



# Measurement of frozen soil–pile dynamic properties: A system identification approach

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## ABSTRACT

A free-decay response signal approach is proposed for reliable estimation of frozen soil–pile dynamic properties of a partially embedded pile. Theoretical consideration and approximations are given for the free vibration of pile structures, with 20% of the pile cantilevered aboveground and the remaining embedded in Fairbanks silt. Winter measurements were taken for free-decay response of the pile. A comprehensive frequency spectrum analysis that includes fast Fourier transform, power spectrum density, and spectrograms is used to evaluate the system's vibration properties. Empirical mode decomposition is then used to decompose the signal to extract specific components for parameter identification. Results show that the response exhibits time-variant and nonlinear characteristics in the time–frequency domain. Experimental data show that the tested system exhibits weak nonlinearity. Dominant system parameters, used to characterize frozen soil–pile interactions, are identified. Two dominant frequencies for a stiff pile embedded 20 ft (6.096 m) deep in frozen Fairbanks silt are 97 Hz and 1080 Hz. Damping was found to be approximately 0.016.

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## 1. Introduction

Dynamic measurements of the vibration spectrum of piles embedded in frozen soils are used to estimate pile integrity or stiffness for deep pile foundations. That is, the vibration spectrum is used to estimate the integrity or interaction stiffness of a pile in deep soil foundations (Chau et al., 2009; Chau and Yang, 2005; Hua et al., 2008; Ku et al., 2003; Maheshwaria et al., 2004; Masoumi et al., 2009; Naggar and Novak, 1995; Ni et al., 2008; Puri and Braja, 1993; Tahghighi and Konagai, 2007; Takewaki, 2005; Teguh, 2008; Xiong and Yang, 2008). When piles in deep soil foundations are subjected to an earthquake, response depends on stiffness and damping. In the past, most studies have focused on the behavior of piles in unfrozen soils. The response of such structures subjected to earthquakes is influenced by the season. Seismic loading can cause strains in the soil to increase to a point where the soil shear modulus and stiffness decrease while damping increases. A change in modal parameters is dependent on structure boundary condition, material deterioration, or damage. Modal parameter identification is a well-known method for system identification and condition monitoring. Traditional methods for modal parameter identification are commonly used to fulfill general identification tests in the laboratory or in well-controlled field tests.

In classical experimental modal analysis, the modal parameters (resonance frequencies, damping ratios, etc.) for a structure are identified via forced excitation experiments. However, for many structures, implementation of measured input is not conveniently available. Impact response measurements in these circumstances are probably the most popular method of modal parameter identification. For a soil–pile structure with little or no lumped mass, the test signals acquired from the soil–pile system tend to be complicated. Many factors—for example, the nonlinearity of the soil–pile structure—affect the captured signal, resulting in nonlinear stiffness and nonlinear damping. A hysteresis condition occurs for a pile and soil that interacts as an inelastic material. For example, when the pile pushes against the soil, a gap will likely form between the soil and the pile at ground line. So, the response is changed by soil hardening or weakening as the soil deforms and a gap occurs between the pile and soil at the ground surface. This interaction affects the soil–pile deformation or stiffness.

Analytical and experimental procedures that account for nonlinear soil behavior are described in the literature (Chau et al., 2009; Chau and Yang, 2005; Hua et al., 2008; Naggar and Novak, 1995; Tahghighi and Konagai, 2007). Generally, pile response under dynamic loads can be analyzed using spring–mass models. Soil springs are obtained from the shear modulus of the soil. Soil nonlinear effects can be accounted for by using strain-dependent values from laboratory shear modulus data. To account accurately for soil nonlinearity, seismic response analysis for the pile foundation should be conducted in the time domain. The proper representation of damping and inertia effects for the adjacent continuum (soil media) is needed, and the effects of plasticity and soil hardening or softening are usually required. Seasonally

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frozen soil causes significant change in the stiffness and damping ratio of the soil–pile system (Xiong and Yang, 2008). Despite previous related research, an effective method does not appear available for describing actual in situ dynamic properties and nonlinear response for frozen soil–structure interaction systems. Therefore, a high level of uncertainty in characterizing frozen soil adjacent to the pile continues, causing the prediction of structure system integrity to be elusive.

We propose an approach to reliable estimation of the dynamic properties of a partially embedded pile in frozen soil. The proposed method relies on using the free-decay response signal of the pile. Theoretical considerations and approximations of the free vibration of the frozen soil–pile structure interaction are presented. Measurements are taken for the free-decay response of the pile, and a comprehensive frequency spectrum analysis is conducted. Conventional spectrum results such as fast Fourier transform (FFT), power spectrum density, and spectrograms are used to give preliminary evaluation of the system's vibrational properties in the context of a linear system. Besides employing conventional approaches for identification of system response, the empirical mode decomposition (EMD) method (Huang et al., 1998, 1999; Yang et al., 2004; Yang and Lei, 1999) is adopted to enhance the characteristics of the testing signal, which is done to improve identification. Empirical mode decomposition is a method of decomposing a nonlinear, nonstationary signal into a series of zero-mean amplitude modulation-frequency modulation (AM-FM) components that represent the characteristic time scale of the observation.

Based on this approach, the nonlinearity of the frozen soil–pile system is estimated. An analysis of the acquired data shows that the tested frozen soil–pile system exhibits weak nonlinearity and that the dominant portion can be approximated by a linear model. A system identification approach is used to extract modal and damping parameters, which are used to characterize the frozen soil–pile interactions under varied conditions. A distinct linear phenomenon between pile and soil is observed in a specific frequency range. Non-linear effects are within a wide frequency range. Bouncing phenomena, caused by the development of ground surface separation (gap) between the frozen soil and the pile, are observed.

