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Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized Internal Model Control approaches

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ABSTRACT

In this paper, a two-degrees-of-freedom Internal Model Control structure is incorporated in production inventory control for a supply chain system. This scheme presents an intuitive and simple parametrization of controllers, where inventory target tracking and disturbance (demand) rejection in the inventory level problems are treated separately. Moreover, considering that the lead times are known, this scheme presents a perfect compensation of the delay making the stabilization problem easier to handle. This control structure is formulated for a serial supply chain in two ways (by using a centralized and a decentralized control approach). The behavior of these inventory control strategies is analyzed in the entire supply chain. Analytical tuning rules for bullwhip effect avoidance are developed for both strategies. The results of controller evaluations demonstrate that centralized control approach enhances the behavior with respect to the inventory target tracking, demand rejection and bullwhip effect in the supply chain systems.

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1. Introduction

A common Supply Chain (SC) includes the necessary entities to provide the customer with goods from production centers. There are a large number of participants, processes and randomness in the information flow of a supply chain. Therefore, the coordination of the supply chain becomes a key point in order to optimize the use of its resources and compete on a global scale. There are many aspects to look at in this complex network. One of these focuses on the improvement of inventory management policies. The aim of inventory management is to maintain the inventory level of each element of the supply chain in order to satisfy the demands of its customers. It is carried out ordering products from its immediate supplier of the supply chain. Thus, the supply chain is modelled as a serial process where each element orders goods to its immediate supplier. In this way, each echelon may obtain enough goods to supply the orders of its immediate customer of the chain.

Once an order is placed on the immediate supplier, there is a time to satisfy it; this is known as the replenishment lead time and consists of a time period ordering delay and a time period of physical production or distribution delay. Each participant of the supply chain stores the goods received from its immediate supplier which implies integrative dynamics. Moreover, since the entire supply chain works as a serial process whose elements are only related to its immediate downstream and upstream elements, this kind of processes can also be described as a Multiple Inputs–Multiple Outputs (MIMOs) system represented by a matrix with a block-diagonal structure.

Lead times and integrative dynamics make the inventory control task harder. Therefore, they play an important role in the design of the inventory replenishment strategies. Moreover, the MIMO nature of the supply chain system increases the complexity of the inventories control task and helps many undesirable effects appear when an inadequate inventory control policy is applied. Among these ones, instability represents the main problem, which implies that signals describing the inventory and orders can diverge as time goes on Hoberg et al. (2007).

Another inconvenient associated to an inadequate inventory policy is that the variability in the ordering patterns often increases as we move up into the chain, from the customer towards the suppliers and factory. This phenomenon is known as the *bullwhip effect*. Zhang and Burke (2011) investigate compound causes of the bullwhip effect by considering an inventory system with multiple price-sensitive demand streams.

Besides the stability and bullwhip effect issues, another major problem is the possible existence of an inventory deficit (difference between inventory target and actual inventory level), usually called inventory drift (Aggelogiannaki and Sarimveis, 2008).

In order to overcome these problems regarding production inventory and supply chain inventory management, replenishment policies based on process control theory have been successfully applied. Among them, Hoberg et al. (2007) apply linear control theory

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to study the effect of various inventory policies on order and inventory variability while their conditions for stability are examined by the Jury criteria. Dejonckheere et al. (2003), Disney and Towill (2003), and Hoberg et al. (2007), have analyzed the effect of the replenishment policies focused on the bullwhip effect estimation and suppression. Moreover, Lin et al. (2004) present approaches based on Control Engineering, including proportional–integral (PI) controllers and cascade control as inventory replenishment policies. Balan et al. (2009), Kristianto et al. (2011), and Deshpande et al. (2011) apply a soft computing approach (fuzzy logic based control) to generate inventory replenishment policies. The design of these controllers is also focused on the mitigation of the bullwhip effect. All these approaches present an acceptable inventory control performance with smooth information flow when is implemented in a single echelon. Nevertheless, the analysis of the inventory control performance of these inventory control policies on the entire supply chain is not taken into account.

Schwartz and Rivera (2010) introduce the two degrees of freedom feedback and three degrees of freedom feedback–feedfordward Internal Model Control as well as the model predictive control as a novel inventory replenishment policy in the supply chain. The simulation results of these schemes are compared with smoothing replenishment rule presented by Dejonckheere et al. (2003) showing an important improvement in the performance of manufacturing systems with a long lead time and significant uncertainty. However, analytical tuning to guarantee bullwhip effect avoidance is omitted. Moreover, this work is oriented to inventory control of a single echelon instead of a complete supply chain composed by multiple echelons.

Since the supply chain is naturally described as a multivariable system, the generalization of the inventory control strategies to the entire supply chain can yield insights to improve the inventory control in an overall scale. In this way, Yanfeng and Xiaopeng (2010) have analyzed the bullwhip effect in supply chain networks operated with linear and time-invariant inventory management policies. Perea-López et al. (2003) and Schwartz et al. (2006) propose the predictive control as replenishment inventory policy. These works show that inventory control in the entire supply chain is a current subject of research. Henceforth, in this paper we focus on the development of an inventory control strategy for the complete supply chain. The inventory control will be based on Internal Model Control (Schwartz and Rivera, 2010; Morari and Zafiriou, 1989). An advantage of the multi-degrees-of-freedom IMC topologies is that their performance to set point tracking (i.e. meeting an inventory target), measured disturbance rejection (i.e. meeting forecasted demand), unmeasured disturbance rejection (i.e. satisfying unforecasted demand) is improved by using three independent controllers avoiding trade-off between these problems. Moreover, the design guidelines for a single echelon can be extended to the design of the general controller of the MIMO system. Nevertheless, the guidelines to tune these controllers for the MIMO case so as to avoid the bullwhip effect have not been explored yet. Therefore, we advocate to design a multiple degrees-of-freedom IMC scheme (Schwartz and Rivera, 2010) for the entire supply chain (MIMO system) where the bullwhip effect is taken into account. Generally, since the demand is considered completely stochastic, a feedforward degree of freedom based in the forecast of the demand does not contribute an improvement in the behavior of the system respect to the feedback configuration (Schwartz and Rivera, 2010).

Therefore, in this work a two-degrees-of-freedom-feedback IMC scheme to tackle the set point tracking (i.e. meeting an inventory target) and unmeasured disturbance rejection (i.e., satisfying unforecasted demand) is performed for the entire supply chain.

There are two ways to perform an IMC based inventory control strategy for the entire supply chain: by using decentralized control, where an independent controller is applied to each echelon of the supply chain and by using centralized control, where a single controller is applied to the entire supply chain. The decentralized control approach is suitable for supply chains where its elements belong to different companies and do not share each others' information. On the other hand, when all or most of the supply chain elements belong to the same company or share internal information the centralized control approach would be applied.

In this work the two-degrees-of-freedom feedback IMC design for a complete supply chain is performed by applying both decentralized and centralized control strategies. Analytical guidelines to tune the controllers for bullwhip effect avoidance in the entire supply chain under centralized and decentralized inventory control strategies are also provided, which are not considered in previous works. Moreover, a comparison between the performance of both approaches is included and discussed.

The rest of the paper is formulated as follows: Section 2 presents the complete supply chain model using z-transform. As a result, a discrete Multiple Input–Multiple Output (MIMO) system is obtained. Section 3 presents the formulation of the Internal Model Control as a delay compensation scheme to control the inventory level in a single echelon of supply chains. After that, the generalization of the IMC scheme for an entire supply chain by using decentralized control strategy is presented. Section 4 presents the generalization of the IMC design for the entire supply chain with centralized control. The paper ends with the discussion and concluding remarks in Section 6.

