Applied Soft Computing 10 (2010) 520–528

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Fuzzy AHP to determine the relative weights of evaluation criteria and Fuzzy TOPSIS to rank the alternatives

Fatemeh Torfi [a], Reza Zanjirani Farahani [b,c,*], Shabnam Rezapour [d]

[a] Department of Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran
[b] Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran
[c] Centre for Maritime Studies, National University of Singapore, Singapore
[d] Department of Industrial Engineering, Urmia University of Technology, Urmia, Iran

ARTICLE INFO

Article history:
Received 14 January 2008
Received in revised form 18 May 2009
Accepted 23 August 2009
Available online 28 August 2009

Keywords:
MADM
Fuzzy decision-making
Fuzzy AHP
Fuzzy TOPSIS

ABSTRACT

The aim of this study is to propose a Fuzzy multi-criteria decision-making approach (FMCDM) to evaluate the alternative options in respect to the user's preference orders. Two FMCDM methods are proposed for solving the MCDM problem: Fuzzy Analytic Hierarchy Process (FAHP) is applied to determine the relative weights of the evaluation criteria and the extension of the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) is applied to rank the alternatives. Empirical results show that the proposed methods are viable approaches in solving the problem. When the performance ratings are vague and imprecise, this Fuzzy MCDM is a preferred solution.

1. Introduction

MCDM refers to finding the best opinion from all of the feasible alternatives in the presence of multiple, usually conflicting, decision criteria. Priority-based, outranking, distance-based and mixed methods could be considered as the primary classes of the current methods [1]. Multiple objective decision-making (MODM) consists of a set of conflicting goals that cannot simultaneously be achieved. It concentrates invariably on the continuous decision spaces, and can be solved by mathematical programming techniques. MODM deals generally with (i) preferences relating to the decision maker's objectives and (ii) the relationships between objectives and attributes. An alternative could be described either in terms of its attributes or in terms of the attainment of the decision maker's objectives [11].

MADM deals with the problem of choosing an option from a set of alternatives, which are characterized in terms of their attributes. MADM is a qualitative approach due to the existence of the criteria subjectivity. It requires information on the preferences among the instances of an attribute, and the preferences across the existing attributes. The decision maker may express or define a ranking for the attributes in terms of importance/weights. The aim of the MADM is to obtain the optimum alternative that has the highest degree of satisfaction for all of the relevant attributes [11].

One of the most outstanding MCDM approaches is the Analytic Hierarchy Process (AHP) [2,39], which has its roots in obtaining the relative weights among the factors and the total values of each alternative based on these weights. In comparison with other MCDM methods, the AHP method has widely been used in multi-criteria decision-making and has been applied successfully in many practical decision-making problems [40]. In spite of AHP method popularity, this method is often criticized because of its inability in handling the uncertain and imprecise decision-making problems [3]. TOPSIS, another MCDM method, is based on choosing the best alternative, which has the shortest distance from the positive-ideal alternative and the longest distance from the negative-ideal alternative. More detailed information about the TOPSIS method can be found in Hwang and Yoon [4].

In the primitive forms of the AHP and TOPSIS methods, experts' comparisons about the criteria, sub-criteria, and alternatives are represented in the form of exact numbers. However, in many practical cases, the experts' preferences are uncertain and they are reluctant or unable to make numerical comparisons. Fuzzy decision-making is a powerful tool for decision-making in fuzzy environment. Classical decision-making methods work only with exact and ordinary data, so there is no place for fuzzy and vague data. Human has a good ability for qualitative data processing, which helps him/her to make decisions in fuzzy environments.

* Corresponding author. Tel.: +98 65 6516 5356; fax: +98 65 6775 6762.
E-mail addresses: farahan@aut.ac.ir, cmszfr@nus.edu.eg (R.Z. Farahani).

1568-4946/$ – see front matter
doi:10.1016/j.asoc.2009.08.021

The main objective of this paper is to propose new AHP and TOPSIS methods' frameworks for dealing with the evaluations' uncertainty and imprecision in which the expert's comparisons are represented as fuzzy numbers. In this paper, we will use Fuzzy Analytic Hierarchy Process (FAHP) method for determining the final weights of alternatives, also by using a group of experts' comparisons. However, for evaluating the alternatives of multi-criteria problems, different attribute weights have an important role in the decision-making process.

Stam and Silva [41] use multiplicative priority rating methods for the AHP. Saaty [2] showed that eigenvector of one pairwise comparison matrix represents the local priority weights of the compared elements (criteria, sub-criteria, and alternatives). Abo-sinna and Amer [42] extend TOPSIS approach to solve multi-objective large-scale nonlinear programming (MOLSNLP) problems. Aguaron et al. [5] focused on evaluating the consistency of the experts' judgments in AHP decision support system (DSS). Holloway and White [43], with uncertainty, used the question–response process as a sequential decision-making method and developed a dynamic programming for it. One of the most diffused approaches in MCDM is the simple additive weight method (SAW), in which all the criteria is weighted by a suitable real number, representing the importance of them. In spite of its simplicity, the SAW method has some problems: no interaction among the attributes is admitted, so the preferential independence axiom is required. Moreover, some difficulties exist about the weights assignment. So later, some new methods such as AHP are suggested [2], and other tools such as Fuzzy logic, and the theory of aggregation operators [6] have been used to improve multi-objective decision-making methods.

For evaluating airlines' service quality, Tsaur et al. [7] use AHP method to calculate the criteria weights and use TOPSIS method to determine the alternatives' ranking. Feng and Wang [19] uses TOPSIS method for evaluating the performance of different airlines. TOPSIS and Fuzzy TOPSIS methods have been applied in different applications, and are commonly used in solving multiple attribute decision problems (MADM) [8,9].

Isiklar and Buyukozkan [10] use a multi-criteria decision-making approach to evaluate mobile phone alternatives. In this paper, we try to solve their problem but in a fuzzy environment. Yang and Hung [11] focuses on the evaluation of alternative layout designs. We use their problem but we solve it with FAHP and FTOPSIS procedures. Our proposed methodology consists of two steps: in the first step, Fuzzy Analytic Hierarchy Process (FAHP) is applied to determine the relative weights of the evaluation criteria. In the second step, fuzzy TOPSIS method (FTOPSIS) is applied to rank the alternatives.