## 2. Test setup and measurements

Full-scale pile dynamics tests were conducted on a 16 in. (406 mm) diameter steel-jacketed reinforced concrete pile (Davis, 2010). A 20 ft (6.096 m) pile was imbedded in a soil profile of uniform Fairbanks

silt, with 5 ft (1,524 mm) of the pile exposed aboveground. The horizontal acceleration of the top end of the pile was measured by applying a horizontal impulse load. Fig. 1 is a schematic diagram of the test setup.

Free horizontal vibration tests were conducted by first applying incremental static loads to about 5000 lb (22 kN). During incremental loading, pile strains, displacements, and applied load were monitored. An accelerometer was used to monitor free vibrations that occurred after suddenly removing the applied load using a quick release. An accelerometer was also used to record free-decay response. A data acquisition system was used to store the data. The resultant information was transferred to a computer for processing. The soil at the site is classified as Fairbanks silt; its properties were determined by conducting in situ and laboratory tests. Others have conducted laboratory tests for the soils at this test site to evaluate the soil properties such as dynamic shear moduli were determined by laboratory tests (Czajkowski and Vinson, 1980; Wilson, 1982). Details of the soil properties, described in the research reports, are not included in this paper. Field experiments were conducted during December and January (the winter season).

## 3. Spectrum analysis

Fig. 2(a) shows the FFT of a measured acceleration signal. Two specific peaks are visible, corresponding to  $f_1 = 98$  Hz and  $f_2 = 1080$  Hz. Fig. 2(b) and (c) show the FFT of three measured acceleration signals recorded on the same day. Fig. 3 shows the power spectrum density of three measured acceleration signals for the same day. Note in Fig. 3 that the first specific frequency is less sensitive to test history and the first specific peak of all three tests is identical, whereas the second specific frequency is sensitive to test history and the second specific peak of the three tests is within the range of  $f_2 = 1065$ –1080 Hz. These data suggest that tests conducted on the same day could slightly change the soil boundary condition, which is reflected by the change in the second specific frequency.

If the system is idealized as a linear system, then we may presume that the two peaks correspond to the first two modes of the system. Thus, the first-order natural frequency is a horizontal vibration of  $f_1 = 97$  Hz, and the second-order natural frequency is a horizontal vibration of  $f_2 = 1080$  Hz. The validity of this assumption will be demonstrated later in the paper. Fig. 4 shows a spectrogram of one of the measured acceleration signals determined from the field test. Fig. 4(a) is a contour plot showing that four kinds of components exist. The first component is the dominant one, corresponding to the specific frequency of  $f_1 = 97$  Hz. The second component consists

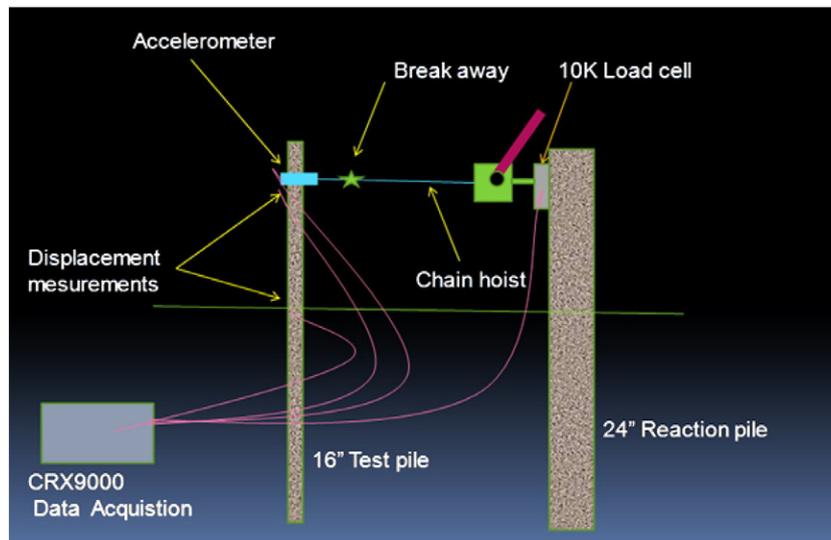


Fig. 1. Schematic diagram of the test setup.