2. Supply chain model

The model for a general supply chain is developed in this section. For the sake of simplicity, it is assumed a period base of time $T_m = 1$ which can be 1 day, 1 week or 1 month according to the dynamics of the supply chain. In this model there are N logistic echelons between the factory and the customer. The customer is considered the base while the factory is on the top of the supply chain. Thus, $j = 1, 2, \ldots, N$ (where N is a finite integer) denotes each one of the intermediate logistic echelons of the supply chain, while $j = 1$ represents the retailer, $j = N + 1$ represents the factory. According to this notation, $(j + 1)$ represents an immediate supplier and $(j - 1)$ represents an immediate customer of the jth echelon. A summarized list of variables is shown below:

- $\beta_{a,b}(t)$ denotes the amount of goods delivered by each logistic node a to the node b.
- $y_j(t)$ is the inventory level of the jth echelon at any discrete time instant $t = nT_m$ where n is a natural number.
- $o_{j,j+1}(t)$ represents the order placed by the jth echelon to its immediate supplier $j, j + 1$.
- $o_{j-1,j}(t)$ represents the order perceived by the jth echelon from its immediate downstream echelon $j - 1$, where $j - 1 > 0$.
- $d_j(t)$ is the demand perceived by the jth echelon from external customers.

Thus, the inventory balance in each echelon is given by the difference between the goods received from the immediate supplier and the goods delivered to the immediate customer as follows:

$$y_j(t) = y_j(t-1) + \beta_{j+1,j}(t) - \beta_{j,j-1}(t), \quad j = 1, 2, \ldots, N \tag{1}$$

A lead time $L_j \in \mathbb{N}$ is considered between the time when an order is placed by node jth and the time when the goods are received from the immediate supplier (Amini and Li, 2011; Aggelogiannaki and Sarimveis, 2008; Dejonckheere et al., 2003). It is also assumed that each node has enough existences to satisfy the demand of its immediate customer. In this way, the amount of goods ordered to an immediate supplier at time t will arrive at time $t+L_j$ i.e. $\beta_{j+1,j}(t) = o_{j,j+1}(t-L_j)$. Therefore, the sequence of events in the supply chain is the following:

i. At each discrete time t, the echelon jth receives the goods ordered L_j periods ago.
ii. The demand $d_j(t)$ is observed and satisfied immediately i.e. $\beta_{j,j-1}(t) = d_j(t)$ (i.e. there is no backlogged orders).
iii. The new inventory level of each echelon $y_j(t)$, is observed.
iv. Finally, an order $o_{j,j+1}(t)$ is placed on the $(j+1)$th level (upstream) according to the values of the inventory levels, $y_j(t)$. The order-up-to-level replenishment policy based on the Two-degrees-of-freedom feedback IMC scheme is stated in Section 3.

Thus, the Eq. (1) that relates the inventory balance with the demand $d_j(t)$ and order $o_{j,j+1}(t)$ at node j becomes now:

$$y_j(t) = y_j(t-1) + o_{j,j+1}(t-L_j) - d_j(t), \quad j = 1,2,\ldots,N \tag{2}$$

Eq. (2) is a difference equation which can be solved directly in the time domain or by using transformation techniques. In particular, the z-transform is the most extended one among transformations because it transforms Eq. (2) into an algebraic equation. Then, applying the time shifting property of the z-transform, $Z\{x[t-k]\} = z^{-k}Z\{x[t]\} = z^{-k}X(z)$ to Eq. (2), where k is a finite integer, Eq. (2) becomes:

$$y_j(z) = y_j(z)z^{-1} + o_{j,j+1}(z)z^{-L_j} - d_j(z), \quad j = 1,2,\ldots,N \tag{3}$$

Now, isolating $y_j(z)$ from (3) we get:

$$y_j(z) = \overbrace{\left[\frac{1}{1-z^{-1}}\right]z^{-L_j}}^{p_j(z)} o_{j,j+1}(z) - \overbrace{\left[\frac{1}{1-z^{-1}}\right]}^{p^m(z)} d_j(z) \quad j = 1,2,\ldots,N \tag{4}$$

which relates the z-transform of the inventory level, $y_j(z)$, with the order and the demand only. For IMC design, $p_j(z)$ must be factored into a minimum-phase portion:

$$p^m(z) = \frac{1}{1-z^{-1}} \tag{5}$$

and a portion $p_j^a(z)$ that includes the delays of the system (Morari and Zafiriou, 1989):

$$p_j^a(z) = z^{-L_j} \tag{6}$$

The model for an echelon presented in Eq. (4) is amenable to implement some controllers that exist in literature as is shown in the last works (Hoberg et al., 2007; Dejonckheere et al., 2003; Disney and Towill, 2003). Moreover, an equivalent model to Eq. (4), but in continuous time, is presented in Schwartz and Rivera (2010). However, these works only consider the inventory control of one echelon while in the present work the model is extended to the complete supply chain.

A model for the complete supply chain can be obtained considering that an order $o_{j-1,j}(z)$ generated by a downstream echelon $j-1$ is perceived and supplied by the immediate supplier j. In this way, the multivariable model described by Eq. (7) is obtained:

$$\begin{aligned}
y_1(z) &= \tfrac{z^{-L_1}}{1-z^{-1}} o_{1,2}(z) - \tfrac{1}{1-z^{-1}} d_1(z) \\
y_2(z) &= \tfrac{z^{-L_2}}{1-z^{-1}} o_{2,3}(z) - \tfrac{1}{1-z^{-1}} o_{1,2}(z) - \tfrac{1}{1-z^{-1}} d_2(z) \\
&\vdots \qquad\qquad \vdots \qquad\qquad \forall j = 1,2,\ldots,N \\
y_j(z) &= \tfrac{z^{-L_j}}{1-z^{-1}} o_{j,j+1}(z) - \tfrac{1}{1-z^{-1}} o_{j-1,j}(z) - \tfrac{1}{1-z^{-1}} d_j(z)
\end{aligned} \tag{7}$$

Remark 1. In the decentralized control strategy all echelons take independent decisions and there is no information sharing. Therefore, each echelon must consider this input as a disturbance. Since in the centralized control strategy a single controller generates all orders of the supply chain, this input becomes a control action.

The model expressed in Eq. (7) is a linear system of equations that can be represented in a matrix form. Let the vector $\mathbf{Y}(z) = [y_1(z), y_2(z), \ldots, y_N(z)]^T$ represent the set of inventories, which are the controlled variables, and $\mathbf{O}(z) = [o_{1,2}(z), o_{2,3}(z), \ldots, o_{N,N+1}(z)]^T$ represent the vector of orders, which are the manipulated variables of the supply chain. Finally, the unknown demand signals perceived by each echelon are represented by the vector $\mathbf{D}(z) = [d_1(z), d_2(z), \ldots, d_N(z)]^T$.

Thus, the complete supply chain is modelled by the matrix Eq. (8):

$$\mathbf{Y}(z) = \mathbf{P}(z)\mathbf{O}(z) - \mathbf{P}^d(z)\mathbf{D}(z) \tag{8}$$

where the transfer function matrix that relates the set of inventories $\mathbf{Y}(z)$ with the orders vector $\mathbf{O}(z)$ is given by:

$$\mathbf{P}(z) = \begin{pmatrix} p^m(z)p_1^a(z) & 0 & 0 & \cdots & 0 \\ -p^m(z) & p^m(z)p_2^a(z) & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -p^m(z) & p^m(z)p_N^a(z) \end{pmatrix} \tag{9}$$

and the transfer function matrix that relates the set of inventories $\mathbf{Y}(z)$ with the set of demands $\mathbf{D}(z)$ is represented by:

$$\mathbf{P}^d(z) = p^m(z)\mathbf{I} \tag{10}$$

where $\mathbf{I}$ is the identity matrix.

