

# The Method Research of Membership Degree Transformation in Multi-indexes Fuzzy Decision-Making

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**Abstract.** The conversion of membership degree is the key computation of fuzzy evaluation for multi-indexes fuzzy decision-making. But the method should be discussed, because redundant data in index membership degree is also used to compute object membership degree, which is not useful for object classification. The new method is: based on data mining of entropy, mining knowledge information about object classification hidden in every index, affirming the relation of object classification and index membership, eliminating the redundant data in index membership for object classification by defining distinguishable weight, extracting valid values to compute object membership. Thus constructing a new membership degree conversion method that can not be effected by redundant data and it is used for fuzzy decision for multi-indexes.

**Indexterms:** fuzzy decision-making; membership degree transformation; distinguishable weight; valid values; comparable values.

## 1 Introduction

There are many factors that effect decision goal in relatively decision system, among these effective factors, selecting the more important factors called as indexes; these different indexes are divided into some levels, decision-making index system is a hierarchical structure: the top level contain one factor  $Q$ , called as general goal; base level contains some base levels that are controllable indexes, so every base index (quantitative or qualitative) does not has its index; there are some intermediate levels between top and base level; and except base index, other levels have some index; in order to descript simplify, let hierarchical structure only have one intermediate level, because there is not difference between two intermediate levels or more and one intermediate level by computation.

If the question is simplified, for example, decision-making goal is that determining the importance order of base indexes about top goal (such as simplify plans scheduling). Saaty provides analytic hierarchy process based on "multiple comparison": under the condition of general goal, scheduling the importance of base indexes, and after the importance of base indexes are normalized, obtaining the importance weights of base indexes of top goal. Although the analytic hierarchy process is not perfect, it can solve above questions scheduling.

But the multi-index decision-making is complex, it does not only need obtain the importance scheduling of indexes, for example,  $m_i$  base index belonging to  $i$  index

of intermediate level is quantitative index, and when  $j(j = 1 \sim m_i)$  index is continuous in intervals  $[a_j, b_j]$ , so  $i$  index changes continuously, which leads to variation of top general goal  $Q$ . The goal of decision-making is that: what status is the top general goal when the value of base index  $j(1 \leq j \leq m)$  is  $x_j \in [a_j, b_j]$ .

Obviously, if wanting to solve above question, first, it must discrete continuous status of  $i$  index into  $P$  different classes (also called kinds), let  $C_k(k = 1 \sim p)$  represents  $k$  th class of  $i$  index. Generally, let  $\{C_1, C_2, \dots, C_p\}$  is a division of state-space  $C$ , and satisfies:

$$C_i \cap C_j = \emptyset \quad (i \neq j) \quad \bigcup_{k=1}^P C_k = C \tag{1}$$

Correspondingly, the value intervals of  $j$  index also is divided into  $P$  sub-intervals, let the value of  $j$  index in  $k$  th sub-interval represents that  $i$  index is  $C_k$  class, called the value of  $j$  index in  $k$  th sub-interval belongs to  $C_k$  class. Following to this division method, although the values of  $j$  index in interval  $[a_j, b_j]$  between two boundary points are so near, they belong to two different classes, which is unreasonable, the reason is that let gradually variational membership degree represents that the value  $x_j$  of  $j$  index belongs to  $C_k$  class which is superior to mutational “belongs to ” (represents number 1) or “not belongs to”(represents number 0). So let fuzzy membership degree  $\mu_k(x_j)$  represents the value  $x_j$  of  $j$  index belongs to  $C_k$  class, which is great contribution of Zadeh [2]. When let fuzzy membership degree represents one index belonging to  $C_k$  class, so providing the following questions:

If  $m_i = 1$ , that is  $i$  index only has one base index  $j$ , doubtlessly, the membership degree  $\mu_k(x_j)$  of the value  $x_j$  of  $j$  index belonging to  $C_k$  class is the membership degree of  $i$  index belonging to  $C_k$  class.

But when  $m_i \geq 2$ , the status changes: how to determine the membership degree of  $i$  index according to the membership degree of  $m_i$  base indexes? That is how to realize membership degree transformation from membership degree of  $j$  index to membership degree of  $i$  index. Because it is inevitable in any multi-indexes decision-making, must be answered explicitly.

