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Decision Support

A bi-objective weighted model for improving the discrimination power in MCDEA

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ABSTRACT

Lack of discrimination power and poor weight dispersion remain major issues in Data Envelopment Analysis (DEA). Since the initial multiple criteria DEA (MCDEA) model developed in the late 1990s, only goal programming approaches; that is, the GPDEA-CCR and GPDEA-BCC were introduced for solving the said problems in a multi-objective framework. We found GPDEA models to be invalid and demonstrate that our proposed bi-objective multiple criteria DEA (BiO-MCDEA) outperforms the GPDEA models in the aspects of discrimination power and weight dispersion, as well as requiring less computational codes. An application of energy dependency among 25 European Union member countries is further used to describe the efficacy of our approach.

1. Introduction

Data envelopment analysis (DEA) was first proposed by Charnes, Cooper, and Rhodes (1978) and remained the leading technique for measuring the relative efficiency of decision-making units (*DMUs*) based on their respective multiple inputs and outputs. DEA has been the fastest growing discipline in the past three decades covering easily over a thousand papers in the *Operations Research* and *Management Science* discipline (Emrouznejad, Parker, & Tavares, 2008; Hatami-Marbini, Emrouznejad, & Tavana, 2011). The efficiency of a *DMU* is defined as a weighted sum of its outputs divided by the weighted sum of its inputs on a bounded ratio scale.

One of the drawbacks of DEA is the lack of discrimination among efficient decision making units (*DMUs*), hence yielding many *DMUs* to be efficient. The problem is highlighted when the number of *DMUs* evaluated is significantly lesser than the number of inputs and outputs used in the evaluation. The weights derived from a DEA analysis may reveal that some inputs or outputs have zero values. This is counter-intuitive especially in a decision making exercise, where one expects to use all the inputs and output values that are rated for the *DMUs*. Hence, it further implies that some of the variables were not used in the evaluation judgment in achieving the final ranking. On the contrary, the unrealistic weight distribution for DEA also occurs when some *DMUs* are rated as efficient due to extremely large weights in a single output and/or extremely small weights in a single input.

Thompson, Singleton, Thrall, and Smith (1986) are among the first authors to propose the use of weight restriction to increase the discrimination power of *DMUs*. The issue was immediately picked up by many authors, including Dyson and Thanassoulis (1988), Charnes, Cooper, Huang, and Sun (1990), Thanassoulis and Allen (1998). Hence, several methods such as assurance region (AR) procedure (Khalili, Camanho, Portela, & Alirezaee, 2010; Mecit & Alp, 2013; Sarrico & Dyson, 2004; Thompson, Langemeier, Lee, & Thrall, 1990) and cone ratio envelopment (Cao & Kong, 2010; Charnes et al., 1990) were addressed in the literature as strategies to solve problems arising from unrealistic weight distribution. However, there are some drawbacks to the methods – AR and cone ratio techniques are highly dependent on the measurement of the inputs-outputs units, which may lead to infeasible solutions. In other words, both the methods incorporate extra constraints to the model; thus, making it harder to solve the problem.

Subsequently, other DEA models were introduced in the literature to overcome the discriminant power problems, such as the super-efficiency model (Andersen & Petersen, 1993; Chen, 2005; Chen, Du, & Huo, 2013; Lee, Chu, & Zhu, 2011) and cross-efficiency evaluation technique (Anderson, Hollingsworth, & Inman, 2002; Doyle & Green, 1995; Green, Doyle, & Cook, 1996; Sexton, Silkman, & Hogan, 1986; Wang & Chin, 2010, 2011). The super-efficiency DEA model may obtain infeasible solutions for efficient *DMUs*; particularly, under variable returns to scale (VRS) model. However, attempts had been made to solve the infeasibility problem in super

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efficiency methods. Chen (2005) proposed an approach in which both input-oriented and output-oriented super-efficiency models are used to fully characterize the super-efficiency model, thus claiming that the approach kept infeasibility to a rare occasion. However, Soleimani-damaneh, Jahanshahloo, and Foroughi (2006) presented some counter examples to negate Chen's (2005) claims without any proposed alternative. Drawing from two main sources (i.e. Chen, 2005; Cook, Liang, Zha, & Zhu, 2009), Lee et al. (2011) later provided a solution by a two-stage process catering to adjustments in input saving and output surpluses. Chen and Liang (2011) subsequently formulated a one-model solution to the two-stage process. Lee and Zhu (2012) found that the solution can still be infeasible should some of the input variables have zero values.

With regards to cross-efficiency evaluation technique, the non-uniqueness of the DEA weights could provide a large number of multiple optimal solutions for DEA models. Although recent improvements of cross-efficiency evaluation techniques were proposed (Angiz & Sajedi, 2012), the solution is computationally expensive with the need to solve a series of linear programming problems. The suggestion of imposing secondary goals to improve variability of cross efficiency scores still leaves the non-uniqueness problem looming (see Cook & Zhu, 2013).

Drawing from a multiple objective decision making framework, the multiple criteria (or multi-objective) DEA model (Chen, Larbani, & Chang, 2009; Foroughi, 2011; Li & Reeves, 1999) was suggested as a means to overcome discriminant power and weight dispersion problems. However, the original formulation of Li and Reeves (1999) does not promise complete ranking but merely presupposes the decision maker to use its model's 3 objectives interactively. Thus, in the MCDEA model, the three objectives are analyzed separately; one at a time, and no preference order was set for those objectives. Bal, Örkcü, and Çelebioglu (2010) recently proposed the goal programming approach for solving all 3 objectives of the MCDEA model simultaneously. Their GPDEA models (i.e. constant returns to scale and variable returns to scale) were claimed to improve the dispersion of weights and discriminatory power in a MCDEA framework. This paper highlights that those claims were unfounded, and goes onto show a new bi-objective multiple criteria DEA (BiO-MCDEA) model that could solve those drawbacks.

The focus of this paper is to introduce a weighted model for improving the discrimination power and weight dispersion in the domain of Multiple Criteria Data Envelopment Analysis (MCDEA). The rest of the paper is organized as follows. Section 2 gives a brief description of the multiple criteria data envelopment analysis (MCDEA) and the more recent goal programming data envelopment analysis (GPDEA) as a procedure for MCDEA. Section 3 highlights the drawbacks of using GPDEA to represent MCDEA analysis. We therefore introduce an alternative bi-objective multiple criteria model (BiO-MCDEA) to improve the discrimination power of MCDEA in Section 4. An application of energy dependency among 25 EU member countries demonstrates the efficacy of the model in Section 5. Concluding remarks are given in Section 6.

2. Improving discrimination power in DEA: Recent developments

2.1. Multiple criteria data envelopment analysis (MCDEA)

Li and Reeves (1999) first proposed the MCDEA model as a means to improve the discrimination power of the classical DEA model. In their solution procedure, Li and Reeves (1999) suggested an interaction approach for solving three objectives. The first objective or criterion considers the classical definition of relative efficiency, thus capturing the classical DEA solution within the set of MCDEA solutions. The other two objectives, Minimax and Minsum objectives provide a more restrictive or lax efficiency solutions, respectively. This implies that a wider solution is possible with MCDEA, so as to gain more reasonable input and output weights.