For IMC design, $\mathbf{P}(z)$ must be factored into a portion $\mathbf{P}^A(z)$ that includes the delays of the system (Morari and Zafiriou, 1989):

$$\mathbf{P}^A(z) = \begin{pmatrix} p_1^a(z) & 0 & 0 & \cdots & 0 \\ p_2^a(z)-1 & p_2^a(z) & 0 & \ddots & \vdots \\ p_3^a(z)-1 & p_3^a(z)-1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ p_N^a(z)-1 & \cdots & p_N^a(z)-1 & p_N^a(z)-1 & p_N^a(z) \end{pmatrix} \tag{11}$$

and a minimum-phase portion given by:

$$\mathbf{P}^M(z) = p^m(z)\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix} \tag{12}$$

such that

$$\mathbf{P}(z) = \mathbf{P}^A(z)\mathbf{P}^M(z) \tag{13}$$

For control purposes, the following assumption is made in the rest of the paper:

Assumption 1. The rational part of the system, i.e. Eq. (12), and all the delays between each pair output/input, i.e. Eq. (11) are known. □

Since the rational part of the system describes the balance of material carried out in each echelon and the lead times can be determined by (on-line) identification algorithms (Aggelogiannaki and Sarimveis, 2008; Garcia et al., 2012), the Assumption 1 is feasible in practice for the inventory control problem in the supply chain.

The objective of this work is to design a decentralized IMC control using the model described by Eq. (4), and a centralized IMC control using the MIMO model represented by Eqs. (8)–(10) for a complete supply chain in order to compare the two approaches.

There are two goals for this control system: The inventory target tracking and the demand rejection (i.e., the changes in the demand should not affect the inventory tracking). Moreover, the control system must satisfy these objectives avoiding aggressiveness in the orders (bullwhip effect).

Therefore, in the following section a two degrees-of-freedom Internal Model Control (IMC) structure (Schwartz et al., 2006; Schwartz and Rivera, 2010) for inventory control in a single echelon of the supply chain is formulated in discrete-time. After that, the design is extended to the complete supply chain. Moreover, novel guidelines for the controllers design and bullwhip effect formulation for an entire supply chain not taken into account in previous works are presented.

3. Decentralized control based in the two-degrees-of-freedom feedback IMC

The Two-degrees-of-freedom feedback IMC is shown in Fig. 1, where $r_j(z)$ denotes the inventory target for the control system of each echelon, $q_j^t(z)$ and $q_j^d(z)$ represent the two feedback controllers of the scheme, $p_j(z) = p^m(z)p_j^a(z)$ is the actual dynamics of the supply chain and $\hat{p}_j(z) = p^m(z)\hat{p}_j^a(z)$ represents the nominal model of the system. Each echelon may perceive demand from an external customer of the supply chain $d_j(z)$ and orders from the downstream echelon of the supply chain $o_{j-1,j}(z)$ as is shown in Fig. 1. Since in the decentralized control approach each echelon has no control on the downstream orders, $o_{j-1,j}(z)$ are added to $d_j(z)$ in a single disturbance input $\upsilon_j^m(z) = (o_{j-1,j}(z) + d_j(z))$. Within this structure, the problems of inventory target tracking (Inventory target tracking) and disturbance rejection (demand rejection) can be tackled by separate controllers as it will pointed out.

When the model is exact, $p_j(z) = \hat{p}_j(z)$, (i.e. under Assumption 1), the lead time becomes external in closed-loop. Under these circumstances, the scheme compensates the delay and makes the control problem easier. In order to point out this property, the equation of inventory balance for a single echelon j under this scheme is obtained and represented by Eq. (14):

$$y_j(z) = \left\{\frac{p^m(z)p_j^a(z)q_j^t(z)}{1 + p^m(z)\left[p_j^a(z) - \hat{p}_j^a(z)\right]q_j^d(z)}\right\}r_j(z) - \left\{1 - \frac{p^m(z)p_j^a(z)q_j^d(z)}{1 + p^m(z)\left[p_j^a(z) - \hat{p}_j^a(z)\right]q_j^d(z)}\right\}p^d(z)\upsilon_j^m(z) \quad j = 1, 2, \ldots, N \tag{14}$$

It can be seen in Eq. (14), that if the lead time model is known $p_j^a(z) = \hat{p}_j^a(z)$ (i.e. under Assumption 1), we get:

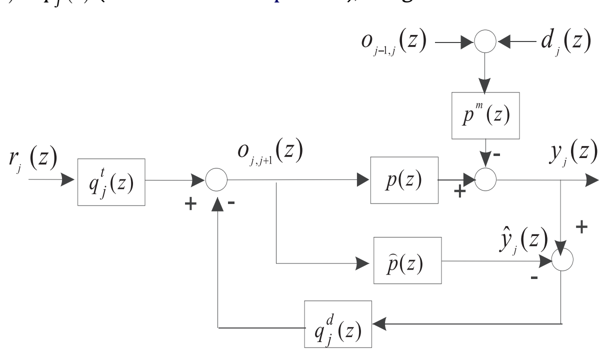

Fig. 1. Two-degrees-of-freedom-feedback IMC scheme.

$$y_j(z) = q_j^t(z)p^m(z)p_j^a(z)r_j(z) - \left(1 - p^m(z)p_j^a(z)q^d(z)\right)p^d(z)\upsilon_j^m(z) \quad j = 1, 2, \ldots, N \tag{15}$$

Eq. (15) shows that the lead-time has disappeared from the denominator of Eq. (14), converting the time-delay into external in closed-loop.

The main objective of the control system is to avoid the error between the inventory target and the inventory level, i.e. $r_j(z) - y_j(z) = 0$. Therefore, this error for the two degree of freedom IMC control system when $p_j^a(z) = \hat{p}_j^a(z)$ is calculated as follows:

$$e_j(z) = r_j(z) - y_j(z) = [1 - p^m(z)p_j^a(z)q_j^t(z)]r_j(z) - \left[1 - p^m(z)p_j^a(z)q_j^d(z)\right]p^d(z)\upsilon_j^m(z) \quad j = 1, 2, \ldots, N \tag{16}$$

Thereby, Eq. (16) shows that the $q_j^t(z)$ controller only affects the transfer function that relates the inventory target $r_j(z)$ with the error $e_j(z)$ (the first term of Eq. (16)). Then, the design of this controller is oriented to minimize this term (inventory target tracking). Similarly, $q_j^d(z)$ only affects the transfer function that relates the disturbance signal $\upsilon_j^m(z)$ with the inventory level (the second term of Eq. (16)). Therefore, these two controllers can be designed separately since both of them have a distinctive use and influence on the overall closed loop response:

- $q_j^t(z)$ is an IMC controller designed for inventory target tracking.
- $q_j^d(z)$ is designed mainly to achieve the internal stability and to satisfy the disturbance rejection objective (rejection to the demand perceived by each echelon $\upsilon_j^m(z)$).

3.1. Bullwhip effect formulation (effect of the disturbance on the order signal)

Besides the equation of inventory balance, the relation between the demand perceived by the echelon and the generated orders must be taken into account in the control design, since this relation determines the well known bullwhip effect constraint. The bullwhip effect can be characterized as an amplification of demand fluctuations ($\upsilon_j^m(z)$ in the decentralized control case) as one move upwards in the supply chain. This propagation of demand fluctuations is only possible when every node has sufficient stock. If there are neither changes in the set point nor model mismatch, the relation between demand and orders to successive nodes under the two-degrees-of-freedom-feedback-IMC is given by:

$$o_{j,j+1}(z) = q_j^d(z)p^d(z)\upsilon_j^m(z) \tag{17}$$

and the ratio of orders to successive nodes can be expressed as:

$$|\gamma_j(z)| = \frac{|o_{j-1,j}(z)|}{\left|\upsilon_j^m(z)\right|} = \left|q_j^d(z)p_j^d(z)\right| = \left|q_j^d(e^{i\omega})p_j^d(e^{i\omega})\right| \quad \omega \in [0, 2\pi) \tag{18}$$

where i is the imaginary unity. Lin et al. (2004) have stated that the amplitude of demand fluctuations will not be amplified if

$$|\gamma_j(e^{j\omega})| \leqslant 1 \quad \forall\omega \in [0, 2\pi) \tag{19}$$

Notice from Eq. (18) that if there are neither changes in the inventory target nor model mismatch, the q_j^d controller must be designed such that the bullwhip effect restriction Eq. (19) is satisfied. The IMC controllers design is shown in Section 3.2.