For a hierarchical structure, if obtaining membership degree of  $i$  index belonging to  $C_k$  class, it can obtain membership degree from intermediate level to top general goal  $Z$  belonging to  $C_k$  class. And every membership degree transformation in every level can be summarized in the following membership transformation model:

Suppose that there are  $m$  indexes affecting object  $Q$ , where importance weight of  $j(j = 1 \sim m)$  index about object  $Q$  is  $\lambda_j(Q)$  that satisfies:

$$0 \leq \lambda_j(Q) \leq 1, \quad \sum_{j=1}^m \lambda_j(Q) = 1 \tag{2}$$

Every index is classified into P classes.  $C_K$  represents the  $K$  th class and  $C_K$  is prior to  $C_{K+1}$ . If the membership  $\mu_{jK}(Q)$  of  $j$  th index belonging to  $C_K$  is given, where  $K = 1 \sim P$  and  $j = 1 \sim m$ , and  $\mu_{jK}(Q)$  satisfies:

$$0 \leq \mu_{jK}(Q) \leq 1, \sum_{K=1}^P \mu_{jK}(Q) = 1 \tag{3}$$

What is the membership  $\mu_K(Q)$  of object  $Q$  belonging to  $C_K$ ?

Obviously, if the above conversion method is correct or not, which determines that the evaluation result is credible or not.

For the above membership transformation, there are 4 transformation methods in fuzzy comprehensive evaluation:  $M(\wedge, V)$ ,  $M(\bullet, V)$ ,  $M(\wedge, \oplus)$  and  $M(\bullet, +)$ . However through a long-time research on the application, only  $M(\bullet, +)$  is accepted by most researchers, which regards object membership as “weighted sum”:

$$\mu_k(Q) = \sum_{j=1}^m \lambda_j(Q) \cdot \mu_{jk}(Q), (k = 1 \sim p) \tag{4}$$

And the “ $M(\bullet, +)$ ” method as the mainstream membership transformation algorithm is widely used [4-9]. And above method is basic method realizing membership transformation from universe  $U$  fuzzy set to universe  $V$  fuzzy set in fuzzy logical system.

But  $M(\bullet, +)$  method is in dispute in academic circles especially in application field. For example, Ref. [10,11] pointed out that the “weighted sum” method was too simple and did not use information sufficiently. The authors proposed a “subjective and objective comprehensive” method based on evidence deduction and rough sets theory to realize membership transformation. In [11], in the improved fuzzy comprehensive evaluation, a new “comprehensive weight” is defined to compute “weighted sum” instead of index importance weight.

Ref. [12~14] define over proof weight to compute “weighted sum”; Ref. [15] avoid membership degree transformation from index to goal, compute goal membership degree by optimal weight in fuzzy pattern recognition.

However, including these mentioned methods, many existing membership transformation methods are not designed for object classification, thus they can’t indicate “which parts in index membership are useful for object classification and which parts are useless”. The redundancy of membership degree transformation shows that: the correct method realizing membership degree transformation is not found, which need further study.

For the redundant data in existing membership transformation methods, based on data mining of entropy, mining knowledge information about object classification hidden in every index, affirming the relation of object classification and index membership, eliminating the redundant data in index membership for object classification by defining distinguishable weight, therefore, exploring the concrete way to compute object membership degree without the interference of redundant data.

## 2 Distinguishable Weight and Effective Value of $K$ th Class Index Membership

From the viewpoint of classification, what are concerned most are these following questions: Dose every index membership play a role in the classification of object  $Q$ ?

Are there redundant data in index membership for the classification of object  $Q$ ? These questions are very important. Because their answers decide which index membership and which value are qualified to compute membership of object  $Q$ . To find the answers, we analyze as follows.

**2.1 Distinguishable Weight**

(1) Assume that  $\mu_{j1}(Q) = \mu_{j2}(Q) = \dots = \mu_{jp}(Q)$ , then  $j$  th index membership implies that the probability of classifying object  $Q$  into every grade is equal. Obviously, this information is of no use to the classification of object  $Q$ . Deleting  $j$  th index will not affect classification. Let  $\alpha_j(Q)$  represent the normalized and quantized value describing  $j$  th index contributes to classification, then in this case  $\alpha_j(Q) = 0$ .

