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A REAL-TIME INVENTORY MODEL TO MANAGE VARIANCE OF DEMAND FOR DECREASING INVENTORY HOLDING COST

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Re-order point model (amount driven) transformed into a time driven model, which is a real-time inventory model.

Developed real-time inventory model eliminates the safety stock.

Some formula is proven considering the uncertainty for calculating re-order time.

A simulation experiment is arranged to determine the characteristics of real-time inventory model, and both models are compared.

ABSTRACT

Variance of demand is one of the inevitable problems in the manufacturing environment. Market conditions and competition force companies reducing costs. Different approaches and methods have been developed to remedy these problems. Common points of these approaches are utilizing resources and responding in the shortest time possible. To achieve better inventory management, inventory-holding cost is tried to be decreased under the same service level. Re-order point is arranged dynamically regarding couple of factors in some studies. We try to arrange the re-order point in a time based manner by transforming the re-order point into re-order time, which eliminates the safety stock; and it makes the inventory model real-time.

Recent production planning studies focus on real-time planning and dynamic scheduling to increase utilization and robustness. In these studies, real-time data is used for planning, but in most of them manufacturing systems and planning methodology are not transformed into a real-time system approach. The novel aspect of this study is the presentation of a model in which the manufacturing system has been designed as a real-time system that consists of real-time planning activities working with real-time data, and eliminating the safety stock.

Keywords: real-time inventory model, inventory management, inventory holding cost, safety stock

1 INTRODUCTION

Increasing variance of demand causes businesses to hold more inventory. Re-order point model offers safety stock against the variance of demand, which causes additional inventory holding cost. However, while competition obliges companies to decrease costs and move faster, product variety is increasing to adapt to changes in the market. Achieving rapid response times by holding inventory is inefficient, while decreasing lead-time is efficient. Thus, agile manufacturing, adaptive manufacturing, and lean
manufacturing studies focus on solving utilization and lead-time problems. In light of this, we aim to develop a planning methodology that enhances planning strategy precise against time dependent changes.

In current approaches, attempts are made to overcome variance of demand through holding safety stock and alternative strategies. Yet these strategies (utilizing multiple suppliers, just – in- time manufacturing and so forth) require significant infrastructure investment and cause additional costs. Instead of applying classic strategies or variations of them, we designed a model that allows the planning process to begin as early as possible, avoiding safety stock. This early planning approach eliminates safety stock by setting the time at the order given, which is the difference between proposed model and re-order point. To set the order time, inventory depletion time is calculated on every inventory transaction. When calculated time is equal to supply lead-time, then the supply order is given.

Time based planning of inventory, makes the approach real-time, in order to make it amount based, because real-time systems are systems working under time constraints to achieve time goals (Zhang, 2010). Real-time systems must provide responses within a set timeframe, as Paul et. al. explain (Paul, et al., 2015). Proposed method provides the time window to be calculated for giving an order, which is the significant point making the model a real-time system.

Real-time systems are divided into two categories: hard real–time systems break down when any failure occurs, while soft real–time systems carry on when any failure occurs. Failure can be incorrect output or any output produced after the time allowed expires. Our model is a soft real-time system, as the model can be run under stock out conditions and with extra inventory holding cost related to variations of demand in a particular period.

2 LITERATURE REVIEW

Various strategies have been developed and adapted to the manufacturing environment to manage uncertainty, which is one of the inevitable problems in the manufacturing environment. The uncertainty problem is investigated by studying the variance of orders, variance of lead-time and backorder. (Chatfield & Pritchard, 2013; Sodhi, et al., 2014) Lee et al. recommend the application of order batching, considering order trends, and sharing information to lessen the bullwhip effect and decrease costs. (Lee, et al., 1997) Information sharing lowers cost and manufacturing lead-time by decreasing uncertainty, as Lee et al. show in the two-level supply chain. (Lee, et al., 2000) However, it does not decrease the complexity of manufacturing and is invalid in some cases.

