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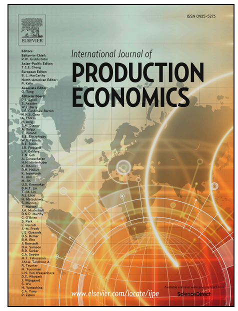

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Financing and Ordering Strategies for a Supply Chain under the Option Contract

Abstract

We study a two-echelon supply chain consisting of a capital-constrained retailer ordering via the option contract to satisfy uncertain demand from a single supplier. The retailer can apply for either a bank loan or trade credit from the supplier whenever necessary. In addition to economic revenue, the supplier has a relationship concern and takes the retailer's revenue into consideration. By developing a Stackelberg game, we analyze the ordering and financing problems in the supply chain. The results show that in the presence of the retailer's bankruptcy risk, the supplier should always finance the retailer at the risk-free interest rate. Given the supplier's offer, the retailer will always prefer to raise money from the supplier due to the lower interest rate. In contrast, under trade credit, the supply chain's efficiency is improved when the production cost is high but decreases when the production cost is low. Furthermore, our results show that the supplier's relationship concern can improve the supply chain's efficiency and the retailer's revenue most of the time, but increases the retailer's bankruptcy risk when the production cost is high, implying that the supplier's attempt to help the retailer eventually harms its long-run survival.

Key words: supply chain; option contract; financing strategy; relationship concern

1 Introduction

In business, many firms face the capital constraint, especially small and medium-sized enterprises (SMEs). According to a credit survey of 3,459 SMEs in the U.S., 47% applied for financing in 2015, while only half of those that did not apply had sufficient cash flow (Barkley et al., 2016). A similar situation exists in developing countries. For example, about half of the SMEs in developing countries applied for financing in 2011, for a total amount of US$2.1 to $2.6 trillion (Owens and Wilhelm, 2017). The capital constraint can severely stymie firms' operations, so it has been considered as the biggest obstacle to firms' growth (Xu and Birge, 2004; Ayyagari et al., 2017). However, many classic studies of operations management (OM) are based on the assumption of sufficient capital (e.g., Petruzzi and Dada, 1999; Lariviere and Porteus, 2001; Cachon, 2003; Bernstein and Federgruen, 2005; Perakis and Roels, 2007), which means that their results may be less relevant to current business practice.

Commercial bank loans are a popular way for firms to deal with the capital constraint. However, due to complex application procedures and strict collateral requirements, SMEs are usually ruled out from financing via bank loans. For example, only 4.7% of working capital loans and 23.3% of bank loans are issued to SMEs in China (Tsai, 2015). Given this situation, trade credit is widely accepted by firms in various industries, especially for SMEs where such credit is extended by upstream partners within the same supply chain (Jing et al., 2012). For instance, in 2004, trade credit accounted for 22.9% of the liabilities in the non-financial industries in Canada (Chandler, 2009). In China, this ratio was 20% in 2012 (Lin and Chou, 2015). In 2007, 90% of worldwide merchandise trade, amounting to US$14 trillion, was underpinned by trade credit (Williams, 2008). To facilitate operations in SMEs and to improve the efficiency of the entire supply chain, large capital-rich upstream suppliers often provide trade credit financing services to downstream SMEs in practice. For example, Ford Motor Company provides wholesale loans to dealers to finance the purchase of vehicle inventory, as well as loans to dealers to finance working capital and improvements to dealership facilities, finance the purchase of dealership real estate, and finance other dealer vehicle programs via Ford Motor Credit Company LLC, a wholly-owned subsidiary by Ford Motor Company (Ford Motor Company, 2018). Gree Electric, a Chinese major appliances manufacturer, contracts with many small-sized dealers who suffer from severe budget constraints yet usually are not eligible for bank loans. Therefore, Gree Electric cooperates with insurance and bonding companies to provide loans to these small-sized dealers (Zhuhai Gree Group Finance Company, 2018), in order to develop and protect its distribution channels. After obtaining the loans, these dealers are able to make better operational decisions, especially ordering decisions with Gree. Through IBM Global Financing, a wholly-owned subsidiary of IBM, IBM helps its clients to get access to IBM software by offering them short and long term loans (IBM Global Financing, 2016). GE Capital offers various financing products exclusively to GE customers in capital intensive industries like healthcare and energy (GE Capital, 2018).

Building on the trust of the partnership, the trade credit financing service provided by large upstream suppliers not just helps the downstream SMEs smooth out their operations, but also creates potential revenues/profits for the suppliers. From the above examples, the upstream manufacturers' behaviour of offering financial solutions to their small-sized downstream partners implies the manufacturers' relationship concern as they try to improve those minuscule dealers' situations, which is in return beneficial to these suppliers by maintaining an ideal market share and a sustainable supply chain cooperation. Many models of supply chain management implicitly assume that the decision makers are only concerned for their own earnings and ignore the interests of their partners. However, behavioural economics shows that the decision makers may also have a relationship concern in addition to focusing on the economic benefits (Bolton and Ockenfels, 2000; Charness and Rabin, 2002). For example, automobile manufacturer like Ford, Toyota, Nissan, and Honda maintain good relationships with their suppliers. They send their own engineers to suppliers' factories or provide training courses for suppliers' employees to solve operations problems; in return, they get a steady supply of high-quality components (Sako, 2004). In China, major appliances manufacturers like Gree, offer training and direct customer service for their dealers. In the U.K., big brands like Procter & Gamble and Imperial Tobacco provide support to small local retailers, and these retailers communicate consumer needs to them (Baron et al., 2001). Some researchers (Uzzi, 1996; Huberman et al., 2004; Loch and Wu, 2008) have concluded that relationship concern does exist among supply chain members, which means that the decision makers have incentives to maintain good relationships with others by improving the latter's economic benefits out of long-run consideration. Motivated by the aforementioned examples, we study the joint decision on ordering and financing, with suppliers' behavioural concerns taken into consideration. Specifically, in our study the supplier is a core firm with sufficient capital, while the retailer is an SME suffering from capital constraint. We consider the scenario in which the retailer cares only about its own economic welfare to achieve short-run survival, while the supplier cares about not only its own revenue, but also the retailer's revenue because of the relationship concern.

By considering a two-echelon supply chain consisting of a single supplier and a single SME retailer, this research also aims to explore the financing problem in a distribution channel when both bank loans and trade credit are available. However, our work differs from recent research on financing the supply chain (Kouvelis and Zhao, 2012; Yang and Birge, 2017; Tunca and Zhu, 2017) as we adopt an option contract as the purchasing contract and consider behavioural factors embodied by the decision makers. An option contract is a financial derivative instrument widely used to hedge different kinds of risks in financial markets. From the operations perspective, it helps firms to hedge the risks due to price variability and demand uncertainty (Barnes-Schuster et al., 2002; Burnetas and Ritchken, 2005; Breiter and Huchzermeier, 2015). For example, in the semi-conductor industry, Intel Corporation saved tens of millions of dollars by implementing a dual-mode equipment procurement framework that used the option contract (Peng et al., 2012). In the retail industry, Suning Commerce Group avoids holding too much inventory by adopting the option contract (Wang and Liu, 2007). The option contract is used widely in various industries such as petroleum, natural gas, electricity, and agriculture (Hale et al., 2002; Kang and Mahajan, 2006). Therefore, we depart from the literature on the supply chain financing problem by considering the option contract instead of the wholesale price contract, and studying the interaction between the financing and ordering decisions.

To better understand the current business practice, we investigate the following questions in this paper. First, concerning the financing decisions, does the supplier have an incentive to provide trade credit to the retailer when a bank loan is available? If yes, what would be the optimal interest rate for trade credit? From the retailer's perspective, which financing channel is preferred, i.e., a bank loan or trade credit? What is the preferred size of the loan under either financing channel? Second, considering the operations decisions, what is the supplier's pricing decision for the option contract? What is the retailer's optimal order quantity? Finally, considering the behavioural impacts, how does the supplier's relationship concern impact the retailer's decisions and the performance of the entire supply chain?

The results of our inquiry can be summarized as follows. First, if the retailer chooses a bank loan, both the supplier and retailer make the same decisions as in the traditional case where the retailer has no budget constraint. By contrast, if the retailer chooses trade credit, both the supplier's and retailer's decisions are influenced by the budget constraint. Second, from both the supplier's and retailer's perspectives, trade credit weakly dominates bank loans. Third, when the retailer's budget constraint is considered, the option contract cannot coordinate the supply chain. However, compared with the wholesale price contract, the option contract encourages the retailer to order more when the manufacturing cost of the product is low; hence, supply chain efficiency is improved. Finally, although the supplier's relationship concern can increase the retailer's payoff in most cases, it may also increase the retailer's bankruptcy risk when the production cost is sufficiently high.

Our work contributes to the literature in the following ways. First, it bridges the gap between operations and financing decisions in a supply chain when the option contract is adopted by the supplier and retailer. Second, our results show that with a downstream capital-constrained retailer, all the stakeholders in the supply chain can benefit from the use of the option contract; thus, higher efficiency can be achieved when the manufacturing cost is low. Third, it is always optimal for the supplier to provide trade credit to the retailer, as the former can influence the retailer's decisions both operationally and financially to realize a higher utility for the former. Finally, to the best of our knowledge, this study is the first attempt to take relationship concern into account in a budget-constrained supply chain. Our study reveals that this behavioural factor has a significant influence on the ordering and financing decisions, and on the payoffs for each stakeholder in a supply chain with limited capital.

We organize the rest of this paper as follows. In §2 we review the related literature. In §3 we present the model and assumptions in detail. In §4 and §5 we consider the scenarios in which the retailer chooses a bank loan and trade credit, respectively. We derive the optimal decisions for the retailer, supplier, and bank. We compare the optimal outcomes between bank-loan and trade-credit financing in §6. In §7 we investigate the impact of the supplier's relationship concern. We conclude the paper and suggest topics for future research in §8. We provide all the proofs in the Appendix.

2 Literature Review

Our work is related to the literature on financing supply chains, option contract, and behavioural concerns in supply chains. We jointly study the operational and financial problems in a supply chain with the capital constraint from the perspectives of both the supplier and retailer.

Classical studies of supply-chain ordering decisions usually assume that all the members in the supply chain have sufficient liquidity (e.g., Petruzzi and Dada, 1999; Lariviere and Porteus, 2001; Cachon, 2003), so short-term financing issues are rarely covered. In contrast, corporate financing problems have been extensively studied in finance and economics. A number of sources of financing, such as bank loans, trade credit, debt financing, and venture investment, are commonly used in practice, especially for SMEs (Chemmanur and Fulghieri, 1994; Jordan et al., 1998; Hellmann and Puri, 2000). We refer to Brennan et al. (1988), Cuñat (2007), and Fabbri and Menichini (2010) for theoretical studies on trade credit, which is the main focus of this paper. Most research on trade credit focuses on problems of information asymmetry about default risk, price discrimination by suppliers, customized products, and the advantage of liquidation. However, none of this research has considered the financing problems in a supply chain, particularly when the retailer, i.e., the SME, faces the capital constraint and is in need of financing.

The interaction between financing and ordering decisions has received a great deal of attention in recent years (e.g., Yang et al., 2015; Tunca and Zhu, 2017; Tang et al., 2018). Most of these studies approach the financing problem in the framework of the classical newsvendor problem. Dada and Hu (2008) analyzed the newsvendor problem under the assumption that the retailer has the capital constraint and bank loans are viable. Kouvelis and Zhao (2011) extended their work by taking bankruptcy cost into consideration and assuming that the bank loan is fairly priced. They found that the retailer's ordering decision is influenced by its wealth level, and the equilibrium order size is smaller than that in the traditional newsvendor model. In Jing et al. (2012), the retailer, which is a start-up firm, completely relies on loans to purchase from the supplier. When only trade credit is available to the retailer, the supplier will set the wholesale price high to absorb all of the profit in the supply chain. When both a bank loan and trade credit are available to the retailer, the retailer's choice between them is dependent on the supplier's production cost. Considering a similar problem, Kouvelis and Zhao (2012) relaxed the assumption that the retailer has no initial capital. They demonstrated that the retailer's optimal financing choice is trade credit because the supplier has an incentive to set the trade credit interest rate no greater than the risk-free interest rate. When the retailer can use a bank loan and trade credit simultaneously, Cai et al. (2014) explored the retailer's optimal order size and financing portfolio. Against the same background, Yang and Birge (2017) showed the role of trade credit in risk sharing and supply chain efficiency improvement. One common feature of all these studies is the adoption of the wholesale price contract in the framework of the newsvendor problem. However, few researches have ever considered the joint decisions on financing and ordering based on the adoption of option contract. Hence, our paper contributes to the literature by adopting the option contract as the purchasing contract to investigate the interaction between financing and ordering decisions in a capital-constrained supply chain. We also investigate whether the option contract can improve supply chain members' payoffs compared with the wholesale price contract in the capital-constrained supply chain, which has not been explored before. Our analyses show that option contract can improve the supply chain efficiency to a higher level when the production cost is low. On the other hand, the aforementioned papers do not take behavioural factor into account, instead, they assume all supply chain members to be self-interested. Nevertheless, as behavioural concern becoming more common in practice, our work is based on the consideration of supplier's relationship concern. We show that it indeed has significant influence on the interaction of financing and ordering decisions.

Our research is closely related to the literature on the option contract in supply chains. As a financial derivative instrument, options have been explored by a large body of literature on finance (e.g., Black and Scholes, 1973; Muravyev, 2016; Andersen et al., 2017). In the field of OM, Ritchken and Tapiero (1986) were the first to bring the option contract into inventory management to hedge the risks due to price variability and demand uncertainty. Golovachkina (2003) considered a two-echelon supply chain where a retailer purchases from a capacity-constrained supplier via the option contract in the presence of a spot market. They demonstrated that the option contract can significantly improve supply chain efficiency. Considering a similar problem, Pei et al. (2011) correlated the option price with the retailer's order size. Schummer and Vohra (2003), Wu and Kleindorfer (2005), Fu et al. (2012), and Andersen et al. (2017) studied supply chains with multiple suppliers and a single retailer, where the retailer decides the optimal purchasing portfolio from different suppliers via the option contract. Nevertheless, none of these studies has ever considered a capital-constrained supply chain. While, our work fills this research gap by taking the retailer's capital constraint into account. The phenomenon of capital shortage is not infrequent to witness in practice, especially for small and medium-sized enterprises. Actually, even though an enterprise knows it's better to adopt the option contract to manage demand risk, without enough cash flow, it cannot achieve its goal. Hence, compared with prior literature, our work is more consistent with practice. Furthermore, our study enriches this stream of literature by incorporating the supplier's relationship concern since none of the aforementioned studies has considered before. More importantly, we show that the supplier's relationship concern can further improve the supply chain efficiency when using option contract in a supply chain with budget-constrained retailer.

Some research has revealed that the option contract can coordinate the supply chain in proper parameter settings (e.g., Barnes-Schuster et al., 2002; Zhao et al., 2010; Chen et al., 2014). However, our results show that with the capital constraint, the option contract cannot coordinate the supply chain even though multiple financing sources are available, i.e., bank loans and trade credit. Although a study of supply chain coordination is beyond the scope of this paper, our analyses show that the combination of trade credit and the option contract does improve supply chain efficiency.

This study is also closely related to the literature on behavioural concerns, especially relationship concern in supply chains. Research on economics and sociology suggests that decision makers may account for social preferences, such as fairness, reciprocity, and status, in addition to economic payoffs (e.g., Bolton and Ockenfels, 2000; Charness and Rabin, 2002; Loch et al., 2006). Assuming both the supplier and retailer have preferences for reciprocity, Du et al. (2014) found that their kindness/unkindness intention plays an important role in their decisions concerning the equilibrium retail price and wholesale price. Loch and Wu (2008) incorporated the social preference of relationship concern and status seeking in a supply chain through an experimental study. They demonstrated that relationship (status-seeking) concern has a positive (negative) impact on the individual’s payoff and supply chain efficiency. By focusing on the impact of behavioural factors on pricing contracts, Özer and Zheng (2012) concluded that when supply chain members care about one another, i.e., have a relationship concern, the effect of double marginalization is decreased. Similarly, Griffith et al. (2006) and Yang (2009) empirically showed that a closer supply chain relationship can decrease conflict between supply chain members and improve their performance. Our work differs from this area of the literature by focusing on the scenario where the retailer is a capital-constrained SME and cares only about short-term profit, while the supplier is a large firm with a relationship concern that cares about the long-term development of the supply chain. Consistent with the findings in prior literature, we further confirm the positive impact of supplier’s relationship concern on retailer’s payoff in such a new context. However, our study also show that the relationship concern may increase the retailer’s bankruptcy risk when the production cost is sufficiently high, indicating the negative impact of relationship concern. This result is in stark contrast with the literature on behavioural concerns.

3 Model Description

We study the Stackelberg game between a small-sized downstream retailer and a large upstream supplier in two periods, indexed as $t = 0$ and $t = 1$. At time $t = 0$, the supplier offers an option contract to the retailer and determines the option price c_o. Each option gives the retailer the right but not the obligation to buy a unit of the product from the supplier at a predetermined exercise price c_e at time $t = 1$. Following Burnetas and Ritchken (2005), Li et al. (2009), Chen and Shen (2012), and Liu et al. (2014), we assume c_e is exogenous to keep our model tractable.[1] Given this contract, the retailer determines the quantity of options to buy, denoted by q, and pays the supplier $c_o q$ in advance. The supplier then executes its production at the unit cost c.

As an SME, the retailer is potentially capital-constrained, i.e., its initial capital, denoted by y, may not be sufficient to cover its ordering cost. The supplier has enough capital, denoted by Y, making it able to provide trade credit to the retailer. Whenever the retailer is capital constrained, it chooses to take either trade credit (if it is offered by the supplier) or a bank loan (which is always available). Note that by offering trade credit, we assume the supplier provides the SME retailer with a short-term loan to purchase from the supplier to reflect the common practice in reality, as illustrated in the examples presented in the Introduction. We use the subscript $i = b, s$ to represent the two financing channels, i.e., the bank and supplier, respectively. Denote r as the interest rate on the loan, then r_b and r_s represent the bank’s and supplier’s interest rate, respectively. Both the supplier and retailer, after reserving enough money for operations, invest their leftover money at the risk-free interest rate r_f. We summarize the notation used throughout the paper in Table 1.