In MCDEA, the three objectives are analyzed separately; one at a time, with no preference order set for those objectives. The solutions derived from each run are considered non-dominated in the multi-objective linear programming (MOLP) sense. Li and Reeves (1999) note that generally the Minimax criterion is more restrictive than the Minsum criterion, while the first criterion (i.e. Classical DEA objective) is considered to be the least restrictive of the three. Since the Minimax and Minsum criteria tend to provide less number of efficient *DMUs* as compared to the first criterion, it is said to provide better discrimination power than a classical DEA model. As such, the Minimax and Minsum criteria are helpful when the number of *DMUs* is not sufficiently larger than the number of inputs and outputs used for evaluation.

Consider the relative efficiency of n *DMUs* which use m inputs (x_{ij}, $i = 1, \ldots, m$, $j = 1, \ldots, n$) to produce s outputs (y_{rj}, $j = 1, \ldots, s$, $j = 1, \ldots, n$). The MCDEA model proposed by Li and Reeves (1999) which considers three objective functions: (i) minimizing d_o (or maximizing θ_o), (ii) minimizing the maximum deviation, and (iii) minimizing the sum of deviations, is defined as follows in Model 1:

2.2. Model 1: Multiple criteria data envelopment analysis

$$
\begin{aligned}
&\min \quad d_o\left(or \ \max \ \theta_o = \sum_{r=1}^{s} u_r y_{ro}\right)\\
&\min \quad M,\\
&\min \quad \sum_{j=1}^{n} d_j\\
&\quad \sum_{i=1}^{m} v_i x_{io} = 1,\\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \ldots, n,\\
&\quad M - d_j \geqslant 0, \quad j = 1, \ldots, n,\\
&\quad u_r \geqslant 0, \quad r = 1, \ldots, s,\\
&\quad v_i \geqslant 0, \quad i = 1, \ldots, m,\\
&\quad d_j \geqslant 0, \quad j = 1, \ldots, n.
\end{aligned} \tag{1}
$$

The quantity d_0 in the first objective function is bounded on an interval [0, 1) and is regarded as a measure of inefficiency. Thus, DMU_0 is efficient at $h_0 = 1 - d_0$ where h_0 is the efficiency measure in a classical DEA. In short, the first objective function $\left[\text{i.e. } \min \ d_0 \ \left(or \ \max \sum_{r=1}^{s} u_r y_{ro}\right)\right]$ is equivalent to the objective function of the classical DEA. The M in the second objective function (Minmax criterion) represents the maximum quantity of all deviation variables $d_j (j = 1, \ldots, n)$. The third objective function is a Minsum of all deviation variables. Another noteworthy point is the introduction of the $M - d_j \geqslant 0$, $(j = 1, \ldots, n)$ constraint in MCDEA, which does not alter the feasible region of the solution but merely ensure that max $d_j \geqslant 0$.

2.3. Goal programming DEA models (GPDEA)

Li and Reeves (1999) did not suggest a solution for their proposed MCDEA model that optimizes all objectives simultaneously. The aim of their proposed MCDEA model solution process is not to extract an optimal solution; but instead, to find a series of non-dominated solutions that is left to the analyst in selecting the most preferred one, if need be. Therefore, goal programming can be seen as a natural progression in converting the multi-objective programming of the MCDEA model into a single objective problem.

Goal programming is a type of multi-objective optimization, which can provide a way of striving towards several such objectives simultaneously. The basic approach of goal programming is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of unwanted deviations of these objective functions from their respective goals.

Bal et al. (2010) recently proposed the following goal programming to solve the formulation proposed by Li and Reeves (1999). The former adopted the non-weighted approach in their solution design and claimed to be an equivalent single objective form to the latter's three objectives.

2.4. Model 2: Goal programming data envelopment analysis under CRS (GPDEA-CCR)

$$\min \quad a=\left\{d_1^-+d_1^++d_2^++\sum_j d_{3j}^-+\sum_j d_j\right\}$$

$$\begin{aligned}
&\sum_{i=1}^m v_i x_{io}+d_1^--d_1^+=1,\\
&\sum_{r=1}^s u_r y_{ro}+d_2^--d_2^+=1,\\
&\sum_{r=1}^s u_r y_{rj}-\sum_{i=1}^m v_i x_{ij}+d_j=0,\quad j=1,\dots,n,\\
&M-d_j+d_{3j}^--d_{3j}^+=0,\quad j=1,\dots,n,\\
&u_r\geqslant 0,\quad r=1,\dots,s,\\
&v_i\geqslant 0,\quad i=1,\dots,m,\\
&d_j\geqslant 0,\quad j=1,\dots,n,\\
&d_1^-,d_1^+,d_2^-,d_2^+\geqslant 0,\\
&d_{3j}^-,d_{3j}^+\geqslant 0,\quad j=1,\dots,n.
\end{aligned} \tag{2}$$

The above model is with the assumption of constant returns to scale (CRS) (Bal et al., 2010), where d_1^- and d_1^+ are the unwanted deviations for the goal which the weighted sum of inputs equal to unity, d_2^- and d_2^+ are the wanted and unwanted deviation variables which make the weighted sum of outputs less than or equal to one, whereas d_{3j}^- and $d_{3j}^+ (j=1,\dots,n)$ are the unwanted and wanted deviation variables for the goal $M-d_j\geqslant 0$, $(j=1,\dots,n)$. remains as the maximum deviation, for $DMU_j(j=1,\dots,n)$, which is also an unwanted deviation. A similar model under the VRS assumption is placed in Appendix A.

The achievement objective function $\{d_1^-+d_1^++d_2^++\sum_j d_{3j}^-+\sum_j d_j\}$ states that all deviations have been given equal weights. In the GPDEA's case, minimizing the unwanted deviations from the goal values are to be desired (Ignizio, 1976; Lee, 1972). However, there are fundamental flaws associated with the GPDEA models, ranging from the interpretation of a goal programming method to the reported results. We highlight some of these issues separately in the next section.

3. The drawbacks on GPDEA models

The purpose of this section is to highlight the drawbacks of the GPDEA models, which will help us to further develop the new bi-objective multiple criteria DEA (BiO-MCDEA) model in Section 4.

3.1. The validity of GPDEA and the issue of zero weights for all variables in some DMUs

We were initially intrigued by the use of goal programming as a means to achieve greater weight dispersion and discrimination power among criteria in DEA. When attempting to reproduce the analysis in Bal et al. (2010), we have noted some methodological and formulation problems. We found some of these problems to be consistent for all datasets in Bal et al. (2010). However, for the purpose of illustrating the inappropriateness of the GPDEA models, we only explain the solutions of 'dataset 1' and 'university dataset' in Bal et al. (2010).

Table 1
Example 1 dataset.