3.2. Controllers design

Once the control system requirements are stated, the Two-degrees-of-freedom-feedback IMC design is oriented to satisfy them.

The IMC design for these controllers is comprised of the following two procedures.

3.2.1. Inventory target tracking design

For this control purpose $q^t(z)$ is designed for H_2-optimal set point tracking where the control policy is determined such that the sum of the square error

$$\|e\|_2^2 = \sum_{t=0}^{\infty} e^2(t) \tag{20}$$

is minimized.

The H_2-optimal problem for inventory tracking is formulated in Schwartz et al. (2006) and Schwartz and Rivera (2010) as:

$$min_{q_j^t(z)} \left\| \frac{y_j(z)}{r_j(z)} \right\|_2 min_{q_j^t(z)} \|[1 - \underbrace{p^m(z)p_j^a(z)}_{p_j(z)} q_j^t(z)]\|_2 \tag{21}$$

which is the first term of Eq. (16).

The IMC solution for this problem is given by Schwartz et al. (2006) and Schwartz and Rivera (2010):

$$\tilde{q}_j^t(z) = z(p^m(z)r_j(z))^{-1}\{z^{-1}(p_j^a(z))^{-1}r_j(z)\}_* \tag{22}$$

where the $\{\cdot\}_*$ operator denotes that after a partial fraction expansion of the operand $\{\cdot\}$, all the terms involving the poles of $(p_j^a(z))^{-1}$ are omitted. Assuming a step change in the inventory target $r_j(z)$, the optimal controller obtained by applying this procedure is Schwartz et al. (2006) and Schwartz and Rivera (2010):

$$\tilde{q}_j^t(z) = (p^m)^{-1} \tag{23}$$

This controller provides a well inventory target tracking but as a result, orders are aggressive, which is unacceptable for factory managers. Therefore, the optimal controller is augmented with a low-pass filter in order to detune this optimal performance of the controller by a parameter $\lambda^t \in [0,1)$. In counteraction, this filter avoids the aggressive orders. Since step changes in the inventory target are considered, the optimal controller $\tilde{q}_j^t(z)$ for inventory target tracking at each echelon, j, is augmented with a type-1 filter (Morari and Zafiriou, 1989) defined as:

$$f_j^t(z) = \frac{(1-\lambda_j^t)z}{z-\lambda_j^t} \tag{24}$$

The final controller is given by:

$$q_j^t(z) = \tilde{q}_j^t(z) f_j^t(z) \tag{25}$$

3.2.2. Controller design to step disturbance rejection in the inventory signal

In this control problem, the $q_j^d(z)$ controller is designed specifically to provide a fast response of inventory level to step demand changes (abrupt changes in the demand). As a result of the integrative nature of the disturbance model $p^m(z)$ (Eq. (4)), a step change in demand becomes a Type-2(ramp) disturbance. Therefore the design procedure relies on solving the H_2-optimal control given by

$$min_{q_j^d(z)} \left\| \frac{y_j(z)}{\upsilon_j^m(z)} \right\|_2 = min_{q_j^d(z)} \|[1 - \underbrace{p^m(z)p_j^a(z)}_{p_j(z)} q_j^d(z)]p^m(z)\|_2 \tag{26}$$

which is related with the second term of Eq. (16) where $\upsilon_j^m(z)$ is consider as an step signal. For disturbance rejection, the optimal controller is generally calculated as (Morari and Zafiriou, 1989):

$$\tilde{q}_j^d(z) = z(p^m(z)\upsilon_j^m(z))^{-1}\{z^{-1}(p_j^a(z))^{-1}(z)\upsilon_j^m(z)\}_* \tag{27}$$

The optimal IMC controller obtained for a ramp disturbance rejection is:

$$\tilde{q}_j^d(z) = (p^m(z))^{-1}\frac{(L_j+1)z - L_j}{z} \tag{28}$$

Notice that Eqs. (18) and (19) imply that a good bullwhip effect avoidance needs $|p_j(z)q_j^d(z)| \ll 1$ while Eq. (26) requires $|p_j(z)q_j^d(z)| \approx 1$ for step disturbance rejection in the inventory signal. Thus, the $q_j^d(z)$ controller must be designed taking into account two opposite objectives: step disturbance rejection in the inventory level and bullwhip effect avoidance. Therefore, an analytical detuning of the $\tilde{q}_j^d(z)$ optimal controller to obtain a trade-off between these two objectives is performed below. This analytical detuning for IMC controllers for bullwhip effect is not explored in previous works (Schwartz et al., 2006; Schwartz and Rivera, 2010).

- *Detuning of $q_j^d(z)$ for bullwhip effect avoidance*

In the IMC formulation for control systems, $q_j^d(z)$ is augmented with low-pass filter $f_j^d(z)$ to detune the nominal performance in order to satisfy a grade of stability robustness to uncertainty in the plant. However, in the supply chain case, provided that Assumption 1 holds, these filters will be used instead to counteract the bullwhip effect. Thereby, the nominal performance to an step change in the demand is deteriorated but, in exchange, high component frequencies of the demand are rejected in order to satisfy a grade of bullwhip effect avoidance i.e. to satisfy Eq. (19). In the disturbance rejection case, a generalized Type-2 filter is used to guarantee no asymptotic offset for both step and ramp disturbances. Moreover, two type-2 filters connected in series will be used generating a filter of order 4:

$$f_j^d(z) = \frac{((\alpha_j^1 z - \alpha_j^2)(1-\lambda_j^d)z)^2}{(z-\lambda_j^d)^4} \tag{29}$$

Thus, the final controller is given by:

$$q_j^d(z) = \tilde{q}_j^d(z) f_j^d(z) = (p^m(z))^{-1}\frac{((L_j+1)z - L_j)\left(\left(\alpha_j^1 z - \alpha_j^2\right)\left(1-\lambda_j^d\right)z\right)^2}{z\left(z-\lambda_j^d\right)^4} \tag{30}$$

Thereby, the bullwhip restriction for the two-degrees-of-freedom-feedback IMC scheme is given by:

$$|\gamma_j(z)| = \left| \frac{((L_j+1)z - L_j)\left(\left(\alpha_j^1 z - \alpha_j^2\right)\left(1-\lambda_j^d\right)z\right)^2}{z\left(z-\lambda_j^d\right)^4} \right| \leqslant 1 \tag{31}$$

Notice that, the bullwhip effect depends on the lead time L_j and the $q_j^d(z)$ parameters (λ_j^d, α_j^1 and α_j^2). Since the λ_j^d parameter modifies the bandwidth, this is selected so as to satisfy the bullwhip effect condition for a determined L_j value while the parameters α_j^1 and α_j^2 are adjusted to guarantee internal stability for this λ_j^d value. The $q_j^d(z)$ controller must be tuned such that the system has a fast response to low frequency demand changes. Thereby, the inventory level can be maintained. On the other hand, this controller must limit the ratio of orders less than 1 at high frequency to guarantee bullwhip effect mitigation.