The market demand D is uncertain at time $t = 0$, with probability density function (p.d.f.)

$f(\cdot)$, cumulative distribution function (c.d.f.) $F(\cdot)$, and support $[0, N]$. We assume that the distribution has an increasing and convex failure rate $h(D) = f(D)/(1 - F(D))$.[2] At time $t = 1$, the market demand D is realized, and the supplier and retailer both learn of this demand. After observing the demand information, the retailer chooses to exercise $\min\{q, D\}$ units of the option and pays the supplier $c_e \min\{q, D\}$. The product is then delivered from the supplier to the retailer and sold to customers at the retail price p. At the end of the selling season, the retailer has the obligation to repay the loan principal and interest. If it does not have enough money to fully repay the loan, bankruptcy will occur.

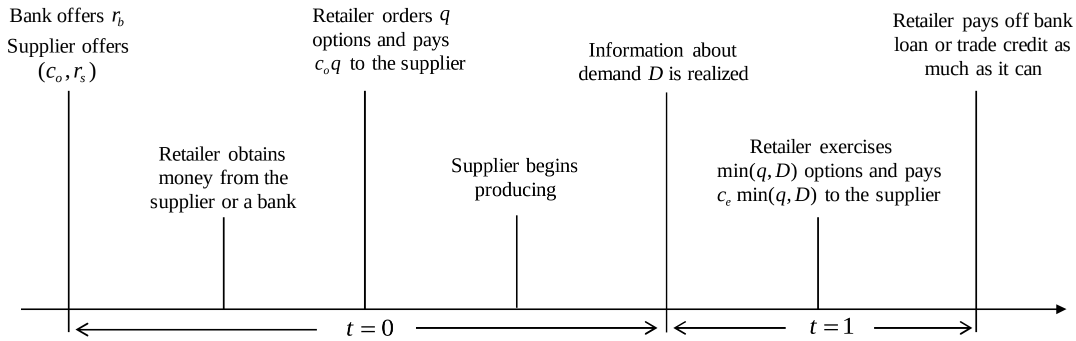


Figure 1. Sequence of events

Table 1. Summary of notation

D	Market demand with p.d.f. $f(\cdot)$ and c.d.f. $F(\cdot)$
$[0, N]$	Support of D
$h(\cdot)$	Increasing failure rate of the market demand, $h(D) := f(D)/\bar{F}(D)$
p	Retail price per unit of the product
c	The supplier's production cost per unit
y	The retailer's initial working capital
Y	The supplier's initial working capital
q_i	The retailer's order size
c_{oi}	The unit option price
c_e	Exercise price of the option
r_f	The risk-free interest rate
r_i	The interest rate of bank loan ($i = b$) or trade credit ($i = s$)
B_i	The amount of money the retailer borrowed from the bank ($i = b$) or supplier ($i = s$)
z_i	The minimum market demand for the retailer to fully repay the principal and interest
π_i	The retailer's expected ending cash level
Π_i	The supplier's expected ending cash level
θ	The supplier's relationship concern parameter
U_i	The supplier's expected utility

The retailer, as a short-run profit seeker, determines the channel by which it should take the loan and the order quantity to maximize its expected ending cash level. By contrast, in addition to attempting to maximize the economic payoff, the supplier tries to improve the retailer's ending cash level out of long-run consideration, i.e., the relationship concern. Therefore, to maximize its utility, including its own and the retailer's ending cash level, the supplier determines not only whether to offer trade credit and (if yes) the interest rate, but also the price of the option contract. Figure 1 shows the sequence of events corresponding to the game model.

We assume that the retailer can only apply for financing at $t=0$, meaning that the retailer must make sure it has enough money to cover both the option-buying cost $c_{oi}q_s$ and the product-ordering cost $c_e q_i$. If the retailer chooses not to exercise all of the purchased options at time $t=1$, some money remains in its account until the end of the selling season with no interest. In addition, to make the model feasible, we make the following assumptions: (i) $0<N$, which guarantees the existence of positive market demand; (ii) $(c_{oi}+c_e)(1+r_f)\le p$, which means that the retailer always has an incentive to order from the supplier and then sell to consumers; (iii) $c_{oi}(1+r_f)+c_e\ge c(1+r_f)$, which means that the supplier always has an incentive to produce the product and will obtain a non-negative profit; and (iv) $r_b\ge r_f$ and $r_s\ge r_f$, which guarantee that profit from financing the retailer is no less than that from a risk-free investment by the bank and supplier, respectively.

4 Equilibrium Analysis under Bank Financing

In this section we study the case where the retailer chooses the bank loan, i.e., $i=b$. The supplier determines the option price c_{ob}. Given the option contract and initial capital level, the retailer then decides the order size q_b and the amount of money it needs to borrow from the bank B_b.

4.1 The Bank's Interest Rate Decision

At time $t=0$, the retailer borrows an amount of money $B_b=(c_{ob}q_b+c_e q_b-y)^+$ from the bank. At the end of the selling season, the retailer has an amount of cash L before paying back the loan to the bank, where

$$L=p\min\{q_b,D\}+c_e(q_b-D)^+ +(y-c_{ob}q_b-c_e q_b)^+(1+r_f). \tag{1}$$

The first part is the revenue from selling the product to the customers and the second part is the retailer's leftover money after exercising the purchased units of the option at $t=1$. If the realized demand is no less than the number of options it purchased, the retailer will exercise all units of the option, i.e., $c_e(q_b-D)^+=0$. The last part of (1) is the revenue from the risk-free investment at $t=0$ if the retailer has money in excess of the ordering cost.

Because the retailer is an SME with limited liability, it repays $\min\{L,B_b(1+r_b)\}$ to the bank. The bank loan offered by a commercial bank is in a perfectly competitive financial market, i.e., the bank's expected return from lending money to the retailer is equal to the return of the risk-free investment with the same amount of money. Hence, the bank loan's interest rate r_b^* satisfies the following equation

$$\mathbb{E}[\min\{L,B_b(1+r_b^*)\}]=B_b(1+r_f). \tag{2}$$

Given the retailer's initial capital level y and ordering decision q_b, the bank solves the optimal interest rate r_b^* from (2). Regarding whether the retailer encounters bankruptcy risk, there are two possible cases. When there is no bankruptcy risk for the retailer, i.e., the probability of bankruptcy $\mathrm{P}(L<B_b(1+r_b^*))=0$, the optimal interest rate is $r_b^*=r_f$. When bankruptcy risk exists, i.e., the probability of bankruptcy $\mathrm{P}(L<B_b(1+r_b^*))>0$, it is obvious to set $r_b^*>r_f$ Consequently, we conclude that $r_b^*\ge r_f$. Please refer to Chen and Wan (2011, Proposition 1) for the proof of the existence and uniqueness of an r_b that satisfies (2).

4.2 The Retailer's Ordering and Financing Decisions

At the end of the selling season, the retailer has an amount of cash L before paying back $\min\{L, B_b(1+r_b^*)\}$ to the bank. Given the interest rate r_b^* and the option price c_{ob}, the retailer's expected ending cash level is

$$\pi_b(q_b) = \mathbb{E}[L - \min\{L, B_b(1+r_b^*)\}]. \tag{3}$$

Substituting (1) and (2) into (3), we have

$$\pi_b(q_b) = \mathbb{E}[p\min\{q_b, D\} + c_e(q_b - D)^+] - (c_{ob}q_b + c_e q_b)(1+r_f) + y(1+r_f). \tag{4}$$

Equation (4) demonstrates that the retailer's expected ending cash level is not influenced by the interest rate of the bank loan, which is consistent with the results in Jing et al. (2012) and Kouvelis and Zhao (2012), where the wholesale price contract is adopted in the supply chain. Then, solving the first-order condition of (4), we obtain the retailer's optimal order size as follows:

$$q_b = \bar{F}^{-1}\left(\frac{(c_{ob}+c_e)(1+r_f)-c_e}{p-c_e}\right). \tag{5}$$

Equation (5) shows that the retailer's optimal ordering decision is independent of its initial capital level y and the interest rate of the bank loan r_b^*. In other words, if the retailer chooses to raise money from the bank, it always orders the same quantity of the product from the supplier. The underlying reason is discussed in Corollary 1. In addition, by substituting (5) into B_b, we can determine the amount of money the retailer should borrow from the bank.

4.3 The Supplier's Option Price Decision

At time $t = 0$, after receiving the retailer's order and the corresponding payment, the supplier produces the product at the unit cost c. The supplier then invests the money in excess of the production cost at the risk-free interest rate r_f. At time $t = 1$, the supplier delivers the corresponding quantity of the product to the retailer after receiving payment for the exercised units of the option. Therefore, the supplier's expected ending cash level is

$$\Pi_b(c_{ob}) = (Y + c_{ob}q_b - cq_b)(1+r_f) + \mathbb{E}[c_e\min\{q_b, D\}], \tag{6}$$

where $c_e\min\{q_b, D\}$ is the revenue from selling the product via the option contract, cq_b is the production cost, and $(Y + c_{ob}q_b - cq_b)(1+r_f)$ represents the revenue from the investment at the risk-free interest rate.

In addition to making a profit, the supplier has a relationship concern, i.e., it includes the retailer's ending cash level in its objective function to improve the retailer's economic benefit. Thus, its goal is to maximize its expected utility $U_b(c_{ob})$ instead of its expected ending cash level, which is

$$U_b(c_{ob}) = \Pi_b(c_{ob}) + \theta\pi_b, \tag{7}$$

where θ is the relationship concern parameter. The higher θ is, the more the supplier cares about the relationship with the retailer. In the context of our research, it is natural to assume that the supplier cares more about its own revenue than that of the retailer; thus, we keep $0 \le \theta < 1$ throughout the paper. In particular, when $\theta = 1$, the supply chain turns into a centralized supply chain. Thus, we provide conclusions for the case of $\theta = 1$ separately.

From (5), we know that there is a one-to-one mapping between q_b and c_{ob}. By substituting the inverse function of (5) into (7), we drive the optimal order size q_b^* and option price c_{ob}^*. Then, combined with (2) and (5), the equilibrium under bank financing can be summarized in the following proposition.

Proposition 1. *When the retailer chooses the bank loan, the equilibrium (r_b^*, q_b^*, c_{ob}^*) is characterized by*

$$\begin{cases} B_b\left(1+r_f\right) = \mathbb{E}[\min(L, B_b(1+r_b^*))] & \text{(8a)} \\ (p-c_e)\bar{F}(q_b^*)\left[\frac{p}{p-c_e} - (1-\theta)h(q_b^*)q_b^*\right] = \left(c\left(1+r_f\right)+c_e r_f\right) & \text{(8b)} \\ r_{ob}^* = \frac{(p-c_e)\bar{F}(q_b^*)-c_e r_f}{1+r_f} & \text{(8c)} \end{cases}$$

Proposition 1 implies that if the retailer chooses to take the bank loan, both the supplier's option price decision and the retailer's ordering decision are independent of the interest rate r_b^* and the retailer's initial capital y. In other words, the financing-related decisions have no impact on the supplier's and retailer's operation decisions. Therefore, the following corollary is straightforward.

Corollary 1. *When the capital-constrained retailer purchases via the option contract and borrows via bank loan, the supplier's and retailer's equilibrium decisions are the same as those in the case of no capital constraint.*

The underlying reason for Corollary 1 is as follows: Because the competition in the capital market is perfect, the bank loan interest is competitively priced. Equivalently, from the retailer's perspective, the expected cost of borrowing from the bank equals the cost of using its own money when it is rich enough. This conclusion is obvious when the retailer has no bankruptcy risk, as $r_b^* = r_f$. In contrast, when the retailer's bankruptcy risk exists, it would seem that it has to pay more for the bank loan, as $r_b^* > r_f$. However, note that although the interest rate of the bank loan r_b^* is higher than the risk-free interest rate, there is no need for the retailer to pay all of the principal and interest if bankruptcy occurs, due to limited liability. Consequently, the expected cost of using the bank loan still equals the cost of using its own working capital. In short, the bank can be viewed as the retailer's internal accounting department. Therefore, the retailer can make its ordering decisions without considering the financing problem, and so can the supplier.

5 Equilibrium Analysis under Trade Credit

In this section we study the case where the retailer borrows using trade credit. The supplier determines not only the option price c_{os}, but also the interest rate of the trade credit r_s. Given the supplier's offer (c_{os}, r_s), the retailer then decides the order size q_s and the amount of money it needs to borrow from the supplier B_s.

5.1 The Retailer's Ordering and Financing Decisions

Define z_s as the bankruptcy threshold, such that if and only if the realized demand $D \geq z_s$ can the retailer fully pay back both the loan principal and interest. Hence, $z_s = \{D: pD + c_e(q_s - D)^+ = B_s(1+r_s)\}$ when $B_s(1+r_s) > c_e q_s$, and $z_s = 0$ otherwise, where $B_s = (c_{os}q_s + c_e q_s - y)^+$ is the amount of money it borrows from the supplier. Specifically, $z_s = 0$ implies that the amount of money borrowed by the retailer is no more than $c_e q_s/(1+r_s)$, so that there is no bankruptcy risk for the retailer. The reason is that even when $D = 0$, the retailer's cash level before paying back the supplier is $L = c_e q_s + (y - c_{os}q_s - c_e q_s)^+(1+r_f)$; hence, the probability of bankruptcy $\mathrm{P}(L < B_s(1+r_s)) = 0$ always holds. However, $z_s > 0$ implies that the retailer cannot pay back the supplier in full; hence, there is bankruptcy risk for the retailer, i.e., $\mathrm{P}(L < B_s(1 + r_s)) > 0$.

Lemma 1. *When the capital-constrained retailer uses trade credit to raise money, the bankruptcy threshold is less than its order size, i.e.,* $z_s < q_s$.

Lemma 1 indicates that the retailer always has a chance to fully repay the loan and obtain a positive cash level after the selling season, which is the necessary condition for the retailer to stay in the market. According to Lemma 1, given the supplier's decisions on the option price c_{os} and interest rate r_s, the bankruptcy threshold z_s can be written as

$$z_s = \frac{[B_s(1+r_s) - c_e q_s]^+}{p - c_e}. \tag{9}$$

Based on the relationship between the retailer's initial capital level y and the ordering cost $(c_{os} + c_e)q_s$, the retailer's expected ending cash level is

$$\pi_s(q_s) = \begin{cases} \mathbb{E}[(p\min\{q_s, D\} + c_e(q_s - D)^+ - B_s(1+r_s))^+] & (c_{os}+c_e)q_s > y + \dfrac{c_e q_s}{1+r_s} \quad \text{(10a)} \\ \mathbb{E}[p\min\{q_s, D\} + c_e(q_s - D)^+] - B_s(1+r_s) & y < (c_{os}+c_e)q_s \le y + \dfrac{c_e q_s}{1+r_s} \quad \text{(10b)} \\ \mathbb{E}[p\min\{q_s, D\} + c_e(q_s - D)^+] & (c_{os}+c_e)q_s = y \quad \text{(10c)} \\ \mathbb{E}[p\min\{q_s, D\} + c_e(q_s - D)^+] + (y - (c_{os}+c_e)q_s)(1+r_f) & (c_{os}+c_e)q_s < y \quad \text{(10d)} \end{cases}$$

Concerning the interaction between the retailer's bankruptcy risk and need for financing, there are four possibilities. Specifically, in (10a), the retailer's budget constraint is tight compared with the ordering cost; thus, a bankruptcy risk exists, i.e., $z_s > 0$ or equivalently $(c_{os} + c_e)q_s > y + c_e q_s/(1 + r_s)$. In (10b), there is no bankruptcy risk for the retailer, but it still needs to borrow money from the supplier due to its limited initial working capital, i.e., $z_s = 0$ and $y < (c_{os} + c_e)q_s \le y + c_e q_s/(1 + r_s)$. (10c) implies that the retailer has just enough money to cover the ordering cost, i.e., $(c_{os} + c_e)q_s = y$; thus, financing is not needed. In (10d), the retailer has sufficient money to pay for the order, i.e., $(c_{os} + c_e)q_s < y$, and to invest its leftover money at the risk-free interest rate.

Given the supplier's decisions on the option price c_{os} and the interest rate r_s, the retailer's optimal order size is as follows:

$$q_s = \begin{cases} \bar{F}^{-1}([c_{os}(1+r_s) + c_e r_s]\bar{F}(z_s)/(p - c_e)) & (c_{os}+c_e)q_s > y + \dfrac{c_e q_s}{1+r_s} \quad \text{(11a)} \\ \bar{F}^{-1}([c_{os}(1+r_s) + c_e r_s]/(p - c_e)) & y < (c_{os}+c_e)q_s \le y + \dfrac{c_e q_s}{1+r_s} \quad \text{(11b)} \\ y/(c_{os}+c_e) & (c_{os}+c_e)q_s = y \quad \text{(11c)} \\ \bar{F}^{-1}([c_{os}(1+r_s) + c_e r_s]/(p - c_e)) & (c_{os}+c_e)q_s < y \quad \text{(11d)} \end{cases}$$

Obviously, the retailer's optimal order size q_s depends on c_{os}, r_s, and y. Lemma 2 characterizes the monotonicity of the retailer's order size q_s and expected ending cash level $\pi_s(q_s)$.

Lemma 2. *For any given* r_s*, both* $\pi_s(q_s)$ *and* q_s *monotonically decrease in* c_{os}.