The rest of this paper is organized as follows: The evaluation framework is presented in Section 2. The next section illustrates the methods used to compute the criteria weights and to select the best alternative. Section 4 illustrates these methods in detail for the special defined problem of this paper. Computational results are represented in Section 5 and Section 6 includes the conclusions and future researches.

2. Evaluation framework

MCDM is a powerful tool used widely for solving the problems with multiple, and usually conflicting, criteria [1]. The MCDM techniques generally are enabled to structure the problem clearly and systematically. With this characteristic, decision makers have the possibility to easily examine and scale the problem in accordance with their requirements [10]. The main objective of this paper as mentioned above is to select the best alternative from among a number of mobile phone options in respect to the user's preference orders. For this purpose, we use FAHP to determine the priorities of different criteria, and then choose the best mobile phone alternative by TOPSIS method. Yang and Hung [11] used TOPSIS and fuzzy TOPSIS methods for selecting the best plant layout with respect to several different criteria. The evaluation procedure in this paper consists of three main steps as summarized in Fig. 1.

Fig. 1. The evaluation procedure.

Step 1. Identifying the selection (evaluation) criteria that are considered as the most important criteria for the users.
Step 2. After constructing the evaluation criteria hierarchy, calculating the weights of criteria through applying FAHP method.
Step 3. Conducting FTOPSIS method to achieve the final ranking results.

The detailed descriptions of each step are illustrated in the following sections.

3. The FAHP and FTOPSIS methodology

3.1. Fuzzy AHP model

AHP is a powerful decision-making methodology for determining the priorities among different criteria. The AHP encompasses six basic steps [10]. First, we briefly review the rationale for the Fuzzy Theory before the development of fuzzy AHP and fuzzy TOPSIS as follows:

Definition 3.1. A Fuzzy set $\tilde{a}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{a}}(x)$ which associates with each element x in X, a real number in the interval [0,1]. The function value $\mu_{\tilde{a}}(x)$ is termed the grade of membership of x in $\tilde{a}$ [12]. The present study uses triangular Fuzzy numbers. A triangular Fuzzy number, $\tilde{a}$, can be defined by a triplet (a_1, a_2, a_3). Its conceptual schema and mathematical form are shown by Eq. (1) [13].

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \prec x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \prec x \le a_3 \\ 1 & x \succ a_3 \end{cases} \tag{1}$$

Definition 3.2. Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular Fuzzy numbers, then the vertex method is defined to calculate the distance between them, as Eq. (2):

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2\right]} \tag{2}$$

Definition 3.3. Let a triangular Fuzzy number $\tilde{a}$, then α-cut defined as Eq. (3):

$$\tilde{a}_\alpha = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha] \tag{3}$$

Definition 3.4. Let $\tilde{a} = (a_1, a_2, a_3)$, $\tilde{b} = (b_1, b_2, b_3)$ be two triangular Fuzzy numbers and $\tilde{a}_\alpha$, $\tilde{b}_\alpha$ be α-cut, $\tilde{a}$ and $\tilde{b}$, then the method is defined to calculate the divided between $\tilde{a}$ and $\tilde{b}$, as Eqs. (4)–(7) [14]:

$$\frac{\tilde{a}_\alpha}{\tilde{b}_\alpha} = \left[\frac{(a_2 - a_1)\alpha + a_1}{-(b_3 - b_2)\alpha + b_3}, \frac{-(a_3 - a_2)\alpha + a_3}{(b_2 - b_1)\alpha + b_1}\right] \tag{4}$$

When α = 0,

$$\frac{\tilde{a}_0}{\tilde{b}_0} = \left[\frac{a_1}{b_3}, \frac{a_3}{b_1}\right] \tag{5}$$

When α = 1,

$$\frac{\tilde{a}_1}{\tilde{b}_1} = \left[\frac{(a_2 - a_1) + a_1}{-(b_3 - b_2) + b_3}, \frac{-(a_3 - a_2) + a_3}{(b_2 - b_1) + b_1}\right]$$
$$\frac{\tilde{a}_1}{\tilde{b}_1} = \left[\frac{a_2}{b_2}, \frac{a_2}{b_2}\right] \tag{6}$$

So the approximated value of $\tilde{a}/\tilde{b}$ will be

$$\frac{\tilde{a}}{\tilde{b}} = \left[\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right] \tag{7}$$

Property 3.1. Assuming that both $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are real numbers, the distance measurement $d(\tilde{a}, \tilde{b})$ is identical to the Euclidean distance [15].

The basic operations on Fuzzy triangular numbers are as follows [11]:

$$\text{For approximation of multiplication} : \tilde{a} \times \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \tag{8}$$

$$\text{For addition} : \tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \tag{9}$$

Given the above-mentioned Fuzzy theory, the proposed Fuzzy AHP procedure is then defined as follows:

Step 1. AHP uses several small sub-problems to present a complex decision problem. Thus, the first act is to decompose the decision problem into a hierarchy with a goal at the top, criteria and sub-criteria at levels and sub-levels and decision alternatives at the bottom of the hierarchy (Fig. 3).

Step 2. The comparison matrix involves the comparison in pairs of the elements of the constructed hierarchy. The aim is to set their relative priorities with respect to each of the elements at the next higher level.

$$D = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & & x_{2n} \\ x_{31} & x_{32} & x_{33} & & x_{3n} \\ \vdots & & & & \\ x_{n1} & x_{n2} & x_{n3} & & x_{nn} \end{bmatrix} \end{array} \tag{10}$$

The elements $\{x_{ij}\}$ can be interpreted as the degree preference of ith criterion over jth criterion. It appears that the weight determination of criteria is more reliable when using pairwise comparisons than obtaining them directly, because it is easier to make a comparison between two attributes than make an overall weight assignment. Before all the calculations of vector of priorities, the comparison matrix has to be normalized into the range of [0,1] by Eq. (11):

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} \tag{11}$$

$$R = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & r_{23} & & r_{2n} \\ r_{31} & r_{32} & r_{33} & & r_{3n} \\ \vdots & & & & \\ r_{n1} & r_{n2} & r_{n3} & & r_{nn} \end{bmatrix} \end{array} \tag{12}$$

Step 3. AHP also calculates an inconsistency index (or consistency ratio) to reflect the consistency of the decision maker's judgments during the evaluation phase. The inconsistency index in both the decision matrix and in pairwise comparison matrices could be calculated with Eq. (13) [5]:

$$CI = \frac{\lambda_{\max} - N}{N - 1} \tag{13}$$

where $\lambda_{\max}$ is the principal eigenvalue of the judgement matrix and n is the order of the judgement matrix. The closer the inconsistency index to zero, the greater the consistency. The consistency of the assessments is ensured if the equality ($a_{ij}a_{jk} = a_{ik}$, $\forall i, j, k$) holds for all criteria. The relevant index should be lower than 0.10 to accept the AHP results as consistent. If this is not the case, the decision maker should go back and redo the assessments and comparisons [10].