According to Xu et. al., when market conditions become volatile, companies should decrease product modularity (Xu, et al., 2012), which can be overcome by implementing a robust manufacturing system. Moreover, people attempt to manage manufacturing complexity with dynamic systems; dynamic routing in a just–in–time system is one of the obvious examples of this. (Weng, et al., 2012; Emde & Boysen, 2012) Georgiadis and Michaloudis studied the dynamic adaptation of a desired system’s states in a similar manner (Georgiadis & Michaloudis, 2012), whereas Prince and Key applied group technology via virtual groups to be independent of layout to provide the manufacturing system’s agile and lean production characteristics. (Prince & Kay, 2003) With group technology, the manufacturing schedule is not affected by changing the manufacturing order in-group when an unexpected situation occurs. (Ji, et al., 2014)
Variance of inventory is tried to be controlled via different lot sizing and re-order point strategies in the literature. Chen tried to find the optimal re-order point and lot size by fuzzy membership functions, dynamically. (Chen, 2011) Babai et. al. focused on forecast based dynamic re-order point control policy to reduce computation time and inventory holding cost. (Babai, et al., 2009) Porras and Dekker implemented a bootstrap method to re-order point model to decrease inventory-holding cost, which they modify. (Porras & Dekker, 2008) Gamberinia et. al. analysed different re-order policies and inventory management approaches under irregular and sporadic demand profiles, because assuming current techniques are not simply enough and effectively implementable for different manufacturing environments. (Anon., 2014) These studies show that dynamic re-order point decreases the inventory holding cost under same service level.

One way of changing re-order point is making the inventory management inventory Zhong et al. noted that the real–time monitoring of manufacturing decreases tardiness and increases the immune ability. (Zhong, et al., 2015) In addition, Hung et al. showed that planning with real-time data decreases lead-time. (Hung, et al., 2013) Real-time production planning studies generally focus on collecting real-time data from job shops. Thus, Poon et al., Choi and Shin, and Zhang et al. use real-time data to set the current state of the system as the time at which planning begins. (Poon, et al., 2011; Choi & Shin, 1997; Zhang, et al., 2014) According to Cowling and Johansson, dynamic scheduling with real–time data increases stability and utility. (Cowling & Johansson, 2002)

Real-time systems are generally applied in emergency processes (Jeong, et al., 2010), earthquake monitoring (Brown, et al., 2011), or chemical processes (Zuo & Wu, 2000) that require instant (in a particular time) reaction or response. However, because studies focus on working with real-time data, manufacturing systems usually are not designed as real-time systems. Real–time data is used for planning in a classic system with classic approaches. This study will contribute the literature by a real-time inventory planning approach and eliminating safety stock.

3 REAL-TIME INVENTORY MODEL

Real-time inventory model transforms current inventory planning approach into a time driven perspective, and eliminates the safety stock in its manner. As the time to depletion of remaining inventory, reorder time (ROT) is used in the real–time inventory model instead of reorder point (ROP), as is the case in the current approach. ROT is a particular time, making the system real-time, changes constantly when any variations of demand occur. Eventually, safety stock is unneeded in the real–time inventory model. However, the variance of orders is handled with safety stock in the re-order point model. This significant difference of real-time inventory model eliminates the additional inventory holding cost by eliminating safety stock.

Two inventory models are compared in three cases to investigate the effect of eliminating safety stock in the real-time perspective. The first case is to satisfy all orders with the exact amount of inventory (including safety stock), the second is to satisfy all orders with some inventory left over, and the third case is a stock out. The three cases are designed to represent all situations in inventory management process, which can be faced with. The average inventory holding cost is investigated as a domain to compare both models, because the order points can be different as time and amount. On the other hand, inventory holding cost is a common performance factor for inventory models. Different types of service levels are not considered in this study, although they are common factors.
Orders are given at ROT in the real-time model and at ROP in the current approach, in which safety stock is used, as shown in Figure 1. Because all inventory is consumed in the period regarding to the first case, all safety stock ($Q_s$) is considered as sold. New inventory is assumed to arrive when inventory is consumed.

![Figure 1: Reorder time and reorder point (the first case)](image)

Inventory holding cost is equal to the product of the area in the chart and unit variable cost ($C_V$), with multiplication of the unit constant cost of warehousing ($C_C$) and the cycle time ($L_0$ in the real-time model, $L$ in the re-order point model). The initial inventory amount is $Q$, the total inventory holding cost is $C_T$, and the average inventory holding cost is $C_A$.

\[ T = L_0 C_C + \frac{Q C_V L_0}{2} \Rightarrow C_A = \frac{C_T}{L_0} = C_C + \frac{Q C_V}{2} \tag{1} \]

The initial amount of inventory in the re-order point model is $Q + Q_s$ because of the safety stock. The amount of $Q$ inventory is consumed during $L_0$, and safety stock is consumed during $L - L_0$. $C_T$ and $C_A$ are provided below for the re-order point model.

\[ T = L C_C + \frac{(Q + Q_s) C_V L}{2} \Rightarrow C_A = \frac{C_T}{L} = C_C + \frac{(Q + Q_s) C_V}{2} \tag{2} \]