Lemma 2 shows that a lower option price can always encourage the retailer to order more and then fulfill more potential demand, which is consistent with intuition. In addition, Lemma 2 reveals a one-to-one mapping between q_s and c_{os}. Thus, c_{os} can be expressed as a function of q_s according to (11). Before solving the supplier's decision, for ease of analysis, we convert the constraints regarding $(c_{os}+c_e)q_s$ in (11a)~(11d) into constraints regarding q_s as follows:

$$\begin{cases} q_s\bar{F}(q_s) > \dfrac{y(1+r_s)}{p-c_e} & \text{(12a)} \\ \dfrac{y(1+r_s)}{p-c_e} < q_s\bar{F}(q_s) + \dfrac{c_e q_s}{p-c_e} \le \dfrac{y(1+r_s)}{p-c_e} + \dfrac{c_e q_s}{p-c_e} & \text{(12b)} \\ \dfrac{y(1+r_f)}{p-c_e} \le q_s\bar{F}(q_s) + \dfrac{c_e q_s}{p-c_e} \le \dfrac{y(1+r_s)}{p-c_e} & \text{(12c)} \\ q_s\bar{F}(q_s) + \dfrac{c_e q_s}{p-c_e} < \dfrac{y(1+r_f)}{p-c_e} & \text{(12d)} \end{cases}$$

Given any r_s, (12a)~(12d) are equivalent to four sets of q_s, denoted by $\Omega_1 = (q_1^l, q_1^u)$, $\Omega_2 = (q_2^l, q_1^l] \cup \Delta_1$, $\Omega_3 = [q_3^l, q_2^l] \cup \Delta_2$, and $\Omega_4 = [0, q_3^l) \cup \Delta_3$, respectively (refer to Figure 2 for an illustration and Lemma A.3 in the Appendix for the proof). Δ_1, Δ_2, and Δ_3 are sets of q_s satisfying (12b), (12c), and (12d), respectively, whose values are no less than q_1^u. Please note that because the value of $N\bar{F}(N) + \frac{c_e N}{p-c_e}$ can be greater than $\frac{y(1+r_s)}{p-c_e}$ or less than $\frac{y(1+r_f)}{p-c_e}$, q_2^u and q_3^u may not exist in some cases. Therefore, we use Δ_i, i = 1, 2, 3, to represent the right part of Ω_i, i = 2, 3, 4, respectively.

Corresponding to the four constraints in (12), the four sets of q_s, i.e., Ω_i $(i = 1,2,3,4)$, can be regarded as different levels of the order size compared with the retailer's initial working capital. Specifically, Ω_1 represents the interval of q_s in which the retailer orders significantly more than it can pay with its initial working capital and bankruptcy risk exists; Ω_2 represents the interval of q_s in which the retailer orders slightly more than it can pay and no bankruptcy risk exists; Ω_3 represents the interval in which the retailer has just enough money to pay for the purchase; and Ω_4 represents the interval in which the retailer has more money than it needs for purchasing.

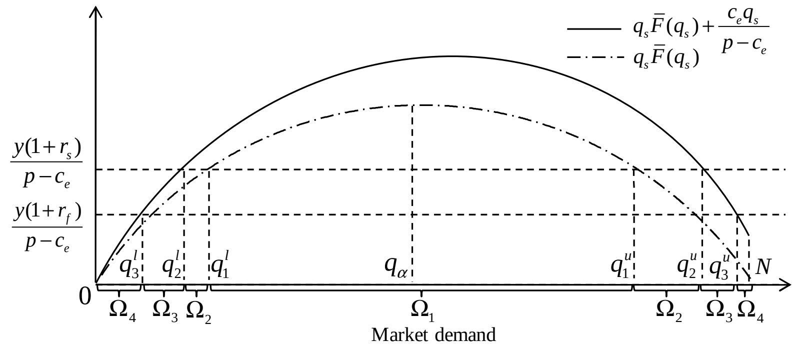


Figure 2 Illustration of four sets of q_s regarding the retailer's initial capital level

Note. q_α is the solution to $\mathrm{d}q_s\bar{F}(q_s)/\mathrm{d}q_s = 0$, which satisfies $q_\alpha h(q_\alpha) = 1$. By definition, the relationship $0 \le q_3^l \le q_2^l \le q_1^l \le q_\alpha \le q_1^u \le b$ always holds.

5.2 The Supplier's Problem

The supplier decides the option price c_{os} and interest rate r_s at time $t = 0$. Then, according to the retailer's financing request, the supplier lends an amount of money $B_s = (c_{os}q_s + c_e q_s - y)^+$ to the retailer and receives the payment for the purchased units of the option. Afterwards, the supplier produces the product and invests its leftover money at the risk-free interest rate r_f. Then, at time $t = 1$, the supplier receives the money for the exercised units of the options from the retailer and delivers the product. Finally, at the end of the selling season, the supplier receives $\min\{L, B_s(1+r_s)\}$ from the retailer as repayment of the loan. Therefore, the supplier's expected ending cash level is

$$\Pi_s(c_{os}, r_s) = \mathbb{E}[(Y - B_s + c_{os}q_s - cq_s)(1+r_f) + c_e \min\{q_s, D\} + \min\{L, B_s(1+r_s)\}]. \quad (13)$$

By substituting B_s and L, we can write (13), with the retailer's order size characterized in (12), as follows:

$$\Pi_s(c_{os}, r_s) = \begin{cases} (Y + y - cq_s)(1+r_f) - c_e q_s r_f + \mathbb{E}[p\min\{q_s, D\}] - \pi_s(q_s) & q_s \in \Omega_1 \quad (14a) \\ (Y + y - cq_s)(1+r_f) - c_e q_s r_f + \mathbb{E}[p\min\{q_s, D\}] - \pi_s(q_s) & q_s \in \Omega_2 \quad (14b) \\ (Y + c_{os}q_s - cq_s)(1+r_f) + \mathbb{E}[c_e \min\{q_s, D\}] & q_s \in \Omega_3 \quad (14c) \\ (Y + c_{os}q_s - cq_s)(1+r_f) + \mathbb{E}[c_e \min\{q_s, D\}] & q_s \in \Omega_4 \quad (14d) \end{cases}$$

Then, the supplier's expected utility function is

$$U_s(c_{os}, r_s) = \Pi_s(c_{os}, r_s) + \theta\pi_s(q_s). \quad (15)$$

To jointly analyze the supplier's decisions on c_{os} and r_s, we first fix r_s and find the optimal c_{os}. By Lemma 2, we know that there is a one-to-one mapping between the retailer's order size q_s and the option price c_{os}, which is characterized by (11). Hence, we derive the optimal order size from the supplier's perspective instead of finding the optimal option price. By substituting the inverse function of (11) into (15), we re-write the supplier's expected utility as a function of q_s instead of c_{os}, i.e., $U_s(q_s, r_s)$, because the supplier can manipulate the retailer's order size decision by changing the option price. Therefore, the first-order derivative of $U_s(q_s, r_s)$ with respect to q_s is

$$\frac{\partial U_s(q_s, r_s)}{\partial q_s} = \begin{cases} \hat{G}(q_s, r_s) & q_s \in \Omega_1 \quad (16a) \\ \bar{G}(q_s, r_s) & q_s \in \Omega_2 \quad (16b) \\ \theta p\bar{F}(q_s) - c_e F(q_s) - c_e r_f - c(1+r_f) & q_s \in \Omega_3 \quad (16c) \\ \bar{G}(q_s, r_s) & q_s \in \Omega_4 \quad (16d) \end{cases}$$

where

$$\hat{G}(q_s, r_s) = \frac{(p - c_e)\bar{F}(q_s)}{1-\delta}\left[\frac{p(1-\theta\delta) - c_e\delta(1-\theta)}{p - c_e} - (1-\theta)q_s h(q_s)\right] - c_e r_f - c(1+r_f),$$

$$\bar{G}(q_s, r_s) = (p - c_e)\bar{F}(q_s)\left[\frac{p}{p - c_e} - (1-\theta)q_s h(q_s)\right] - c_e r_f - c(1+r_f),$$

and $\delta = q_s h(z_s)[c_{os}(1+r_s) + c_e r_s]/(p - c_e)$. It is noteworthy that $\hat{G}(q_s, r_s) \geq \bar{G}(q_s, r_s)$ when $q_s \in [0, q_\alpha]$ and $\hat{G}(q_s, r_s) < \bar{G}(q_s, r_s)$ when $q_s \in (q_\alpha, N)$. The following lemma reveals that the optimal q_s has an upper bound q_β, where q_β satisfies $q_\beta h(q_\beta) = p/[(p - c_e)(1-\theta)]$ and $q_\beta > q_\alpha$.

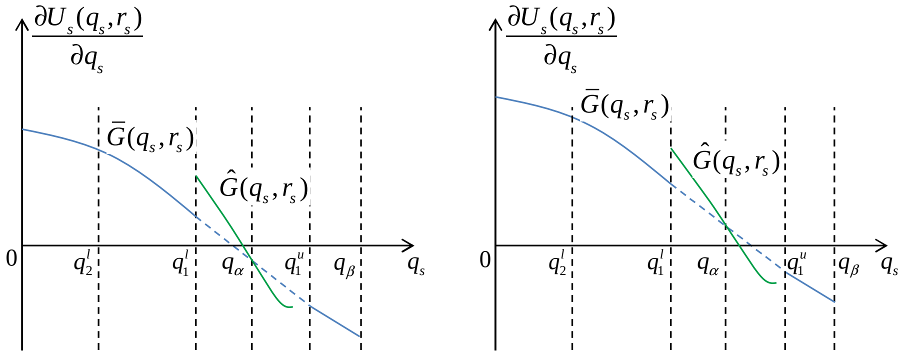
Lemma 3. *For any given r_s, from the supplier's perspective, the optimal order size $q_s^* < q_\beta$.*

Lemma 3 implies that the supplier has no incentive to induce the retailer to order more than q_β. The intuition is as follows: By purchasing via the option contract, the retailer is able to share part of the risk due to demand uncertainty with the supplier; hence, it is encouraged to order more of the product. However, if the retailer orders too much, the risk of overproduction outweighs the benefit from option selling; hence, there is an upper bound on the optimal order size from the supplier's perspective. For convenience of expression, we define $\Theta = \{q_s | q_s \in [0, q_\beta)\}$ as the interval in which q_s^* lies. The following lemma shows the existence of an optimal order size for the supplier.

Lemma 4. *For any given q_s^*, there exists at least one optimal order size from the supplier's perspective, i.e., $q_s^* = \bar{q}$ or $\hat{q}$, where $\bar{q}$ satisfies $\bar{G}(\bar{q}, r_s) = 0$ and $\hat{q}$ satisfies $\hat{G}(\hat{q}, r_s) = 0$. $\bar{q}$ is feasible only if $\bar{q} \in \Omega_2 \cup \Omega_4 \cap \Theta$ and $\hat{q}$ is feasible only if $\hat{q} \in \Omega_1 \cap \Theta$.*

In fact, $\bar{q}$ ($\hat{q}$) is the optimal order size from the supplier's perspective when there is (no) bankruptcy risk for the retailer. By comparing $\bar{G}(q_s, r_s)$ with (8b), we find that $\bar{q} = q_b^*$, meaning that the order sizes under trade credit and bank financing are the same when no bankruptcy risk exists. The reason is as follows: Recall that under bank financing, q_b^* is independent of the interest rate r_b because the operation decisions are independent of the financing problem (refer to Proposition 1 and Corollary 1). A similar property, i.e., $q_s^* = \bar{q}$ being independent of r_s, also occurs in the trade credit case as long as the retailer is not very tightly capital-constrained.

However, when $\bar{q}$ is feasible, it is unique due to the monotonicity of $\bar{G}(q_s, r_s)$ with respect to q_s (refer to Lemma A.4 in the Appendix for the details). Nevertheless, when $\hat{q}$ is feasible, it is unique only when $\hat{G}(q_\alpha, r_s) \le 0$. When $\hat{G}(q_\alpha, r_s) > 0$, $\hat{q}$ is feasible but may not be unique because the monotonicity of $\hat{G}(q_s, r_s)$ is unclear in $(q_\alpha, q_{\beta'})$ and $\hat{G}(q_s, r_s) < 0$ for $q_s \in [q'_\beta, q_\beta)$, where q'_β satisfies $q'_\beta h(q'_\beta) = \frac{p(1-\theta\delta) - c_e\delta(1-\theta)}{(p-c_e)(1-\theta)}$. Hence, in the case that more than one feasible solution exists, i.e., both $\bar{q}$ and $\hat{q}$ are feasible or multiple $\hat{q}$ exist, we need to compare all of the candidates to find the global optimal solution. By definition, it is obvious that both $\bar{q}$ and $\hat{q}$ are influenced by the unit production cost c. The following proposition introduces the range of q_s^* under different scenarios, as shown in Figure 3.



(a) High production cost: $c \ge \tilde{c}$. (b) Low production cost: $c < \tilde{c}$.

Figure 3. Impact of the production cost on the optimal order size

Lemma 5. *For any given r_s, $q_s^* \in (0, q_\alpha]$ when $c \ge \tilde{c}$ and $q_s^* \in (q_\alpha, q_\beta)$ when $c < \tilde{c}$, where $\tilde{c} = \left[(c_e + \theta(p - c_e))\bar{F}(q_\alpha) - c_e r_f\right]/(1 + r_f)$.*

Lemma 5 states a significant distinction between the equilibria when the option contract versus the wholesale price contract is used in the decentralized supply chain. Based on the adoption of the wholesale price contract, Kouvelis and Zhao (2012), and Jing et al. (2012) studied the ordering problem in a supply chain with a budget-constrained retailer and found that the optimal order size is never greater than q_α. Following the definition of the generalized failure rate in Lariviere and Porteus (2001), q_α is the point where the probability of stock-out will decrease by 1% if the order size is increased by 1%. Nevertheless, using the option contract, the optimal order size may exceed q_α when the production cost c is sufficiently low. The underlying intuition is as follows: Recall that when the wholesale price contract is adopted, the order size in a decentralized supply chain is small because the retailer bears all of the risk caused by demand uncertainty. However, by using the option contract, part of the risk is transferred from the retailer to the supplier. When the production cost is low, the cost of overproduction is also low, which means that the supplier is able to bear more risk. Consequently, the order size can be high enough to exceed the upper bound q_α. Thus, higher supply chain efficiency is achieved. In addition, note that this result holds regardless of the supplier's relationship concern. Furthermore, from Figure 3, we can determine the relationship between $\bar{q}$ and $\hat{q}$. If $c \geq \tilde{c}$, then $\bar{q} \leq \hat{q} \leq q_\alpha$; if $c < \tilde{c}$, then $\bar{q} > \hat{q} > q_\alpha$. We elaborate on these relationships in Proposition 4.

After finding the optimal order size from the supplier's perspective, the supplier's decision on the option price can accordingly be obtained by the inverse function of (11a) or (11b). However, note that the previous results (from Lemma 3 to Lemma 5) are based on the assumption that the interest rate r_s is fixed. In what follows, we relax this assumption to explore the supplier's optimal joint decisions on the interest rate and option price. For simplicity of notation, we define $\underline{\mathbb{C}} = (p - c_e)\bar{q}\bar{F}(\bar{q})/(1 + r_f)$ and $\overline{\mathbb{C}} = (p - c_e)\hat{q}\bar{F}(\hat{q})/(1 + r_f)$ as the two thresholds for the retailer's initial capital level, where $\underline{\mathbb{C}} \leq \overline{\mathbb{C}}$ always holds. Combined with the retailer's optimal response in (11), we summarize the equilibrium under trade credit in the following proposition.

Proposition 2. *When the retailer chooses trade credit, the equilibrium* (r_s^*, q_s^*, c_{os}^*) *is characterized by*

$$r_s^* = r_f,$$
$$q_s^* = \begin{cases} \hat{q} & \text{if } y < \underline{\mathbb{C}}, \\ \operatorname{argmax}\{U_s(\hat{q}, r_s^*), U_s(\bar{q}, r_s^*)\} & \text{if } \underline{\mathbb{C}} \leq y \leq \overline{\mathbb{C}}, \\ \bar{q} & \text{if } y > \overline{\mathbb{C}}, \end{cases}$$
$$c_{os}^* = \begin{cases} \dfrac{\bar{F}(\hat{q})(p - c_e)}{\bar{F}(z_s)(1 + r_s^*)} - \dfrac{c_e r_s^*}{1 + r_s^*} & \text{if } q_s^* = \hat{q}, \\ \dfrac{\bar{F}(\bar{q})(p - c_e) - c_e r_f}{1 + r_f} & \text{if } q_s^* = \bar{q}. \end{cases}$$

Proposition 2 concludes the supplier's optimal decisions on trade credit interest rate, order size, and option price when the retailer is in different wealth levels. From Proposition 2, we can see that the supplier always has an incentive to set the interest rate $r_s^* = r_f$. Hence, the interest rate set by the supplier is never greater than that set by the bank. Specifically, $r_s^* = r_b^*$ when there is no bankruptcy risk for the retailer, while $r_s^* < r_b^*$ when bankruptcy risk exists. By setting $r_s^* = r_f$, the supplier can decrease the retailer's financing cost, especially when it has bankruptcy risk, and hence encourage the retailer to choose trade credit instead of a bank loan.

In Lemma 4, for a given r_s, we conclude that both $\bar{q}$ and $\hat{q}$ are the potential optimal order sizes for the supplier. In Proposition 2, after determining the optimal r_s^*, we specify the supplier's decisions on the order size under different scenarios. $y < \underline{\mathbb{C}}$ implies that the retailer's initial capital level is low and bankruptcy risk exists, so only $\hat{q}$ is feasible and $q_s^* = \hat{q}$. $y > \overline{\mathbb{C}}$ means that the retailer's $\bar{q}$ initial capital level is high and no bankruptcy risk exists, so only $\bar{q}$ is feasible and $q_s^* = \bar{q}$. When the retailer's initial capital level is in an intermediate range, i.e., $\underline{\mathbb{C}} \leq y \leq \overline{\mathbb{C}}$, both $\hat{q}$ and $\bar{q}$ are feasible, in which case the supplier needs to compare the expected utility for $\hat{q}$ and $\bar{q}$. Note that if it is optimal for the supplier to choose $\hat{q}$, then the retailer's total ordering cost is higher than that in the case of the supplier choosing $\bar{q}$. In addition, in the former case, there is bankruptcy risk for the retailer, but not in the latter case. After determining the optimal order size, the option price can accordingly be obtained by the inverse function of (11a) or (11b).