Step 4. In the next step, transform the real elements of matrix R into the fuzzy numbers.

Step 5. Before conducting all the calculation of vector of priorities, the comparison matrix D has to be normalized by Eq. (11).

Step 6. To find the criteria weights, calculate the average of the elements of each rows from matrix obtained from step 4, by Definition 3.4.

3.2. Fuzzy membership function

Experts usually use the linguistic variable to evaluate the importance of the criteria and to rate the alternatives with respect to various criteria. The example of the present study has only precise values for the performance ratings and for the criteria weights. In order to illustrate the idea of Fuzzy MACD, we deliberately transform the existing precise values to five levels, Fuzzy linguistic variables very low (VL), low (L), medium (M), high (H), and very high (VH). The purpose of the transformation process is two-folded: (i) to illustrate the proposed Fuzzy MACD method and (ii) to benchmark the empirical results using other precise value methods in the later analysis.

Among the commonly used Fuzzy numbers, triangular and trapezoidal fuzzy numbers are likely to be the adoptive ones due to their simplicity in modeling easy interpretations. Both triangular and trapezoidal fuzzy numbers are applicable to the present study. We assume that a triangular fuzzy number can adequately represent the five-level Fuzzy linguistic variables, thus, is used for the analysis hereafter.

As a rule of thumb, each rank is assigned an evenly spread membership function that has an interval of 0.30 or 0.25. Based on these assumptions, a transformation table can be found as shown in Table 1. For example, the Fuzzy variable, very low has its associated triangular Fuzzy number with the minimum of 0.00 mode of 0.10 and maximum of 0.25. The same definition is then applied to another Fuzzy variable Low, Medium, High, and Very High. Fig. 2 illustrates the Fuzzy membership function [11].

Table 1
Transformation for Fuzzy membership functions.

Rank	Sub-criteria grade	Membership function
Very low (VL)	1	(0.00,0.10,0.25)
Low (L)	2	(0.15,0.30,0.45)
Medium (M)	3	(0.35,0.50,0.65)
High (H)	4	(0.55,0.70,0.85)
Very high (VH)	5	(0.75,0.90,1.00)

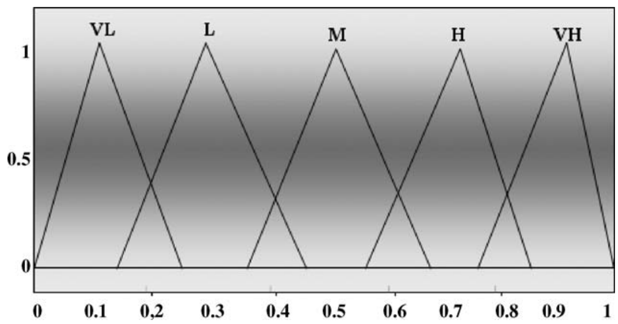

Fig. 2. Fuzzy triangular memebership functions.

3.3. Principles of TOPSIS

TOPSIS method is based on choosing the best alternative, which has the shortest distance from the positive-ideal solution and the longest distance from the negative-ideal solution, more detailed information can be found in Hwang and Yoon [4].

3.4. Fuzzy TOPSIS model

It is often difficult for a decision maker to assign a precise performance rating to an alternative for the criteria under consideration. The merit of using a Fuzzy approach is to assign the relative importance of the criteria using Fuzzy numbers instead of precise numbers. This section extends the TOPSIS to the Fuzzy environment. The Fuzzy MCDM can be concisely expressed in matrix format as Eqs. (14) and (15).

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & & \tilde{x}_{2n} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & & \tilde{x}_{3n} \\ \vdots & & & & \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{14}$$

$$\tilde{W} = [\tilde{w}_1, \quad \tilde{w}_2, \quad \ldots, \quad \tilde{w}_n] \tag{15}$$

where $\tilde{x}_{ij}, i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ and $\tilde{w}_j$, $j = 1, 2, \ldots, n$ are linguistic triangular Fuzzy numbers, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (a_{j1}, b_{j2}, c_{j3})$. Note that $\tilde{x}_{ij}$ is the performance rating of the ith alternative, A_i, with respect to the jth criterion, C_j and $\tilde{w}_j$ represents the weight of the jth criterion, C_j. The normalized Fuzzy decision matrix denoted by $\tilde{R}$ is shown as Eq. (16):

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \tag{16}$$

The weighted Fuzzy normalized decision matrix is shown in Eq. (17):

$$V = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \ldots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \tilde{v}_{m1} & \tilde{v}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{n2} & \tilde{v}_{m1} & \tilde{v}_{mn} \end{bmatrix} = \begin{bmatrix} \tilde{w}_1\tilde{r}_{11} & \tilde{w}_2\tilde{r}_{12} & \ldots & \tilde{w}_n\tilde{r}_{1n} \\ \tilde{w}_1\tilde{r}_{21} & \tilde{w}_2\tilde{r}_{22} & \ldots & \tilde{w}_n\tilde{r}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{w}_1\tilde{r}_{m1} & \tilde{w}_2\tilde{r}_{m2} & \ldots & \tilde{w}_n\tilde{r}_{mn} \end{bmatrix}. \tag{17}$$

The merit of using a Fuzzy approach is to assign the relative importance of the criteria using Fuzzy numbers instead of precise numbers. This section extends the TOPSIS to the Fuzzy environment. This method is particularly suitable for solving the group decision maker problem under Fuzzy environment. Given the above Fuzzy theory, the proposed Fuzzy TOPSIS procedure is then defined as follows:

Step 1. Choose the linguistic ratings $(\tilde{x}_{ij})$ $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ for alternatives with respect to criteria and the appropriate linguistic variables ($\tilde{w}_j$, $j = 1, 2, \ldots, n$) for the weight of the criteria. The Fuzzy linguistic rating $(\tilde{x}_{ij})$ preserves the property that the ranges of normalized triangular Fuzzy numbers belong to [0,1]; thus, there is no need for a normalization procedure. For this instance, the $\tilde{D}$ defined by Eq. (14) is equivalent to the $\tilde{R}$ defined by Eq. (16).

