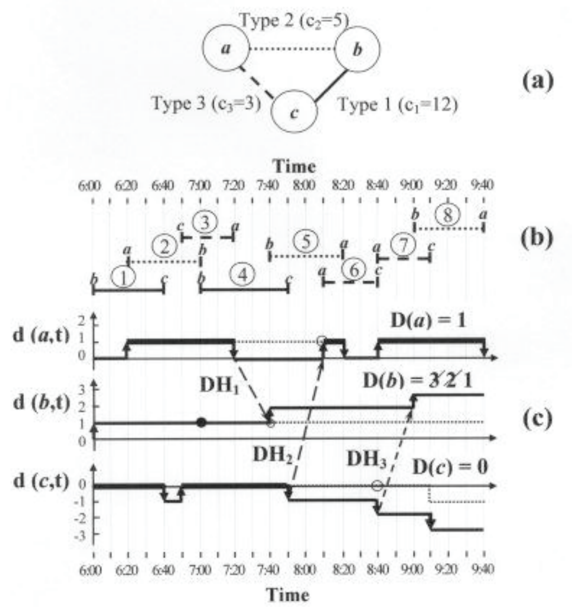

Figure 7. Network, schedule and DFs of the example

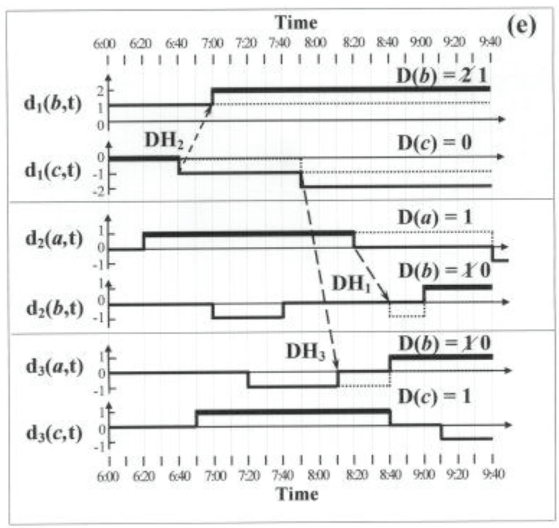

Figure 8. Min-cost solution

For inserting a DH trip, the NT rule (the first hollow is the longest) is applied; this results in the selection of terminal *b*. The URDHC procedure with R=2 (furthest start of a hollow) then results in three DH trips, in which DH_2 is used for the level of D(*a*). Two vehicle chains are then created using the FIFO [1-3-DH_1-5-7] and [2-4-DH_2-6-DH_3-8], and the total cost is C_1=24.

Algorithm VTSP continues with treating vehicle types separately. The maximum DF of Types 1 and 2 are reduced by one, using DH_1 and DH_2, respectively; the number of Type 3 vehicles remains same. Thus, $N_2 = 1 + 1 + 2 = 4$, and the four following chains are derived by using the FIFO rule: [1-DH_1-4] (vehicle type 1), [2-5-DH_2-8] (vehicle type 2), [3-7], and [6] (vehicles of Type 3); this results in a total cost of $C_2 = 23$.

The next step in algorithm VTSP compares $N_1 = 2$ with $N_2 = 4$, and then moves to the next step. Figure 8 (marked as part 'e' of the process) illustrates the process of this step which again applies the NT rule and the URDHC procedure with R=2 (furthest start of a hollow), but this time with the possibility of inserting any DH trip from a DF with a more expensive vehicle type to a DF with a less expensive type. The first terminal selected is *b,* based on $d_2(b,t)$, from which DH_1 is determined from terminal *a*. The DFs are then updated and the next terminal again *b,* but related to $d_1(b,t)$; DH_2 is inserted from terminal *c*. We continue with the next selected terminal *c,* based on $d_3(c,t)$; however, no DH trip can be inserted into its maximum-interval starting point, including. Thus, *a* is selected next, based on $d_3(a,t)$, and DH_3 is inserted to arrive from *c,* based on the updated $d_1(c,t)$. This terminates this step and results in the three following (FIFO) chains: [1-DH_2-4-DH_3-6] (vehicle type 1), [2-5-DH_1-8] (vehicle Type 2), and [3-7] (vehicle Type 3), with a total cost of 12 + 5 + 3 = 20.

6. Concluding remark

This work addresses two transit operations-planning activities: timetable development and vehicle-scheduling with different vehicles types. Alternative timetables are constructed with either even headways, but not necessarily even passenger loads or even average passenger loads, but not even headways. A method to construct timetables with the combination of both even-headway and even-load is developed for multi-vehicle sizes. The vehicle-scheduling problem is based on given sets of trips and vehicle types arranged in decreasing order of vehicle cost using the deficit-function theory. Further analysis of the deficit function theory is to include possible shifting in departure times within bounded tolerances; this was introduced by Ceder and Stern (1985). Basically, the shifting criteria is based on a defined tolerance time $[t_s^i - \Delta_a^i, t_s^i + \Delta_d^i]$ where Δ_a^i is the maximum advance of the trip scheduled departure time (early departure) and Δ_d^i is the maximum delay allowed (late departure). This shifting analysis is included in the VSTP procedures described by Ceder (2011). Finally, it is believed that prudent use of transit vehicles by the consideration of different vehicle sizes can help making the need for travel more economical, thus saving resources.

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