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Design and Analysis of a Vacation Model for Two-Phase Queueing System with Gated Service

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Abstract

This paper mainly deals with a two phase service queueing model with gated service vacation. In this gated service vacation model, only those customers who are present in the queue when the server starts a batch mode service are taken into the batch. We have derived probability generating functions of the system size and the orbit size. We have investigated the conditions under which the steady state exists. Some useful performance measures are also obtained. The service to further arrivals is deferred until the server completes the second phase individual service to the batch. Based on the numerical calculations and graphical representations it reveals the fact that this type of modelling helps in analyzing certain situations in which customers require service in two stages of which the first stage of service is essentially of gated type.

1. INTRODUCTION

In this paper we study the two phase queueing system with gated service and multiple server vacations. The gated service policy to be considered in the present model under consideration is exceptional in the sense that the server's gated mechanism will last until the system becomes empty after which the server is allowed to take vacations. In certain situations it is necessary to perform batch mode service to those customers only who are present in the system at the commencement of the batch service. Customers first enter into a waiting room. They are served in the service room. When the service room becomes empty, all the customers in the waiting room are transferred to

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service room when they served individually. At such epochs of transfer of customers from the waiting room, a random number of overhead customers are also added to the service room.

Notations

N = System size at the commencement of a batch service.

N_1 = System size at the end of the batch service.

N_2 = System size when the server completes the second phase of service to the batch.

M = Number of customer arrivals during batch service.

X = Total number of arrivals when the server is in the individual service mode during a service cycle.

Y = Number of arrivals ≥ 1 during a vacation.

$B(t)$ = Batch service time.

$F(t)$ = Individual service time.

$V(t)$ = Vacation time.

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2. MATHEMATICAL MODEL

Consider a single server two phase service vacation queueing system in which customers arrive according to a poisson process with parameterλ. The customers are served in batch mode at the first phase following the gated service policy. On the completion of the batch mode service, the server starts servicing the customers of the same batch following the first in first out queue discipline. After exhausting the batch the server returns immediately to the first phase and starts the next gated batch service provided at least one customer waiting in the queue.

Busy Cycle - The time interval from the instant of commencement of a batch service at the end of a vacation to the instant of starting the next batch service after availing at least one vacation is a busy cycle.

Service Cycle - The time interval between the commencements of two consecutive batch services when the later starts immediately at the end of the second phase service to the former batch.

The system is in steady state.

$$\gamma = \lambda \, E(B), \ \rho = \lambda \, E(S) < 1, \ N_1 = N + M, \ N_2 = X + M, \tag{1}$$

$$N = \begin{cases} N_2 & \text{when } N_2 > 0 \\ Y & \text{when } N_2 = 0 \end{cases} \tag{2}$$

$$P(N_2 = 0) = P_{20} \tag{3}$$

Let $P^*(z)$, $P_1^*(z)$ and $P_2^*(z)$ denote the PGFs of N, N_1 and N_2 respectively.

$$
\begin{aligned}
P^*(z) &= E(z^N) \\
&= E[E(z^N/N_2)] \\
&= E(z^N/N_2 = 0)P(N_2 = 0) + \sum_{k=1}^{\infty} E(z^N/N_2 = k)P(N_2 = k) \\
&= E(z^\gamma)P_{20} + \sum_{k=1}^{\infty} E(z^k)P(N_2 = k) \\
&= E(z^\gamma)P_{20} + \sum_{k=1}^{\infty} z^k P(N_2 = k) \\
&= \{[\overline{V}(\lambda - \lambda z) - \overline{V}(\lambda)]/[1 - \overline{V}(\lambda)]\} P_2 + P_2^*(z) - P_{20} \\
&= P_2^*(z) - P_{20}\{1 - [\overline{V}(\lambda - \lambda z) - \overline{V}(\lambda)]/[1 - \overline{V}(\lambda)]\}
\end{aligned}
\tag{4}
$$

Let

$b_k = P\{K \text{ customers arrive during a vacation}\}$

$$= \int_0^\infty e^{-\lambda t}\left[\frac{(\lambda t)^k}{k!}\right] dV(t)$$

(5)

P (K customers arrive during the j-th vacation)

$= (\overline{V}(\lambda))^{j-1} b_k$, $j = 1, 2, 3, \dots$.