Step 2. Construct the weighted normalized Fuzzy decision matrix. The weighted normalized value $\tilde{V}$ is calculated by Eq. (17).

Step 3. Identify positive ideal (A^*) and negative ideal (A^-) solutions. The Fuzzy positive-ideal solution (FPIS, A^*) and the Fuzzy negative-ideal solution (FNIS, A^-) are shown in Eqs. (18) and (19):

$$A^* = \{\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*\} = \{(\max_i \tilde{v}_{ij} | i = 1, \ldots, m), j = 1, 2, \ldots, n\}. \tag{18}$$

$$A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\} = \{(\min_i \tilde{v}_{ij} | i = 1, \ldots, m), j = 1, 2, \ldots, n\}. \tag{19}$$

Max and min operations does not give triangular fuzzy number but it is possible to express approximated values of min and max as triangular fuzzy numbers [14], we know that the elements $\tilde{v}_{ij}$ $\forall i,j$ are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0,1]. Thus, we can define the fuzzy positive-ideal solution and the negative-ideal as $\tilde{v}_j^* = (1, 1, 1)$ and $\tilde{v}_j^- = (0, 0, 0)$, $j = 1, 2, \ldots, n$ [10].

Step 4. Calculate separation measures. The distance of each alternative from A^* and A^- can be currently calculated using Eqs. (20) and (21).

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, \ldots, m \tag{20}$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \ldots, m \tag{21}$$

Step 5. Calculate the similarities to ideal solution. This step solves the similarities to an ideal solution by Eq. (22):

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \tag{22}$$

Step 6. Rank preference order. Choose an alternative with maximum CC_i^* or rank alternatives according to CC_i^* in descending order [11].

4. Empirical illustrations and discussions

4.1. Empirical illustrations for AHP method

The hierarchical structure of the decision model of the paper with the alternatives and the criteria is portrayed in Fig. 3. The decision problem consists of three levels: at the highest level, the objective of the problem is situated while in the second level, the criteria are listed, and in the third level, the sub-criteria are listed; the last level belongs to the alternatives.

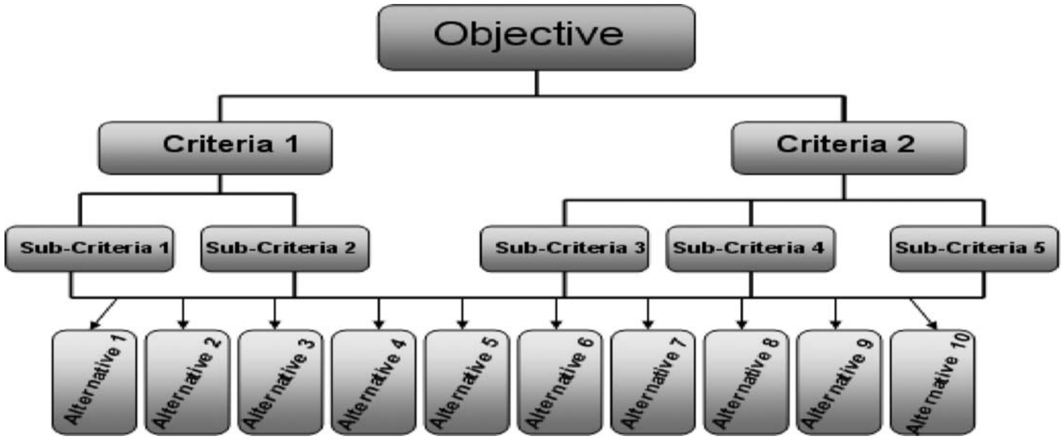

Fig. 3. The decision-making problem's hierarchy.

Table 2
The pairwise comparison matrix of the criteria and sub-criteria.

No.	C_1	C_2	SC_1	SC_2	SC_3	SC_4	SC_5
C_1	1	0.5	–	–	–	–	–
C_2	2	1	–	–	–	–	–
SC_1	–	–	1	1	0.3333	1	1
SC_2	–	–	1	1	0.3333	1	7
SC_3	–	–	3	3	1	3	5
SC_4	–	–	1	1	0.3333	1	3
SC_5	–	–	1	0.1428	0.2	0.3333	1

Table 3
Priority weights in the AHP decision tree.

Criteria1	0.3333333				
Sub-criteria11		0.425	2	0.14166	4
Sub-criteria12		0.575	1	0.19166	2
Criteria2	0.6666666				
Sub-criteria21		0.644835	1	0.42989	1
Sub-criteria22		0.244575	2	0.16305	3
Sub-criteria23		0.11059	3	0.07372	5

As mentioned, the AHP methodology first necessitates the pairwise comparisons of the criteria and sub-criteria in order to determine their weighs. These consistent comparison matrices are shown in Table 2. The normalized priority weights among the two main criteria and five sub-criteria and their ranking have been depicted in Table 3.

4.2. Empirical illustrations for Fuzzy AHP method

Step 1. The comparison matrixes 2 have to be normalized into the range of [0,1] by Eq. (11). Table 2 can be transformed into Table 4.

The comparison matrices 4 are consistent (CI < 0.1) according to Isiklar and Buyukozkan [10]. In the next step, we use the Fuzzy membership function discussed in Section 3.2 to transform Table 4 into Table 5 as explained by the following example. If the numeric rating is 0.45, then its Fuzzy linguistic variable is "*M*". Therefore, the new pairwise comparison matrix will be as in Table 5.

The fuzzy linguistic variables of the above matrix are then transformed into a Fuzzy triangular membership function as shown in Table 6.

In the third step, we calculate the average of the elements of each row; the resulting matrixes are shown in Tables 7 and 8.

Table 4
Normalized pairwise comparison matrix of the criteria and sub-criteria for AHP analysis.