If the inventory holding costs of both models are considered, because $Q$ smaller than $Q + Q_s$ average inventory holding cost is lower in the real-time model. This situation is valid when actual consumption speed and theoretical consumption speed (according to ROP and ROT) $\left(\frac{Q}{L_0}\right)$ are equal. If actual consumption speed is lower, new inventory will arrive before the current inventory is consumed (which is the second case), as shown in Figure 2. In the second case, the areas of the trapezoids in the charts have to be calculated. $Q_o$ at $L_0$ and $L$ is calculated using speed of consumption ($E$) to determine the areas of the trapezoids.
The time inventory consumed is $\frac{Q_o}{E}$ in the real-time model and $\frac{Q+Q_S}{E}$ in the re-order point model. By using these equations, the amounts of inventory at $L_0$, which are $Q_{O_1}$ and $Q_{O_2}$, are given below. $C_T$ and $C_A$ are calculated regarding to $Q_{O_1}$ and $Q_{O_2}$. $Q_{O_1}$ is used for calculating average inventory holding cost of real-time inventory model.

$$Q_{O_1} = E\left(\frac{q}{E} - L_0\right), \quad C_T = L_0 \cdot c + \frac{(Q+Q_{O_1})C_0L_0}{2} = \Rightarrow \quad A = C_c + \frac{(Q+Q_{O_1})C_0}{2} \quad (3)$$

Inventory amount of re-order point model $Q_{O_2}$ at $L_o$ is used for calculation of $C_T$ and $C_A$. Equations are given below.

$$Q_{O_2} = E\left(\frac{Q+Q_S}{E} - L\right), \quad C_T = L \cdot c + \frac{(Q+Q_{O_2})C_0L}{2} = \Rightarrow \quad A = C_c + \frac{(Q+Q_S+Q_{O_2})C_0}{2} \quad (4)$$

With respect to obtained $C_A$ equations of both models, $Q_{O_1}$ and $Q_S + Q_{O_2}$ are enough to be compared to understand the difference of models.

$$Q_{O_1} = E\left(\frac{q}{E} - L_0\right) = Q - EL_0, \quad Q_{O_2} = E\left(\frac{q+Q_S}{E} - L\right) = Q + Q_S - EL \quad (5)$$

$$Q_{O_1} \leq Q_S + Q_{O_2} \Rightarrow Q - EL_0 \leq Q + 2Q_S - EL \Rightarrow EL_0 \geq 2Q_S + EL \quad (6)$$

$EL$ is the amount of inventory consumed during $L$. It can be calculated as $EL = EL_0 + E(L - L_0)$ in the same way to compare both models. Thus, the relationship between $EL_0$ and $Q_S$ is shown below.

$$EL_0 \geq 2Q_S + EL_0 + E(L - L_0) \Rightarrow 2Q_S \geq E(L - L_0) \quad (7)$$

Multiplication of $L - L_0$ and $E$ is equal to safety stock. Because actual consumption speed is slower than theoretical consumption speed in second case, $Q_S > \frac{E(L - L_0)}{2}$ is correct. Therefore, the real-time inventory model is superior to the re-order point.

If actual consumption speed is higher than theoretical consumption speed, there will be a stock out, as shown in Figure 3. Actual consumption speed must be calculated by using the time of inventory is depleted ($L_0$) to compare the amount of stock out ($Q_R$) in both models.
Figure 3: Reorder time and reorder point (the third case)

The equation below has been formulated to compare both models in terms of consumption speed \( E \), regarding to \( L_E \) and amount of inventory.

\[
E = \frac{Q}{L_{E1}} = \frac{Q + Q_S}{L_{E2}} \tag{8}
\]

Equation 1 and equation 2 show that the average inventory holding cost is lesser in real-time model than re-order point model during the consumption period. Another situation occurred in the third case is stock out, which should be investigated. The amount of stock out is represented as \( Q_{R_1} \) in the real-time model and \( Q_{R_2} \) in the current model.

\[
Q_{R_1} = \frac{Q}{L_{E1}} (L_O - L_{E1}) \Rightarrow Q_{R_1} = \frac{QL_O}{L_{E1}} - Q \tag{9}
\]

\[
Q_{R_2} = \frac{Q + Q_S}{L_{E2}} (L - L_{E2}) \Rightarrow Q_{R_2} = \frac{(Q + Q_S)L}{L_{E2}} - (Q + Q_S) \tag{10}
\]

The equation below has been formulated to compare both models in terms of consumption speed \( E \).