In this way, Lin et al. (2004) have suggested to consider the following two factors on the magnitude ratio $|\gamma_j(z)|$.

1. Bandwidth: the frequency at which the magnitude ratio (Eq. (31)) is reduced to below 0.7. A wide bandwidth indicates a faster response but poorer bullwhip mitigation. Note that we are

dealing with a discrete-time system. Therefore, the highest frequency is at $\omega = \pi/T_m = \pi$ since $T_m = 1$. Thus, we can define a term γ_j^{π} as the magnitude ratio given by Eq. (31) at $\omega = \pi$ i.e. $\gamma_j^{\pi} = \gamma_j(\omega = \pi)$. Since a higher γ_j^{π} implies a wider bandwidth and a faster response, it results in more severe bullwhip.
2. Resonance peak (σ_j): the highest value of the amplitude ratio (Eq. (31)). A higher resonance peak indicates a fast response to low frequency demand changes (Disturbance rejection) but the closed-loop response may be more oscillatory. Suitable setting of σ_j ranges from 1.5 to 2.0.

The disadvantage of the controllers (PI and cascade PI) proposed in (Lin et al., 2004) is that there is no direct correspondence between the parameters of the controllers and the bandwidth of the magnitude ratio $|\gamma_j(z)|$. Therefore, that work performed an empirical tuning for bullwhip effect based on trial and error. In the Two-degrees-of-freedom feedback IMC scheme the bandwidth can be manipulated directly by using of the λ_j^d parameter of the $q_j^d(z)$ controller. Therefore, the application of this analytical tuning criterion for bullwhip effect avoidance is simplified. The application of this tuning criterion for the design of the IMC controllers is also novel in supply chain systems.

Fig. 2 shows a tuning example for an echelon with $L = 3$. In this figure, the magnitude ratio $|\gamma_j(z)|$ for several values of λ_j^d is plotted. It can be seen that for λ_j^d values close to 1 the system present strong mitigation of high frequency but low resonance peak σ_j. That means a mitigation of the bullwhip effect but a sluggish response to low frequency demand changes. On the other hand, for λ_j^d values close to 0 the system present poor mitigation of high frequency (severe bullwhip effect) but faster response to low frequency demand changes or step changes (disturbance rejection).

Therefore, the following approximate tuning criterion suggested by (Lin et al., 2004) to find a trade-off between fast inventory tracking and bullwhip effect mitigation can be used:

Choose a controller setting with $\gamma_j^{\pi} < 1$ and σ_j in the range 1.5–2. There are several λ_j^d solutions based on this criteria as is shown in Fig. 2 in solid lines. Therefore, in this work, to perform the simulations we chose a λ_j^d such that $\gamma_j^{\pi} < 1$ and σ_j be close to 1.8. Fig. 3 extends this criteria for delays between 1 and 10 periods of time.

Since $q_j^d(z)$ must satisfy asymptotically tracking and internal stability, the filter has to be designed in such a way that all these requirements hold. Hence, once the λ_j^d parameter is selected for the above commented trade-off, the α_j^1 and α_j^1 parameters must be adjusted so as to make the filter satisfy inventory tracking and internal stability. For this system with a pole of multiplicity 1 at $z = 1$, the filter has to satisfy the following conditions at $z = 1$ (Morari and Zafiriou, 1989):

$$f_j^d(z) = 1, \quad \frac{df_j^d(z)}{dz} = 0 \tag{32}$$

Solving this system, we get a mathematical relation which relates the α_j^1, α_j^2 parameters with λ_j^d as:

$$\alpha_j^2 = 2\lambda_j^d, \quad \alpha_j^1 = 1 + \lambda_j^d \tag{33}$$

After formulating the two degrees of freedom IMC scheme for a particular jth echelon, the control system is now generalized to a entire supply chain in a decentralized control way.

A decentralized control means that a controller is designed for each echelon of the supply chain. The resulting diagonal controller matrix for inventory target tracking is given by

$$\mathbf{Q}^t(z) = diag(q_1^t(z), \quad q_2^t(z), \ldots, q_N^t(z)) \tag{34}$$

In the same way, the resulting diagonal controller matrix for disturbance rejection is given by Eq. (35). When the decentralized control strategy is used, the orders $o_{j-1,j}(z)$ and $d_j(z)$ are considered as the perturbation for the echelon j since each echelon is autonomous to take decisions and these informations are not shared with the rest of entities.

$$\mathbf{Q}^d(z) = diag(q_1^d(z), \quad q_2^d(z), \ldots, q_N^d(z)) \tag{35}$$

Each controller $q_j^t(z)$ and $q_j^d(z)$ is designed independently for each echelon using the guidelines formulated in subSection 3.2.

An alternative to the decentralized control strategy is a full centralized control approach where all information of the supply chain is taken into account. In this approach, the entire orders vector $O(z)$ is designed simultaneously. This formulation is developed in Section 4.

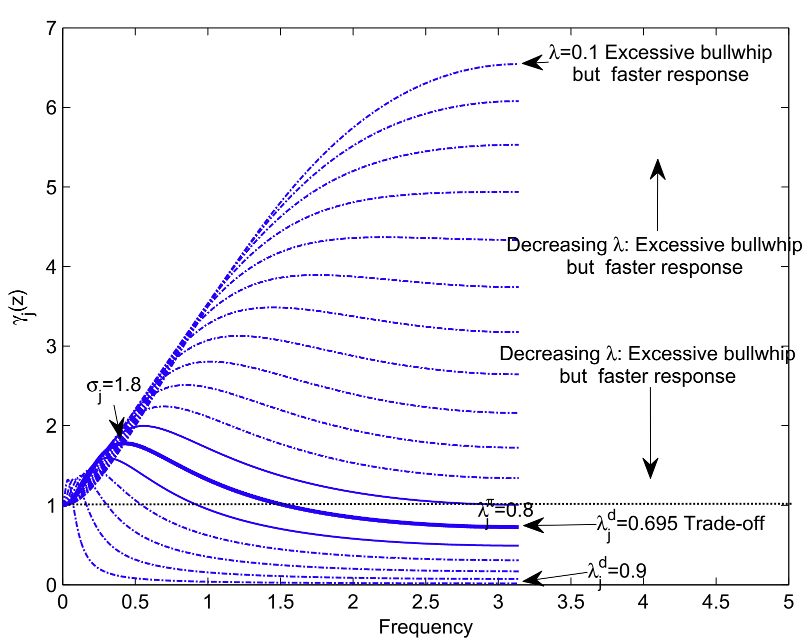

Fig. 2. Frequency response of $|\gamma_j(z)|$ with $L_j = 3$ for various λ_j^d values.

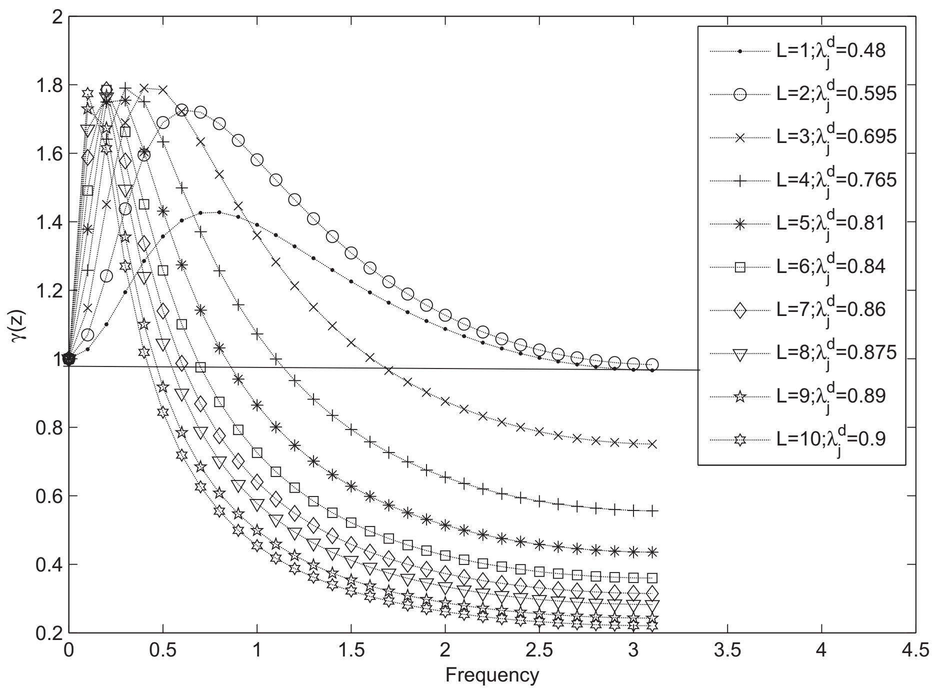

Fig. 3. Frequency response of $|\gamma_j(z)|$ for various L_j values.