DMU	Outputs				Inputs			
	y_1	y_2	y_3	y_4	x_1	x_2	x_3	x_4
1	47	93	54	65	32	50	82	46
2	88	56	92	80	61	56	68	37
3	94	65	80	80	42	58	45	34
4	50	53	93	97	73	39	88	81
5	47	42	70	52	45	38	68	41
6	86	45	100	47	86	62	44	32
7	83	91	62	74	38	74	71	74
8	79	60	72	98	61	54	70	62
9	85	68	51	41	84	52	38	47
10	78	95	70	92	87	47	31	52

Let us first start with the hypothetical dataset consisting of 10 *DMU*s with four inputs and four outputs (see Table 1 which is reproduced from Bal et al. (2010) for ease of reference).

We used Model 1 formulation for both CRS and VRS assumptions to reproduce the results as depicted in Tables 2 and 3. It is easy to observe that the true efficiency values differ significantly from the ones reported in Bal et al. (2010). More importantly, we examined the weights and noticed contrary to what had been claimed in Bal et al. (2010), the input–output weights and efficiency values for some *DMU*s could attain zero values for all variables. For example in this case, zero weights were discovered for all variables for DMU_1 (under CRS) and DMU_5 (under CRS and VRS). This just disproves the "…improvement of the dispersion of input–output weights and the improvement of discrimination power…" as claimed in Bal et al. (2010). This is problematic when some of the efficiency values can be '1' at the same instance, thus confirming the inability for the input and output weights to translate into technical efficiency effectively (see Appendix B for proof).

It is rather quite simple to reason where the problem lies. As one can easily observe in the next section, we impose some restrictions on the weights to avoid this issue. In an input-oriented model, it is necessary to set the constraint $\sum_{i=1}^m v_i x_{io}=1$, and seek to achieve an output that is as high as possible. This is a fundamental aspect of scaling and benchmarking, where one has to fix either the sum of input or the sum of the output to be 1, before proceeding to determine the other. In Bal et al.'s case (Bal et al., 2010), they chose to set both $\sum_{i=1}^m v_i x_{io}+d_1^--d_1^+=1$ and $\sum_{r=1}^s u_r y_{ro}+d_2^--d_2^+=1$. It stands to reason that proper scaling cannot be achieved in this manner as the model is neither input nor output oriented. Even if we eliminate $\sum_{r=1}^s u_r y_{ro}+d_2^--d_2^+=1$, there is a possibility for $0\leqslant\sum_{i=1}^m v_i x_{io}\leqslant 1$ due to the minimization of $d_1^--d_1^+$ in the objective function.

3.2. The validity of GPDEA when compared with the results of MCDEA

To explore the results of MCDEA models we further compared the results of GPDEA with MCDEA. We discovered that the GPDEA models do not conduct nor achieve the same purposes as the MCDEA model. MCDEA model uses non-dominated solutions and each objective is handled one at a time. Unlike GPDEA models, MCDEA does not attempt to get a global optimal value but more towards generating a series of non-dominated solutions interactively. In other words, MCDEA can be used to achieve either a

Table 2
GPDEA-CCR results based on Example 1 dataset.

DMU	Output weights				Input weights				Efficiency (true values)	Efficiency provided by Bal et al. (2010)
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4		
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.968
2	0.00317	0.00434	0.00464	**0**	0.00403	0.01347	0	0	0.948	0.951
3	0.00333	0.00456	0.00488	**0**	0.00424	0.01417	0	0	1	1
4	0	0.00488	0.00797	**0**	0.00336	0.01182	0.00059	0.00298	1	1
5	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.950
6	0.00268	0.00367	0.00392	**0**	0.00341	0.01140	0	0	0.788	0.794
7	0.00070	0.00371	0.00564	**0**	0.00245	0.00990	0	0.00235	0.745	0.779
8	0.00084	0.00446	0.00679	**0**	0.00295	0.01193	0	0.00283	0.823	0.843
9	0.00305	0.00417	0.00446	**0**	0.00388	0.01297	0	0	0.771	0.767
10	0.00322	0.00441	0.00471	**0**	0.00409	0.01370	0	0	1	1

Table 3
GPDEA-BCC results based on Example 1 dataset.

DMU	Output weights				Input weights				Efficiency (true values)	Efficiency provided by Bal et al. (2010)
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4		
1	0.00762	0.00000	0.00172	**0**	0.00155	0.01901	0	0	0.765	0.971
2	0.00340	0.00328	0.00307	**0**	0.00368	0.01385	0	0	0.945	0.951
3	0.00355	0.00343	0.00321	**0**	0.00385	0.01446	0	0	1	1
4	0	0.00500	0.00821	**0**	0.00314	0.01190	0.00032	0.00344	1	1
5	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.961
6	0.00289	0.00279	0.00261	**0**	0.00313	0.01178	0	0	0.788	0.965
7	0.00520	0	0.00118	**0**	0.00106	0.01297	0	0	0.718	0.798
8	0.00349	0.00338	0.00316	**0**	0.00379	0.01424	0	0	0.890	1
9	0.00330	0.00319	0.00298	**0**	0.00358	0.01345	0	0	0.824	0.909
10	0.00350	0.00338	0.00316	**0**	0.00379	0.01426	0	0	1	1

stricter or more lenient solution set, depending on whether a more or less number of efficient *DMUs* are sought by the analyst in the decision making process.

We recomputed the results of the MCDEA model of Li and Reeves (1999) using only the Minsum objective function of $\sum_j d_j$ and reproduce them in Table 4 (CRS) and Table 5 (VRS). If one were to compare the MCDEA-Minsum analysis with the results gathered from analyzing the GPDEA models (see Tables 2 and 3), the efficiency values were found to be similar. It has to be emphasized that the comparison has to be made on the corrected values denoted as 'true values' in Tables 2 and 3 and not the 'values reported in Bal et al. (2010). Given that the GPDEA models only capture the solution set of a single objective in MCDEA (i.e. Minsum), it is trivial to note that the GPDEA models' objective function $\left\{d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j\right\}$ cannot handle all of the three criteria in the MCDEA model.

3.3. The validity of GPDEA when investigating the case of variable returns to scales (VRS)

In classical VRS model (Banker, Charnes, & Cooper, 1984), c_o is a free variable placed in both the objective function and the inequality constraint. We ran the analysis based on a wrongly formulated VRS model on purpose, by considering only c_o in the constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0$ but not in the Minsum objective function of $\sum_j d_j$ (see Appendix C). With the exception of DMU_5, we achieved the same efficiency results as Bal et al. (2010) with the incorrect formulation! This can be observed by comparing the true values in Table 3 against the efficiency values in Table 5. It can therefore be concluded that the GPDEA-BCC model proposed by Bal et al. (2010) is not an acceptable extension of VRS model (Banker et al., 1984) for MCDEA.

3.4. The validity of GPDEA and the issue of zero weights for all DMUs

Table 6 can be found in Bal et al. (2010), which is reproduced here for ease of reference. The data consist of 7 departments (*DMUs*) of a university with the following input and output variables: number of academic staff (x_1), academic staff salaries in thousands of pounds (x_2), support staff salaries in thousands of pounds (x_3), the number of undergraduate students (y_1), number of postgraduate students (y_2), number of research papers (y_3).