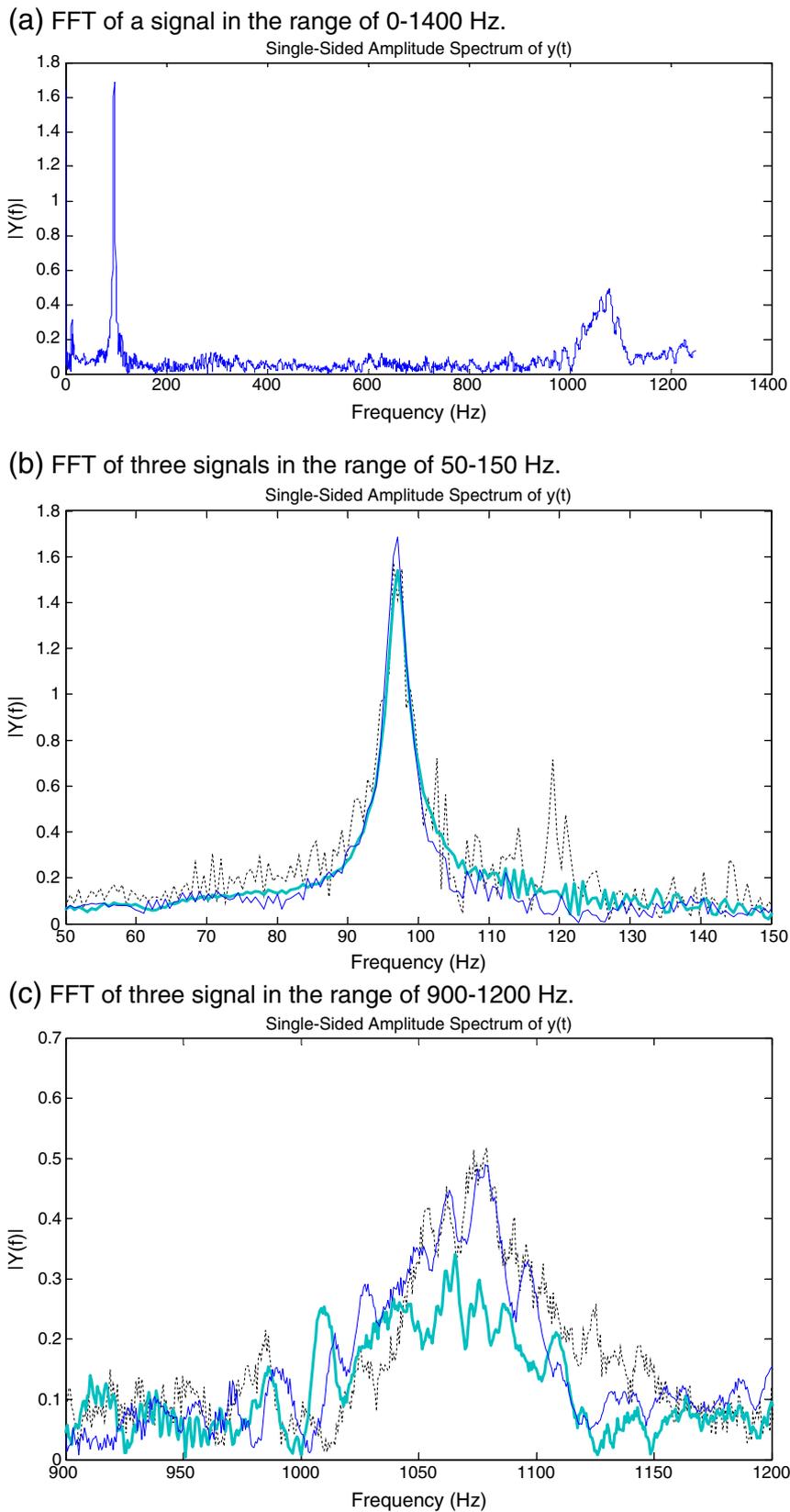


Fig. 2. FFT of a typical measured acceleration signal and the comparison of three signals taken on same day.

of many components extending to high frequencies, which only exists for a short period, decaying to zero after 0.02 s. This decay rate can be attributed to the initial nonlinear interaction of pile and soil. The third component has harmonics and the characteristics of frequency-varying,

which suggests the existence of nonlinear effect. The final component is unidentified.

From the three-dimensional plot of Fig. 4(b), however, we can see that the amplitude of the dominant component with specific frequency

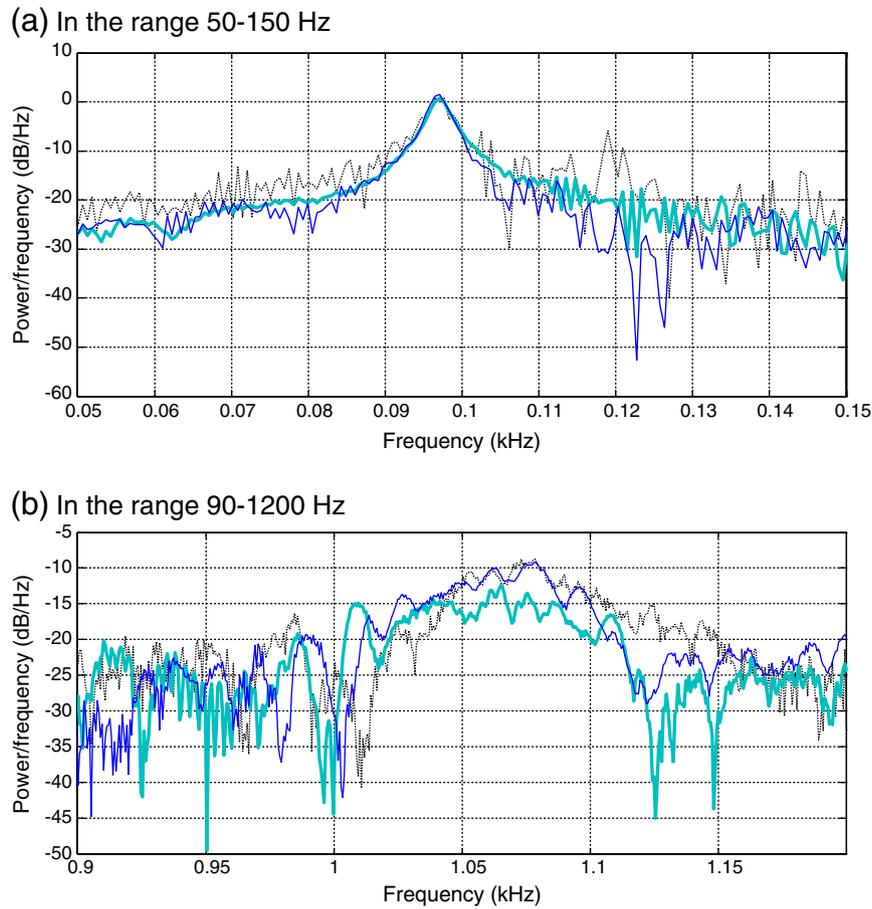


Fig. 3. Power spectrum density of three measured acceleration signals.