4. Centralized control strategy

The centralized control design is based on the IMC scheme shown in Fig. 1 where the model formulated for the entire supply chain, Eqs. (11) and (10), is taken into account. Thus, the vector of inventories, $\mathbf{Y}(z)$, is given by:

$$\mathbf{Y}(z) = \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^t(z)\mathbf{R}(z) - (\mathbf{I} - \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^d(z))\mathbf{P}^d(z)\mathbf{D}(z) \quad (36)$$

In the centralized control strategy $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ can be designed for inventory target tracking and disturbance (demand) rejection respectively by using the IMC guidelines for multivariable (MIMO) systems.

4.1. Bullwhip effect formulation

The bullwhip effect formulation for a single echelon can be generalized for a centralized control considering the transfer function matrix that relates the orders vector $\mathbf{O}(z)$ with the demand vector $\mathbf{D}(z)$. Then, considering no changes in the set point and no model mismatch, the relation between the set of demands and the set of orders is given by:

$$\mathbf{O}(z) = \mathbf{P}^d(z)\mathbf{Q}^d(z)\mathbf{D}(z) \quad (37)$$

The generalization of the magnitude ratio of orders to successive nodes γ for a multivariable (MIMO) system under a centralized strategy can be expressed as:

$$|\Gamma(z)| = |\mathbf{P}^d(z)\mathbf{Q}^d(z)| = |\mathbf{P}^d(e^{i\omega})\mathbf{Q}^d(e^{i\omega})| \quad \omega \in [0, 2\pi) \quad (38)$$

where the magnitude ratio is calculated component-wise. Thus, the demand signals perceived in the supply chain will not be amplified if:

$$|\Gamma_{ij}(e^{i\omega})| \leqslant 1 \quad i = 1, 2, \ldots, N \quad j = 1, 2, \ldots, N \; \omega \in [0, 2\pi) \quad (39)$$

In this case, the bullwhip effect implies that each demand signal represented by d_j introduced in the system is not amplified to subsequents suppliers represented by $o_{j,j+1}(z)$. Therefore, the bullwhip effect can be analyzed component-wise since each component of Γ contains the relation between each pair $d_j(z), o_{j,j+1}(z)$. Thus, in the centralized control approach multiples demand signals are taken into account i.e. $d_j(z) \neq 0$ $j = 1, 2, \ldots, N$. After formulating the inventory control system in a centralized control way, the controller matrices $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ will be designed in subSection 4.2.

4.2. Controllers design

Notice from Eq. (36) that $\mathbf{Q}^t(z)$ only affects the relation between $\mathbf{Y}(z)$ and $\mathbf{R}(z)$ as well as $\mathbf{Q}^d(z)$ only affects the relation between $\mathbf{Y}(z)$ and $\mathbf{D}(z)$. Therefore these controllers can be designed separately. The design procedure is presented below:

4.2.1. Inventory target tracking design

For inventory target tracking $\mathbf{Q}^t(z)$ is designed to solve the H_2-optimal MIMO problem given by

$$min_{Q^t(z)} \| [\mathbf{I} - \mathbf{P}^A(z)\mathbf{P}^M(z)\mathbf{Q}^t(z)]\mathbf{R}(z) \|_2 \quad (40)$$

where the vector $\mathbf{R}(z)$ contains the set of inventory targets $r_j(z)$ for the entire supply chain. Assuming a step change in the inventory target, the optimal IMC controller is:

$$\widetilde{\mathbf{Q}}^t(z) = (p^m(z))^{-1} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & 1 & 1 \end{pmatrix} \quad (41)$$

The optimal controller $\widetilde{\mathbf{Q}}^t(z)$ for inventory target tracking must be enhanced with a low-pass filters bank in order to degrade the fast response of this controller to changes in the inventory targets obtaining less aggressive orders. Thus, $\widetilde{\mathbf{Q}}^t(z)$ is augmented with a low-pass filter bank given by:

$$\mathbf{F}^t(z) = Diag[f_1(z), \quad f_2(z), \ldots, f_N(z)] \quad (42)$$

where each one type 1 filter $f_j(z)$ appearing in Eq. (42) is defined by Eq. (24). Thus, the final controller is:

$$Q^t(z) = \widetilde{Q}^t(z)\mathbf{F}^t \quad (43)$$

4.2.2. Design for disturbance rejection

The two-degrees-of-freedom-IMC scheme allows us to specify the system response to demand changes by using the $\mathbf{Q}^d(z)$ controller. As a result of the integrative nature of the inventory process, a step change in demand becomes a Type-2(ramp) disturbance. Therefore the design procedure relies on solve the H_2-optimal control given by

$$min_{\mathbf{Q}^d(z)}\|[1-\mathbf{P}^M(z)\mathbf{P}^A(z)\mathbf{Q}^d(z)]\mathbf{P}^d(z)\mathbf{D}(z)\|_2 \quad (44)$$

The IMC controller that solve this problem for a ramp disturbance is:

$$\widetilde{\mathbf{Q}}^d(z)=\frac{1}{p^m}\begin{pmatrix} \frac{(1+L_1)z-(L_1)}{z} & 0 & \cdots & 0 \\ \frac{(1+L_1+L_2)z-(L_1+L_2)}{z} & \frac{(1+L_2)z-(L_2)}{z} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \frac{\left(1+\sum_1^N L_j\right)z-\left(\sum_1^N L_j\right)}{z} & \cdots & \frac{(1+L_{N-1}+L_N)z-(L_{N-1}+L_N)}{z} & \frac{(1+L_N)z-(L_N)}{z} \end{pmatrix} \quad (45)$$

Since the optimal controller for disturbance rejection yields strong variability in the orders, the bullwhip effect is also a restriction to be taken into account in the $Q^d(z)$ controller design. *Therefore, novel guidelines to detune $\widetilde{Q}^d(z)$ controller to satisfy bullwhip effect constraint are shown below.*

- *Detuning of $\widetilde{\mathbf{Q}}_j^d(z)$ for bullwhip effect avoidance*

Since each demand signal is related with each order, the detuning must be done component-wise with a low-pass filter. Each one of the low-pass filters appearing in Eq. (46) is defined by Eq. (29). Thus, the filter matrix is given by:

$$\mathbf{F}^d(z)=\begin{pmatrix} f_{1,1}^d(z) & 0 & \cdots & 0 \\ f_{2,1}^d(z) & f_{2,2}^d(z) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ f_{N,1}^d(z) & \cdots & f_{N,N-1}^d(z) & f_{N,N}^d(z) \end{pmatrix} \quad (46)$$

Therefore, the final controller can be obtained using the Schur product (or component-wise product), in the form:

$$\mathbf{Q}^d(z)=\widetilde{\mathbf{Q}}^d(z)\cdot\mathbf{F}^d \quad (47)$$

The tuning criterion applied in a single echelon in Section 3.2 is also considered in the centralized control, in this case component-wise. Thus, we chose λ_{ij}^d values such that each $\Gamma_{ij}(\omega=\pi)<1$ and σ_{ij} close to 1.8 which determines the bandwidth and resonance peak of each component of the magnitude ratio of orders respectively.