When applying the GPDEA models, we first noticed the results reported in Bal et al. (2010) were incorrect. We therefore reported the corrected results in Tables 7–10. It is easy to observe that the input–output weights do not discriminate well and the GPDEA model cannot be representative of the MCDEA model. Based on the corrected weights reported in Tables 7–10 derived from the analysis, it can be noted that the third input is completely ignored by almost all *DMUs* in Tables 7 and 9. Also, the first and third outputs are completely ignored by all *DMUs* in Tables 8 and 10 (i.e. all weights are set to zero). This suggests that the variables have no effect in the efficiency values of the evaluation! We will see that in the proposed model of Section 4; we would impose some restrictions on the weights to avoid this issue.

4. A new bi-objective multiple criteria (BiO-MCDEA) model

The aim of this section is to introduce an alternative MCDEA model which is able to provide better weight dispersion and discrimination power while allowing multiple criteria to be optimised simultaneously. We seek to avoid the earlier issues raised in the GPDEA models through our proposed BiO-MCDEA model.

Although there are a variety of solution procedures for multi-objective or multiple criteria linear programming (MOLP or MCLP), only goal programming had been suggested for optimizing all objectives simultaneously. The difficulty of a multi-objective problem is not just in finding an optimal solution for each objective function but to find an optimal solution that simultaneously optimizes all objectives. In most cases, no single optimal solution would satisfy all the conditions simultaneously, thus requiring a set of efficient or non-dominated solutions. Further details on MOLP problem can be found in (Cohon, 1987; Dimitris, 2003).

Table 4
Minsum DEA-CCR results based on Example 1 dataset.

DMU	Output weights				Input weights				Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	
1	0.00102	0.00543	0.00827	**0**	0.00359	0.01453	0	0.00345	1
2	0.00317	0.00434	0.00464	**0**	0.00403	0.01347	0	0	0.948
3	0.00333	0.00456	0.00488	**0**	0.00424	0.01417	0	0	1
4	0	0.00488	0.00797	**0**	0.00336	0.01182	0.00059	0.00298	1
5	0.00119	0.00636	0.00967	**0**	0.00420	0.01699	0	0.00403	1
6	0.00268	0.00367	0.00392	**0**	0.00341	0.01140	0	0	0.788
7	0.00070	0.00371	0.00564	**0**	0.00245	0.00990	0	0.00235	0.745
8	0.00084	0.00446	0.00679	**0**	0.00295	0.01193	0	0.00283	0.823
9	0.00305	0.00417	0.00446	**0**	0.00388	0.01297	0	0	0.771
10	0.00322	0.00441	0.00471	**0**	0.00409	0.01370	0	0	1

Table 5
Minsum DEA-BCC results based on Example 1 dataset.

DMU	Output weights				Input weights				Efficiency
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4	
1	0.00762	0	0.00172	**0**	0.00155	0.01901	0	0	0.765
2	0.00340	0.00328	0.00307	**0**	0.00368	0.01385	0	0	0.945
3	0.00355	0.00343	0.00321	**0**	0.00385	0.01446	0	0	1
4	0	0.00500	0.00821	**0**	0.00314	0.01190	0.00032	0.00344	1
5	0.00491	0.00475	0.00444	**0**	0.00532	0.02001	0	0	1
6	0.00289	0.00279	0.00261	**0**	0.00313	0.01178	0	0	0.788
7	0.00520	0	0.00118	**0**	0.00106	0.01297	0	0	0.718
8	0.00349	0.00338	0.00316	**0**	0.00379	0.01424	0	0	0.890
9	0.00330	0.00319	0.00298	**0**	0.00358	0.01345	0	0	0.824
10	0.00350	0.00338	0.00316	**0**	0.00379	0.01426	0	0	1

Table 6
Example 2 university dataset.

DMU	Outputs			Inputs		
	y_1	y_2	y_3	x_1	x_2	x_3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

In Li and Reeves (1999) proposed MCDEA model, they used the 'non-dominated' solution approach. Bal et al. (2010) proposed goal programming as an alternative for achieving all objectives simultaneously in the MCDEA model. It has been pointed out in the previous section that the proposed GPDEA models suffer from serious drawbacks. We are compelled therefore to consider an alternative approach to optimize all objectives simultaneously in a MCDEA model, i.e. a bi-objective weighted formulation.

Recalling Li and Reeves (1999) approach, the MCDEA model's objective functions consist of three parts: min d_0, min M, and min $\sum_j d_j$ as defined in model 1. In a weighted method, the MCDEA's tri-objective function can be restated as follows, $w_1 d_o + w_2 M + w_3 \sum_j d_j$ for the single weighted objective equivalent. The weights $w_i (i = 1,2,3)$ can be varied to obtain different efficient solutions.

However, given that the first objective w_1 is in fact the equivalent to a conventional CCR model, it can be eliminated from the MCDEA in the weighted objective sense. Besides, Li and Reeves had demonstrated that the first objective yields lower discrimination power as compared to the other two objectives. Hence, for our proposed BiO-MCDEA model, we solved the bi-objective weighted problem using both the second and third objectives. The value of w_1 is set equal to zero because whenever $\sum_j d_j$ is minimized, d_o will be minimized as well. Thus, we proposed the following model:

4.1. Model 3: A new bi-objective MCDEA (BiO-MCDEA) model under CRS

$$
\begin{aligned}
\min \quad & h = \left(w_2 M + w_3 \sum_j d_j \right) \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \ldots, n, \\
& M - d_j \geqslant 0, \quad j = 1, \ldots, n, \\
& u_r \geqslant \varepsilon, \quad r = 1, \ldots, s, \\
& v_i \geqslant \varepsilon, \quad i = 1, \ldots, m, \\
& d_j \geqslant 0, \quad j = 1, \ldots, n,
\end{aligned}
\tag{3}
$$

Table 7
GPDEA-CCR results of the university dataset.

DMU	Output weights			Input weights			Efficiency true values	Efficiency provided by Bal et al. (2010)
	u_1	u_2	u_3	v_1	v_2	v_3		
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	1
2	0.00333	0.00921	0.00288	0.02019	0.00082	**0**	0.956	0.955
3	0.00160	0.00442	0.00139	0.00970	0.00039	**0**	0.765	0.764
4	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0.576
5	0.00130	0.00361	0.00113	0.00791	0.00032	**0**	1	1
6	0.00339	0.00936	0.00293	0.02053	0.00084	**0**	1	1
7	0.00260	0.00218	0	0	0.00041	**0.00006**	1	1

Table 8
GPDEA-BCC results of the university dataset.

DMU	Output weights			Input weights			Efficiency (true values)	Efficiency provided by Bal et al. (2010)
	u_1	u_2	u_3	v_1	v_2	v_3		
1	**0**	**0**	**0**	**0**	**0**	**0**	**0**	1
2	0.00834	0.00700	**0**	**0**	0.00131	0.00021	1	0.963
3	0.00420	0.00353	**0**	**0**	0.00066	0.00010	0.960	0.813
4	0.01031	0.00866	**0**	**0**	0.00162	0.00025	0.480	0.576
5	0.00311	0.00261	**0**	**0**	0.00049	0.00008	1	1
6	0.00861	0.00722	**0**	**0**	0.00136	0.00021	1	1
7	0.00260	0.00218	**0**	**0**	0.00041	0.00006	1	1

Table 9
Minsum DEA-CCR results of the university dataset.