6 Comparison between Different Financing Channels

6.1 The Preferred Financing Channel

In this section we study the preferred financing channel from both the supplier's and retailer's perspectives. On the one hand, if the supplier prefers a bank loan to trade credit, it will not lend money to the retailer and decide the option price as in §4.3. Consequently, the retailer can only finance itself via a bank loan. On the other hand, if the supplier prefers trade credit to a bank loan, it will make decisions on the option price and interest rate as in §5.2. Then, the retailer needs to choose one financing channel, i.e., either a bank loan or trade credit. In addition, based on the following analyses, we find that the supplier and retailer can reach an agreement on the choice of the financing channel, which is summarized in the following proposition. For simplicity of notation, we denote $z_b = [(c_{ob}^* q_b^* + c_e q_b^* - y)^+ (1 + r_b^*) - c_e q_b^*]^+ / (p - c_e)$ as the retailer's bankruptcy threshold under bank financing.

Proposition 3. *Both the supplier and retailer weakly prefer trade credit to a bank loan. The detailed preferences are summarized in Table 2.*

Table 2. The supplier's and retailer's financing channel preferences

Supplier	$y < \underline{\mathbb{C}}$	$\underline{\mathbb{C}} \le y \le \overline{\mathbb{C}}$	$\overline{\mathbb{C}} < y$
	Trade credit $\succ$ Bank loan	Trade credit $\succcurlyeq$ Bank loan	Trade credit $\sim$ Bank loan
Retailer	$z_b > 0$		$z_b = 0$
	Trade credit $\succ$ Bank loan		Trade credit $\sim$ Bank loan

Proposition 3 implies that trade credit weakly dominates bank financing in a capital-constrained supply chain, from both the supplier's and retailer's perspectives. That means, the supply chain should adopt trade credit instead of a bank loan when the retailer has budget constraint. The underlying reason is as follows: From the supplier's perspective, when the retailer's initial working capital level is low, i.e., $y < \underline{\mathbb{C}}$, by financing the retailer, the supplier is able to maintain stronger control over the supply chain. In other words, by lending money to the retailer, the supplier can influence the retailer's decisions on both operations and financing. However, if the supplier does not offer trade credit, the retailer has to borrow money from the bank, so the supplier can only influence the retailer's decision operationally. When $y > \overline{\mathbb{C}}$, the retailer is not capital-constrained and $r_s^* = r_b^* = r_f$; therefore, its decisions under bank financing and trade credit are the same, and these two financing channels are indifferent to the supplier. When the retailer's initial capital level is within a range, i.e., $\underline{\mathbb{C}} \le y \le \overline{\mathbb{C}}$, in the trade credit case, the supplier can influence the retailer's choice between $\hat{q}$ and $\bar{q}$ by adjusting the option price for a higher expected profit. Because $\bar{q}$ is the supplier's optimal decision under bank financing, the output of trade credit is no worse than that of the bank loan.

On the other hand, from the retailer's perspective, given the supplier's offer (c_{os}^*, r_s^*) and combined it with its optimal response function under bank financing, if $z_b > 0$, then there is bankruptcy risk for the retailer and $r_b^* > r_f = r_s^*$. Therefore, it is better for the retailer to choose trade credit due to the lower interest rate. If $z_b = 0$, then $r_b^* = r_f = r_s^*$, so trade credit and bank financing are indifferent to the retailer.

6.2 The Optimal Order Size

When the retailer faces no bankruptcy risk, the equilibria under bank financing and trade credit are the same; hence, the supplier's expected utility, the retailer's expected profit, and the supply chain's efficiency are the same in these two cases. However, when bankruptcy risk does exist, things are different. Therefore, in this subsection, we mainly focus on the scenario when bankruptcy risk exists. Proposition 4 summarizes the comparison of the optimal order sizes under bank financing and trade credit.

Proposition 4. $q_b^* \le q_s^* \le q_\alpha$ *when* $c \ge \tilde{c}$ *and* $q_\alpha < q_s^* < q_b^*$ *when* $c < \tilde{c}$.

Recall that $q_b^* = \bar{q}$ and $q_s^* = \hat{q}$ in the presence of bankruptcy risk. Referring to Figure 3, the relationship between q_b^* and q_s^* is obvious. The result of Proposition 4 implies that, when the production cost is high, trade credit achieves higher supply chain efficiency than bank financing, in that the order size is higher. In contrast, when the production cost is low, bank financing achieves higher supply chain efficiency than trade credit. The intuition behind this observation is as follows: On the one hand, when the production cost is high, meaning that the option is also expensive, the retailer cannot afford ordering too much. In this case, the supplier can encourage the retailer to order more by providing the retailer with a more economical source of financing, i.e., trade credit. On the other hand, when the production cost is low, meaning that the option is not that expensive, the retailer has an incentive to order more of the product. Under trade credit, the supplier can better adjust the option price, which is beneficial not only because it can obtain a higher price margin, but also because of the decreased order size and the consequential reduction of the risk of over-production.

Proposition 5. *In a decentralized supply chain with a capital-constrained retailer purchasing via the option contract, neither bank financing nor trade credit can coordinate the supply chain, i.e.,* $q_b^* < q_c^o$ *and* $q_s^* < q_c^o$ *always hold, where* $q_c^o = \bar{F}^{-1}\left(\frac{c(1+r_f)}{p}\right)$ *is the order size under supply chain coordination.*

Proposition 5 demonstrates that in a decentralized supply chain with a capital-constrained retailer purchasing via the option contract, neither bank financing nor trade credit can coordinate the supply chain, as the effect of double marginalization cannot be completely eliminated. Nevertheless, as was explained in the paragraph following Lemma 5, when the production cost is low, the order size can exceed q_α, which is an upper bound on the order size in the literature using the wholesale price contract.

7 Impact of Relationship Concern

7.1 Impact of Relationship Concern on Order Size

In this paper we assume that the supplier has relationship concern for the retailer. With $\theta \ge 0$, both the retailer's and supplier's expected ending cash level can influence the supplier's decision due to long-run development consideration. Thus, it is natural to consider the impact of relationship concern θ under different financing channels. The following proposition demonstrates the impact of relationship concern on the optimal order size.

Proposition 6. *Suppose* $0 \leq \theta < 1$.
(i) When using a bank loan, q_b^* *increases in* θ.
(ii) When using trade credit, if $z_s = 0$, q_s^* *increases in* θ*; if* $z_s > 0$, q_s^* *increases in* θ *when* $c \geq \tilde{c}_l$, *where* $\tilde{c}_l = [\bar{F}(\hat{q})(p - 2(p - c_e)(1 - \theta)) - c_e r_f]/(1 + r_f) < \tilde{c}$.

Proposition 6 shows that the supplier's relationship concern always has a positive impact on supply chain efficiency, i.e., order size, under bank financing. The same result can be found when there is no bankruptcy risk for the retailer, i.e., $z_s = 0$, under trade credit. However, when bankruptcy risk exists, supply chain efficiency benefits from the degree of relationship concern as long as the production cost is not very low, i.e., $c \geq \tilde{c}_l$. Here, note that if θ and c_e are small and $p - 2(p - c_e)(1 - \theta) \leq c_e r_f$ is satisfied, then $\tilde{c}_l < 0$. Then, q_s^* consistently increases in θ. From Proposition 6, we know that when the supplier is the core company in a supply chain and cares about the development of the supply chain as a whole, it can improve the performance of the supply chain by taking its partner's payoff into consideration.

Proposition 7. When $\theta = 1$, regardless of the financing channel, the decentralized supply chain reduces to the centralized supply chain and the retailer's optimal order size is

$$q_{b,s}^* = \bar{F}^{-1}\left(\frac{c(1+r_f)+c_e r_f}{p}\right). \tag{17}$$

Note that the result in Proposition 7 holds regardless of which financing channel is used. When $\theta = 1$, the supplier's objective function becomes $U_{b,s}(c_o, r_s) = \Pi_{b,s}(c_o, r_s) + \pi_{b,s}(q)$, which is the expected ending cash level of the entire supply chain. Therefore, the supply chain achieves its highest efficiency under the option contract. However, although the decentralized supply chain becomes the centralized supply chain when $\theta = 1$, supply chain coordination cannot be achieved because the retailer has to keep some money in hand from time $t = 0$ to time $t = 1$ to exercise the purchased units of the option at time $t = 1$. Thus, the time value of this amount of money is lost.

7.2 Impact of Relationship Concern on Retailer's Revenue

In this subsection we investigate the impact of relationship concern on the retailer's welfare under different financing channels.

Proposition 8. *Assume* $0 \leq \theta \leq 1$.
(i) *Both* $\pi_b(\bar{q})$ *and* $\pi_s(\bar{q})$ *increase in* θ, and $\pi_s(\hat{q})$ *increases in* θ *when* $c \geq \tilde{c}_l$.
(ii) *When using a bank loan,* z_b *increases in* θ *if* $c \geq \tilde{c}$ *and decreases in* θ *if* $c < \tilde{c}$. *When using trade credit,* z_s *increases in* θ *if* $c \geq \tilde{c}$ *but decreases in* θ *if* $\tilde{c}_l \leq c < \tilde{c}$.

Proposition 8(i) reveals that the retailer benefits from the supplier's higher level of relationship concern, in the sense of a higher expected ending cash level, under either bank financing or trade credit without bankruptcy risk. However, if bankruptcy risk exists when using trade credit, the retailer benefits from the supplier's higher level of relationship concern as long as the production cost is not too low, i.e., $c \geq \tilde{c}_l$.

The result in Proposition 8(ii) is somewhat surprising. Intuitively, one might expect that the retailer can always benefit from the supplier's relationship concern due to its long-run development and collaboration consideration. Nevertheless, when the production cost is sufficiently high, the bankruptcy risk in equilibrium becomes higher, regardless of which financing channel is used. From the retailer's perspective, a high revenue comes at the cost of high risk.

From the supplier's perspective, its intention to care about the welfare of and long-run collaboration with the retailer eventually harms the retailer's long-run survival and thus the long-run development of the supply chain. Therefore, the supplier's relationship concern is not always beneficial to the retailer. While helping the retailer improving its economic payoff, the supplier should also take care of the risk faced by the retailer.

8 Conclusions

In this paper we investigate the joint ordering and financing problems in a two-echelon supply chain consisting of a small-sized downstream retailer with limited capital and a large upstream supplier with sufficient capital. Facing uncertain market demand, the retailer orders from the supplier based on the option contract, with the price determined by the supplier. Nevertheless, because of limited capital, the retailer may need to raise money via either bank financing or trade credit (if it is provided by the supplier) to maintain a reasonable capital level to pay for option orders. Moreover, the retailer cares more about its own economic welfare to achieve short-run survival, while the supplier includes relationship concern in its objective function to achieve long-run development of the supply chain. By solving the Stackelberg game, we obtain the supplier's optimal pricing and trade credit decisions, and the retailer's optimal financing and ordering decisions. Furthermore, we analyze the supplier's and retailer's preferences between the financing channels and explore the impact of the supplier's relationship concern on the equilibrium outcomes.

We summarize the main results of this research as follows. First, if the retailer chooses bank financing, both the supplier and retailer make the same decisions as those in the traditional case where there is no budget constraint for the retailer. Under this circumstance, the bank can be viewed as the retailer's internal accounting department, due to the perfectly competitive financial market. By contrast, if the retailer chooses trade credit, things become more complicated because both the supplier's and retailer's decisions are influenced by the budget constraint. Specifically, the retailer's optimal order size and choice of the financing channel are dependent on its initial capital level. For the supplier, its decision on the interest rate of the trade credit always equals the risk-free interest rate.

Second, from both the supplier's and retailer's perspectives, we find that trade credit weakly dominates bank financing. To be specific, trade credit dominates bank financing when the retailer's initial working capital is low and bankruptcy risk exists. Otherwise, trade credit is no different from bank financing. This conclusion is consistent with the results of a small business credit survey, which shows that the approval rate of supplier financing at 84% among SMEs is higher than for bank financing at 79% (Barkley et al., 2016).

Third, when the budget constraint on the retailer's side is considered, we find that the option contract cannot coordinate the supply chain, which is in stark contrast with traditional research. However, compared with the wholesale price contract, the option contract can encourage the retailer to order more when the manufacturing cost of the product is low; supply chain efficiency is hence improved.

Finally, the supplier's relationship concern can increase the equilibrium order size. However, if the retailer chooses bank financing, the supplier's relationship concern consistently improves the retailer's ending cash level. If the retailer chooses trade credit, the same conclusion still holds when the production cost is not too low. Furthermore, although the supplier's relationship concern can increase the retailer's payoff in most cases, it may also hurt the retailer by increasing its bankruptcy risk when the production cost is sufficiently high.

By theoretical analyses, we figure out the the optimal decisions on financing, ordering, and option pricing in a capital-constrained supply chain. Also, we explore the impacts of the supplier's relationship concern on the retailer and the entire supply chain efficiency. In what follows, we summarize the managerial insights implied by our theoretical results. First, from the perspective of supply chain financing, our results suggest that the supplier should finance the retailer by providing trade credit when the retailer suffers from budget constraint. Moreover, it is optimal for the supplier to set the interest rate as low as the risk-free interest rate. The insights are two-fold. From the retailer's perspective, it always (weakly) prefers trade credit than bank loan since the former is much cheaper, especially when it has bankruptcy risk. From the supplier's perspective, by lending money to the retailer, the supplier is able to proactively influence the retailer's decisions both operationally and financially. Consequently, the supplier can benefit from trade credit by having stronger control over the supply chain. In sum, our work sheds light on the advantages of internal financing over external financing in a capital-constrained supply chain. Second, regarding the adoption of option contract, our study indicates that option contract can be a better choice than the wholesale price contract for a capital-constrained supply chain when the product manufacturing cost is low. This result provides insights on the selection of purchasing contract in different scenarios. Lastly, our work reveals that in a supply chain consisting of a capital-rich supplier and a capital-constrained retailer, the supplier's relationship concern can further improve the expected profit of the entire supply chain. However, it may increase the retailer's bankruptcy risk when the production cost is high. Therefore, our research implies that the supplier should pay close attention to not only the economic payoff but also the bankruptcy risk in order to achieve a long-run development of the supply chain.

There are several future research directions worth exploring. First, in this paper, based on the assumption that the supplier is a large firm and the retailer is an SME, we only take the supplier's relationship concern into consideration. However, some studies (e.g., Cui et al., 2007; Loch and Wu, 2008) have indicated that other behavioural factors like fairness concern and status concern may also have an impact on decision making. It is thus worth exploring the impacts of other behavioural factors on a supply chain that has the budget constraint. Second, future research could relax the assumption that the bank operates in a perfectly competitive market to explore the bank's interest rate decisions in detail. Finally, it is worth investigating how the retailer's risk attitude influences its ordering and financing decisions in a decentralized supply chain. In our study, we assume that the retailer is risk-neutral and that it will go bankrupt if it cannot fully repay the loan. We conjecture that the retailer's decisions may be significantly different when it is risk-averse and includes bankruptcy risk in its objective.

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Endnotes

[1] In some high-tech or capital-intense industries, option price is transferred to the supplier to build capacity. By paying option price, the retailer can obtain flexibility in ordering and reduce market

demand risk. The supplier can benefit by selling options and obtaining market demand information from the retailer's order. Under this background, we can see option price as the price of flexibility by the supplier to the retailer, and the exercise price as the wholesale price determined by other factors, such as market competition or government.

[2] This assumption can be satisfied by some common distributions, such as truncated normal, uniform and Weibull $f(D) = k\lambda^k D^{k-1} e^{-(\lambda D)^k}$ for $\lambda > 0,\ k \geq 3$.

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Appendices

Proof of Proposition 1. Since we show the analyses for bank loan interest and the retailer's order decision in 4.1 and 4.2, we only provide the proof of Equation (8b) here. From (7), the supplier's expected utility can be written as

$$U_b(c_{ob}) = \mathbb{E}\left[\begin{array}{l}(Y + c_{ob}q_b - cq_b)(1+r_f) + c_e \min\{q_b, D\} + \\ \theta\left[p\min\{q_b, D\} + c_e(q_b - D)^+ - (c_{ob}q_b + c_e q_b - y)(1+r_f)\right]\end{array}\right].$$

Based on the strict increase function $F(\cdot)$, we have the inverse function of (5)

$$c_{ob} = \frac{(p-c_e)\bar{F}(q_b) - c_e r_f}{1+r_f}.$$

Substituting c_{ob} into $U_b(c_{ob})$, the first order derivative of $U_b(q_b)$ with respect to q_b is

$$\frac{dU_s(q_b)}{dq_b} = (p-c_e)\bar{F}(q_b)\left[\frac{p}{p-c_e} - (1-\theta)h(q_b)q_b\right] - c(1+r_f) - c_e r_f$$

Define q_β that satisfies $h(q_\beta)q_\beta = \frac{p}{(p-c_e)(1-\theta)}$. When $q_b \geq q_\beta$, $U_b(q_b)$ decreases in q_b. Thus the optimal order size $q_b^* \in [0, q_\beta)$. In $[0, q_\beta)$, the second order derivative of $U_b(q_b)$ with respect to q_b is

$$\frac{d^2U_b(q_b)}{d{q_b}^2} = -(p-c_e)f(q_b)\left[\frac{p}{p-c_e} - (1-\theta)h(q_b)q_b\right] - (p-c_e)(1-\theta)\bar{F}(q_b)\left(h'(q_b)q_b + h(q_b)\right) < 0,$$

which means $U_b(q_b)$ is concave in the interval $[0, q_\beta)$, and the optimal q_b^* solves $\frac{dU_s(q_b)}{dq_b} = 0$. Combined with (5), the optimal option price under the bank loan case is $c_{ob}^* = [(p-c_e)\bar{F}(q_b^*) - c_e r_f]/(1+r_f)$. □

Proof of Lemma 1. When the retailer chooses to finance via trade credit from the supplier, its expected ending cash level is

$$\pi_s(q_s) = \mathbb{E}\left[\left(p\min\{q_s, D\} + c_e(q_s - D)^+ - B_s(1+r_s)\right)^+\right],$$

where $B_s = (c_{os}q_s + c_e q_s - y)^+$ is the money the retailer borrowed from the supplier. To prove Lemma 1, we firstly suppose $z_s \geq q_s > 0$. From $pz_s + c_e(q_s - z_s)^+ = B_s(1+r_s)$, we have $pz_s = B_s(1+r_s)$. Therefore $\mathbb{E}\left[(p\min\{q_s, D\} + c_e(q_s - D)^+ - B_s(1+r_s))^+\right] = 0$ because $p\min\{q_s, D\} + c_e(q_s - D)^+ \leq pq_s \leq B_s(1+r_s)$, meaning that the retailer has no working capital left at the end of the selling season and even its initial working capital is lost.