Fig. 4 presents an example of the tuning of the λ_{ij}^d for the complete supply chain according to this tuning criterion. Notice that each component of Fig. 4 represents the magnitude ratio of an order $o_{j,j+1}(z)$, $\forall j=1,2,\ldots,N$ respect each demand signal $d_j(z)$, $\forall j=1,2,\ldots,N$. Moreover, all components can be designed at once. In this case, the actual delays are also considered known and $\widehat{L}_1=L_1=3,\ \widehat{L}_2=L_2=3,\ \widehat{L}_3=L_3=3$.

Section 5 evaluates the basic supply chain with three echelons ($N=3$), under a decentralized and centralized control ways in order to show the behavior respect to the inventory target tracking, demand rejection and bullwhip effect avoidance.

5. Controller evaluation

Since the Two-degrees-of-freedom feedback IMC scheme decouples the inventory tracking from the demand rejection and bullwhip effect avoidance, the simulations are performed in two different subsections. SubSection 5.1 is oriented to evaluate the inventory target tracking and subSection 5.2 evaluates the rejection to demand and bullwhip effect mitigation. In this last situation, the behavior of the decentralized control approach is performed assuming no changes in the inventory tracking.

5.1. Evaluation of the inventory target tracking

In this case of study, the performance of the two-degrees-of-freedom-feedback-IMC to inventory tracking, under the decentralized and centralized control strategies is evaluated for a step change in the setpoints (Inventory targets). In the simulations, no customer demand is considered and the actual lead times knowledge is assumed, i.e. $\widehat{L}_1=L_1=3,\ \widehat{L}_2=L_2=3,\ \widehat{L}_3=L_3=3$. It is also considered an initial inventory of 100 unities in each echelon and suddenly a deterministic step change of 100 unities in each inventory target from $t=20$ and onwards is introduced.

For the decentralized and centralized control strategies the λ_j^t parameter can take values in the interval [0,1), where 0 implies that the optimal controllers behavior are not deteriorated while 1 correspond to the worst degradation case of this optimal controllers behavior. In this simulation, the inventory tracking performance in a supply chain composed by three echelons under both strategies is evaluated for $\lambda_j^t=0.2, 0.5$ and 0.8. In general, it can be seen in Figs. 5 and 6 that for values of λ_j^t close to zero, with both

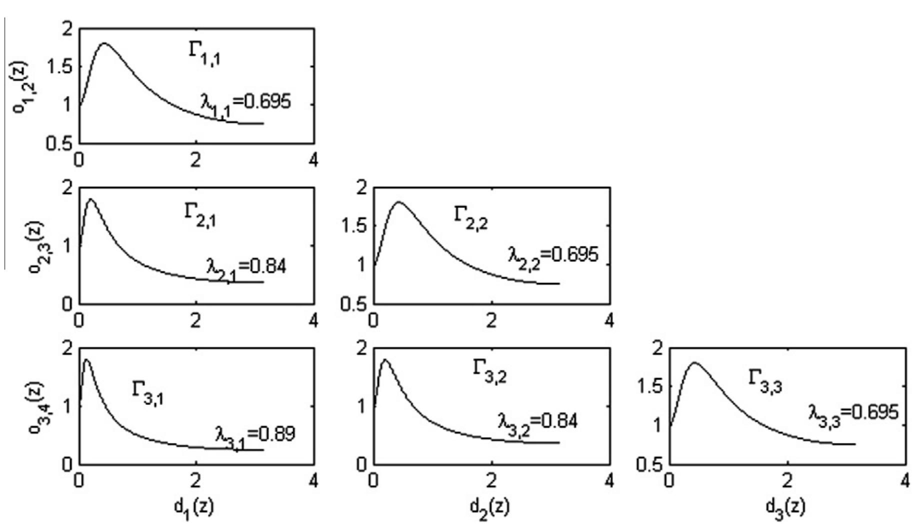

Fig. 4. Tuning of λ_{ij}^d for the entire supply chain.

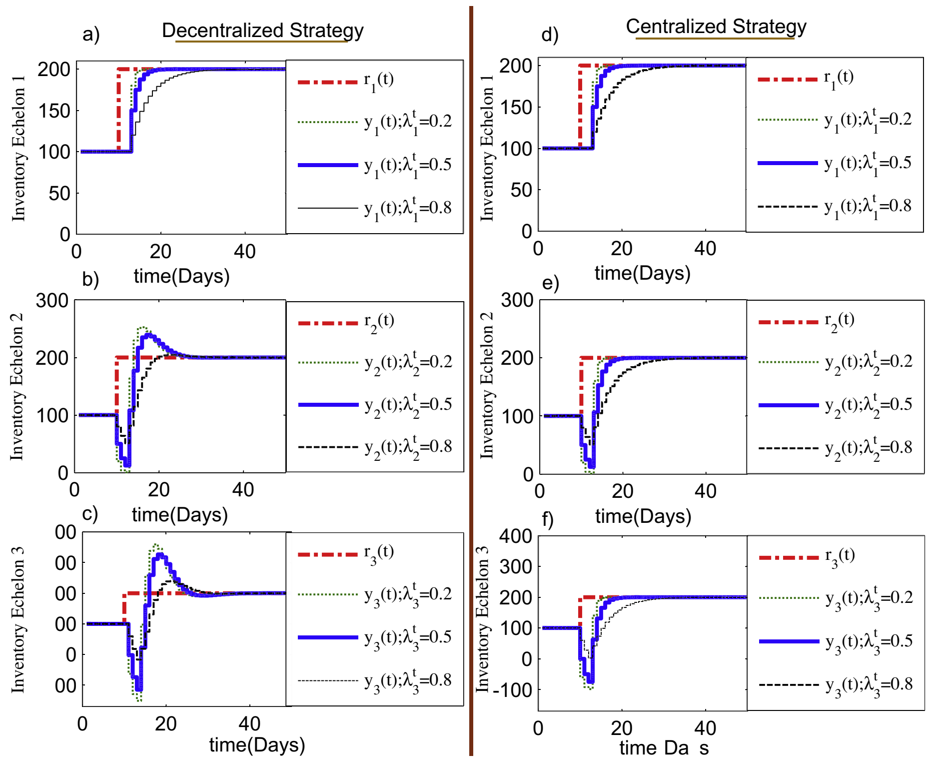

Fig. 5. Inventory responses to step changes in the inventory targets.