DMU	Output weights			Input weights			Efficiency
	u_1	u_2	u_3	v_1	v_2	v_3	
1	0.00583	0.01612	0.00505	0.03536	0.00144	**0**	1
2	0.00333	0.00921	0.00288	0.02019	0.00082	**0**	0.956
3	0.00160	0.00442	0.00139	0.00970	0.00039	**0**	0.765
4	0.00418	0.01157	0.00362	0.02537	0.00103	**0**	0.577
5	0.00130	0.00361	0.00113	0.00791	0.00032	**0**	1
6	0.00339	0.00936	0.00293	0.02053	0.00084	**0**	1
7	0.00121	0.00334	0.00105	0.00732	0.00030	**0**	1

Table 10
Minsum DEA-BCC results of the university dataset.

DMU	Output weights			Input weights			Efficiency
	u_1	u_2	u_3	v_1	v_2	v_3	
1	0.01575	0.01322	**0**	**0**	0.00248	0.00039	0.564
2	0.00834	0.00700	**0**	**0**	0.00131	0.00021	1
3	0.00420	0.00353	**0**	**0**	0.00066	0.00010	0.960
4	0.01031	0.00866	**0**	**0**	0.00162	0.00025	0.480
5	0.00311	0.00261	**0**	**0**	0.00049	0.00008	1
6	0.00861	0.00722	**0**	**0**	0.00136	0.00021	1
7	0.00260	0.00218	**0**	**0**	0.00041	0.00006	1

where d_0 and d_j $(j = 1, \ldots, n)$ are the deviation variables for DMU_0 and the jth DMU respectively. DMU_0 is efficient if and only if $d_0 = 0$, otherwise the efficiency value of DMU_0 is $h_o = 1 - d_o$. The effect of constraints $M - d_j \geqslant 0 (j = 1, \ldots, n)$ does not change the feasible region of the solution but merely to make M the maximum deviation. The values of u_r and v_i are set to be greater than or equal to, thus ensuring this lower bound specification prevents inputs or outputs from being ignored by the $DMUs$.

We analyzed the dataset of 'Example 1' and the 'university dataset' with our proposed BiO-MCDEA approach. The efficiency values in Tables 11 and 12 perform better when compared against the actual efficiency values of the GPDEA-CCR models (Tables 2 and 7, respectively).

5. An application of energy dependency among EU member countries

We further illustrate our proposed model with a 3-input and 2-output dataset of 25 European Union (EU) member countries (except Malta and Estonia) as presented in Appendix D. Data were based on the EU Emissions Trading Scheme of more than 10,000 installations that generate an excess of 20^{MW} each within the country. This is believed to capture about half of the CO_2 emissions within EU. We termed the model as energy dependency as the choice of inputs are based on a set of resources that generate carbon emissions and the output will be the extent of those resources in limiting the carbon effects. The operational definition of the 3 inputs and 2 outputs are as follows (see Table 13):

Although the weights of our proposed BiO-MCDEA can be varied to obtain a set of efficiency scores according to the decision analyst's preference, we have set equal objectives such that $w_2 = w_3 = 0.5$ for the purpose of this study. The results are presented in Table 14 along with Bal et al.'s GPDEA-CCR solution. The results show that the BiO-MCDEA outperforms the GPDEA model; both, in terms of discrimination power and weight dispersion.

Comparing the two (where an infinitesimal value is chosen), it can be easily observed from Table 14 that the efficiency scores from our BiO-MCDEA model could provide ease of ranking without any ties (other than the efficient units). Such is not the case for the GPDEA efficiency scores. All of the efficiency scores and weights for GPDEA appear to be close to zero. Fig. 1 illustrates that BiO-MCDEA outperforms GPDEA. The Nonparametric Levene test confirms that there is a significantly greater weight dispersion for the Bio-MCDEA model (see Appendix E).

In the case of setting $\varepsilon = 0$, the GPDEA model cannot even generate a value to be above zero; that is, all the efficiency values, input and output weights are zeroes (see Table 15). However, the BiO-MCDEA model did not suffer a similar fate and appeared to be robust.

Table 11
BiO-MCDEA model results based on Example 1 dataset ($\varepsilon = 0.0001$).

DMU	Output weights				Input weights				Efficiency	Super Efficiency	Rank
	u_1	u_2	u_3	u_4	v_1	v_2	v_3	v_4			
1	0.00420	0.00481	0.00573	0.00010	0.00453	0.01678	0.00010	0.00016	0.961	0.961	4
2	0.00290	0.00435	0.00480	0.00010	0.00404	0.01324	0.00010	0.00014	0.948	0.948	5
3	0.00358	0.00408	0.00488	0.00010	0.00386	0.01429	0.00010	0.00013	1	1.210	2
4	0.00010	0.00486	0.00782	0.00010	0.00344	0.01191	0.00058	0.00288	1	1.079	3
5	0.00420	0.00624	0.00690	0.00010	0.00576	0.01906	0.00010	0.00024	0.947	0.947	6
6	0.00245	0.00369	0.00408	0.00010	0.00344	0.01123	0.00010	0.00011	0.789	0.789	8
7	0.00116	0.00373	0.00522	0.00010	0.00283	0.01031	0.00010	0.00165	0.767	0.767	9
8	0.00147	0.00445	0.00617	0.00010	0.00339	0.01237	0.00010	0.00190	0.837	0.837	7
9	0.00279	0.00418	0.00463	0.00010	0.00389	0.01275	0.00010	0.00014	0.761	0.761	10
10	0.00294	0.00441	0.00488	0.00010	0.00410	0.01346	0.00010	0.00015	1	1.419	1

Table 12
BiO-MCDEA model results based on the university dataset ($\varepsilon = 0.0001$).

DMU	Output weights			Input weights			Efficiency	Super Efficiency	Rank
	u_1	u_2	u_3	v_1	v_2	v_3			
1	0.00584	0.01619	0.00486	0.03711	0.00138	0.00010	1	1.136	3
2	0.00335	0.00930	0.00270	0.02200	0.00077	0.00010	0.955	0.955	5
3	0.00162	0.00452	0.00120	0.01151	0.00034	0.00010	0.763	0.763	6
4	0.00419	0.01162	0.00343	0.02707	0.00097	0.00010	0.575	0.575	7
5	0.00133	0.00372	0.00095	0.00975	0.00027	0.00010	1	1.171	2
6	0.00341	0.00947	0.00275	0.02236	0.00078	0.00010	1	1.037	4
7	0.00122	0.00342	0.00086	0.00909	0.00024	0.00010	1	1.241	1

Table 13
Model variables and operational definition.

Input variables	Definition
Installation Count (x_1)	An installation is a stationary technical unit where one or more activities are carried out, which could have an effect on emissions and pollution
Allocated Carbon Allowances (x_2)	It is an allowance distributed each year for free to installations according to the national allocation plan, measured in tonnes of carbon dioxide equivalent
Gross Inland energy consumption (GIC), by fuel (x_3)	GIC is the quantity of energy, expressed in oil equivalents, consumed within the borders of a country. It is calculated as total domestic energy production plus energy imports and changes in stocks minus energy exports
Output variables	
Electricity Generated From Renewable Sources (y_1)	Percentage of gross electricity consumed
Share of renewable energy in fuel consumption of transport (y_2)	The degree to which conventional fuels have been substituted by biofuels in transportation

Table 14
BiO-MCDEA model and Bal et al.'s GPDEA-CCR model results of the 25-country dataset ($\varepsilon = 0.00001$).