However, if the retailer chooses to purchase only with all its initial working capital, the expected ending cash level will be $\mathbb{E}\left[p\min\left(\frac{y}{c_{os}+c_e}, D\right) + c_e\left(\frac{y}{c_{os}+c_e} - D\right)^+\right] > 0$, which is strictly greater than the former case. Thus the retailer will not finance itself via trade credit, which contradicts the assumption. Therefore $q_s > z_s$ is proved. □

Lemma A. 1. *$q\bar{F}(q)$ is a quasi-concave function of q and its maximum value is achieved at q_α, where q_α satisfies $q_\alpha h(q_\alpha) = 1$.*

Proof of Lemma A.1. The first derivative of $q\bar{F}(q)$ with respect to q is $\bar{F}(q) - qf(q) = \bar{F}(q)[1 - qh(q)]$. Given increasing failure rate function $h(\cdot)$, $d(q\bar{F}(q))/dq > 0$ when $q < q_\alpha$, $d(q\bar{F}(q))/dq < 0$ when $q > q_\alpha$ and $d(q\bar{F}(q))/dq = 0$ at $q = q_\alpha$. Thus, $q\bar{F}(q)$ is a quasi-concave function of q and the maximum value is achieved at q_α. □

Lemma A. 2. *Define $\delta = \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s h(z_s)$, then $\delta < 1$.*

Proof of Lemma A.2. Based on (11a) and Lemma A.1, we know

$$\begin{aligned}
q_\alpha \bar{F}(q_\alpha) &\geq q_s \bar{F}(q_s) \\
&= \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s \bar{F}(z_s) \\
&= \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s \bar{F}\left(\frac{c_{os}(1+r_s)q_s + c_e r_s q_s - y(1+r_s)}{p-c_e}\right) \\
&\geq \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s \bar{F}\left(\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s\right).
\end{aligned}$$

Meanwhile, since $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} < 1$ and $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s < q_s$, we have $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s < q_\alpha$. Therefore,

$$\begin{aligned}
1 - \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s h(z_s) &\geq 1 - \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s h\left(\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s\right) \\
&> 1 - q_\alpha h(q_\alpha) = 0.
\end{aligned}$$

The ">" holds because $qh(q)$ increases in q and $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s < q_\alpha$. Then Lemma A.2 is proved. □

Proof of Lemma 2. Firstly, we prove q_s in (11) monotonously decreases in c_{os}. In (11a), by implicit function theorem, the first derivative of q_s with respect to c_{os} is

$$\begin{aligned}
\frac{dq_s}{dc_{os}} &= -\frac{(1+r_s)\bar{F}(z_s)\left(1 - \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} q_s h(z_s)\right)}{(p-c_e)\bar{F}(q_s)\left[h(q_s) - \left(\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\right)^2 \frac{f(z_s)}{\bar{F}(q_s)}\right]} \\
&= -\frac{(1+r_s)\bar{F}(z_s)\,(1-\delta)}{(p-c_e)\bar{F}(q_s)\left[h(q_s) - \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} h(z_s)\right]}.
\end{aligned}$$

The second "=" holds because of the retailer's optimal response function $\bar{F}(q_s) = \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} \bar{F}(z_s)$.

Since $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} < 1$ and $z_s < q_s$, $h(q_s) - \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e} h(z_s) > 0$. Meanwhile, we have proved $\delta < 1$ in Lemma A.2, thus $\frac{\partial q_s}{\partial c_{os}} < 0$.

Similarly, in (11b)$\sim$(11d), we can easily figure out that q_s decreases in c_{os}.

Next, for any given r_s, we will prove that the retailer's expected ending cash level $\pi_s(q_s)$ monotonously decreases in c_{os}. From (10a),

$$\begin{aligned}\pi_s(q_s) = &\int_{z_s}^{q_s} [pq_s - p(q_s - x) + c_e(q_s - x) - (c_o q_s + c_e q_s - y)(1 + r_s)]\, f(x)dx \\ &+ \int_{q_s}^{N} [pq_s - (c_{os} q_s + c_e q_s - y)(1 + r_s)]\, f(x)dx.\end{aligned}$$

$$\begin{aligned}\frac{d\pi_s(q_s)}{dc_{os}} =& - (p - c_e)[F(q_s) - F(z_s)]\frac{dq_s}{dc_{os}} + [p - (c_{os} + c_e)(1 + r_s)]\bar{F}(z_s)\frac{dq_s}{dc_{os}} \\ &- q_s(1 + r_s)\bar{F}(z_s) + [(p - c_e)(q_s - z_s) - pq_s + (c_{os} q_s + c_e q_s - y)(1 + r_s)]\, f(z_s)\frac{dz_s}{dc_{os}}.\end{aligned}$$

Substituting $z_s = \frac{(c_{os}q_s + c_e q_s - y)(1+r_s) - c_e q_s}{p-c_e}$ into the last part,

$$\begin{aligned}\frac{d\pi_s(q_s)}{dc_{os}} &= -(p - c_e)[F(q_s) - F(z_s)]\frac{dq_s}{dc_{os}} + [p - (c_{os} + c_e)(1 + r_s)]\bar{F}(z_s)\frac{dq_s}{dc_{os}} - q_s(1 + r_s)\bar{F}(z_s) \\ &= \frac{dq_s}{dc_{os}}\left[(p - c_e)\bar{F}(q_s) - (c_{os}(1 + r_s) + c_e r_s)\bar{F}(z_s)\right] - q_s(1 + r_s)\bar{F}(z_s) \\ &= -q_s(1 + r_s)\bar{F}(z_s) < 0.\end{aligned}$$

Similarly, we can prove that $\frac{d\pi_s(q_s)}{dc_{os}} < 0$ in (10b) $\sim$ (10d). □

Lemma A. 3. *The four constraints of* $(c_{os} + c_e)q_s$ *in (11) can be converted into sets* $\Omega_i, i = 1, 2, 3, 4$*, in Figure 2 respectively.*

Proof of Lemma A.3. Before the proof, we need to emphasize that Δ_2 and Δ_3 might be empty sets in some cases. Particularly, when $N\bar{F}(N) + c_e N/(p - c_e) < y(1 + r_f)/(p - c_e)$, i.e., $N > q_3^u$, $\Delta_1 = [q_1^u, q_2^u)$, $\Delta_2 = [q_2^u, q_3^u]$, $\Delta_3 = (q_3^u, N]$; when $y(1 + r_f)/(p - c_e) \le N\bar{F}(N) + c_e N/(p - c_e) < y(1 + r_s)/(p - c_e)$, i.e., $q_2^u \le N < q_3^u$, $\Delta_1 = [q_1^u, q_2^u)$, $\Delta_2 = [q_2^u, N]$, $\Delta_3 = \varnothing$; when $y(1 + r_s)/(p - c_e) \le N\bar{F}(N) + c_e N/(p - c_e)$, $q_1^u \le N < q_2^u$, $\Delta_1 = [q_1^u, N]$ and $\Delta_2 = \Delta_3 = \varnothing$. N is not less than q_1^u because $N\bar{F}(N) = 0$.

Now we prove Lemma A.3. According to Figure 2 and (11d), when the retailer is rich in capital, $q_s\bar{F}(q_s) + c_e q_s/(p - c_e) = (c_{os} + c_e)q_s(1 + r_f)/(p - c_e)$ and $(c_{os} + c_e)q_s < y$. Therefore, $q_s\bar{F}(q_s) + c_e q_s/(p - c_e) < y(1 + r_f)/(p - c_e)$, i.e., $q_s \in \Omega_4$.

In (11b), when the retailer needs to borrow money and has no bankruptcy risk, $q_s\bar{F}(q_s) + c_e q_s/(p - c_e) = (c_{os} + c_e)q_s(1 + r_s)/(p - c_e)$ and $y < (c_{os} + c_e)q_s < y + c_e q_s/(1 + r_s)$. Therefore, $y(1 +$

$r_s)/(p-c_e) < q_s\bar{F}(q_s) + c_e q_s/(p-c_e) \leq [y(1+r_s) + c_e q_s]/(p-c_e)$, i.e., $q_s \in \Omega_2$.

In (11c), when the retailer just has enough money to pay, i.e., $(c_{os}+c_e)q_s = y$, $q_s \in \Omega_3$.

From (11a), we have $q_s\bar{F}(q_s) = \frac{(c_{os}+c_e)q_s(1+r_s)-c_e q_s}{p-c_e}\bar{F}(z_s) > \frac{y(1+r_s)}{p-c_e}\bar{F}(z_s)$. When $z_s = 0$, $q_s = q_1^l$ or q_1^u; when $z_s > 0$, $q_s \in \Omega_1$. Thus, Lemma A.3 is proved. □

Proof of Lemma 3. From (16a),

$$\frac{\partial U_s(q_s, r_s)}{\partial q_s} = \frac{(p-c_e)\bar{F}(q_s)}{1-\delta}\left[\frac{p(1-\theta\delta) - c_e\delta(1-\theta)}{p-c_e} - (1-\theta)q_s h(q_s)\right] - c_e r_f - c(1+r_f).$$

When $q_s h(q_s) \geq [p(1-\theta\delta) - c_e\delta(1-\theta)] \,/\, [(p-c_e)(1-\theta)]$, it is obvious that $\frac{\partial \Pi_s(q_s,r_s)}{\partial q_s} < 0$. Thus the optimal order size q_s^* satisfies $q_s^* h(q_s^*) < \frac{p(1-\theta\delta)-c_e\delta(1-\theta)}{(p-c_e)(1-\theta)} < \frac{p}{(p-c_e)(1-\theta)}$, i.e., $q_s^* < q_\beta$.

Similarly, if $q_s \geq q_\beta$, $\frac{\partial U_s(q_s,r_s)}{\partial q_s}$ in (16b) $\sim$ (16d) will be negative. Thus for any given $r_s \geq r_f$, the optimal order size q_s^* for the supplier is less than q_β. □

Lemma A. 4. *For a given r_s and increasing convex failure rate $h(\cdot)$, $\bar{G}(q_s, r_s)$ monotonously decreases in q_s in $[0, q_\beta)$ with $\bar{G}(0, r_s) > 0$ and $\bar{G}(q_\beta, r_s) < 0$; $\hat{G}(q_s, r_s)$ monotonously decreases in q_s in $[0, q_\alpha]$ with $\hat{G}(0, r_s) > 0$ and has unclear monotonicity in (q_α, q'_β), but $\hat{G}(q_s, r_s) < 0$ for $q_s \in [q'_\beta, q_\beta)$, where q'_β satisfies $q'_\beta h(q'_\beta) = [p(1-\theta\delta) - c_e\delta(1-\theta)]/[(p-c_e)(1-\theta)]$ and $q'_\beta < q_\beta$.*

Proof of Lemma A.4. For $q_s \in [0, q_\beta)$, in $\bar{G}(q_s, r_s)$, both $\bar{F}(q_s)$ and $\frac{p}{p-c_e} - (1-\theta)q_s h(q_s)$ are positive and monotonously decrease in q_s. Hence $\bar{G}(q_s, r_s)$ is a monotonous decrease function of q_s with $\bar{G}(q_s = 0, r_s) = p - c_e r_f - c(1+r_f) > 0$ and $\bar{G}(q_s = q_\beta, r_s) = -c_e r_f - c(1+r_f) < 0$.

As for $\hat{G}(q_s, r_s)$, we can rewrite it as

$$\hat{G}(q_s, r_s) = (1-\theta)(p-c_e)\bar{F}(q_s)\frac{1-q_s h(q_s)}{1-\delta} + (c_e + \theta(p-c_e))\bar{F}(q_s) - c_e r_f - c(1+r_f).$$

Based on the definition of δ and z_s, we have

$$\begin{aligned}
&\frac{\partial \delta}{\partial q_s} = \left[\frac{1+r_s}{p-c_e}q_s\frac{\partial c_{os}}{\partial q_s} + \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\right]\left[h(z_s) + \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}q_s h'(z_s)\right],\\
&\frac{1+r_s}{p-c_e}q_s\frac{\partial c_{os}}{\partial q_s} = \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\left[q_s h(z_s)\frac{\partial z_s}{\partial q_s} - q_s h(q_s)\right],\\
&\frac{\partial z_s}{\partial q_s} = \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\frac{1-q_s h(q_s)}{1-\delta}.
\end{aligned}$$

Then

$$\begin{aligned}
&\frac{\partial}{\partial q_s}\ln\left(\frac{1-q_s h(q_s)}{1-\delta}\right)\\
&=\frac{\partial}{\partial q_s}\ln(1-q_s h(q_s))-\frac{\partial}{\partial q_s}\ln(1-\delta)\\
&=-\frac{h(q_s)+q_s h'(q_s)}{1-q_s h(q_s)}+\frac{\frac{\partial\delta}{\partial q_s}}{1-\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}q_s h(z_s)}\\
&<\frac{-[h(q_s)+q_s h'(q_s)]+\frac{\partial\delta}{\partial q_s}}{1-q_s h(q_s)}\\
&=\frac{-[h(q_s)+q_s h'(q_s)]+\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\frac{1-q_s h(q_s)}{1-\delta}\left[h(z_s)+\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}q_s h'(z_s)\right]}{1-q_s h(q_s)}\\
&<0
\end{aligned}$$

The first "<" holds because $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}q_s < q_s$ and $h(z_s) < h(q_s)$; the second "<" holds because $\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\frac{1-q_s h(q_s)}{1-\delta} < 1$ and the increasing convex failure rate $h(\cdot)$.

Therefore, when $q_s \in [0, q_\alpha]$, $\bar{F}(q_s)$ and $\frac{1-q_s h(q_s)}{1-\delta}$ in $\hat{G}(q_s, r_s)$ are both positive and monotonously decrease in q_s. Thus $\hat{G}(q_s, r_s)$ is a monotonous decrease function in q_s for $q_s \in [0, q_\alpha]$.

When $q_s \in [q'_\beta, q_\beta)$, it is obvious that $\hat{G}(q_s, r_s) < 0$.

When $q_s \in (q_\alpha, q'_\beta)$,

$$\begin{aligned}
\frac{\partial\hat{G}(q_s,r_s)}{\partial q_s} = -\frac{(1-\theta)(p-c_e)\bar{F}(q_s)}{(1-\delta)^3}&\Bigg[(1-\delta)^2 q_s h'(q_s)-(1-q_s h(q_s))^2\left(\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}\right)^2 q_s h'(z_s)\\
&+\left(1+\frac{p(1-\theta\delta)-c_e\delta(1-\theta)}{(1-\theta)(p-c_e)}-q_s h(q_s)\right)h(q_s)(1-\delta)^2-\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}h(z_s)\left(1-q_s h(q_s)\right)^2\Bigg]
\end{aligned}$$

Since the relationship between $(1-\delta)^2$ and $(1-q_s h(q_s))^2$ is unclear, the sign of the square brackets is unclear. Then the the monotonicity of $\hat{G}(q_s, r_s)$ is ambiguous for $q_s \in (q_\alpha, q'_\beta)$.

Additionally, since $\hat{G}(q_s, r_s) - \bar{G}(q_s, r_s) = (p-c_e)\bar{F}(q_s)\frac{(1-\theta)\delta}{1-\delta}[1-q_s h(q_s)]$, we have $\hat{G}(q_s, r_s) > \bar{G}(q_s, r_s)$ when $q_s \in [0, q_\alpha)$, $\hat{G}(q_s, r_s) < \bar{G}(q_s, r_s)$ when $q_s \in (q_\alpha, q_\beta]$, and $\hat{G}(q_s, r_s) = \bar{G}(q_s, r_s)$ when $q_s = q_\alpha$. □

Proof of Lemma 4. For any given r_s, we firstly define the left and right derivatives of $U_s(q_s, r_s)$ at q_1^l

and q_1^u with respect to q_s as follows:

$$
\begin{aligned}
G^-(q_1^l, r_s) &= (1-\theta)(p-c_e)\bar{F}(q_1^l)(1-q_1^l h(q_1^l)) + (c_e+\theta(p-c_e))\bar{F}(q_1^l) - [c_e r_f + c(1+r_f)],\\
G^+(q_1^l, r_s) &= \frac{(1-\theta)(p-c_e)}{1-\delta}\bar{F}(q_1^l)(1-q_1^l h(q_1^l)) + (c_e+\theta(p-c_e))\bar{F}(q_1^l) - [c_e r_f + c(1+r_f)],\\
G^-(q_1^u, r_s) &= \frac{(1-\theta)(p-c_e)}{1-\delta}\bar{F}(q_1^u)(1-q_1^u h(q_1^u)) + (c_e+\theta(p-c_e))\bar{F}(q_1^u) - [c_e r_f + c(1+r_f)],\\
G^+(q_1^u, r_s) &= (1-\theta)(p-c_e)\bar{F}(q_1^u)(1-q_1^u h(q_1^u)) + (c_e+\theta(p-c_e))\bar{F}(q_1^u) - [c_e r_f + c(1+r_f)].
\end{aligned}
$$

It is obvious that $G^-(q_1^l, r_s) \le G^+(q_1^l, r_s)$, $G^-(q_1^u, r_s) \le G^+(q_1^u, r_s)$. Based on the definitions, $\bar{q}$ and $\hat{q}$ are the potential optimal order size for the supplier. However, when $q_\beta \le q_1^u$ as shown in Figure A1, $\bar{q}$ is feasible only if $\bar{q} \in [0, q_3^l) \cup (q_2^l, q_1^l]$ and $\hat{q}$ is feasible only if $\hat{q}(r_s) \in (q_1^l, q_\beta)$; when $q_\beta > q_1^u$ as shown in Figure A2, $\bar{q}$ is feasible only if $\bar{q} \in [0, q_3^l) \cup (q_2^l, q_1^l] \cup [q_1^u, q_\beta)$, and $\hat{q}$ is feasible only if $\hat{q}(r_s) \in (q_1^l, q_1^u)$.