\[
\frac{QL_O}{L_{E1}} - Q ? \frac{(Q + Q_S)L}{L_{E2}} - (Q + Q_S) \Rightarrow \frac{QL_O}{L_{E1}} + Q_S ? \frac{(Q + Q_S)L}{L_{E2}} \tag{11}
\]

\[
EL_O + Q_S ? EL \Rightarrow Q_S ? E(L - L_O) \tag{12}
\]

\[
Q_S = E(L_{E2} - L_{E1}) \Rightarrow Q_S = \frac{(L_{E2} - L_{E1})Q}{L_{E1}} \tag{13}
\]

\( L - L_O \) is multiplied with a number greater than theoretical consumption speed. If it were multiplied with theoretical consumption speed, the result would be equal to safety inventory \( Q_S \). Because it is multiplied with a larger number, \( Q_S < E(L - L_O) \) is correct. In this case, the real-time model is superior to the re-order point.

4 CALCULATION OF REORDER TIME

The real-time inventory model relies on predicting the depletion time of inventory considering consumption speed and variance of orders. Variance of demand is attempted to be tolerated by changing ROT in the real-time model instead of by using safety stock, as in the current model.
calculation of ROT is similar to the calculation of the amount of safety stock, but the ROT calculation searches for time instead of amount. \( Q_C \) is consumed inventory at the moment a transaction occurs, remaining inventory is \( Q_0 \), and the remaining time to order is \( L_C \) as shown in Figure 4. In addition to these parameters, the time at inventory consumed \( (L_D) \) is calculated with variance of orders in the current cycle \( (\sigma_C) \), average demand \( (D) \), and variance of demands \( (\sigma) \).

\[
\frac{Q_C - DL_C}{\sigma_C} = \frac{Q_0 - DL_0}{\sigma_0} \quad (14)
\]

\( DL_C \) and \( DL_0 \) are consumed inventory during \( L_C \) and \( L_0 \). Because \( \sigma_C \) is not known, the \( \sigma_C = \sigma \sqrt{L_C} \) equation is used to calculate \( L_0 \). When we expand equation 14, the equation given below is obtained.

\[
Q_0 \sigma_C + DL_C \sigma_0 - DL_0 \sigma_C = Q_C \sigma_0 \Rightarrow DL_C \sigma_0 + \sigma \sqrt{L_C} (Q_0 - DL_0) = Q_C \sigma_0 \quad (15)
\]

When \( \sqrt{L_C} \) is considered as \( x \), equation 15 can be formed as \( ax^2 + bx + c = 0 \). The root of the equation is given below.

\[
L_C = \left( -\frac{a(Q_0 - DL_0) \pm \sqrt{(a(Q_0 - DL_0))^2 + 4DQ_C \sigma_0^2}}{2D \sigma_0} \right)^2 \quad (16)
\]

The minimum positive value of \( L_C \) is the time when inventory is consumed. ROT is calculated by subtracting lead-time from \( L_C \) and ROT is calculated under variable orders and constant lead-time.

When lead-time \( (L) \) is variable, \( \sqrt{L^2 + D^2S^2_0} \) is used instead of \( \sigma \sqrt{L_C} \) in equation 15 for calculating \( L_C \) which includes variance of lead-time \( (S_0) \). The new \( L_C \) is calculated as shown below.

\[
L_C = \frac{Q_C \sigma_0 - \sqrt{L^2 + D^2S^2_0(Q_0 - DL_0)}}{D \sigma_0} \quad (17)
\]
ROT is calculated by subtracting average lead-time from $L_C$. When ROT is calculated, two failures related to variance of demand may be faced, namely stock out and holding extra inventory. The cause of these failures is that the average demand and variance ($\sigma_C$) of demand ($D_C$) in $L_C$ are unknown and predicted using $D$ and $\sigma$. Information sharing should be used to prevent these errors and to determine $D_C$ and $\sigma_C$.