DMU	Outputs				Inputs						Performance			
	u_1	u_1[b]	u_2	u_2[b]	v_1	v_1[b]	v_2	v_2[b]	v_3	v_3[b]	Eff.	Rank	Eff.[b]	Rank[b]
Austria	0.00913	0.00047	0.01671	0.00321	0.00001	0.00001	0.88495	0.01627	0.00002	0.00001	0.718	8	0.052	3
Belgium	0.02173	0.00047	0.06671	0.00321	0.81837	0.00001	1.29635	0.01627	0.00013	0.00001	0.352	15	0.013	18
Bulgaria	0.00832	0.00047	0.05677	0.00321	0.00001	0.00001	0.28790	0.01627	0.00018	0.00001	0.116	25	0.007	24
Cyprus	0.00989	0.00047	0.06750	0.00321	0.00001	0.00001	0.34229	0.01627	0.00021	0.00001	0.136	24	0.006	25
Czech Republic	0.00549	0.00047	0.03745	0.00321	0.00001	0.00001	0.18991	0.01627	0.00012	0.00001	0.165	22	0.014	17
Denmark	0.00783	0.00047	0.04149	0.00321	8.36858	0.00001	0.58502	0.01627	0.00001	0.00001	0.231	19	0.014	16
Finland	0.00590	0.00160	0.02152	0.00502	2.05593	0.00001	0.40883	0.09925	0.00002	0.00001	0.202	21	0.053	2
France	0.02749	0.00047	0.10460	0.00321	0.00001	0.00001	2.16515	0.01627	0.00016	0.00001	1.000	1	0.026	8
Germany	0.01974	0.00047	0.06204	0.00321	0.00001	0.00001	1.22543	0.01627	0.00012	0.00001	0.673	9	0.026	7
Greece	0.00806	0.00047	0.05500	0.00321	0.00001	0.00001	0.27892	0.01627	0.00017	0.00001	0.159	23	0.009	22
Hungary	0.03370	0.00047	0.11127	0.00321	10.55950	0.00001	2.06097	0.01627	0.00014	0.00001	0.580	11	0.013	19
Ireland	0.01038	0.00047	0.06628	0.00321	1.31826	0.00001	0.29005	0.01627	0.00021	0.00001	0.271	18	0.013	20
Italy	0.02631	0.00047	0.08266	0.00321	0.00001	0.00001	1.63285	0.01627	0.00017	0.00001	0.854	6	0.022	11
Latvia	0.01269	0.00047	0.01703	0.00321	0.00001	0.00001	1.41464	0.01627	0.00001	0.00001	0.645	10	0.027	6
Lithuania	0.01541	0.00047	0.08855	0.00321	19.49392	0.00001	1.17342	0.01627	0.00001	0.00001	0.457	13	0.016	15
Luxembourg	0.02199	0.00047	0.06911	0.00321	0.00001	0.00001	1.36516	0.01627	0.00014	0.00001	0.226	20	0.008	23
Netherlands	0.02308	0.00047	0.07254	0.00321	0.00001	0.00001	1.43276	0.01627	0.00014	0.00001	0.516	12	0.018	14
Poland	0.00849	0.00047	0.05788	0.00321	0.00001	0.00001	0.29353	0.01627	0.00018	0.00001	0.327	16	0.018	13
Portugal	0.01953	0.00047	0.07430	0.00321	0.00001	0.00001	1.53809	0.01627	0.00011	0.00001	0.917	3	0.027	5
Romania	0.02721	0.00047	0.08550	0.00321	0.00001	0.00001	1.68894	0.01627	0.00017	0.00001	0.896	4	0.018	12
Slovakia	0.01982	0.00047	0.06086	0.00321	0.74654	0.00001	1.18257	0.01627	0.00012	0.00001	0.878	5	0.036	4
Slovenia	0.01699	0.00047	0.05611	0.00321	5.32473	0.00001	1.03927	0.01627	0.00007	0.00001	0.732	7	0.023	9
Spain	0.02721	0.00047	0.08552	0.00321	0.00001	0.00001	1.68928	0.01627	0.00017	0.00001	1.000	1	0.023	10
Sweden	0.00576	0.00160	0.00984	0.00502	0.00001	0.00001	0.57960	0.09925	0.00001	0.00001	0.397	14	0.127	1
United Kingdom	0.01283	0.00047	0.08750	0.00321	0.00001	0.00001	0.44371	0.01627	0.00027	0.00001	0.322	17	0.012	21
$F(1,49)$	21.962**		21.941**		116.694**		21.941**		75.106**					

[b] Assigned to indicate Bal et al. (2010) model results. All inputs of the raw data were scaled by the population size of their respective countries. See Appendix D for the raw data.

** All 5 input–output variables registered significantly higher variation for BiO-MCDEA model as compared to the GPDEA-CCR model at $p < 0.01$.

6. Concluding remarks

With the exception of this study; to date, only GPDEA models were proposed as a solution method to the MCDEA model. We have demonstrated that the GPDEA models are not alternatives to the MCDEA model. It has major drawbacks in both discrimination power and weight dispersion, aside from the misreported efficiency values of all the tests. Hence, the fair basis of comparison would be between our proposed BiO-MCDEA model and the GPDEA models, given that the MCDEA model merely provided a mathematical formulation with an interactive solution procedure without any emphasis placed on the issues of discrimination

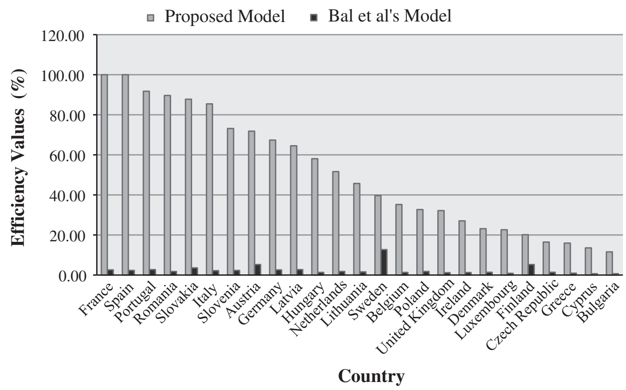

Fig. 1. Performance comparison: BiO-MCDEA vs. GPDEA.

power and weight dispersion. In short, we have illustrated that the BiO-MCDEA model outperforms the GPDEA models in terms of both weight dispersion and discrimination power.

Although we have proposed a bi-objective weighted method for solving the MCDEA model, we stress that there may be other solution procedures that can be used to extract solutions under multi-objective LP environment. We merely showed a procedure that performs better than the GPDEA in terms of ease of formulation and mathematical programming (i.e. less computational codes). We hope that future researchers in DEA will explore other methods such as metaheuristic solution procedures for the MCDEA model.