Here we need to note that when $\hat{q}$ is feasible, if $\hat{G}(q_\alpha, r_s) \le 0$, i.e., $c \ge \tilde{c}$, $\hat{q}$ is unique; if $\hat{G}(q_\alpha, r_s) > 0$, i.e., $c < \tilde{c}$, $\hat{q}$ may be not unique.

Intuitively, Ω_3 may also include potential optimal solutions for the supplier in the cases of Figure A1 (b) $\sim$ (d) and Figure A2 (b) $\sim$ (e). But next, we prove that the potential optimal candidate in Ω_3 can be ignored. Ω_3 is the interval where the retailer uses all of its money for ordering without raising money from the supplier. Firstly, we define

$$\widetilde{G}(q_s, r_s) = \theta p \bar{F}(q_s) - c_e F(q_s) - c_e r_f - c(1+r_f).$$

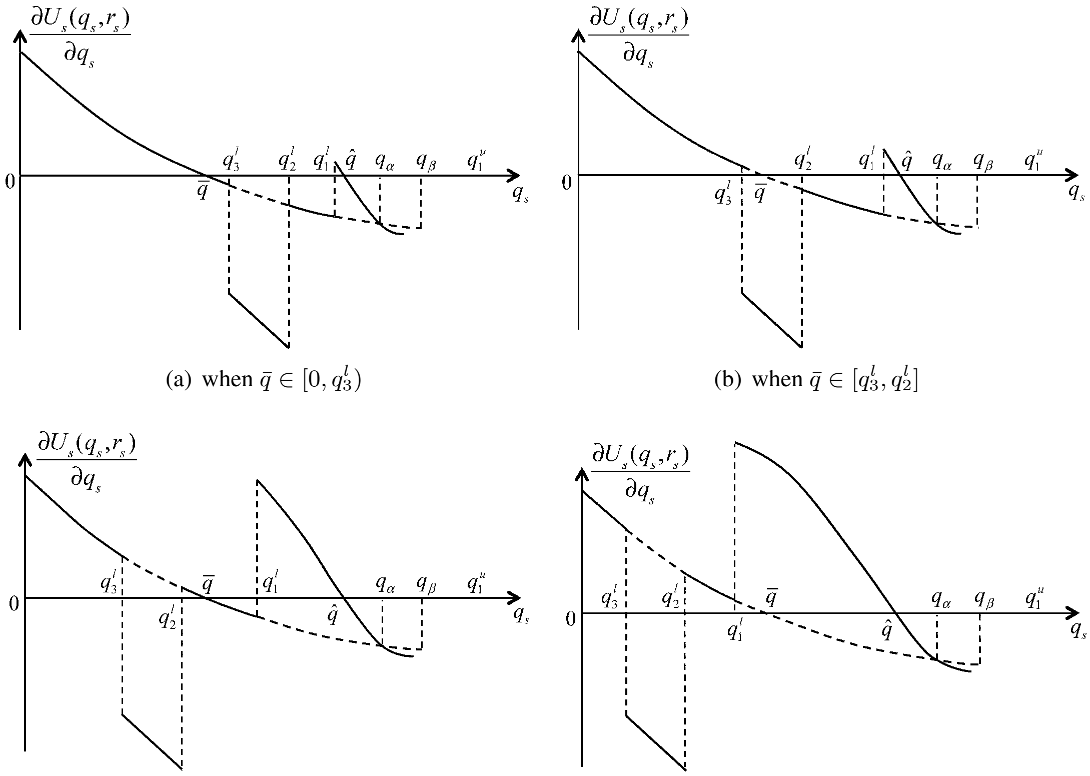
Then $\partial \widetilde{G}(q_s, r_s)/\partial q_s = -\theta p f(q_s) - c_e f(q_s) < 0$. Let $\tilde{q}$ be the potential candidate in $\Omega_3 \cap \Theta$, then $\tilde{q}$ is either one of the end points of this interval or a point that satisfies $\widetilde{G}(q_s, r_s) = 0$. According to the defination, $\tilde{q}$ is independent of r_s. Therefore, we can consider the special case $r_s = r_f$. Then $q_3^l = q_2^l$, $q_2^u = q_3^u$ and $\partial U_s(q_s, r_s)/\partial q_s$ in continued in $[0, q_1^l]$ and $[q_1^u, N]$. When $\bar{q} \in [0, q_1^l] \cup [q_1^u, N]$, $\bar{q}$ is feasible and it is a solution no worse than $\tilde{q}$; when $\bar{q} \in (q_1^l, q_1^u)$, $\bar{q}$ is not feasible but $\hat{q}$ is feasible and it is a solution no worse than q_1^l which is no worse than $\tilde{q}$. Consequently, for $r_s = r_f$, $\tilde{q}$ is not a better solution than $\bar{q}$ or $\hat{q}$. Hence we can ignore the discussion about q_3^l in our following analyses.

Furthermore, we will prove that there always exists at least one potential optimal solution for the supplier.

When $q_\beta \le q_1^u$, if $G^+(q_1^l, r_s) > 0$, $\hat{G}(q_s, r_s)$ intersects the horizontal axis in (q_1^l, q_β), then at least one $\hat{q}$ exists; if $G^+(q_1^l, r_s) \le 0$, since $\bar{G}(q_s = 0, r_s) > 0$ and $G^-(q_1^l, r_s) \le G^+(q_1^l, r_s) \le 0$, $\bar{G}(q_s, r_s)$ intersects the horizontal axis in $[0, q_1^l]$ and $\bar{q}$ is a potential optimal solution.

Similarly, the same conclusion can be easily proved when $q_\beta > q_1^u$.

Therefore, we can say that there always exists at least one feasible optimal sales volume for the



(a) when $\bar{q} \in [0, q_3^l)$ (b) when $\bar{q} \in [q_3^l, q_2^l]$

(c) when $\bar{q} \in (q_2^l, q_1^l]$ (d) when $\bar{q} \in (q_1^l, q_\beta)$

Figure A1. Value of $\frac{\partial U_s(q_s,r_s)}{\partial q_s}$ when $q_\beta \leq q_1^u$

supplier in different cases. □

Lemma A. 5. *$U_s(\hat{q}, r_s)$ decreases in r_s in interval $\Omega_1 \cap \Theta$, $U_s(\bar{q}, r_s)$ decreases in r_s in interval $\Omega_2 \cap \Theta$, $U_s(\bar{q}, r_s)$ is independent of r_s in interval $\Omega_4 \cap \Theta$;*

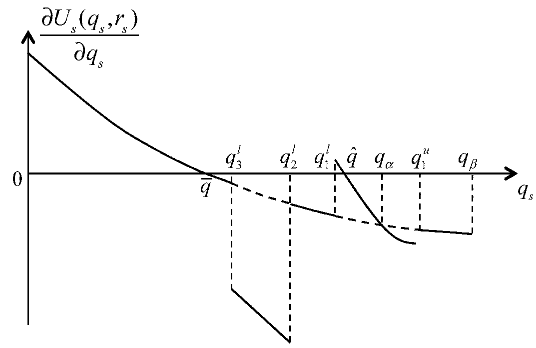
Proof of Lemma A.5. When $\bar{q} \in \Omega_2 \cap \Theta$, combined with (14b) and (15), the supplier's expected utility function is

$$\begin{aligned} U_s(\bar{q}, r_s) = \Bigg[&(Y + y - cq_s)(1 + r_f) - c_e q_s r_f + p\int_0^{q_s} xf(x)dx + pq_s\bar{F}(q_s) \\ &-(1-\theta)\left[pq_s - (p - c_e)\int_0^{q_s}(q_s - x)f(x)dx\right] + (1-\theta)\left((c_{os}q_s + c_e q_s)(1 + r_s) - y(1 + r_s)\right)\Bigg] \end{aligned} \tag{A1}$$

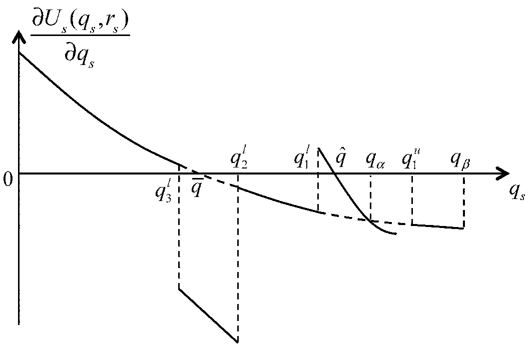
Substituting $q_s = \bar{q}$ and $(c_{os} + c_e)(1 + r_s) = (p - c_e)\bar{F}(\bar{q}) + c_e$ into it,

$$\begin{aligned} U_s(\bar{q}, r_s) = \Bigg[&(Y + y - c\bar{q})(1 + r_f) - c_e\bar{q}r_f + p\int_0^{\bar{q}} xf(x)dx + p\bar{q}\bar{F}(\bar{q}) \\ &-(1-\theta)\left[p\bar{q} - (p - c_e)\int_0^{\bar{q}}(\bar{q} - x)f(x)dx\right] + (1-\theta)\left((p - c_e)\bar{q}\bar{F}(\bar{q}) + c_e - y(1 + r_s)\right)\Bigg]. \end{aligned}$$

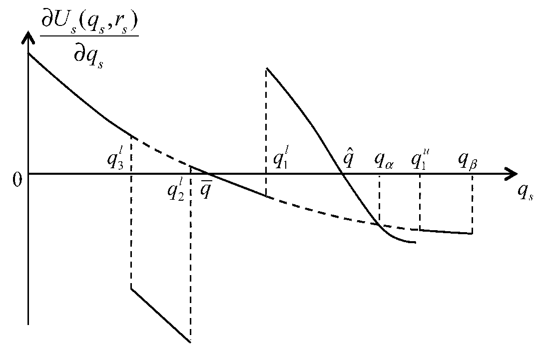
Recall that $\bar{q} = q_b^*$ is the solution of (8b) and independent of r_s. Meanwhile, since $-y(1 + r_s)$ decreases in r_s, $U_s(\bar{q}, r_s)$ is a decrease function of r_s.



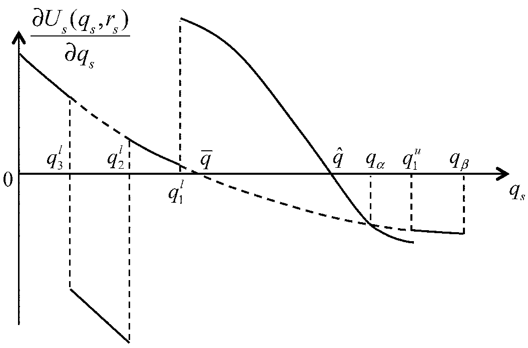
(a) when $\bar{q} \in [0, q_3^l)$



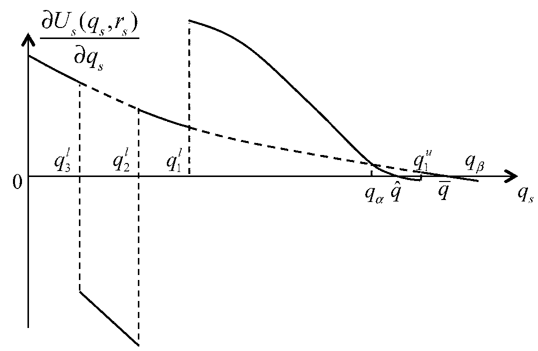
(b) when $\bar{q} \in [q_3^l, q_2^l]$



(c) when $\bar{q} \in (q_2^l, q_1^l]$



(d) when $\bar{q} \in (q_1^l, q_1^u)$



(e) when $\bar{q} \in [q_1^u, q_\beta)$

Figure A2. Value of $\frac{\partial U_s(q_s,r_s)}{\partial q_s}$ when $q_\beta > q_1^u$

When $\bar{q} \in \Omega_4 \cap \Theta$, bases on (14d) and (15), the supplier's expected utility is

$$\begin{aligned} U_s(\bar{q}, r_s) =& (Y + c_{os}q_s - cq_s)(1 + r_f) + c_e q_s \bar{F}(q_s) + c_e \int_0^{q_s} x f(x) dx \\ &+ \theta \left[p q_s \bar{F}(q_s) + p \int_0^{q_s} x f(x) dx + y(1 + r_f) - (c_{os} + c_e) q_s (1 + r_f)) \right]. \end{aligned}$$

By setting $q_s = \bar{q}$ and the fact that $(c_{os} + c_e)(1 + r_f) = (p - c_e)\bar{F}(\bar{q}) + c_e$,

$$\begin{aligned} U_s(\bar{q}, r_s) =& (Y + c_{os}\bar{q} - c\bar{q})(1 + r_f) + c_e \bar{q} \bar{F}(\bar{q}) + c_e \int_0^{\bar{q}} x f(x) dx \\ &+ \theta \left[p \bar{q} \bar{F}(\bar{q}) + p \int_0^{\bar{q}} x f(x) dx + y(1 + r_f) - (p - c_e) \bar{q} \bar{F}(\bar{q}) - c_e \right]. \end{aligned}$$

Therefore $U_s(\bar{q}, r_s)$ is independent of r_s.

To explore the impact of r_s on $U_s(\hat{q}, r_s)$, Based on (11a) and (16a), we define

$$\begin{aligned} V^1 &= \bar{F}(q_s)(h(q_s)q_s - 1) + \frac{c(1 + r_f) + c_e r_f - (c_e + \theta(p - c_e))\bar{F}(q_s)}{(1 - \theta)(p - c_e)}(1 - \delta), \\ V^2 &= \bar{F}(q_s) - \frac{c_{os}(1 + r_s) + c_e r_s}{p - c_e} \bar{F}(z_s). \end{aligned}$$

$\hat{q}_s(r_s)$ and $c_{os}(r_s)$ are both functions of r_s and satisfy $V^1 = 0$, $V^2 = 0$. For the purpose of simplicity, we denote $\hat{q}_s(r_s)$ and $c_{os}(r_s)$ by q and c_{os} respectively in the following proof of this Lemma.

Next, we take the first order derivatives of V^1 and V^2 with respect to q, c_{os} and r_s and denote them by V^1_q, $V^1_{c_{os}}$, $V^1_{r_s}$ and V^2_q, $V^2_{c_{os}}$, $V^2_{r_s}$ respectively.

$$\begin{aligned} V^1_q =& \bar{F}(q) \left[q \left(h'(q) - \frac{1 - qh(q)}{1 - \delta} \left(\frac{c_{os}(1 + r_s) + c_e r_s}{p - c_e} \right)^2 h'(z_s) \right) \right. \\ &\left. + \left(h(q) - \frac{1 - qh(q)}{1 - \delta} \frac{c_{os}(1 + r_s) + c_e r_s}{p - c_e} h(z_s) \right) + h(q)(1 - qh(q)) + \frac{(c_e + \theta(p - c_e))(1 - \delta)}{(1 - \theta)(p - c_e)} h(q) \right], \\ V^1_{c_{os}} =& -\frac{1 - qh(q)}{1 - \delta} q \bar{F}(q)(1 + r_s) \left[\frac{h(z_s)}{p - c_e} + \frac{(c_{os}(1 + r_s) + c_e r_s)q}{(p - c_e)^2} h'(z_s) \right], \\ V^1_{r_s} =& -\frac{1 - qh(q)}{1 - \delta} q \bar{F}(q) \left[\frac{c_{os} + c_e}{p - c_e} h(z_s) + \frac{(c_{os}(1 + r_s) + c_e r_s)(c_{os}q + c_e q - y)}{(p - c_e)^2} h'(z_s) \right], \\ V^2_q =& -\bar{F}(q) \left[h(q) - \frac{c_{os}(1 + r_s) + c_e r_s}{p - c_e} h(z_s) \right], \\ V^2_{c_{os}} =& -\frac{1 + r_s}{p - c_e} \bar{F}(z_s) \left[1 - \frac{c_{os}(1 + r_s) + c_e r_s}{p - c_e} q h(z_s) \right], \\ V^2_{r_s} =& -\bar{F}(z_s) \left[\frac{c_{os} + c_e}{p - c_e} - \frac{[c_{os}(1 + r_s) + c_e r_s](c_{os}q + c_e q - y)}{(p - c_e)^2} h(z_s) \right]. \end{aligned}$$

Next, we define $W = V^1_q V^2_{c_{os}} - V^1_{c_{os}} V^2_q$, $W_{c_{os}} = -V^1_q V^2_{r_s} + V^1_{r_s} V^2_q$ and $W_q = -V^1_{r_s} V^2_{c_{os}} + V^1_{c_{os}} V^2_{r_s}$.