$f_1 = 97$  Hz is substantially higher than the amplitude of the remaining three kinds of components. The system's response can be approximated by the dominant component. Fig. 5 shows a spectrogram of the measured acceleration signal from a different angle. The envelope of the amplitude of the dominant component exhibits an obvious exponential decay profile, suggesting that the dominant component with a specific frequency of  $f_1 = 97$  Hz can be used as a characteristic quantity or index for integrity monitoring or can be used for a soil–pile interaction

investigation. To use this dominant component for future analysis, it must be extracted from the whole signal. The selecting filter technique can be implemented to extract the dominant component from the original signal. However, the EMD method was used to extract the specific signal and identify the system parameters because of its applicability to nonlinear problems. Before conducting EMD, we completed a theoretical analysis to correlate the analytical results with the experimental data.

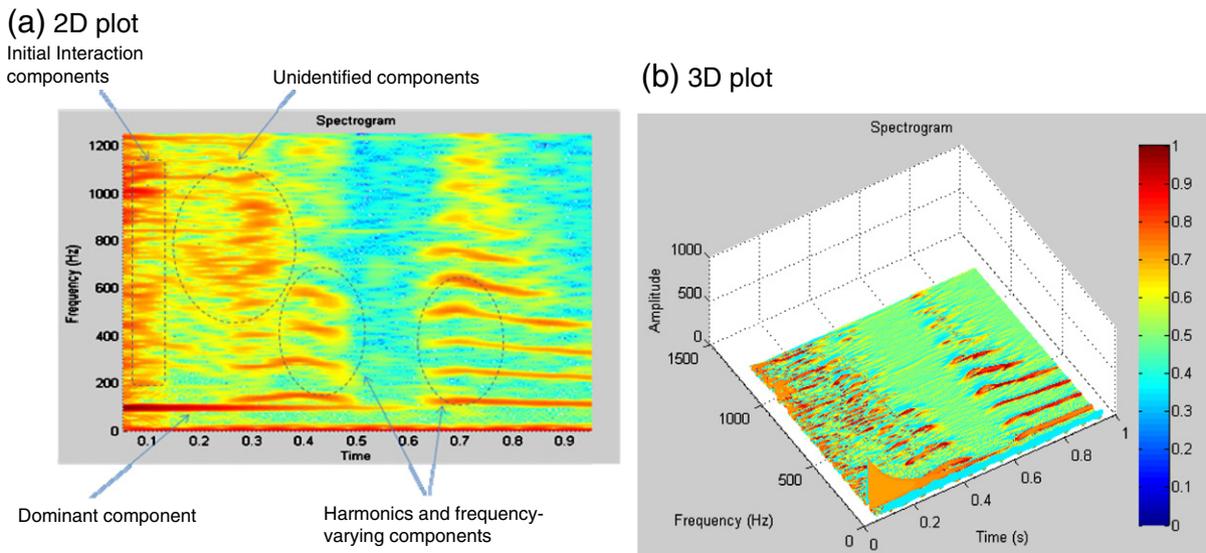


Fig. 4. Spectrogram of a measured acceleration signal.

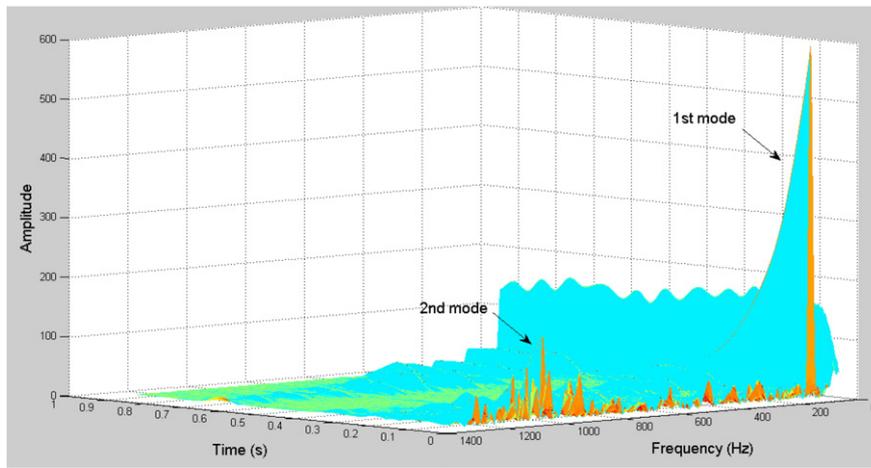


Fig. 5. Spectrogram of a measured acceleration signal (3D plot).

4. Theoretical consideration

In this section, a mathematical model is presented to correlate with the experimental results. Both geometry and material nonlinearities may exist in a soil–pile interaction problem. Since the soil interface has minimal tensile strength, where the soil–pile interface exceeds geostatic stresses around the pile, a gap will open at the ground surface. Material nonlinearity is accounted for by incorporating an advanced plasticity-based soil model. The inelastic model provides a hysteretic rule that is needed to model the cyclic behavior. This model is used to relate displacements and tractions. The displacement rate (increment) is used as a variable for the model. An approximate model (Leissa and Qatu, 2011) was used to correlate with

experimental data. The schematic of the system is shown in Fig. 6, an Euler–Bernoulli beam partially resisted by an elastic foundation.