No.	C_1	C_2	SC_1	SC_2	SC_3	SC_4	SC_5
C_1	0.333333	0.333333	–	–	–	–	–
C_2	0.666666	0.666666	–	–	–	–	–
SC_1	–	–	0.5	0.5	0.5	0.5	0.125
SC_2	–	–	0.5	0.5	0.5	0.5	0.875
SC_3	–	–	0.6	0.724138	0.652174	0.692308	0.55555
SC_4	–	–	0.2	0.241379	0.217391	0.230769	0.33333
SC_5	–	–	0.2	0.034483	0.130435	0.076923	0.11111

4.3. Empirical illustrations for TOPSIS method

The decision matrix of Table 9 is used for the TOPSIS analysis. Based on the first step of the TOPSIS procedure, each element is normalized by Eq. (11). The resulting normalized decision matrix for the TOPSIS analysis is shown in Table 10. The second step required the criteria weight information to calculate the weighted normalized rating. These criteria weights calculated former with AHP. The third step finds the weighted normalized decision matrix. The analysis then proceeds to steps 4 and 5. The results are summarized in Table 11.

Finally, the fifth step rank the alternative according to Table 11 results as follows:

$A_8 > A_2 > A_7 > A_6 > A_1 > A_{10} > A_9 > A_3 = A_4 > A_5$

4.4. Empirical illustrations for Fuzzy TOPSIS method

Table 9, numeric performance ratings are adopted again for Fuzzy TOPSIS analysis. In order to transform the performance ratings to Fuzzy linguistic variables, the performance ratings in Table 9 are normalized into the range of [0 1] by Eqs. (23) and (24):

(i) The larger, the better type [11]:

$$r_{ij} = \frac{[x_{ij} - \min\{x_{ij}\}]}{[\max\{x_{ij}\} - \min\{x_{ij}\}]}. \qquad (23)$$

Table 5
Pairwise comparison matrix of the criteria and sub-criteria using Fuzzy linguistic variables.

No.	C_1	C_2	SC_1	SC_2	SC_3	SC_4	SC_5
C_1	L	L	–	–	–	–	–
C_2	H	H	–	–	–	–	–
SC_1	–	–	M	M	M	M	VL
SC_2	–	–	M	M	M	M	VH
SC_3	–	–	M	H	H	H	M
SC_4	–	–	VL	L	L	L	L
SC_5	–	–	VL	VL	VL	VL	VL

Table 6
Fuzzy pairwise comparison matrix of the criteria and sub-criteria.

No.	C_1	C_2	SC_1	SC_2	SC_3	SC_4	SC_5
C_1	(0.15,0.30,0.45)	(0.15,0.30,0.45)	–	–	–	–	–
C_2	(0.55,0.70,0.85)	(0.55,0.70,0.85)	–	–	–	–	–
SC_1	–	–	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.00,0.10,0.25)
SC_2	–	–	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.75,0.90,1.00)
SC_3	–	–	(0.35,0.50,0.65)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.35,0.50,0.65)
SC_4	–	–	(0.00,0.10,0.25)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.15,0.30,0.45)
SC_5	–	–	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.10,0.25)

Table 7
Fuzzy pairwise comparison matrix of the sub-criteria and Fuzzy criteria weights.

No.	SC_1	SC_2	SC_3	SC_4	SC_5	W	Rank
SC_1	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.05,0.15,0.29)	4
SC_2	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.08,0.21,0.38)	2
SC_3	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.41,0.63,0.85)	1
SC_4	(0.15,0.30,0.45)	(0.00,0.10,0.25)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.08,0.21,0.38)	3
SC_5	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.10,0.25)	(0.00,0.70,0.21)	5

Table 8
Fuzzy pairwise comparison matrix of the criteria and Fuzzy criteria weights.

No.	C_2	C_1	W	Rank
C_1	(0.15,0.30,0.45)	(0.15,0.30,0.45)	(0.15,0.30,0.45)	1
C_2	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	2

Table 9
Decision matrix.

No.	C_1	C_2	C_3	C_4	C_5
A_1	185.9500	3.7500	0.0119	8.0000	0.0575
A_2	206.3800	7.8500	0.0596	9.0000	0.0345
A_3	211.4600	7.7100	0.0714	8.0000	0.0690
A_4	228.0000	14.0000	0.0357	8.0000	0.0920
A_5	185.8500	6.2500	0.0476	8.0000	0.0575
A_6	183.1800	7.8500	0.0595	9.0000	0.0575
A_7	225.2600	2.0000	0.0714	5.0000	0.0690
A_8	202.8200	13.3000	0.0952	10.0000	0.0920
A_9	216.3800	7.7100	0.0476	8.0000	0.0345
A_{10}	185.7500	10.1600	0.0595	9.0000	0.0230

(ii) The smaller, the better type:

$$r_{ij} = \frac{[\max\{x_{ij}\} - x_{ij}]}{[\max\{x_{ij}\} - \min\{x_{ij}\}]} \quad (24)$$

For the present study, C_4 and C_5 are the smaller-the-better type; the others belong to the larger-the-better type. Then, this table can be transformed into Table 12. The next step uses the Fuzzy membership function discussed in Section 3.2 to transform Table 12 into Table 13.

Table 10
Normalized decision matrix for TOPSIS analysis.

No.	C_1	C_2	C_3	C_4	C_5
A_1	0.29013	0.304997	0.062859	0.134672	0.288593
A_2	0.174078	0.343122	0.314823	0.281912	0.3203
A_3	0.348156	0.304997	0.377153	0.276885	0.328184
A_4	0.464208	0.304997	0.188577	0.502774	0.353854
A_5	0.29013	0.304997	0.251435	0.224453	0.288437
A_6	0.29013	0.343122	0.314294	0.281912	0.284294
A_7	0.348156	0.190623	0.377153	0.071825	0.349601
A_8	0.464208	0.381246	0.502871	0.477635	0.314775
A_9	0.174078	0.304997	0.251435	0.276885	0.33582
A_{10}	0.116052	0.343122	0.314294	0.36487	0.288282
W	0.141667	0.191666	0.42989	0.16305	0.075727

The Fuzzy linguistic variable is then transformed into a Fuzzy triangular membership function as shown in Table 14. This is the first step of the Fuzzy TOPSIS analysis. The Fuzzy criteria weight is also collected in Table 14. The second step in the analysis is to find the weighted Fuzzy decision matrix. Using Eq. (8), the Fuzzy multiplication equation, the resulting Fuzzy weighted decision matrix is shown in Table 15. According to Table 15, we can define the Fuzzy positive-ideal solution and the Fuzzy negative-ideal solution as: $\tilde{v}_j^* = (1, 1, 1)$ and $\tilde{v}_j^* = (0, 0, 0), j = 1, 2, \ldots, n$. This is the third step of the Fuzzy TOPSIS analysis. For the fourth step, the distance of each alternative from A^* and A^- can be calculated using Eqs. (19) and (21). The fifth step solves the similarities to an ideal solution by Eq. (22). The resulting Fuzzy TOPSIS analyses are summarized in Table 16.