Hence P (K customers are present in the system at the end of vacation)

$$= \sum_{j=1}^\infty \overline{V}(\lambda)^{j-1} b_k$$
$$= \frac{b_k}{1 - \overline{V}(\lambda)}$$

(6)

$$E(z^\gamma) = \sum_{k=1}^\infty z^k [b_k/(1 - \overline{V}(\lambda))]$$
$$= [1/(1 - \overline{V}(\lambda))] \sum_{k=1}^\infty \left[z^k \int_0^\infty e^{-\lambda t}\left[\frac{(\lambda t)^k}{k!}\right] d(V(t))\right]$$
$$= [1/(1 - \overline{V}(\lambda))] \int_0^\infty e^{-\lambda t} \sum_{k=1}^\infty \left[\frac{(\lambda t)^k}{k!}\right] d(V(t))$$
$$= (\overline{V}(\lambda - \lambda z) - \overline{V}(\lambda)/(1 - \overline{V}(\lambda))$$

(7)

$$P_1^*(z) = E(z^{N+M})$$
$$= E(z^N) E(z^M)$$
$$= P^*(z)\overline{B}(\lambda - \lambda z)$$

(8)

$$P_2^*(z) = E(z^{X+M})$$
$$= E(z^X) E(z^M)$$

(9)

$$E(z^X) = E[E(z^X/N)]$$
$$= P^*(\overline{F}(\lambda - \lambda z))$$

(10)

$$P_2^*(z) = \overline{B}(\lambda - \lambda z) P^*(\overline{F}(\lambda - \lambda z))$$

(11)

Combine all the equations

$$P^*(z) = \overline{B}(\lambda - \lambda z) P^*(\overline{F}(\lambda - \lambda z))$$
$$- P_{20}\{1 - [(\overline{V}(\lambda - \lambda z) - \overline{V}(\lambda))/1 - \overline{V}(\lambda)]\}$$

(12)

Define

$g(z) = \overline{F}(\lambda - \lambda z)$
$h(z) = \overline{B}(\lambda - \lambda z)$
$L(z) = 1 - 1 - [(\overline{V}((\lambda - \lambda z) - \overline{V}(\lambda))/1 - \overline{V}(\lambda)]$

(13)

$$P^*(z) = P*(g(z))h(z) - P_{20}L(z) \tag{14}$$

Let

$$h^{(0)}(z) = z$$
$$h^{(1)}(z) = h(z)$$
$$\qquad = \overline{B}(\lambda - \lambda z) \tag{15}$$

$$g^{(0)}(z) = z$$
$$g^{(1)}(z) = g(z) = \overline{F}(\lambda - \lambda z)$$
$$g^{(n)}(z) = g(g^{(h-1)}(z))$$
$$\qquad = g^{(n-1)}(g(z)), \quad n > 1 \tag{16}$$

By repeat, we get

$$P^*(z) = -P_{20}\left\{ \sum_{k=0}^{\infty}\left(\prod_{j=0}^{k-1} h(g^{(j)}(z))L(g^{(k)}(z)) \right) + \prod_{j=0}^{\infty} h(g^{(j)}(z)) \right\} \tag{17}$$

Since P*(0) = 0, P*(g(0)) h(0) − P20 = 0,

$$g(0) = \overline{F}(\lambda)$$
$$h(0) = \overline{B}(\lambda) \tag{18}$$

$$P_{20} = \frac{h(0)\prod_{j=1}^{\infty} h(g^{(j)}(0))}{1 + h(0)\sum_{k=0}^{\infty}\left(\prod_{j=1}^{\infty} h(g^{(j)}(0)) \right)L(g^{(k+1)}(0))} \tag{19}$$

Let the random variable QS denote the system size when a random customer leaves the system after his service completion. Let $Q_S^*(z)$ denote the PGF of QS. K-denote the position of the random customer in his batch.

X_1 − number of customers left behind the random customer.
X_2 − number of customers arrivals that occur during the K individual service completions in the second phase.
$QS = M + X_1 + X_2.$
M is independent of X_1 and X_2 where as X_1 and X_2 are dependent random variables.

$$Q_S^*(z) = E(z^{Q_s})$$
$$= E(z^{M+X_1+X_2})$$
$$= E(z^M) + E(z^{X_1+X_2}) \tag{20}$$

$$E(z^{X_1+X_2}) = E[E(z^{X_1+X_2}/N,K)]$$
$$= \sum_{n=1}^{\infty}\sum_{k=1}^{n} z^{n-k}(\overline{F}(\lambda-\lambda z))^k P(N=n,K=k)$$
$$= \sum_{n=1}^{\infty}\sum_{k=1}^{n} z^n (g(z)/z)^k (1/n)(nP(N=n/E(n))$$
$$= \{g(z)/E(N)\}\{[P^*(z) - P^*(g(z))]/[z-g(z)]\} \tag{21}$$

$Q_S^*(z) = g(z)[P^*(z)(1-h(z)-P20L(z)/E(N)(g(z)-z))]$ When the server is not allowed to take any vacation and there is no batch service so that $\overline{V}(0) = 1$ and $\overline{B}(0) = 1$.