Since stiffness and damping of soil are a function of environmental conditions such as temperature and moisture, whereas natural modes are functions of stiffness and damping, we can derive the natural frequency dependency on temperature and/or moisture. The sensitivity of natural modes with respect to temperature can be expressed as,

$$\frac{\partial f_1}{\partial T} = F_1(k(T), c(T)) \tag{1}$$

in which  $T$  is temperature. Obviously, frozen soil has a substantial impact on stiffness and damping of soil. Accordingly, frozen soil will have a substantial effect on specific natural frequency. If the system is further simplified as a cantilevered beam, then the following relationship develops between first-order natural frequency and second-order natural frequency, regardless of the geometry and material parameters of the pile:

$$\frac{f_2}{f_1} = 6.27 \tag{2}$$

In view of the first specific frequency of 97 Hz, identified earlier, and the second specific frequency of 1080 Hz, the frequencies are

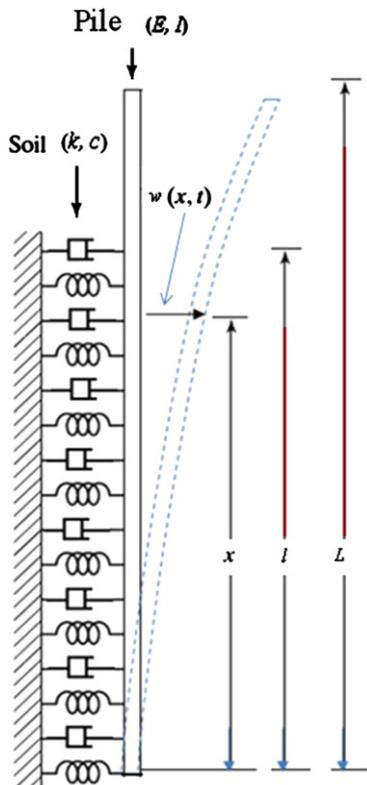


Fig. 6. Frozen soil–pile vibration model.

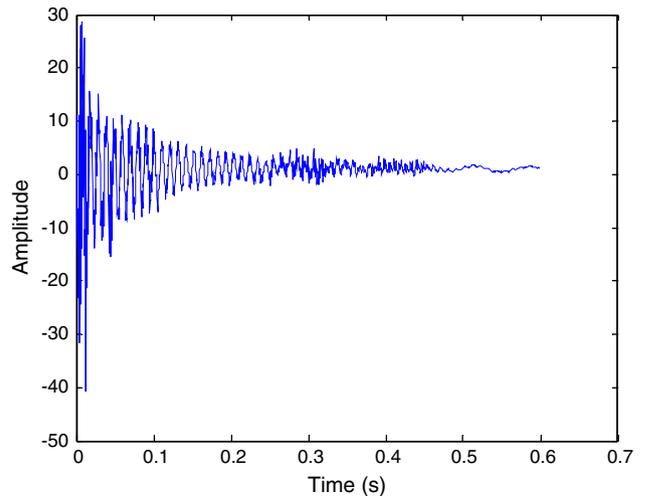


Fig. 7. Time history of a measured acceleration signal.

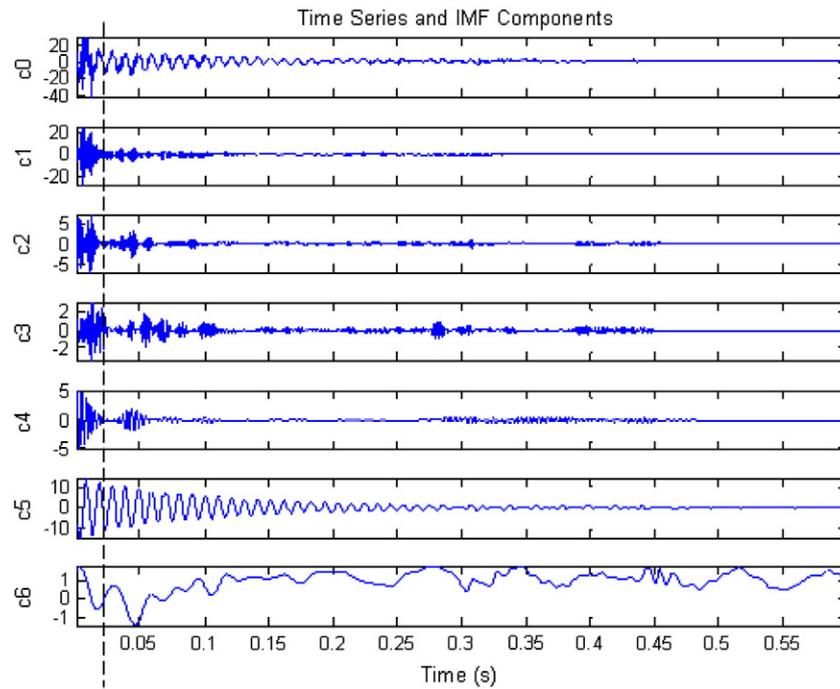


Fig. 8. Decomposed components of a measured acceleration signal.

not complying with the relationship given in Leissa and Qatu, 2011. As such, the specific frequency of 1080 Hz should not be considered as the second-order natural frequency of the assumed linear system.