Table 11
TOPSIS analysis results.

No.	v_{i1}	v_{i2}	v_{i3}	v_{i4}	v_{i5}	S_i^*	S_i^-	C_i^*
A_1	0.041102	0.058458	0.027022	0.021958	0.021854	0.191591	0.068669	0.522671
A_2	0.024661	0.065765	0.135339	0.045966	0.024255	0.097256	0.118143	0.683672
A_3	0.049322	0.058458	0.162134	0.045146	0.024852	0.067332	0.145524	0.33287
A_4	0.065763	0.058458	0.081067	0.081977	0.026796	0.153081	0.076381	0.33287
A_5	0.041102	0.058458	0.10809	0.036597	0.021843	0.114562	0.098714	0.002525
A_6	0.041102	0.065765	0.135112	0.045966	0.021529	0.091689	0.120293	0.567469
A_7	0.049322	0.036536	0.162134	0.011711	0.026474	0.067457	0.094549	0.583613
A_8	0.065763	0.073072	0.216179	0.077878	0.023837	0.085465	0.198931	0.699485
A_9	0.024661	0.058458	0.10809	0.045146	0.025431	0.121324	0.092078	0.431478
A_{10}	0.016441	0.065765	0.135112	0.059492	0.021831	0.106495	0.114315	0.517709
v_j^*	0.065763	0.073072	0.216179	0.011711	0.021529			
v_j^-	0.016441	0.036536	0.027022	0.081977	0.026796			
W	0.141667	0.191666	0.42989	0.16305	0.075727			

Table 12
Normalized decision matrix for Fuzzy TOPSIS analysis.

No.	C_1	C_2	C_3	C_4	C_5
A_1	0.5	0.6	0	0.854167	0.938197
A_2	0.166667	0.8	0.572629	0.5125	0.482374
A_3	0.666667	0.6	0.714286	0.524167	0.369032
A_4	1	0.6	0.285714	0	0
A_5	0.5	0.6	0.428571	0.645833	0.940428
A_6	0.5	0.8	0.571429	0.5125	1
A_7	0.666667	0	0.714286	1	0.061133
A_8	1	1	1	0.058333	0.561803
A_9	0.166667	0.6	0.428571	0.524167	0.259259
A_{10}	0	0.8	0.571429	0.32	0.94266
W	0.141667	0.191666	0.42989	0.16305	0.075727

Table 13
Decision matrix using Fuzzy linguistic variables.

No.	C_1	C_2	C_3	C_4	C_5
A_1	M	H	VL	VH	VH
A_2	VL	VH	M	M	M
A_3	H	H	H	M	M
A_4	VH	H	L	VL	VL
A_5	M	H	M	H	VH
A_6	M	VH	M	M	VH
A_7	H	VL	H	VH	VL
A_8	VH	VH	VH	VL	H
A_9	VL	H	M	M	L
A_{10}	VL	VH	M	L	VH
W	L	H	VH	M	VL

The last step found the preference for the 10 alternatives as follows:

$A_8 > A_3 > A_6 > A_5 > A_2 > A_7 > A_{10} > A_1 > A_9 > A_4$

The Fuzzy TOPSIS analysis of the alternatives is summarized in Fig. 4.

5. Computational results

The top three alternatives, according to the two proposed methods, as well as the results by data envelopment analysis (DEA) [16] are summarized in Table 17. All methods lead to choice of A_8. The Fuzzy TOPSIS concludes with the same top three alternatives

Table 14
Fuzzy decision matrix and Fuzzy criteria weights.

No.	C_1	C_2	C_3	C_3	C_4
A_1	(0.35,0.50,0.65)	(0.55,0.70,0.85)	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.75,0.90,1.00)
A_2	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.35,0.50,0.65)
A_3	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.35,0.50,0.65)
A_4	(0.75,0.90,1.00)	(0.55,0.70,0.85)	(0.15,0.30,0.45)	(0.00,0.10,0.25)	(0.00,0.10,0.25)
A_5	(0.35,0.50,0.65)	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.55,0.70,0.85)	(0.75,0.90,1.00)
A_6	(0.35,0.50,0.65)	(0.75,0.90,1.00)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.75,0.90,1.00)
A_7	(0.55,0.70,0.85)	(0.00,0.10,0.25)	(0.55,0.70,0.85)	(0.75,0.90,1.00)	(0.00,0.10,0.25)
A_8	(0.75,0.90,1.00)	(0.75,0.90,1.00)	(0.75,0.90,1.00)	(0.00,0.10,0.25)	(0.55,0.70,0.85)
A_9	(0.00,0.10,0.25)	(0.55,0.70,0.85)	(0.35,0.50,0.65)	(0.35,0.50,0.65)	(0.15,0.30,0.45)
A_{10}	(0.00,0.10,0.25)	(0.75,0.90,1.00)	(0.35,0.50,0.65)	(0.15,0.30,0.45)	(0.75,0.90,1.00)
W	(0.15,0.30,0.45)	(0.55,0.70,0.85)	(0.75,0.90,1.00)	(0.35,0.50,0.65)	(0.00,0.10,0.25)

Table 15
Fuzzy weighted decision matrix.

No.	C_1	C_2	C_3	C_4	C_5
A_1	(0.05,0.15,0.29)	(0.30,0.49,0.72)	(0.00,0.09,0.25)	(0.26,0.45,0.65)	(0.00,0.90,0.25)
A_2	(0.00,0.03,0.11)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.12,0.25,0.42)	(0.00,0.05,0.16)
A_3	(0.08,0.50,0.65)	(0.30,0.49,0.72)	(0.41,0.63,0.85)	(0.12,0.25,0.42)	(0.00,0.05,0.16)
A_4	(0.11,0.27,045)	(0.30,0.49,0.72)	(0.11,0.27,045)	(0.00,0.05,0.16)	(0.00,0.01,0.06)
A_5	(0.05,0.15,0.29)	(0.30,0.49,0.72)	(0.26,0.45,0.65)	(0.19,0.35,0.55)	(0.00,0.90,0.25)
A_6	(0.05,0.15,0.29)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.12,0.25,0.42)	(0.00,0.90,0.25)
A_7	(0.08,0.50,0.65)	(0.00,0.07,0.21)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.00,0.01,0.06)
A_8	(0.11,0.27,045)	(0.41,0.63,0.85)	(0.56,0.81,1.00)	(0.00,0.05,0.16)	(0.00,0.07,0.21)
A_9	(0.00,0.03,0.11)	(0.30,0.49,0.72)	(0.26,0.45,0.65)	(0.12,0.25,0.42)	(0.00,0.03,0.11)
A_{10}	(0.00,0.03,0.11)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.05,0.15,0.29)	(0.00,0.09,0.25)

Table 16
Fuzzy TOPSIS analysis.