(2) If there exists an integer  $K$  satisfying  $\mu_{jk}(Q) = 1$  and other memberships are zero, then  $j$  th index membership implies that  $Q$  can be only classified into  $C_k$ . In this case,  $j$  th index contributes most to classification and  $\alpha_j(Q)$  should obtain its maximum value.

(3) Similarly, if  $\mu_{jk}(Q)$  is more concentrated for  $K$ ,  $j$  th index contributes more to classification, i.e.,  $\alpha_j(Q)$  is larger. Conversely, if  $\mu_{jk}(Q)$  is more scattered for  $K$ ,  $j$  th index contributes less to classification, i.e.,  $\alpha_j(Q)$  is smaller.

The above (1)~(3) show that  $\alpha_j(Q)$ , reflecting the value that  $j$  th index contributes to classification, is decided by the extent  $\mu_{jk}(Q)$  is concentrated or scattered for  $K$ . And it can be described quantitatively by the entropy  $H_j(Q)$ . Therefore,  $\alpha_j(Q)$  is a function of  $H_j(Q)$ :

$$H_j(Q) = - \sum_{k=1}^p \mu_{jk}(Q) \cdot \log \mu_{jk}(Q) \tag{5}$$

$$v_j(Q) = 1 - \frac{1}{\log p} H_j(Q) \tag{6}$$

$$\alpha_j(Q) = v_j(Q) / \sum_{t=1}^m v_t(Q) \quad (j = 1 \sim m) \tag{7}$$

**Definition 1.** If  $\mu_{jk}(Q)$  ( $k = 1 \sim p, j = 1 \sim m$ ) is the membership of  $j$  th index belonging to  $C_k$  and satisfies Eq. (1); Given by (4) (5) (6),  $\alpha_j(Q)$  is called distinguishable weight of  $j$  th index corresponding to  $Q$ . Obviously,  $\alpha_j(Q)$  satisfies

$$0 \leq \alpha_j(Q) \leq 1, \quad \sum_{j=1}^m \alpha_j(Q) = 1 \tag{8}$$

**2.2 Effective Value of Index Membership**

The significance of  $\alpha_j(Q)$  lies in its “distinguishing” function, i.e., it is a measure that reveals the exactness of object  $Q$  being classified by  $j$  th index membership and even

the extent of the exactness. If  $\alpha_j(Q) = 0$ , from the properties of entropy, then  $\mu_{j1}(Q) = \mu_{j2}(Q) = \dots = \mu_{jp}(Q)$ . This implies  $j$  th index membership is redundant and useless for classification. Naturally the redundant index membership can't be utilized to compute membership of object  $Q$ .

**Definition 2.** If  $\mu_{jk}(Q)$  ( $k = 1 \sim p, j = 1 \sim m$ ) is the membership of  $j$  th index belonging to  $C_k$  and satisfies Eq. (1), and  $\alpha_j(Q)$  is the distinguishable weight of  $j$  th index corresponding to  $Q$ , then

$$\alpha_j(Q) \cdot \mu_{jk}(Q) \quad (k = 1 \sim p) \tag{9}$$

is called effective distinguishable value of  $K$  th class membership of  $j$  th index, or  $K$  th class effective value for short.

If  $\alpha_j(Q) = 0$ , it indicates that  $j$  th index membership is redundant and useless for the classification of object  $Q$ , so it can not be utilized to compute membership of object  $Q$ . Note that if  $\alpha_j(Q) = 0$ , then  $\alpha_j(Q) \cdot \mu_{jk}(Q) = 0$ . So in fact computing  $K$  th class membership  $\mu_k(Q)$  of object  $Q$  isn't to find  $\mu_{jk}(Q)$  but to find  $\alpha_j(Q) \cdot \mu_{jk}(Q)$ . This is a crucial fact.

When index membership is replaced by effective value to compute object membership, distinguishable weight is a filter. In the progress of membership transformation, it can delete the redundant index memberships that are useless in classification and the redundant values in index membership.

### 3 Comparable Value of $K$ th Class Index Membership and Membership Transformation

Undoubtedly,  $\alpha_j(Q) \cdot \mu_{jk}(Q)$  is necessary for computing  $\mu_k(Q)$ . However the problem is in general  $K$  th class effective values of different indexes aren't comparable and can't be added directly. Because, for determining  $K$  th class membership of object  $Q$ , in most cases these effective values are different in "unit importance". The reason is, generally, index membership doesn't imply relative importance of different indexes. So when using  $K$  th class effective value to compute  $K$  th class membership,  $K$  th effective value must be transformed into  $K$  th class comparable effective value.