**5. Parameter estimations**

To decompose the signal as being characterized in the time–frequency expression (see Figs. 4 and 5), an empirical mode decomposition method was used instead of filtering to evaluate the response signal. Empirical mode decomposition (EMD) is a method of decomposing a

nonlinear and nonstationary signal into a series of zero-mean AM-FM components that represent the characteristic time scale for the observation. A multicomponent AM-FM model for a nonlinear and nonstationary signal,  $x(t)$ , can be represented as,

$$x(t) = \sum_{j=1}^n a_j(t) \cos[\varphi_j(t)] \tag{3}$$

where  $a_j(t)$  and  $\varphi_j(t)$  represent the instantaneous amplitude and the instantaneous phase of the  $j$ th component, and  $n$  is the number of

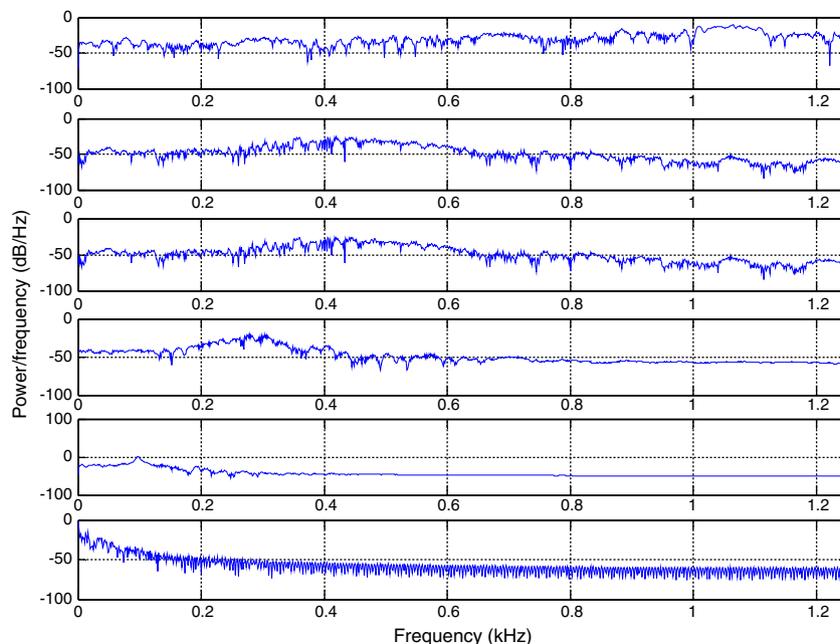


Fig. 9. Power spectrum of the decomposed components of a measured acceleration signal.

components. In the EMD approach, the solution is performed by iteratively conducting a sifting process. Zero-mean AM-FM components are called intrinsic mode functions (IMFs), which must satisfy certain requirements: (1) The number of extreme and the number of zero crossings in the IMF must equal or differ by no more than one; and (2) at any point, the mean value of the envelopes defined by the local maxima and local minima must be zero. In short, the signal is locally symmetric around the time axis.

The sifting process to find IMFs for the signal  $x(t)$  consists of the following steps:

- (1) Find positions and amplitudes of all local maxima and all local minima in the input signal  $x(t)$ . Create an upper envelope by cubic spline interpolation of the local maxima, and a lower envelope by cubic spline interpolation of the local minima. Calculate the mean of the upper and lower envelopes; this is defined as  $m_1(t)$ . Subtract the envelope mean signal,  $m_1(t)$ , from the original input signal,  $h(t) = x(t) - m_1(t)$ . Check whether  $h(t)$  meets the requirements to be an IMF. If not, treat  $h(t)$  as new data and repeat the previous process. Then set  $h_{11}(t) = h_1(t) - m_{11}(t)$ . Repeat this sifting procedure  $k$  times until  $h_{1k}(t)$  is an IMF; this is designated as the first IMF or  $c_1(t)$ .
- (2) Subtract  $c_1(t)$  from the input signal and define the remainder,  $r_1(t)$ , as the first residue. Since the residue,  $r_1(t)$ , still contains information related to longer period components, treat it as a new data stream and repeat the above-described sifting process. This procedure can be repeated  $j$  times to generate  $j$  residues,  $r_j(t)$ . The sifting process is stopped when either of two criteria is met: Either the component,  $c_n(t)$ , or the residue,  $r_n(t)$ , becomes so small that it is considered inconsequential, or the residue,  $r_n(t)$ , becomes a monotonic function from which an IMF cannot be extracted. We finally obtain

$$x(t) = \sum_{i=1}^n c_{imfi}(t) + r_n(t) \quad (4)$$

In other words, the original signal can now be represented as the sum of a set of IMFs plus a residue. Next, apply the Hilbert transform to all IMFs,  $c_j(t)$ , to derive model parameters including frequency and damping:

$$H[c_j(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{c_j(\tau)}{t-\tau} d\tau \quad (5)$$

After the Hilbert transform,  $H[c_j(t)]$  and  $c_j(t)$  form a complex signal  $Z_j(t)$ :

$$Z_j(t) = c_j(t) + iH[c_j(t)] = a_j(t)e^{i\varphi_j(t)} \quad (6)$$

Then the envelop of every IMF can be given by

$$a_j(t) = \sqrt{[c_j(t)]^2 + H[c_j(t)]^2}, \varphi_j(t) = \arctan\{H[c_j(t)]/c_j(t)\} \quad (7)$$

in which  $a_j(t)$ , the instantaneous amplitude of  $x(t)$ , reflects how the energy of  $x(t)$  varies with time. The term  $\varphi_j(t)$  is the instantaneous phase of  $x(t)$ . The instantaneous frequency  $\omega_j(t)$  is defined as the time derivative of the instantaneous phase  $\varphi_j(t)$  as follows:

$$\omega_j(t) = \frac{d\varphi_j(t)}{dt} \quad (8)$$

Then the original signal  $x(t)$  can be expressed as

$$x(t) = \sum_{j=1}^n a_j(t) \exp\left[i \int \omega_j(t) dt\right] \quad (9)$$

In principle, the measured acceleration response of the pile,  $\ddot{w}$ , can be approximately decomposed by the EMD as follows:

$$\ddot{w}(t) = \sum_{j=1}^k \ddot{w}_j(t) + \sum_{i=1}^{n-k} c_i(t) + r_n(t) \quad (10)$$

where  $\ddot{w}_j(t)$  is the  $j$ th modal acceleration response and  $c_i(t)$  is the  $i$ th IMF.

$$\begin{aligned} \omega_j(t) &= \omega_{dj}t - \theta_j \\ \ln a_{ij} &= -\zeta_1 \omega_j t + \ln r_{ij} \end{aligned} \quad (11)$$

Thus, the damped natural frequency,  $\omega_{dj}$ , can be obtained from the slope of the phase angle plot in  $\omega_j(t)$  versus  $t$ , and  $\zeta_1$  can be obtained from the slope of the plot in  $\ln a_{ij}$  versus time.