No.	$\tilde{v}_{i1}$	$\tilde{v}_{i2}$	$\tilde{v}_{i3}$	$\tilde{v}_{i4}$	$\tilde{v}_{i5}$	d_i^+	d_i^-	CC_i
A_1	(0.05,0.15,0.29)	(0.30,0.49,0.72)	(0.00,0.09,0.25)	(0.26,0.45,0.65)	(0.00,0.90,0.25)	3.71	1.5	0.28790
A_2	(0.00,0.03,0.11)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.12,0.25,0.42)	(0.00,0.05,0.16)	3.61	1.57	0.30308
A_3	(0.08,0.50,0.65)	(0.30,0.49,0.72)	(0.41,0.63,0.85)	(0.12,0.25,0.42)	(0.00,0.05,0.16)	3.38	1.81	0.34874
A_4	(0.11,0.27,045)	(0.30,0.49,0.72)	(0.11,0.27,045)	(0.00,0.05,0.16)	(0.00,0.01,0.06)	3.9	1.25	0.24271
A_5	(0.05,0.15,0.29)	(0.30,0.49,0.72)	(0.26,0.45,0.65)	(0.19,0.35,0.55)	(0.00,0.90,0.25)	3.47	1.74	0.33397
A_6	(0.05,0.15,0.29)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.12,0.25,0.42)	(0.00,0.90,0.25)	3.46	1.79	0.34095
A_7	(0.08,0.50,0.65)	(0.00,0.07,0.21)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.00,0.01,0.06)	3.63	1.53	0.29651
A_8	(0.11,0.27,045)	(0.41,0.63,0.85)	(0.56,0.81,1.00)	(0.00,0.05,0.16)	(0.00,0.07,0.21)	3.27	1.97	0.37595
A_9	(0.00,0.03,0.11)	(0.30,0.49,0.72)	(0.26,0.45,0.65)	(0.12,0.25,0.42)	(0.00,0.03,0.11)	3.74	1.42	0.27519
A_{10}	(0.00,0.03,0.11)	(0.41,0.63,0.85)	(0.26,0.45,0.65)	(0.05,0.15,0.29)	(0.00,0.09,0.25)	3.66	1.53	0.29479
A^+	(0,0,0)	(0,0,0)	(1,1,1)	(1,1,1)	(1,1,1)			
A^-	(1,1,1)	(1,1,1)	(0,0,0)	(0,0,0)	(0,0,0)			
W	(0.15,0.30,0.45)	(0.55,0.70,0.85)	(0.75,0.90,1.00)	(0.35,0.50,0.65)	(0.00,0.10,0.25)			

Table 17
Top three alternatives from different methods.

Preference order	Fuzzy AHP and Fuzzy TOPSIS	AHP and TOPSIS	DEA
1	A_8	A_8	A_8
2	A_3	A_2	A_3
3	A_6	A_7	A_6

Table 18
Comparison proposed method with several paper.

Ref.	Isiklar and Buyukozkan [10]	Yang and Hung [11]	Chiou et al. [37]	Wang et al. [38]	Tsvetinov and Mikhailov [34]	Febriamansyah [18]	*Proposed method*
Problem type	MCDM	MADM	MCDM	MCDM	MCDM	MCDM	MCDM
Application area (proposed method without some aspects)	Mobile phone selection (without Fuzzification)	Plant layout design problem (without FAHP)	Construct the roadmap of R&D consortia (without TOPSIS and FTOPSIS)	Evaluating the 64-bits dual core Notebook (without FAHP and FTOPSIS)	Reasoning under uncertainty during pre-negotiations (without TOPSIS and FTOPSIS)	Irrigation water allocation (without TOPSIS and FTOPSIS)	Fully Fuzzy decision problem
Level	4	3	4	3	4	3	4
Criteria	2	6	5	7	3	6	2
Sub-criteria	6	–	20	30	6	–	5
Alternative	3	18	10	8	3	6	5
Solution method	AHP and TOPSIS	AHP and TOPSIS and FTOPSIS	FAHP	TOPSIS	FAHP	AHP and Fuzzy dominance method	FAHP and FTOPSIS
Top three alternatives	$A_2 > A_1 > A_3$	$A_{11} > A_{15} > A_{18}$	$A_6 > A_9 > A_{10}$	$A_4 > A_5 > A_7$	$A_1 > A_3 > A_2$	$A_3 > A_6 > A_4$	
Solution through proposed method	$A_2 > A_1 > A_3$	$A_{11} > A_{15} > A_{18}$	$A_6 > A_9 > A_{10}$	$A_4 > A_5 > A_7$	$A_1 > A_3 > A_2$	$A_3 > A_6 > A_4$	
Fuzzification ability %	0	50	50	0	50	50	100

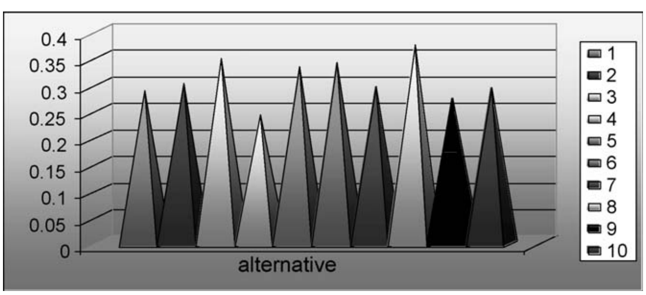

Fig. 4. Ranking of alternatives.