Meanwhile, after some algebra and simplification,

$$W_q = \frac{1-qh(q)}{(1-\delta)(p-c_e)^2}(1+r_s)yq\bar{F}^2(q)\left[h'(z_s)+h^2(z_s)\right].$$

According to the total derivatives of V^1 and V^2, we have $V_q^1 dq + V_{c_{os}}^1 dc_{os} + V_{r_s}^1 dr_s = 0$ and $V_q^2 dq + V_{c_{os}}^2 dc_{os} + V_{r_s}^2 dr_s = 0$. Thus

$$\frac{\partial q}{\partial r_s} = \frac{V_{c_{os}}^1 V_{r_s}^2 - V_{c_{os}}^2 V_{r_s}^1}{V_q^1 V_{c_{os}}^2 - V_{c_{os}}^1 V_q^2} = \frac{W_q}{W},$$

$$\frac{\partial c_{os}}{\partial r_s} = \frac{V_{r_s}^1 V_q^2 - V_q^1 V_{r_s}^2}{V_q^1 V_{c_{os}}^2 - V_{c_{os}}^1 V_q^2} = \frac{W_{c_{os}}}{W},$$

The first derivative of z_s with respect to r_s is

$$\begin{aligned}\frac{\partial z_s}{\partial r_s} &= \frac{1}{p-c_e}\left[\frac{\partial c_{os}}{\partial r_s}q(1+r_s) + \frac{\partial q}{\partial r_s}[(c_{os}+c_e)(1+r_s)-c_e] + (c_{os}q + c_e q - y)\right] \\ &= \frac{1}{(p-c_e)W}\left[-\frac{V_q^2(1+r_s)q\bar{F}(q)yh(z_s)}{p-c_e}\frac{1-qh(q)}{1-\delta} + \frac{V_q^1(1+r_s)\bar{F}(z_s)y}{p-c_e} + (c_{os}(1+r_s)+c_e r_s)W_q\right].\end{aligned} \tag{A2}$$

In (14a), take the first derivative of $U_s(c_{os}, r_s)$ with respect to r_s, we have

$$\begin{aligned}\frac{\partial U_s(c_{os}, r_s)}{\partial r_s} &= \left[p\bar{F}(q) - c_e r_f - c(1+r_f)\right]\frac{\partial q}{\partial r_s} + (1+\theta)q(1+r_s)\bar{F}(z_s)\frac{\partial c_{os}}{\partial r_s} \\ &\quad + (1-\theta)(c_{os}q + c_e q - y)\bar{F}(z_s).\end{aligned}$$

Combined with the first line of (A2), we have

$$\frac{\partial U_s(c_{os}, r_s)}{\partial r_s} = \left[\theta p\bar{F}(q) + (1-\theta)c_e\bar{F}(q) - c_e r_f - c(1+r_f)\right]\frac{\partial q}{\partial r_s} + (p-c_e)\bar{F}(z_s)\frac{\partial z_s}{\partial r_s}.$$

Furthermore,

$$\begin{aligned}\frac{\partial U_s(c_{os}, r_s)}{\partial r_s} =& \frac{W_q}{W}\left[\theta p\bar{F}(q) + (1-\theta)c_e\bar{F}(q) - c_e r_f - c(1+r_f)\right] \\ &+ \frac{\bar{F}(z_s)}{W}\left[\begin{array}{l} -\dfrac{(1+r_s)q\bar{F}(q)yh(z_s)}{p-c_e}\dfrac{1-qh(q)}{1-\delta}V_q^2 \\ +\dfrac{(1+r_s)\bar{F}(z_s)y}{p-c_e}V_q^1 + (c_{os}(1+r_s)+c_e r_s)W_q \end{array}\right] \\ =& \frac{\bar{F}(z_s)}{W}\left[\begin{array}{l} \dfrac{(1+r_s)\bar{F}(z_s)y}{p-c_e}V_q^1 - \dfrac{(1+r_s)q\bar{F}(q)yh(z_s)}{p-c_e}\dfrac{1-qh(q)}{1-\delta}V_q^2 \\ +\dfrac{\theta p\bar{F}(q) + (1-\theta)c_e\bar{F}(q) - c_e r_f - c(1+r_f)}{\bar{F}(z_s)}W_q + (c_{os}(1+r_s)+c_e r_s)W_q \end{array}\right].\end{aligned}$$

Since $\bar{F}(q) = \frac{(c_{os}(1+r_s)+c_e r_s)\bar{F}(z_s)}{p-c_e}$ and $\bar{F}(q)(1-qh(q)) = \frac{c(1+r_f)+c_e r_f-(c_e+\theta(p-c_e))\bar{F}(q)}{(1-\theta)(p-c_e)}(1-\delta)$,

$$
\begin{aligned}
&\frac{\theta p\bar{F}(q) + (1-\theta)c_e\bar{F}(q) - c_e r_f - c(1+r_f)}{\bar{F}(z_s)} W_q + (c_{os}(1+r_s) + c_e r_s)W_q \\
=&W_q\left[-\frac{(1-qh(q))(c_{os}(1+r_s)+c_e r_s)}{1-\delta} + c_{os}(1+r_s) + c_e r_s\right] \\
=&W_q\frac{c_{os}(1+r_s)+c_e r_s}{1-\delta} q\left[h(q) - \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}h(z_s)\right] \\
=&-W_q V_q^2 \frac{c_{os}(1+r_s)+c_e r_s}{(1-\delta)\bar{F}(q)} q.
\end{aligned}
$$

Therefore,

$$
\frac{\partial U_s(c_{os}, r_s)}{\partial r_s} = \frac{\bar{F}(z_s)}{W}\left[\frac{(1+r_s)\bar{F}(z_s)y}{p-c_e}V_q^1 - \frac{(1+r_s)q\bar{F}(q)yh(z_s)}{p-c_e}\frac{1-qh(q)}{1-\delta}V_q^2 - W_q\frac{c_{os}(1+r_s)+c_e r}{(1-\delta)\bar{F}(q)}qV_q^2\right].
$$

Meanwhile, since $\frac{1+r_s}{p-c_e}\bar{F}(z_s) = -\frac{V_{c_{os}}{}^2}{1-\delta}$,

$$
\frac{\partial U_s(c_{os}, r_s)}{\partial r_s} = \frac{\bar{F}(z_s)}{W}\left[-\frac{yV_{c_{os}}^2 V_q^1}{1-\delta} - \frac{(1+r_s)q\bar{F}(q)yh(z_s)}{p-c_e}\frac{1-qh(q)}{1-\delta}V_q^2 - W_q\frac{c_{os}(1+r_s)+c_e r_s}{(1-\delta)\bar{F}(q)}qV_q^2\right].
$$

Based on the expression of W_q,

$$
\begin{aligned}
&-\frac{(1+r_s)q\bar{F}(q)yh(z_s)}{p-c_e}\frac{1-qh(q)}{1-\delta} - W_q\frac{c_{os}(1+r_s)+c_e r_s}{(1-\delta)\bar{F}(q)}q \\
=&-\frac{(1+r_s)yq\bar{F}(q)}{p-c_e}\frac{1-qh(q)}{1-\delta}\left[h(z_s) + \frac{\frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}q\left[h'(z_s)+h^2(z_s)\right]}{1-\delta}\right] \\
=&-\frac{(1+r_s)yq\bar{F}(q)}{p-c_e}\frac{1-qh(q)}{1-\delta}\frac{h(z_s) + \frac{c_{os}(1+r_s)+c_e r_s}{p-c_e}qh'(z_s)}{1-\delta}.
\end{aligned}
$$

Combined with

$$
V_{c_{os}}^1 = -\frac{1-qh(q)}{1-\delta}q\bar{F}(q)(1+r_s)\left[\frac{h(z_s)}{p-c_e} + \frac{(c_{os}(1+r_s)+c_e r_s)q}{(p-c_e)^2}h'(z_s)\right],
$$

finally we get

$$
\frac{\partial U_s(\hat{q}, r_s)}{\partial r_s} = \frac{\partial U_s(c_{os}, r_s)}{\partial r_s} = \frac{\bar{F}(z_s)}{W}\left[-\frac{yV_{c_{os}}^2 V_q^1}{1-\delta} + \frac{yV_{c_{os}}^1 V_q^2}{1-\delta}\right] = -\frac{y\bar{F}(z_s)}{1-\delta} \le 0.
$$

Then Lemma A.5 is proved. □

Proof of Proposition 2. For the first part of Proposition 2, according to Lemma A.5, we can easily figure out that the supplier's optimal interest rate under trade credit case is $r_s^* = r_f$.

As for the supplier's optimal decision on the order size, according to the right part of (12b), at q_1^l and q_1^u, $y = (p - c_e)q_1^l \bar{F}(q_1^l)/(1 + r_f) = (p - c_e)q_1^u \bar{F}(q_1^u)/(1 + r_f)$.

(i) If $\hat{G}(q_\alpha, r_s) = \bar{G}(q_\alpha, r_s) > 0$, as shown in Figure 3(b), we just need to focus on interval (q_α, q_β). Since $q_s \bar{F}(q_s)$ is a decrease function in q_s when $q_s \in (q_\alpha, q_\beta)$, thus q_1^u decreases in y.

When $q_\beta \leq q_1^u$, i.e., $q_1^u > \bar{q} > \hat{q}$ and $y < \underline{\mathbb{C}}$, $\hat{q}$ is feasible but $\bar{q}$ is not, thus the optimal order size $q_s^* = \hat{q}$.

When $q_\beta > q_1^u$, both $\hat{q}$ and $\bar{q}$ could be feasible. But if $y < \underline{\mathbb{C}}$, i.e., $q_1^u > \bar{q}$, then only $\hat{q}$ is feasible and thus $q_s^* = \hat{q}$; if $y > \overline{\mathbb{C}}$, i.e., $q_1^u < \hat{q}$, only $\bar{q}$ is feasible and $q_s^* = \bar{q}$; if $\underline{\mathbb{C}} \leq y \leq \overline{\mathbb{C}}$, i.e., $\hat{q} \leq q_1^u \leq \bar{q} < q_\beta$, both $\hat{q}$ and $\bar{q}$ are feasible, so $q_s^* = \arg\max\{\Pi_s(\bar{q}, r_s^*), \Pi_s(\hat{q}, r_s^*)\}$.

(ii) Similarly, in the case of Figure 3(a), we only need to focus on interval $(0, q_\alpha)$. In this interval, q_1^l increases with y. Therefore, $q_1^l < \bar{q}$ and only $\hat{q}$ is feasible when $y < \underline{\mathbb{C}}$; $q_1^l > \hat{q}$ and only $\hat{q}$ is feasible when $y > \overline{\mathbb{C}}$; $\bar{q} \leq q_1^l \leq \hat{q}$ and both $\bar{q}$ and $\hat{q}$ are feasible when $\underline{\mathbb{C}} \leq y \leq \overline{\mathbb{C}}$.

Therefore, Proposition 2 is proved. □

Proof of Proposition 3. We firstly prove the supplier's preference.

When $y > \overline{\mathbb{C}}$, the optimal sales volume of options for the supplier $q_s^* = q_b^* = \bar{q}$. Meanwhile, by the one-to-one mapping between the option price and the order size, we know the optimal option price $c_{ob}^* = c_{os}^* = \frac{(p-c_e)\bar{F}(\bar{q}) - c_e r_f}{1+r_f}$. According to the supplier's expected utility under bank loan case

$$\begin{aligned} U_b(c_{ob}^*) =& \mathbb{E}\left\{(Y + c_{ob}^* \bar{q} - c\bar{q})(1 + r_f) + c_e \min\{\bar{q}, D\}\right. \\ &\left. +\theta\left[p\min\{\bar{q}, D\} + c_e(\bar{q} - D)^+ + (y - c_{ob}^* \bar{q} - c_e\bar{q})(1 + r_f)\right]\right\}, \end{aligned}$$

and trade credit case

$$\begin{aligned} U_s(c_{os}^*, r_s^*) =& \mathbb{E}\left\{(Y + c_{os}^* \bar{q} - c\bar{q})(1 + r_f) + c_e \min\{\bar{q}, D\}\right. \\ &\left. +\theta\Big[p\min\{\bar{q}, D\} + c_e(\bar{q} - D)^+ + (y - c_{os}^* \bar{q} - c_e\bar{q})(1 + r_f)\Big]\right\}, \end{aligned}$$

it is intuitive that $U_b(c_{ob}^*) = U_s(c_{os}^*, r_s^*)$.

When $\underline{\mathbb{C}} \leq y \leq \overline{\mathbb{C}}$, both $\hat{q}$ and $\bar{q}$ should be considered. Firstly, when $q_s^* = \bar{q}$, the retailer has no bankruptcy risk and $c_{ob}^* = c_{os}^*$, thus $U_b(c_{ob}^*) = U_s(c_{os}^*, r_s^*)$. However, since the supplier's optimal decision is $q_s^* = \arg\max\{U_s(\bar{q}, r_s^*), U_s(\hat{q}, r_s^*)\}$, $U_s(c_{os}^*, r_s^*) \geq U_b(c_{ob}^*)$.

When $y < \underline{\mathbb{C}}$, under the bank loan case, combined with $c_{ob}^* = \frac{(p-c_e)\bar{F}(\bar{q}) - c_e r_f}{1+r_f}$, the supplier's expected utility can be written as

$$
\begin{aligned}
U_b(c^*_{ob}) =&\mathbb{E}\Big\{(Y-c\bar{q})(1+r_f)+c_e\min\{\bar{q},D\}+(p-c_e)\bar{F}(\bar{q})\bar{q}-c_er_f\bar{q}\\
&+\theta\left[p\min\{\bar{q},D\}+c_e(\bar{q}-D)^+-(c^*_{ob}\bar{q}+c_e\bar{q}-y)(1+r_f)\right]\Big\}.
\end{aligned}
\tag{A3}
$$

Under the trade credit case, when the bankruptcy threshold level $z_s > 0$ and optimal order size $q^*_s = \hat{q}$, the supplier's expected utility is

$$
\begin{aligned}
U_s(c^*_{os}, r^*_s) =&\mathbb{E}\Big\{(Y+y-c_e\hat{q}-c\hat{q})(1+r_f)+c_e\min\{\hat{q},D\}+\min\{L,B_s(1+r^*_s)\}\\
&+\theta\Big[p\min\{\hat{q},D\}+c_e(\hat{q}-D)^+-(c^*_{os}\hat{q}+c_e\hat{q}-y)(1+r^*_s)\Big]\Big\}.
\end{aligned}
$$

Since

$$
\begin{aligned}
\min\{L,B_s(1+r^*_s)\} &= p\min\{\hat{q},D\}+c_e(\hat{q}-D)^+-(p-c_e)[\min\{\hat{q},D\}-z_s]^+\\
&=(p-c_e)\min\{z_s,D\}+c_e\hat{q},
\end{aligned}
$$

we have

$$
\begin{aligned}
U_s(c^*_{os}, r^*_s) =&\mathbb{E}\Big\{(Y-c\hat{q})(1+r_f)+c_e\min\{\hat{q},D\}+(p-c_e)\min\{z_s,D\}+y(1+r_f)-c_e\hat{q}r_f\\
&+\theta\Big[p\min\{\hat{q},D\}+c_e(\hat{q}-D)^+-(c^*_{os}\hat{q}+c_e\hat{q}-y)(1+r^*_s)\Big]\Big\}.
\end{aligned}
$$

When the retailer has no bankruptcy risk under bank loan case, $z_b = 0$, $r^*_b = r_f$ and $y(1+r_f) \geq (c^*_{ob}+c_e)(1+r_f)\bar{q}-c_e\bar{q}$,

$$
\begin{aligned}
U_s(c^*_{os}, r^*_s) \geq&\mathbb{E}\Big\{(Y-c\hat{q})(1+r_f)+c_e\min\{\hat{q},D\}+(p-c_e)\min\{z_s,D\}+(c^*_{ob}+c_e)(1+r_f)\bar{q}-c_e\bar{q}-c_e\hat{q}r_f\\
&+\theta\Big[p\min\{\hat{q},D\}+c_e(\hat{q}-D)^+-(c^*_{os}\hat{q}+c_e\hat{q}-y)(1+r^*_s)\Big]\Big\}
\end{aligned}
$$

Then, since $c^*_{ob} = \frac{(p-c_e)\bar{F}(\bar{q})-c_er_f}{1+r_f}$,

$$
\begin{aligned}
U_s(c^*_{os}, r^*_s) >&\mathbb{E}\Big\{(Y-c\hat{q})(1+r_f)+c_e\min\{\hat{q},D\}+(p-c_e)\min\{z_s,D\}+(p-c_e)\bar{F}(\bar{q})\bar{q}-c_e\hat{q}r_f\\
&+\theta\Big[p\min\{\hat{q},D\}+c_e(\hat{q}-D)^+-(c^*_{os}\hat{q}+c_e\hat{q}-y)(1+r^*_s)\Big]\Big\}
\end{aligned}
$$

Now, we define

$$
\begin{aligned}
U'_s(c^*_{os}, r^*_s) =&\mathbb{E}\Big\{(Y-c\hat{q})(1+r_f)+c_e\min\{\hat{q},D\}+(p-c_e)\min\{z_s,D\}+(p-c_e)\bar{F}(\bar{q})\bar{q}-c_e\hat{q}r_f\\
&+\theta\Big[p\min\{\hat{q},D\}+c_e(\hat{q}-D)^+-(c^*_{os}\hat{q}+c_e\hat{q}-y)(1+r^*_s)\Big]\Big\}.
\end{aligned}
\tag{A4}
$$

Then (A3) and (A4) have similar formats.

Referring to Figure 3(a), we know $q_s^* = \hat{q} > \bar{q} = q_b^*$. In (A4), we can increase c_{os}^* to $c_{os}^*{}'$ so that $\hat{q}\,(c_{os}^*{}', r_s^*) = \bar{q}$. Meanwhile, since $r_b^* = r_s^* = r_f$, we know $c_{os}^*{}' = c_{ob}^*$ and $z_s(c_{os}^* = c_{os}^*{}') = 0$. Substituting $c_{os}^*{}'$ into (A4), we have

$$\begin{aligned} U_s'(c_{os}^*{}', r_s^*) =& \mathbb{E}\Big\{(Y - c\bar{q})(1 + r_f) + c_e \min\{\bar{q}, D\} + (p - c_e)\bar{F}(\bar{q})\bar{q} - c_e\bar{q}r_f \\ & + \theta\Big[p\min\{\bar{q}, D\} + c_e(\bar{q} - D)^+ - (c_{os}^*{}'\bar{q} + c_e\bar{q} - y)(1 + r_s^*)\Big]\Big\} \\ =& U_b(c_{ob}^*). \end{aligned}$$

However, since $U_s\,(c_{os}^*{}', r_s^*) \geq U_s{}'\,(c_{os}^*{}', r_s^*)$ and (c_{os}^*, r_s^*) is the optimal strategy for the supplier, finally we get the conclusion that $U_s\,(c_{os}^*, r_s^*) > U_b\,(c_{ob}^*, r_s)$.