The linear least-squares method can be used to fit the plots of  $\ln a_{ij}$  versus time and  $\omega_j(t)$  versus time.

In the next step, we illustrate how signal decomposition and parameter identification are performed by using a typical measured

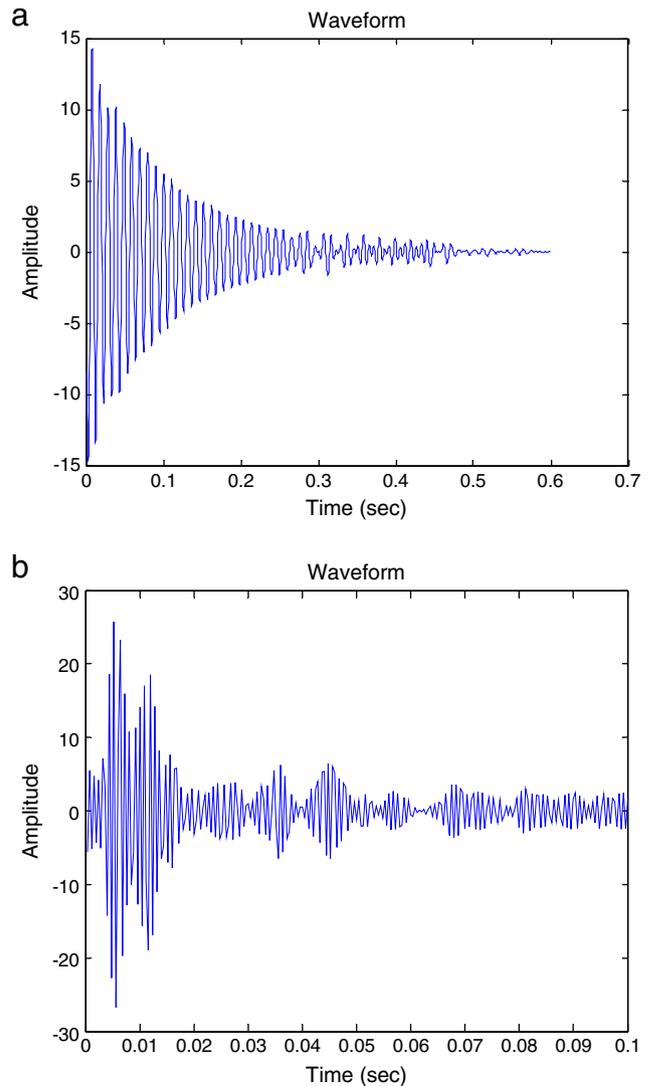


Fig. 10. Decomposed signals corresponding to two specific frequencies.

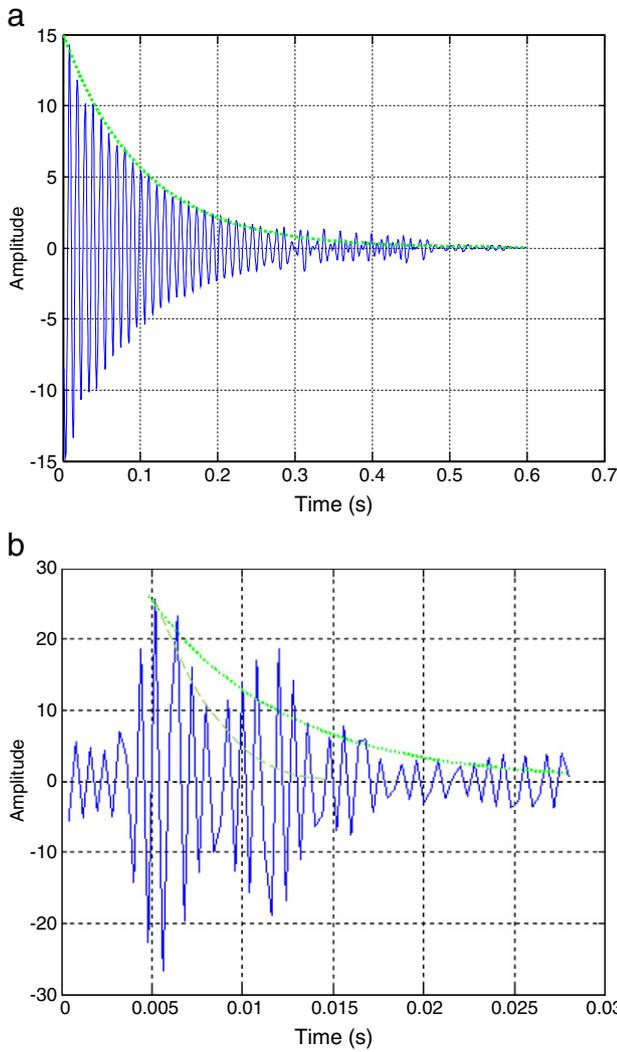


Fig. 11. Envelopes of the decomposed signals corresponding to two specific frequencies.

acceleration signal (shown in Fig. 7). Fig. 8 shows the decomposed components of a measured acceleration signal. From Fig. 8 we can see that the magnitude of  $c_1$  and  $c_5$  are much higher than the rest of the components. To characterize the decomposed signal, the power spectrum of the decomposed signal is calculated (shown in Fig. 9). From Fig. 9 we see that the decomposed component  $c_5$  has a specific frequency of 97 Hz and the decomposed component  $c_1$  has a specific frequency of 1080 Hz. The relationship between the two frequency values is consistent with the above analysis. From Fig. 8 we see that after 0.02 s,  $c_5$  is almost the only component. All of the rest decay nearly to zero. The vibration associated with  $c_5$  lasts until 0.3 sec. In the following discussion, we focus on  $c_5$  and  $c_1$  to identify the corresponding characteristic parameters. Fig. 10 shows the waveforms of the decomposed signals,  $c_5$  and  $c_1$ .