In terms of the application, we have used an energy dependency context for the performance comparison between BiO-MCDEA and GPDEA. Given that environmental context has larger policy implications, future researches should look into the structural differences between the two disposability conditions of natural and managerial disposability when comparing DEA models (see Sueyoshi & Goto, 2012). Natural disposability refers to decreases in input vectors that correspond to decreases in undesirable output vectors (such as CO_2emissions). On the other hand, managerial disposability refers to increases in input vectors that can simultaneously increase desirable output vectors and decrease undesirable output vectors.

In our case, we have only considered natural disposability. By further exploring the aspects of managerial disposability, technological innovation can be uncovered, especially in the context of environmental modeling. For example, a country's growth or productivity (i.e. desirable output) is closely tied to its level of industrialization, which is positively correlated to the by-product of CO_2 emissions (i.e. undesirable output). The implication is such that environmental strategies that are focused on curbing CO_2 emissions should not end up decreasing productivity due to the high correlation between desirable and undesirable outputs. On the methodological forefront, treating such problems may involve negative data, and interested readers may refer to the following papers (see Emrouznejad, Amin, Thanassoulis, & Anouze, 2010; Emrouznejad, Anouze, & Thanassoulis, 2010) for a solution.

Future researchers may also explore our BiO-MCDEA model in a new light by applying the COOPER-framework (see Emrouznejad & De Witte, 2010) which gives much thought to the structuring of data. All of our variables are scaled by the population sizes of the respective countries in order to eliminate the potential bias, given that larger countries would naturally produce more emissions. Nonetheless, it would be interesting to observe a different method such as the clustering approach (see Amin, Emrouznejad, & Rezaei, 2011) for handling data from non-homogenous sets. This would help in generating unbiased results while formulating energy policies that strike a balance between controlling CO_2 emissions and ensuring that the control measures do not impede on the productivity level, owing to a more comparable within-group effects. Therefore, it is more feasible to investigate operational elements that improve the output-input ratio for desirable outputs while controlling for the output-input ratio for undesirable outputs. This can only be achieved through innovative means in the process that augment the production frontier.

Table 15
BiO-MCDEA model and Bal et al.'s GPDEA-CCR model results of the 25-country dataset ($\varepsilon = 0$).

DMU	Outputs				Inputs						Performance		
	u_1	$u_1{}^b$	u_2	$u_2{}^b$	v_1	$v_1{}^b$	v_2	$v_2{}^b$	v_3	$v_3{}^b$	Eff.	Rank	Eff.[b]
Austria	0.00913	0.00000	0.01671	0.00000	0.00001	0.00000	0.88495	0.00000	0.00002	0.00000	0.718	8	0.00000
Belgium	0.02173	0.00000	0.06671	0.00000	0.81837	0.00000	1.29635	0.00000	0.00013	0.00000	0.352	14	0.00000
Bulgaria	0.00832	0.00000	0.05677	0.00000	0.00000	0.00000	0.28790	0.00000	0.00018	0.00000	0.116	25	0.00000
Cyprus	0.00989	0.00000	0.06750	0.00000	0.00000	0.00000	0.34229	0.00000	0.00021	0.00000	0.136	24	0.00000
Czech Republic	0.00549	0.00000	0.03745	0.00000	0.00000	0.00000	0.18991	0.00000	0.00012	0.00000	0.165	22	0.00000
Denmark	0.00652	0.00000	0.04052	0.00000	9.57808	0.00000	0.50657	0.00000	0.00000	0.00000	0.195	21	0.00000
Finland	0.00590	0.00000	0.02151	0.00000	2.05592	0.00000	0.40883	0.00000	0.00002	0.00000	0.202	20	0.00000
France	0.02749	0.00000	0.10460	0.00000	0.00000	0.00000	2.16515	0.00000	0.00015	0.00000	1.000	1	0.00000
Germany	0.01974	0.00000	0.06204	0.00000	0.00000	0.00000	1.22543	0.00000	0.00012	0.00000	0.673	9	0.00000
Greece	0.00806	0.00000	0.05500	0.00000	0.00000	0.00000	0.27892	0.00000	0.00017	0.00000	0.159	23	0.00000
Hungary	0.03370	0.00000	0.11126	0.00000	10.55950	0.00000	2.06097	0.00000	0.00014	0.00000	0.580	11	0.00000
Ireland	0.01038	0.00000	0.06628	0.00000	1.31826	0.00000	0.29005	0.00000	0.00021	0.00000	0.271	18	0.00000
Italy	0.02630	0.00000	0.08266	0.00000	0.00000	0.00000	1.63285	0.00000	0.00016	0.00000	0.854	6	0.00000
Latvia	0.01192	0.00000	0.01236	0.00000	0.00000	0.00000	1.43718	0.00000	0.00000	0.00000	0.602	10	0.00000
Lithuania	0.01412	0.00000	0.08776	0.00000	20.74351	0.00000	1.09708	0.00000	0.00000	0.00000	0.446	13	0.00000
Luxembourg	0.02199	0.00000	0.06911	0.00000	0.00000	0.00000	1.36516	0.00000	0.00014	0.00000	0.226	19	0.00000
Netherlands	0.02308	0.00000	0.07253	0.00000	0.00000	0.00000	1.43276	0.00000	0.00014	0.00000	0.516	12	0.00000
Poland	0.00848	0.00000	0.05788	0.00000	0.00000	0.00000	0.29352	0.00000	0.00018	0.00000	0.327	15	0.00000
Portugal	0.01953	0.00000	0.07430	0.00000	0.00000	0.00000	1.53809	0.00000	0.00011	0.00000	0.917	3	0.00000
Romania	0.02721	0.00000	0.08550	0.00000	0.00000	0.00000	1.68894	0.00000	0.00017	0.00000	0.896	4	0.00000
Slovakia	0.01982	0.00000	0.06086	0.00000	0.74654	0.00000	1.18257	0.00000	0.00012	0.00000	0.878	5	0.00000
Slovenia	0.01699	0.00000	0.05611	0.00000	5.32473	0.00000	1.03927	0.00000	0.00007	0.00000	0.732	7	0.00000
Spain	0.02721	0.00000	0.08552	0.00000	0.00000	0.00000	1.68928	0.00000	0.00017	0.00000	1.000	1	0.00000
Sweden	0.00492	0.00000	0.00510	0.00000	0.00000	0.00000	0.59294	0.00000	0.00000	0.00000	0.314	17	0.00000
United Kingdom	0.01283	0.00000	0.08750	0.00000	0.00000	0.00000	0.44371	0.00000	0.00027	0.00000	0.322	16	0.00000

[b] Assigned to indicate Bal et al. (2010) model results. All inputs of the raw data were scaled by the population size of their respective countries. See Appendix D for the raw data.