#### 3.1 Comparable Value

**Definition 3.** If  $\alpha_j(Q) \cdot \mu_{jk}(Q)$  is  $K$  th class effective value of  $j$  th index, and  $\beta_j(Q)$  is importance weight of  $j$  th index related to object  $Q$ , then

$$\beta_j(Q) \cdot \alpha_j(Q) \cdot \mu_{jk}(Q) \quad (k = 1 \sim p) \tag{10}$$

is called comparable effective value of  $K$  th class membership of  $j$  th index, or  $K$  th class comparable value for short.

Clearly,  $K$  th class comparable values of different indexes are comparable between each other and can be added directly.

### 3.2 Membership Transformation

**Definition 4.** If  $\beta_j(Q) \cdot \alpha_j(Q) \cdot \mu_{jk}(Q)$  is  $K$  th class comparable value of  $j$  th index of  $Q$ , where  $(j = 1 \sim m)$ , then

$$M_k(Q) = \sum_{j=1}^m \beta_j(Q) \cdot \alpha_j(Q) \cdot \mu_{jk}(Q) \quad (k = 1 \sim p) \tag{11}$$

is named  $K$  th class comparable sum of object  $Q$ .

Obviously, the bigger  $M_k(Q)$  is, the more possibly that object  $Q$  belongs to  $C_K$ .

**Definition 5.** If  $M_k(Q)$  is  $K$  th class comparable sum of object  $Q$ , and  $\mu_k(Q)$  is the membership of object  $Q$  belonging to  $C_K$ , then

$$\mu_k(Q) = \frac{M_k(Q)}{\sum_{t=1}^p M_t(Q)} \quad (k = 1 \sim p) \tag{12}$$

Obviously, given by Eq.(11), membership degree  $\mu_k(Q)$  satisfies:

$$0 \leq \mu_k(Q) \leq 1, \quad \sum_{k=1}^p \mu_k(Q) = 1 \tag{13}$$

Up to now, supposing that index membership and index importance weight are given, by Eq. (5) (6) (7)(11) (12), the transformation from index membership to object membership is realized. And this transformation needs no prior knowledge and doesn't cause wrong classification information.

The above membership transformation method can be summarized as “effective, comparison and composition”, which is denoted as  $M(1,2,3)$ .

## 4 Case

Reinforced concrete beam bridge is consist of 7 components including main beam, pier platform, foundation et al. So the reliability is decided by 7 components; and the reliability of every component is effected by concrete factors including carrying capacity, distortion, fracture et al. therefore, the reliability evaluation of defect status of beam bridge is a three levels hierarchical structure [20]. Such as Fig.1.

### 4.1 Fuzzy Evaluation Matrix

By Fig.1, the reliability evaluation of defect status of Beam Bridge is a three levels hierarchical structure. Ref.[20] determines the importance weights of 7 sub-indexes belonging to the reliability evaluation of defect status of beam bridge and importance weights of indexes belonging to every intermediate level by analytic hierarchy process; and according to one beam bridge, determining the membership degree vector of every base index in 5 evaluation classes {good, relatively good, medium, poor, very poor}. At last, obtain the fuzzy evaluation matrix as Table 1.