5 SYSTEM TEST

A simulation experiment was designed to understand the characteristics of the real-time inventory model. The benchmark study of Harish C. Bahl and Neelam Bahl is reviewed to determine the context of simulation model, because benchmark studies are mostly available for lot-sizing rules but safety stock. (Bahl & Bahl, 2009) Supply lead-time is added as new parameter, and the range of other parameters are modified (mostly expanded). According to the study, the real-time inventory model was tested under variable demand, constant lead-time, constant inventory holding and ordering costs. The parameters of simulation are given below, in Table 1.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>RANGE</th>
<th>STEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE DEMAND</td>
<td>240-600</td>
<td>40</td>
</tr>
<tr>
<td>STANDART DEVIATION OF DEMANDS</td>
<td>10-240</td>
<td>10</td>
</tr>
<tr>
<td>SUPPLY LEAD-TIME (DAYS)</td>
<td>1, 2, 3, 4, 5, 7, 9, 11, 13, 15, 18, 21, 24, 27, 30, 35, 40, 45, 55</td>
<td>-</td>
</tr>
<tr>
<td>ORDERING COST</td>
<td>50-1000</td>
<td>50</td>
</tr>
<tr>
<td>INVENTORY HOLDING COST</td>
<td>1-5</td>
<td>1</td>
</tr>
</tbody>
</table>

Number of 501600 simulation experiments are run to compare both models regarding the range of given parameters. In all experiments, the amount of supply order is calculated as satisfying the demand during the lead-time in real-time model. It is calculated as handling safety stock during the lead-time in re-order point model. Any lot sizing models is not used in simulation experiment for both inventory models. All experiments are run for 1000 days. Later on, the difference between real-time model and re-order point model is analysed according to the results of simulation experiments. As a result of simulation experiment, 160006 experiments are convenient regarding the service level. Others will be investigated in a future study, and the reason of stock-out will be considered in the aspect of lot sizing and the simulation method. Our simulation model is a discrete-event simulation, timer of simulation is increased daily. A real-time simulation model should be designed for comparison that is more precise.

The average of difference between total costs, and the average of difference between held inventory amounts of both models are compared regarding some parameters (average demand, supply lead-time, and standard deviation of demand), as presented in Figure 5. The difference represents the exclusion of real-time data from re-order point data. More than expected stock-out is occurred in others, hence we analysed results of the convenient experiments. While variation of demand is getting higher, the real-time model provides mostly a better result than re-order point. Although the amount of average demand has a negative influence on the amount of held inventory, real-time model is better
on all cases. Increment of the supply lead-time increases the performance of real-time model, because re-order point model requires more safety stock regarding the lead-time. The cost of ordering has influence on the total cost, but, it has no influence on the difference between real-time and re-order point models. The inventory holding cost has no influence on the difference between amounts of held inventory, but, in this case, it has linear influence on the total cost.

![Figure 5: Difference Between Re-Order Point Model and Real-Time Model](image)

6 CONCLUSION

The real-time inventory model aims to decrease the amount of inventory held and the system’s response time to unexpected situations. The difference between the real-time inventory model and the re-order point lies in the true time when orders must be given. Most real-time planning studies in scientific literature use real-time data in current planning models. The novel aspect of this study is the updating of inventory model as real-time model working with real-time data to determine true time. By this means, safety stock and additional costs caused to fulfil the variance of demand are eliminated. The time (point in the re-order point model) at the order given is changed according to variance and actual variance of demand in the period, which provide the elimination of safety stock.

Some mathematical proofs have been presented to compare real-time and current inventory models, but a simulation experiment was carried out to understand the characteristics of the real-time inventory model and to compare it to the re-order point inventory model. Effects of standard deviation of orders, supply lead-time, average demand, ordering cost and inventory carrying cost are investigated. Results of the investigation are discussed in “System Test” section. The simulation
experiment confirmed that the real-time model decreases amount of inventory-holding cost. However, the real-time model caused an increment of the stock out amount. Then we eliminated the data causing different service levels for both models. Number of data is decreased to 160006 in which the ratio of stock out is 0,006 for real-time model, and 0,004. The two-sample t-test is applied to total cost data in the data set. The null hypothesis is determined as $\mu_{\text{rop}} = \mu_{\text{real-time}}$, and alternative hypothesis is determined as $\mu_{\text{rop}} > \mu_{\text{real-time}}$ under 99% significance level. Because p-value is $1.5 \times 10^{-25}$ alternative hypothesis is accepted, so real-time model is better under same service level. As the result of experiments, it is 4% better, yet the other data in which ratio of stock out is increased for real-time model should be analysed. The increment will be investigated as the aspect of lot sizing, and the influences of lot-sizing rules to the real-time model is analysed by a real-time simulation model to develop a real-time lot-sizing rule as an optimal strategy in the future study.

7 REFERENCES


• Re-order point model (amount driven) transformed into a time driven model, which is a real-time inventory model.
• Developed real-time inventory model eliminates the safety stock.
• Some formula is proven considering the uncertainty for calculating re-order time.
• A simulation experiment is arranged to determine the characteristics of real-time inventory model, and both models are compared.