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Appendix A. Goal programming DEA model under VRS as proposed in Bal et al. (2010) – GPDEA-BCC

$$\min\ a = \left\{ d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j \right\}$$

$$\sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1,$$

$$\sum_{r=1}^{s} u_r y_{ro} + c_o + d_2^- - d_2^+ = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + c_o + d_j = 0, \quad j = 1, \ldots, n,$$

$$M - d_j + d_{3j}^- - d_{3j}^+ = 0, \quad j = 1, \ldots, n,$$

$$u_r \geqslant 0, \quad r = 1, \ldots, s,$$

$$v_i \geqslant 0, \quad i = 1, \ldots, m,$$

$$d_j \geqslant 0, \quad j = 1, \ldots, n,$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geqslant 0,$$

$$d_{3j}^-, d_{3j}^+ \geqslant 0, \quad j = 1, \ldots, n,$$

c_o *free in sign*

Appendix B. Proof of logical invalidity of GPDEA formulation

From Bal et al.'s GPDEA-CCR model (2):

$$\sum_{r=1}^{s} u_r y_{ro} + d_2^- - d_2^+ = 1 \tag{I}$$

$$\sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1 \tag{II}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \tag{III}$$

Multiplying equality (II) by -1:

$$-\sum_{i=1}^{m} v_i x_{io} - (d_1^- - d_1^+) = -1 \tag{IV}$$

adding (I) and (IV) yields:

$$\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + d_2^- - d_2^+ - (d_1^- - d_1^+) = 0 \tag{V}$$

Suppose that $j = o$ in equality (III):

$$\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + d_o = 0 \tag{VI}$$

By considering (V) and (VI), it can be concluded:

$$d_o = d_2^- - d_2^+ - (d_1^- - d_1^+) \tag{VII}$$

Since the efficiency value for *DMU* under evaluation, h_o, must be equal to $\sum_{r=1}^{s} u_r y_{ro}$, (I) can be restated as:

$$h_o = \sum_{r=1}^{s} u_r y_{ro} = 1 - (d_2^- - d_2^+)$$

Since $h_o = 1 - d_o$ in classical DEA, $d_o = d_2^- - d_2^+$ in (*viii*) therefore, the value of $d_1^- - d_1^+$ in (VII) must be equal to zero to render correctness. Nonetheless, in Bal et al.'s GPDEA models, the weighted sum of inputs for *DMU* under evaluation $\sum_{i=1}^{m} v_i x_{io}$, can be zero or less than unity, which is highly problematic. Without loss of generality, the same problem applies to the GPDEA-BCC model.

Appendix C. Minsum BCC-DEA model under variable returns to scale, a wrongly formulated VRS model

$$\min\ \sum_{j=1}^{n} d_j$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + c_o + d_j = 0, \quad j = 1, \ldots, n,$$

$$M - d_j \geqslant 0, \quad j = 1, \ldots, n,$$

$$u_r \geqslant 0, \quad r = 1, \ldots, s,$$

$$v_i \geqslant 0, \quad i = 1, \ldots, m,$$

$$d_j \geqslant 0, \quad j = 1, \ldots, n,$$

c_o *free in sign*

Appendix D. The energy dependency dataset

Dataset of 25 countries.

Countries	Population (thousands)	Outputs		Inputs		
		Y_1	Y_2	X_1	X_2	X_3
Austria	8394	66.793	6.5	225	8810	31,887,710
Belgium	10,712	6.083	3.3	362	2242	56,797,576
Bulgaria	7494	9.808	0.6	146	1087	40,591,231
Cyprus	1104	0.073	2.0	13	98	5,089,082
Czech Republic	10 493	6.783	3.4	425	2425	85,968,002
Denmark	5550	27.390	0.4	408	3242	23,912,314
Finland	5365	25.777	2.3	661	7887	37,069,940

Appendix D. (*continued*)

Countries	Population (thousands)	Outputs		Inputs		
		Y_1	Y_2	X_1	X_2	X_3
France	62,787	13.547	6.0	1125	19,811	128,660,709
Germany	82,302	16.200	5.7	1997	27,693	391,714,624
Greece	11,359	12.276	1.1	162	1861	63,246,705
Hungary	9984	6.988	3.1	270	1854	23,844,843
Ireland	4470	13.925	1.9	124	641	19,951,911
Italy	60,551	20.536	3.8	1201	16,026	208,982,856
Latvia	2252	49.232	1.2	111	1567	3,532,491
Lithuania	3324	5.505	4.2	114	874	7,573,712
Luxembourg	507	3.678	2.1	15	121	2,488,229
Netherlands	16,613	9.152	4.2	443	3148	83,834,170
Poland	38,277	5.804	4.8	943	6265	202,011,597
Portugal	10,676	33.267	3.6	280	4734	30,902,050
Romania	21,486	27.916	1.6	275	5270	73,956,515
Slovakia	5462	17.880	8.6	201	1214	32,140,581
Slovenia	2030	36.783	1.9	100	887	8,216,051
Spain	46,077	25.747	3.5	1143	12,091	150,707,494
Sweden	9380	56.378	7.3	821	15,819	21,103,878
United Kingdom	62,036	6.664	2.7	1140	6214	217,404,830

Note: The data are taken from four databases: European commision's Eurostat, Carbonmarketdata.com, www.i-insights.com and United Nations, Department of Economic and Social Affairs.

Appendix E. Statistical Analysis (BiO-MCDEA vs. GPDEA)

A number of researches (e.g. Bal et al., 2010; Bal, Örkcü, & Çelebioğlu, 2008) wrongly assume that the Mann–Whitney *U* test is considered to be a non-parametric test because it does not rely on homogeneity of variances (see Mann & Whitney (1947)). In other words, a significant difference in scores between both groups is presumed to indicate support for higher heterogeneity in the data. This misinterpretation may stem from equating homogeneity of data to a normally distributed data. Although there is no assumption of normality, the Mann–Whitney *U* test still requires data from both groups to be distributed in the same form (Golany & Storbeck, 1999; Ward, Storbeck, Mangum, & Byrnes, 1997).

Zimmerman (2006) showed that type 1 error rates and the power of two significance tests are distorted in the Mann–Whitney *U* test under the influence of heterogeneity of variances. The problem exists even when sample sizes are equal, and are more pronounced as sample sizes increase. Hence, studies (e.g. Bal et al., 2010; Bal et al., 2008) that attempt to test whether input–output weights are significantly dispersed in one DEA model as compared to another should not be using the Mann–Whitney *U* test. In short, Mann–Whitney *U* test is more appropriate when one wants to compare two groups of decision making units with the same distribution form through the use of order statistics (see Banker, Zheng, & Natarajan, 2010).

To ascertain whether weight dispersion in one DEA model performs better than another, a test for equality (or inequality) of variances must be conducted. Lim and Loh (1996) found Levene's test to be the most robust and powerful for small to medium sample sizes when compared to other equality of variance tests such as the Box-Andersen, Bartlett and Jacknife tests. The version of Levene's test mentioned in Lim and Loh (1996) is the One-way ANOVA *F*-test based on the median.

This study uses the nonparametric version of the Levene's test, which was found to have greater statistical power under skewed population distributions, while maintaining the performance of type 1 error rates (see Nordstokke & Zumbo, 2010). A one-way ANOVA is conducted on the absolute value of the mean ranks for each group, such that the null hypothesis is that the populations are identical in shape (but not necessarily location). The results indicate that all of the 5 input–output variables are dispersed significantly greater in the BiO-MCDEA model as compared to the GPDEA model.

Appendix F. Mathematical Programming Codes

The Mathematica codes associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ejor.2013.08.041.

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