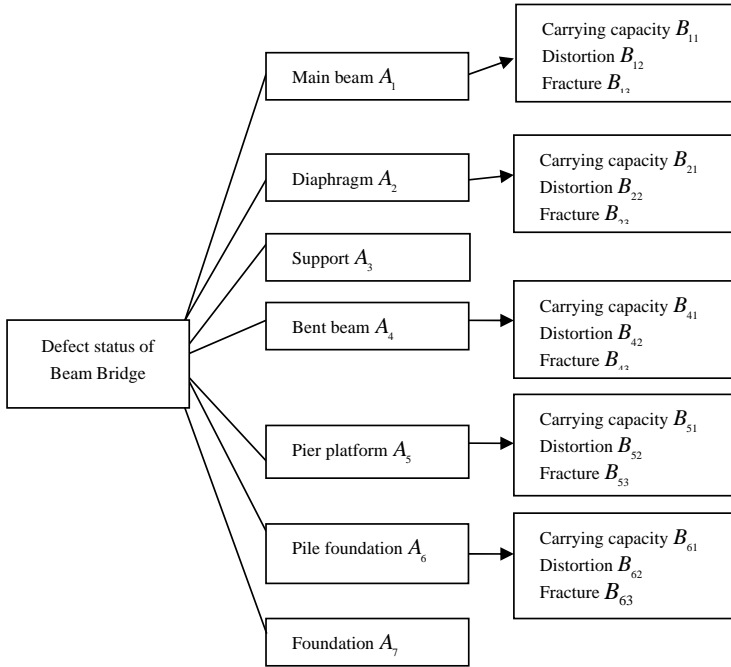


Fig. 1. The reliability evaluation of defect status of Beam Bridge

In Table 1, the figures in parentheses corresponding to the indexes are their importance weights, The vectors behind the lower indexes are their membership vectors including 5 grades. The figures in table are from Ref.[20].

#### 4.2 Steps in the $M(1,2,3)$ Method

As data in table 1, evaluation process as following

(1) base evaluation

Taking the membership degree transformation from Carrying capacity  $B_{11}$  , Distortion  $B_{12}$  、 Fracture  $B_{13}$  to Main beam  $A_1$  for example, steps as following:

①By the evaluation matrix of  $A_1$

$$U(A_1) = \begin{pmatrix} 0.1 & 0.3 & 0.6 & 0 & 0 \\ 0 & 0.4 & 0.5 & 0.1 & 0 \\ 0 & 0.2 & 0.4 & 0.4 & 0 \end{pmatrix}$$

By the  $j$  th row ( $j=1 \sim 3$ ) of  $U(A_1)$ , the distinguishable weights of  $B_{1j}$  are obtained and the distinguishable weight vector is

$$\alpha(A_1) = (0.3682, 0.3447, 0.2871)$$

**Table 1.** Fuzzy evaluation of the reliability evaluation of defect status of Beam Bridge

Goal	Component level	Factor level	Class membership degree {good, relatively good, medium, poor, very poor}
The reliability of Defect status of Beam Bridge <i>Z</i>	Main beam $A_1$ (0.21)	Carrying capacity $B_{11}$ (0.680)	(0.1,0.3,0.6,0,0)
		Distortion $B_{12}$ (0.170)	(0,0.4,0.5,0.1,0)
		Fracture $B_{13}$ (0.150)	(0,0.2,0.4,0.4,0)
	Diaphragm $A_2$ (0.06)	Carrying capacity $B_{21}$ (0.850)	(0,0.3,0.7,0,0)
		Distortion $B_{22}$ (0.075)	(0,0.2,0.7,0.1,0)
		Fracture $B_{23}$ (0.075)	(0,0.2,0.5,0.3,0)
	Support $A_3$ (0.03)		(0,0.5,0.5,0,0)
	Bent beam $A_4$ (0.15)	Carrying capacity $B_{41}$ (0.700)	(0.1,0.5,0.4,0,0)
		Distortion $B_{42}$ (0.150)	(0.2,0.5,0.3,0,0)
		Fracture $B_{43}$ (0.150)	(0.1,0.6,0.3,0,0)
	Pier platform $A_5$ (0.23)	Carrying capacity $B_{51}$ (0.800)	(0.4,0.3,0.3,0,0)
		Distortion $B_{52}$ (0.130)	(0.3,0.5,0.2,0,0)
		Fracture $B_{53}$ (0.070)	(0.4,0.4,0.2,0,0)
	Pile foundation $A_6$ (0.24)	Carrying capacity $B_{61}$ (0.860)	(0.5,0.3,0.2,0,0)
Distortion $B_{62}$ (0.070)		(0.4,0.5,0.1,0,0)	
Fracture $B_{62}$ (0.070)		(0.5,0.4,0.1,0,0)	
Foundation souring $A_7$ (0.08)		(0.6,0.4,0,0,0)	