The envelope of waveform of  $c_5$  exhibits an exponential shape that can be readily fitted by an exponential curve, suggesting that this oscillation can be treated as a single-degree-of-freedom system. The damping coefficient can be identified as  $\zeta_1 = 0.016$  (illustrated in Fig. 11a). However, the envelope of waveform of  $c_1$  does not exhibit an exponential shape; instead, it exhibits modulations of both amplitude and frequency nonlinear oscillation characteristics, suggesting strong nonlinear properties of soil–pile interaction in this mode. Frequency change can be attributed to stiffening of soil stress-strain properties resulting in a change of system stiffness. Furthermore, multiple harmonics may be attributed to bouncing across the gap

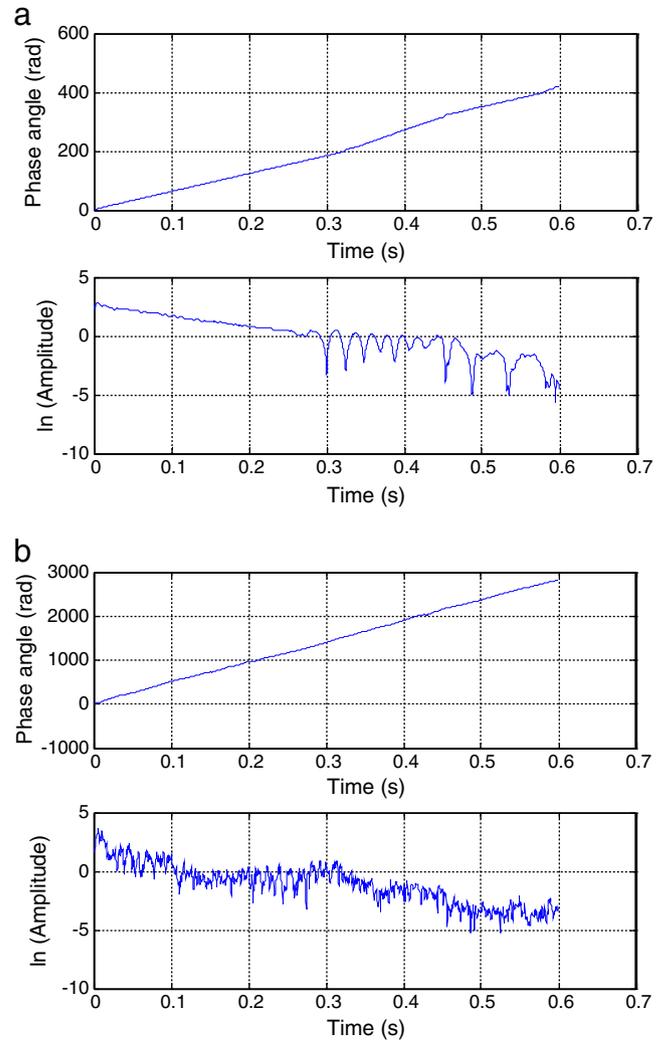


Fig. 12. Characteristic plots of the decomposed signals corresponding to two specific frequencies.

near the ground surface between the soil and the pile. Identification can be implemented by correlating advanced processing, nonlinear modeling, and analysis.

This finding is consistent with the analysis in Section 2, where we illustrate that the corresponding oscillation is unlikely the second mode of the assumed linear system. If we approximate the data with exponential curve fitting, damping ratio could be in the range of 0.07–0.02, but subject to implementations. The specific parameter is obtained by Eq. (11). Fig. 12 shows the characteristic plots of decomposed signals  $c_5$  and  $c_1$  corresponding to the two specific frequencies.

## 6. Conclusions

A free-decay response signal was used to describe the experimental free-vibration-response data for frozen soil–pile structure interaction (test was conducted using an impulse load). The proposed method can be used to evaluate the nonlinearity of the system and to conduct system identification. The specific mode signal can be extracted and the specific parameters can be identified. This approach provides a fast way not only to estimate the extent of nonlinearity of the real system, but also to approximate modal parameters for the linear and nonlinear modes. This approach helps to guarantee the accuracy and reliability of employing a vibration signal to monitor the interaction

response of the frozen soil–pile structure and to investigate frozen soil–pile interaction under various environmental and seismic conditions.

Results show that the response for frozen soil–pile structure interaction exhibits time-variant and nonlinear characteristics in the time–frequency domain. Results also show that the tested system exhibits weak nonlinearity. Dominant system parameters, used to characterize frozen soil–pile interactions, are identified. A 16 in. (406 mm) diameter steel-jacketed concrete pile, with 5 ft (1,524 mm) aboveground and 20 ft (6.096 m) embedded in Fairbanks silt, had two predominant natural frequencies for the winter months of December and January: 97 Hz and 1080 Hz. Damping for a pile embedded in frozen Fairbanks silt was approximately 0.016.

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