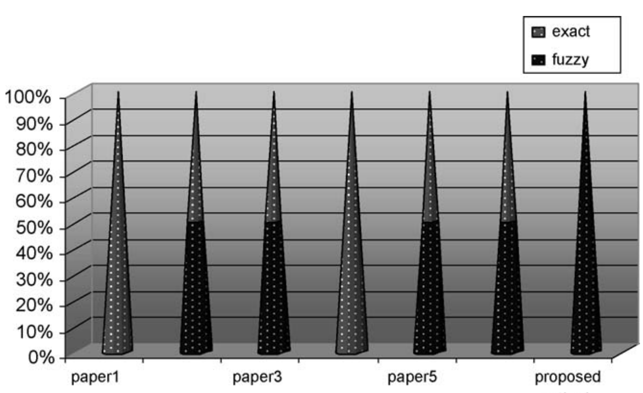

Fig. 5. Comparison proposed method with several paper from Fuzzification ability %.

as those DEA, but, the TOPSIS method concludes with the same top one alternatives as the ones from DEA and Fuzzy TOPSIS. Therefore, the proposed method by fuzzy systematic evaluation of the MCDM problem can reduce the risk of a poor decision in management.

When precise performance ratings are available, the TOPSIS method is considered a viable approach in solving a decision problem. The DEA method is a viable approach. However, it has the constraints in the number of decision-making units and in the limitation to the discrepancy between performance frontiers. For the instance of imprecise or vague performance rating, the fuzzy TOPSIS is a preferred choice [11].

Comparison proposed method with several papers from a fuzzy input data have been shown in Table 18 and Fig. 5.

6. Conclusions

The present study explored the use of AHP, Fuzzy AHP, TOPSIS and Fuzzy TOPSIS in solving a MCDM problem. This study aimed at searching an improved solution to MCDM problems. When the criteria weights and performance ratings are vague and inaccurate, then the Fuzzy AHP and Fuzzy TOPSIS are the preferred techniques. In addition, there exists other worth investigating MADM methods for a MADM problem. This becomes one of the future research opportunities in this classical, yet important, research area.

References

[1] J.C. Pomerol, S.B. Romero, Multicriterion Decision in Management: Principles and Practice, (C. James, Trans.), 1st edition, Kluwer Academic Publishers, Norwell, 2000 (from French).
[2] T.L. Saaty, The Analytic Hierarchy Process, McGraw Hill Company, New York, 1980.
[3] C.H. Cheng, Evaluating weapon systems using ranking fuzzy numbers, Fuzzy Systems 107 (1999) 25–35.
[4] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making—Methods and Applications, Springer-Verlag, Heidelberg, 1981.
[5] J. Aguaron, M.T. Escobar, J.M. Moreno-Jimenez, Consistency stability intervals for a judgment in AHP decision support system, European Journal of Operational Research 145 (2003) 382–393.
[6] S.J. Chen, C.L. Hwang, F.P. Hwang, Fuzzy multiple attribute decision making, 1992 Available at http://www.springerlink.com/index/9056211m22959555.pdf.
[7] S.H. Tsaur, T.Y. Chang, C.H. Yen, The evaluation of airline service quality by fuzzy MCDM, Tourism Management 23 (2002) 107–115.
[8] T. Yang, P. Chou, Solving a multiresponse simulation-optimization problem with discrete variables using a multiple-attribute decision-making method, Mathematics and Computers in Simulation 68 (2005) 9–21.
[9] K.P. Yoon, C.L. Hwang, Multiple Attribute Decision Making, Sage Publication, Thousands Oaks, CA, 1995.

[10] G. Isiklar, G. Buyukozkan, Using a multi-criteria decision making approach to evaluate mobile phone alternatives, Computer Standards and Interfaces 29 (2006) 265–274.
[11] T. Yang, C.C. Hung, Multiple-attribute decision-making methods for plant layout design problem, Robotics and Computer-integrated manufacturing 23 (2007) 126–137.
[12] L.A. Zadeh, Fuzzy sets, Information Control 8 (1965) 338–353.
[13] A. Kaufmann, M.M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Applications, Van Nostrand Reinhold, New York, 1985.
[14] H.L. Kwang, First Course on Fuzzy Theory and Applications, Springer-Verlag, Berlin-Heidelberg/New York, 2005, pp. 137–144.
[15] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (2000) 1–9.
[16] T. Yang, C. Kuo, A hierarchical AHP/DEA methodology for the facilities layout design problem, European Journal of Operational Research 147 (2003) 128–136.
[18] Febriamansyah, R., 1893. The use of AHP (the Analytic Hierarchy Process) method for irrigation water allocation in a small river BASIN (Case Study in Tampo River Basin in West Sumatra, Indonesia), dlc.dlib.indiana.edu/archive/00001893/00/Febriamansyah_Rudi.pdf.
[19] C.M. Feng, R.T. Wang, Performance evaluation for airlines including the consideration of financial ratios, Journal of Air Transport Management 6 (2000) 133–142.
[34] P. Tsvetinov, L. Mikhailov, Reasoning Under Uncertainty During Pre-negotiations Using a Fuzzy AHP, 2004 Available at http://www.expertguide.qut.com/guide/expert-details.jsp.
[37] H.K. Chiou, C.C. Wan, G.H. Tzeng, Fuzzy AHP with MCDA to Construct the Roadmap of R&D Consortia in TAIWAN'S M&S Enterprises, 2005 Available at http://www.superdecisions.com/∼saaty/ISAHP2005/Papers/ChiouHK_WanCC_TzengGH_FuzzyAHPR&DConsortia.pdf.
[38] C.H. Wang, L.P. Wang, Y.C. Hu, Using GC-TOPSIS of Combination Weight for Evaluating the 64-bits Dual Core Notebook, 2007 Available at http://www.yzu.edu.tw/admin/rd/index.php/content/view/166/399/-61k.
[39] T.L. Saaty, Decision Making with Dependence and Feedback: The Analytic Network Process, 2nd edition, PRWS Publications, Pittsburgh PA, 2001.
[40] T.L. Saaty, Multicriteria Decision Making: The Analytic Hierarchy Process, RWS Publications, Pittsburgh, PA, 1988.
[41] A. Stam, A.P. Duarte Silva, On multiplicative priority ratings method for the AHP, European Journal of Operational Research 145 (2003) 92–108.
[42] M.A. Abo-Sinna, A.H. Amer, Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems, Applied Mathematics and Computation 162 (1) (2005) 243–256.
[43] H.A. Holloway, C.C. White III, Question selection for multi-attribute decision-aiding, European Journal of Operational Research 148 (3) (2003), 525–533(9).