②The importance weight vector of  $B_{11} \sim B_{13}$  is given as

$$\beta(A_1) = (0.680, 0.170, 0.150)$$

③Calculate the  $K$  th comparable value of  $B_{1j}$  ( $j = 1, 2, \dots, 4$ ) and obtain the comparable value matrix of  $A_1$  :

$$N(A_1) = \begin{pmatrix} 0.0250 & 0.0751 & 0.1502 & 0 & 0 \\ 0 & 0.234 & 0.0293 & 0.0059 & 0 \\ 0 & 0.0086 & 0.0172 & 0.0172 & 0 \end{pmatrix}$$

④Compute the comparable sum of main beam  $A_1$  and obtain the comparable sum vector

$$M(A_1) = (0.0250, 0.1072, 0.1968, 0.0231, 0)$$

⑤Compute the membership vector of main beam  $A_1$

$$\mu(A_1) = (0.0711, 0.3044, 0.5589, 0.0656, 0)$$

Similarly, obtain membership degree vectors of Diaphragm  $A_2$ 、Bent beam  $A_4$ 、Pier platform  $A_5$ 、Pile foundation  $A_6$  that are  $\mu(A_2)$ 、 $\mu(A_4)$ 、 $\mu(A_5)$ 、 $\mu(A_6)$ 、and the membership degree vectors of Support  $A_3$ 、Foundation souring  $A_7$  is given, as  $\mu(A_3)$ 、 $\mu(A_7)$ 、the fuzzy evaluation matrix  $U(Z)$  of the reliability evaluation of



defect status of Beam Bridge  $Z$  is consist of  $\mu(A_1)$ 、  $\mu(A_2)$ 、  $\mu(A_3)$ 、  $\mu(A_4)$ 、  $\mu(A_5)$ 、  $\mu(A_6)$ 、  $\mu(A_7)$ ,as following:

$$U(Z) = \begin{pmatrix} \mu(A_1) \\ \mu(A_2) \\ \mu(A_3) \\ \mu(A_4) \\ \mu(A_5) \\ \mu(A_6) \\ \mu(A_7) \end{pmatrix} = \begin{pmatrix} 0.0711 & 0.3044 & 0.5589 & 0.0656 & 0 \\ 0 & 0.2891 & 0.6909 & 0.0200 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0.1132 & 0.5162 & 0.3707 & 0 & 0 \\ 0.3858 & 0.3357 & 0.2785 & 0 & 0 \\ 0.4921 & 0.3236 & 0.1842 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 \end{pmatrix}$$

(2) Top evaluation

By matrix  $U(Z)$  and the importance weight vector  $(0.21, 0.06, 0.03, 0.05, 0.23, 0.24, 0.08)$ , the membership vector  $U(Z)$  of  $Z$  can be obtained using the similar algorithm in<1>:

$$\mu(Z) = (0.2840, 0.3660, 0.3362, 0.0138, 0)$$

(3) class of reliability evaluation

Let the class  $C_1$  (good),  $C_2$  (relatively good),  $C_3$  (medium),  $C_4$  (poor),  $C_5$  (very poor) quantitative vector is  $(m_1, m_2, m_3, m_4, m_5) = (5, 4, 3, 2, 1)$ , the reliability of defect status of Beam Bridge  $Z$  is

$$\eta(Z) = \sum_{k=1}^5 m_k \cdot \mu_k(Z) \tag{13}$$

In this study  $\eta(Z) = 3.9200$ , because the  $\eta(Z)$  is near to 4, then  $Z$  belongs to “relatively good” class.

## 5 Conclusions

The conversion of membership degree is the key computation of fuzzy evaluation for multi-indexes fuzzy decision-making, but the transformation method has question, analysis the reason of the question, obtain the solving method, at last build the  $M(1, 2, 3)$  model without the interference of redundant data, which is different from  $M(\bullet, +)$  and is nonlinear model.

$M(1, 2, 3)$  provides the general method for membership transformation of multi – indexes decision-making in application fields. The theory value is that it provides transformation method which is comply to logics to realize the transformation universe  $U$  fuzzy set to universe  $V$  fuzzy set in fuzzy logical system.

From index membership degree of base level, after obtain one index membership degree vector in adjacent upper level by  $M(1,2,3)$ , thus, by the same computation, obtaining membership degree vector of top level. Because of normalization of computation,  $M(1,2,3)$  is suitable for membership transformation which contains multi-levels, multi-indexes, large data.

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