2.1 Performance measures

In this section, we examine some useful performance measures of the system.
(a) The expected number of customers in the system is given by

$$E(L) = \lim_{z \to 1} \frac{d}{dz} K(z) =$$

$$2\lambda(\beta + \mu)\left[1 - R^*(\lambda)\right]$$

$$\frac{+\lambda^2\left[\beta^2 + \mu^2 + 2\beta\mu\right]}{2\left[R^*(\lambda) - \lambda(\beta + \mu)\right]} + \lambda\mu \tag{22}$$

(b) The expected number of customers in the system

$$E(L_q) = F'(1) =$$

$$2\lambda(\beta + \mu)\left[1 - R^*(\lambda)\right]$$

$$\frac{+\lambda^2\left(\beta^{(2)} + \mu^{(2)} + 2\beta\mu\right)}{2\left[R^*(\lambda) - \lambda(\beta + \mu)\right]} \tag{23}$$

The steady state distribution of the server state is given by

q_0 =Prob {server is idle} = $P_{0,0} + P_0(1) = 1 - \lambda(\beta + \mu)$.

q_1 =Prob {server is busy} = $P_1(1) = \lambda\mu$.

q_2 =Prob {server is on vacation} = $P_2(1) = \lambda\beta$

2.2 The System Characteristics

We obtain the first two moments of the system size and system time as follows.

Differentiate $Q_S^*(z)$ with respect to z at z = 1.

$E(Q_S) = \rho + [2(1-\rho)]^{-1}\lambda^2\{E(S^2) + 2E(B) + E(B^2)/E(N)$
$+ P_{20}E(V^2)E(N)(1 - \overline{V}(\lambda))\}$

Differentiate $Q_S^*(z)$ with respect to z twice at z = 0

$E(Q_S^2) = \rho + \dfrac{\lambda^2(1+2\rho)}{2(1-\rho)}\{E(S^2) + 2E(B)\lambda$
$+ E(B^2)/E(N) + P_{20}E(V^2)/[E(N)(1-\overline{V}(\lambda))]\}$
$+ \dfrac{\lambda^3}{3(1-\rho)}\{E(S^3) + E(B^3)/E(N)$
$+ P_{20}E(V^2)/[E(N)(1-\overline{V}(\lambda)))\}$
$+ \dfrac{\lambda^4 E(S^2)}{2(1-\rho)^2}\{E(S^3) + 2E(B)E(S^3)/\lambda + E(V^2)/E(N)\}$
$+ \dfrac{\gamma E(N(N-1))}{E(N)(1-\rho)} + \dfrac{\lambda^2 E(B^2)}{1-\rho} + \dfrac{P_{20}\lambda^3 E(S^2)E(V)}{E(N)(1-\rho)(1-\overline{V}(\lambda))}$
$E(N) = \gamma/1 - \rho + P_{20}\lambda E(V)/(1-\rho)(1-\overline{V}(\lambda))$ \hfill (24)

Similarly obtain E(N(N−1)) and $V(Q_S) = E(Q_S^2) - (E(Q_S))^2$ \hfill (25)

Now we consider two special cases. In the first case we assume that the vacation times are exponentially distributed and for the second case we take exponential distributions for the batch service, individual service and vacation times

Case-1 : Vacation times are exponentially distributed
$\overline{V}(\theta) = \dfrac{1}{(1+E(V)\theta)}$, $E(V^2) = 2 E^2(V)$, The expanded system size is now given by the Little's formula

$E(Q_s) = \lambda\, E(W_s)$, $U<U^*$

Case -2: Batch service times, individual service times are vacation times are exponentially distributed

Let $\overline{B}(\theta) = \frac{1}{(1+E(B)\theta)}$, $F(\theta) = \frac{1}{1+E(S)\theta}$, $V(\theta) = \frac{1}{(1+E(V)\theta)}$, $E(B^2) = 2\,E(B)$, $E(S^2) = 2\,E(S)$ and $E(V^2) = 2\,E(V)$

3. NUMERICAL CALCULATION

$\lambda = 5$, $E(B) = 1/15$, $E(B) = 1/10$, $E(B) = 1/2$

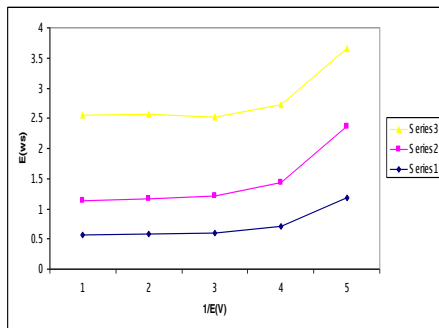| 1/E(V)	E(Ws)	E(Ws)	E(Ws)
5	0.56	0.58	1.41
4	0.58	0.59	1.4
3	0.6	0.61	1.31
2	0.71	0.72	1.3
1	1.18	1.18	1.3

3.1 Graphical Representation

Figure 1

4. CONCLUSION

Two phase service queueing model is designed with gated service vacation. In this gated service vacation model, only those customers who are present in the queue when the server starts a batch mode service are taken into the batch. The service to further arrivals is deferred until the server completes the second phase individual service to the batch. Based on the numerical calculations and graphical representations it reveals the fact that this type of modelling helps in analyzing certain situations in which customers require service in two stages of which the first stage of service is essentially of gated type. In this paper, a new concept is introduced for seclusion using stochastic seclusion to reduce reckoning effort to find a path. The stochastic seclusion technique is to maximizing the scalability and minimizing the complexity in large networks. Graphical representation shows that the stochastic seclusion speed up the routing functions.

5. REFERENCES

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