Similarly, in the case of Figure 3(b), we can get the same conclusion.

When there is bankruptcy risk for the retailer under bank loan case, i.e., $z_b > 0$ and $y(1 + r_f) < (c_{ob}^* + c_e)(1 + r_f)\bar{q} - c_e\bar{q}$. Since $\min\{L, B_b(1 + r_b^*)\} = L - (L - B_b(1 + r_b^*))^+$ and $B_b(1 + r_b^*) = z_b(p - c_e) + c_e\bar{q}$, the amount of money that the retailer can repay the bank can be rewritten as

$$\begin{aligned} \min\{L, B_b(1 + r_b^*)\} &= p\min\{\bar{q}, D\} + c_e(\bar{q} - D)^+ - (p - c_e)(\min\{\bar{q}, D\} - z_b)^+ \\ &= (p - c_e)\min\{z_b, D\} + c_e\bar{q}. \end{aligned}$$

Then combined with (2), $c_{ob}^*\bar{q}(1 + r_f) = (p - c_e)\min\{z_b, D\} - c_e\bar{q}r_f + y(1 + r_f)$. Substituting it into the supplier's expected utility in (7), we have

$$\begin{aligned} U_b(c_{ob}^*) =& \mathbb{E}\Big\{(Y + y - c\bar{q})(1 + r_f) + c_e \min\{\bar{q}, D\} + (p - c_e)\min\{z_b, D\} - c_e\bar{q}r_f \\ & + \theta\left[p\min\{\bar{q}, D\} + c_e(\bar{q} - D)^+ - (c_{ob}^*\bar{q} + c_e\bar{q} - y)(1 + r_f)\right]\Big\}. \end{aligned} \tag{A5}$$

Under the trade credit case, based on (14a) and (15), the supplier's expected utility can be rewritten as

$$\begin{aligned} U_s(c_{os}^*, r_s^*) =& \mathbb{E}\Big\{(Y + y - c_e\hat{q} - c\hat{q})(1 + r_f) + c_e \min\{\hat{q}, D\} + \min\{L, B_s(1 + r_s^*)\} \\ & + \theta\Big[p\min\{\hat{q}, D\} + c_e(\hat{q} - D)^+ - (c_{os}^*\hat{q} + c_e\hat{q} - y)(1 + r_s^*)\Big]\Big\}. \end{aligned}$$

Similarly, since $\min\{L, B_s(1 + r_s^*)\} = (p - c_e)\min\{z_s, D\} - c_e\hat{q}$,

$$\begin{aligned} U_s(c_{os}^*, r_s^*) =& \mathbb{E}\Big\{(Y + y - c\hat{q})(1 + r_f) + c_e \min\{\hat{q}, D\} + (p - c_e)\min\{z_s, D\} - c_e\hat{q}r_f \\ & + \theta\Big[p\min\{\hat{q}, D\} + c_e(\hat{q} - D)^+ - (c_{os}^*\hat{q} + c_e\hat{q} - y)(1 + r_s^*)\Big]\Big\}. \end{aligned} \tag{A6}$$

Now, as shown in (A5) and (A6), the supplier's expected utility functions under bank loan and trade credit cases have the same formats.

Next, we prove that for a given option price c_o, when $z_b > 0$ and $r_s = r_b^*$, the retailer's optimal order size (denoted by ${q_s}'$) under the trade credit case is greater than $q_b^* = \bar{q}$ under the bank loan case.

Under the bank loan case, since $B_b(1+r_f) = \mathbb{E}[\min\{L, B_b(1+r_b^*)\}]$,

$$(c_o q_b^* + c_e q_b^* - y)(1+r_f) = (c_o q_b^* + c_e q_b^* - y)(1+r_b^*)\bar{F}(z_b) + \int_0^{z_b} (px + c_e(q_b^* - x))\, f(x)dx,$$

i.e.,

$$1 + r_f = (1+r_b^*)\bar{F}(z_b) + \frac{\int_0^{z_b} (px + c_e(q_b^* - x))\, f(x)dx}{c_o q_b^* + c_e q_b^* - y}. \tag{A7}$$

Under the trade credit case, in (10a), take the first derivative of $\pi_s(q_s)$ with respect to q_s,

$$\frac{d\pi_s(q_s)}{dq_s} = (p - c_e)\bar{F}(q_s) - [(c_o + c_e)(1+r_s) - c_e]\bar{F}(z_s). \tag{A8}$$

When $q_s = q_b^*$ and $r_s = r_b^*$, substitute $(p-c_e)\bar{F}(q_b^*) = (c_o + c_e)(1+r_f) - c_e$ and (A7) into (A8), we have

$$\begin{aligned} \left.\frac{d\pi_s}{dq_s}\right|_{q_s = q_b^*} &= \frac{(c_o + c_e)\left[c_e q_b^* F(z_b) + \int_0^{z_b} [px - c_e x] f(x)dx\right]}{c_o q_b^* + c_e q_b^* - y} - c_e F(z_b) \\ &> \frac{(c_o + c_e) c_e q_b^* F(z_b)}{c_o q_b^* + c_e q_b^*} - c_e F(z_b) = 0. \end{aligned}$$

For a given c_o and $r_s = r_b^*$, π_s is a concave function in q_s, and ${q_s}'$ is the optimal solution. Thus we have ${q_s}' > q_b^*$.

Get back to the supplier's preference, since ${q_s}'$ decreases in c_{os}, we can increase c_{os} to c_{os}^1 so that ${q_s}' = q_b^*$ is satisfied and get

$$z_b = \frac{(c_{ob} q_b^* - y)(1+r_b^*) + c_e q_b^* r_b^*}{p - c_e}, \; {z_s}' = \frac{(c_{os}^1 q_b^* - y)(1+r_b^*) + c_e q_b^* r_b^*}{p - c_e} > z_b.$$

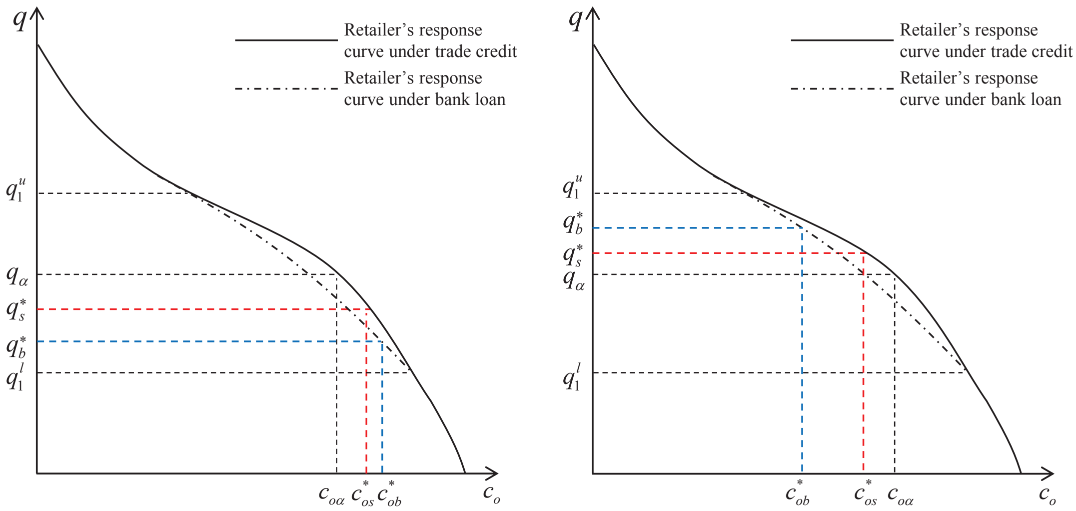
Then, according to (A5) and (A6), it is obvious that $U_b(c_{ob}^*) < U_s(c_{os}^1, r_b^*)$. However, since $r_s^* = r_f$ and $q_s^* = \hat{q}$ are the supplier's optimal decisions in equilibrium, finally we have $U_b(c_{ob}^*) < U_s(c_{os}^*, r_s^*)$.

The proof for the retailer's preference is omit. $\square$

Proof of Proposition 4. According to their definitions,

$$\bar{F}(\bar{q}) = \frac{c(1+r_f) + c_e r_f}{p\left(1 - \frac{(1-\theta)(p-c_e)}{p}\bar{q}h(\bar{q})\right)}, \; \bar{F}(\hat{q}) = \frac{c(1+r_f) + c_e r_f}{\frac{p - c_e\delta - (p-c_e)\theta\delta}{1-\delta} - \frac{(1-\theta)(p-c_e)}{1-\delta}\hat{q}h(\hat{q})}, \; \bar{F}(q_c^o) = \frac{c(1+r_f)}{p}.$$

Since $0 < 1 - \frac{(1-\theta)(p-c_e)}{p}\bar{q}h(\bar{q}) < 1$, we have $\bar{F}(\bar{q}) > \bar{F}(q_c^o)$, i.e., $\bar{q} < q_c^o$. Meanwhile as we know



(a) The production cost is high: $c \geq \tilde{c}$ (b) The production cost is high: $c < \tilde{c}$.

Figure A3. The retailer's optimal response curve in different cases

$z_s < \hat{q}$ and

$$
\begin{aligned}
&\hat{q}h(\hat{q}) > \delta = \hat{q}h(z_s)\frac{c_o(1+r_f)+c_e r_f}{p-c_e} \\
\Rightarrow &(1-\theta)(p-c_e)\hat{q}h(\hat{q}) > (1-\theta)(p-c_e)\delta \\
\Rightarrow &p(1-\delta) > p - c_e\delta - (p-c_e)\theta\delta - (1-\theta)(p-c_e)\hat{q}h(\hat{q}) \\
\Rightarrow &p > \frac{p - c_e\delta - (p-c_e)\theta\delta}{1-\delta} - \frac{(1-\theta)(p-c_e)\hat{q}h(\hat{q})}{1-\delta}.
\end{aligned}
$$

Thus, $\hat{q} < q_c^o$. The first part of Proposition 4 is proved.

When the supplier sets $r_s^* = r_f$, according to the one-to-one mapping between the option price and the retailer's order size in (5) and (11), we have the retailer's optimal response curves in different cases as shown in Figure A3. Then the second part of Proposition 4 can be easily figured out. But please note that in Figure A3(a), the relationship between c_{os}^* and c_{ob}^* is uncertain, i.e., $c_{os}^* = c_{ob}^*$ or $c_{os}^* > c_{ob}^*$ may happen. □

Proof of Proposition 6. Under the bank loan case, $q_b^* = \bar{q}$. In (8b), by taking the first derivative of $\bar{q}$ with respect to θ we obtain

$$
\left[\frac{c(1+r_f)+c_e r_f}{p-c_e}h(\bar{q}) + (1-\theta)(h(\bar{q})+\bar{q}h'(\bar{q}))\bar{F}(\bar{q})\right]\frac{d\bar{q}}{d\theta} = f(\bar{q})\bar{q}.
$$

Then it is obvious that $\frac{d\bar{q}}{d\theta} > 0$.

Under the trade credit case, in the presence of the bankruptcy risk, $q_s^* = \hat{q}$. Based on (16a), we obtain the following by taking the first derivative of $\hat{q}$ with respect to θ

$$\left[(c(1+r_f)+c_e r_f)h(\hat{q})+\frac{(1-\theta)(p-c_e)\bar{F}(\hat{q})}{1-\delta}\left(h(\hat{q})+\hat{q}h'(\hat{q})\right)\right]\frac{d\hat{q}}{d\theta}$$
$$-\frac{(1-\theta)(p-c_e)\bar{F}(\hat{q})}{1-\delta}\frac{c^*_{os}(1+r_f)+c_e r_f}{p-c_e}\left(\frac{1-\hat{q}h(\hat{q})}{1-\delta}\right)^2\left(h(z_s)+\frac{c^*_{os}(1+r_f)+c_e r_f}{p-c_e}\hat{q}h'(z_s)\right)\frac{d\hat{q}}{d\theta}$$
$$=(p-c_e)\bar{F}(\hat{q})\frac{\hat{q}h(\hat{q})-\delta}{1-\delta} \tag{A9}$$

When the production cost is relatively high, i.e., $c \geq \tilde{c}$, $\hat{q}h(\hat{q}) \leq 1$. Then, in (A9), we know $\frac{d\hat{q}}{d\theta} > 0$ because $\frac{c^*_{os}(1+r_f)+c_e r_f}{p-c_e} < 1$, $\frac{1-\hat{q}h(\hat{q})}{1-\delta} < 1$ and $h(\cdot)$ is increasing and convex. When the production cost is low, i.e., $c < \tilde{c}$, since the relationship between $\left(\frac{1-\hat{q}h(\hat{q})}{1-\delta}\right)^2$ and 1 is uncertain, the sign of $\frac{d\hat{q}}{d\theta}$ is uncertain. However, if $\frac{\hat{q}h(\hat{q})-1}{1-\delta} < 1$, $\frac{d\hat{q}}{d\theta} > 0$ for sure. According to (16a), we know

$$\frac{\hat{q}h(\hat{q})-1}{1-\delta}=\frac{(c_e+\theta(p-c_e))\bar{F}(\hat{q})-c(1+r_f)-c_e r_f}{(1-\theta)(p-c_e)\bar{F}(\hat{q})},$$

and $\frac{\hat{q}h(\hat{q})-1}{1-\delta} < 1$ is equivalent to $c > \tilde{c}_l$, where $\tilde{c}_l = \frac{\bar{F}(\hat{q})(p-2(p-c_e)(1-\theta))-c_e r_f}{1+r_f} < \tilde{c}$. In summary, when $z_s > 0$, $\hat{q}$ increases in θ as long as $c \geq \tilde{c}_l$.

Under the trade credit case, when the retailer has no bankruptcy risk, $q^*_s = \bar{q}$. We can obtain the same conclusion as in the bank loan case. □

Proof of Proposition 7. When $\theta = 1$, no matter which financing choice is followed, the first derivative of the supplier's expected utility with respect to q is

$$\frac{\partial U_{b,s}(q,r_s)}{\partial q}=p\bar{F}(q)-c(1+r_f)-c_e r_f.$$

Therefore, $q^*_{b,s}=\bar{F}^{-1}\left(\frac{c(1+r_f)+c_e r_f}{p}\right)$. □

Proof of Proposition 8. For the first part of Proposition 8, under both the bank loan case and the trade credit case without bankruptcy risk, we can easily figure out that

$$\frac{d\pi(q)}{dc_o}=-q(1+r_f)<0, \text{and} \frac{dq}{dc_o}=-\frac{1+r_f}{(p-c_e)f(q)}<0.$$

Then we know $\pi(q)$ increases in q. Since θ improves q under the condition that $c \leq \tilde{c}_l$, it also improves the retailer's expected ending cash level.

Under the trade credit case and in the presence of bankruptcy risk, combined with Lemma 2, we obtain the same conclusion.

For the second part of Proposition 8, we firstly focus on the supplier's problem. We have shown that $\frac{\partial z_s}{\partial \hat{q}} = \frac{c_{os}(1+r^*_s)+c_e r^*_s}{p-c_e}\frac{1-\hat{q}h(\hat{q})}{1-\delta}$ in the proof of Lemma 3 under the trade credit case when there is bankruptcy risk for the retailer. When $c \leq \tilde{c}$, $1 - \hat{q}h(\hat{q}) \geq 0$. Since $\frac{\partial z_s}{\partial \hat{q}} \geq 0$ and $\hat{q}$ increases in θ, we know z_s increases in θ. When $\tilde{c}_l \leq c < \tilde{c}$, $\frac{\partial z_s}{\partial \hat{q}} < 0$, therefore z_s decreases in θ.

Next, under the bank loan case, when there is bankruptcy risk for the retailer,

$$z_b = \frac{(c_{ob}^* \bar{q} + c_e \bar{q} - y)(1 + r_b^*) - c_e \bar{q}}{p - c_e}.$$

Taking the first derivative of z_b with respect to θ, we have

$$(p - c_e)\frac{dz_b}{d\theta} = \bar{q}(1 + r_b^*)\frac{dc_{ob}^*}{d\theta} + (c_{ob}^* + c_e)(1 + r_b^*)\frac{d\bar{q}}{d\theta} + (c_o^* \bar{q} + c_e \bar{q} - y)\frac{dr_b^*}{d\theta} - c_e \frac{d\bar{q}}{d\theta}.$$

From (8a) and (8c) we know

$$\frac{dc_{ob}^*}{d\theta} = -\frac{p - c_e}{1 + r_f} f(\bar{q})\frac{d\bar{q}}{d\theta},$$

$$\begin{aligned}(c_{ob}^* \bar{q} + c_e \bar{q} - y)\bar{F}(z_b)\frac{dr_b^*}{d\theta} =& -(p - c_e)\bar{q}f(\bar{q})\frac{d\bar{q}}{d\theta} + (p - c_e)\bar{q}\frac{1 + r_b^*}{1 + r_f}\bar{F}(z_b)f(\bar{q})\frac{d\bar{q}}{d\theta} + (c_{ob}^* + c_e)(1 + r_f)\frac{d\bar{q}}{d\theta} \\ & - c_e F(z_b)\frac{d\bar{q}}{d\theta} - (c_{ob}^* + c_e)(1 + r_b^*)\bar{F}(z_b)\frac{d\bar{q}}{d\theta}.\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}(p - c_e)\bar{F}(\bar{q})\frac{dz_b}{d\theta} =& -(p - c_e)\bar{q}f(\bar{q})\frac{d\bar{q}}{d\theta} + (c_{ob}^*(1 + r_f) + c_e r_f)\frac{d\bar{q}}{d\theta} \\ =& (p - c_e)\bar{F}(\bar{q})(1 - \bar{q}h(\bar{q}))\frac{d\bar{q}}{d\theta}.\end{aligned}$$

In Proposition 6 we have shown that $\frac{d\bar{q}}{d\theta} \geq 0$. Hence, when $c \geq \tilde{c}$, $\frac{dz_s}{d\theta} \geq 0$; when $c < \tilde{c}$, $\frac{dz_s}{d\theta} < 0$. $\square$