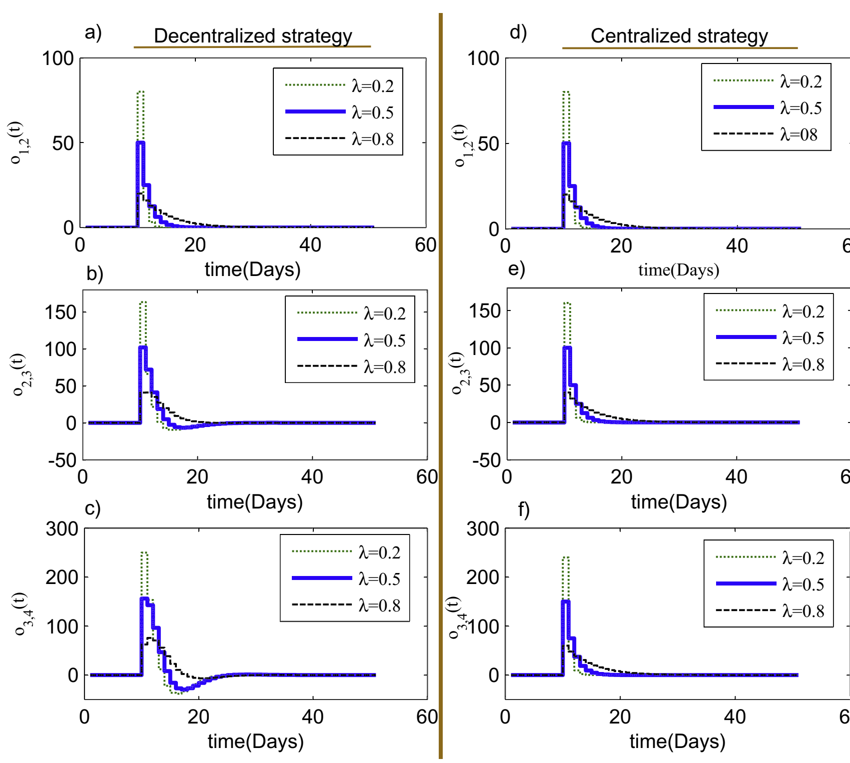

Fig. 6. Orders of the echelons in response to changes in the inventory target. (a) Orders of echelon 1. (b) Orders of echelon 2. (c) orders of echelon 3.

strategies, the first echelon presents a fast response to the changes in the setpoint but aggressive orders are generated, which is unacceptable for factory managers. For λ_j^t values close to 1 the system presents a slow response to the changes in the setpoint but the orders are smoothed. Therefore, a trade-off between these two behaviors is possible if the λ_j^t parameter is moved from zero to one.

In the second and third echelons, although the same $\lambda_j^t = 0.5, 0.5$ and 0.8 values are applied, the behavior is deteriorated significantly in the decentralized control strategy. That is, considerable overshoot in the inventory response to a setpoint change is presented as is shown in Fig. 5b and c. This is a result of the application of decentralized control strategy since the echelon controlled has no information of the dynamics of the chain. On the other hand, by modifying λ_j^t parameters of the controller $Q^t(z)$ from 0 to 1, the trade-off between a faster response of inventory to a step change in the inventory target and a smoothing of orders can be done for all supply chain. Fig. 5d–f shows that, the centralized control approach does not exhibit overshoot for any λ^t values when it is subjected to inventory target changes in the entire supply chain. Orders changes in response to inventory target changes can be less abrupt, consequently decreasing inventory holding costs, smoothing factory operations, and improving profitability.

In order to provide a quantitative evaluation of strategies performance to inventory tracking, the Integral Absolute Error (IAE) which is the cumulative difference between the controlled variable (inventory level) and its set point value (inventory target) is introduced below:

$$\mathrm{IAE} = \sum_{t=0}^{\infty}(r_j(t) - y_j(t)) \tag{48}$$

Values of IAE close to zero means improvement in the performance to inventory tracking. In this way, Table 1, contains the values of IAE for the decentralized and centralized control strategies with the $\lambda_j^t = 0.2, 0.5$ and 0.8 values. This table shows that the centralized control obtains lower values of IAE when a supply chain with more than 1 echelon is considered. These results confirm that the centralized control strategy improves the performance to inventory tracking in the entire supply chain.

Table 1
IAE for the decentralized and centralized control strategies.

Strategy	Decentralized			Centralized		
	$\lambda_j^t = 0.2$	$\lambda_j^t = 0.5$	$\lambda_j^t = 0.8$	$\lambda_j^t = 0.2$	$\lambda_j^t = 0.5$	$\lambda_j^t = 0.8$
Echelon 1	325	400	700	325	400	700
Echelon 2	1127	1164	1201	625	700	1000
Echelon 3	2136	2154	2034	925	1000	1300

5.2. Evaluation of disturbance (demand) rejection and bullwhip effect avoidance

As it is stated in Sections 3 and 4 the design of the controllers for disturbance rejection ($q_j^d(z)$ and $Q^d(z)$) are restricted by the bullwhip effect condition. In this work the IMC scheme uses the λ_j^d parameter to degrade the optimal disturbance rejection obtain a trade-off between the disturbance rejection and bullwhip effect mitigation objectives. In order to show the systems performance to disturbance rejection, a step change in the demand is applied from time instant $t = 20$ and onwards. On the other hand, for bullwhip effect evaluation, a stochastic variability in the customer demand $d_j(z)$ is applied to the systems from time instant $t = 60$ and onwards. The customer demand is formulated as a normal function, with a average equal to 20 and a variance equal to 1, i.e. $\in N(20, 1)$.

On the one hand, for the decentralized control strategy the $\lambda_j^d = 0.695$ value is used according to the bullwhip tuning proposed in Section 3.2. Figs. 7 and 8 show the system behavior to the disturbance input. Again, the analytical tuning works for the first echelon, as is shown in Figs. 7 and 8a since the bullwhip effect is mitigated and an acceptable inventory response to step change in the demand is obtained. For the subsequent echelons 2 and 3 the performance to disturbance rejection and bullwhip effect avoidance are deteriorated. Fig. 7b and c shows that an overshoot in the inventory level appears for the second echelon and onwards. The demand fluctuations in the echelons 2 and 3 are amplified (bullwhip effect) as is shown in Fig. 8b and c. Therefore, the application of the detuning for bullwhip effect mitigation in each particular echelon proposed in Section 3 is inefficient to mitigate the bullwhip effect in the entire supply chain with a considerable number of echelons. This behavior is result of the application of decentralized control strategy. An alternative to solve this problem

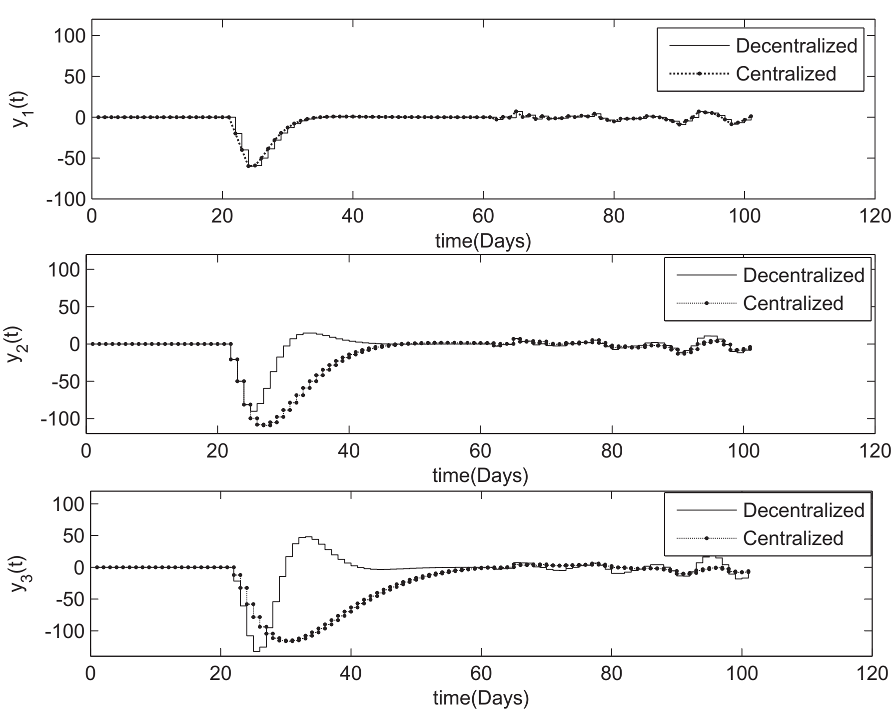

Fig. 7. Inventory behavior to changes in the demand assuming no changes in the inventory target.

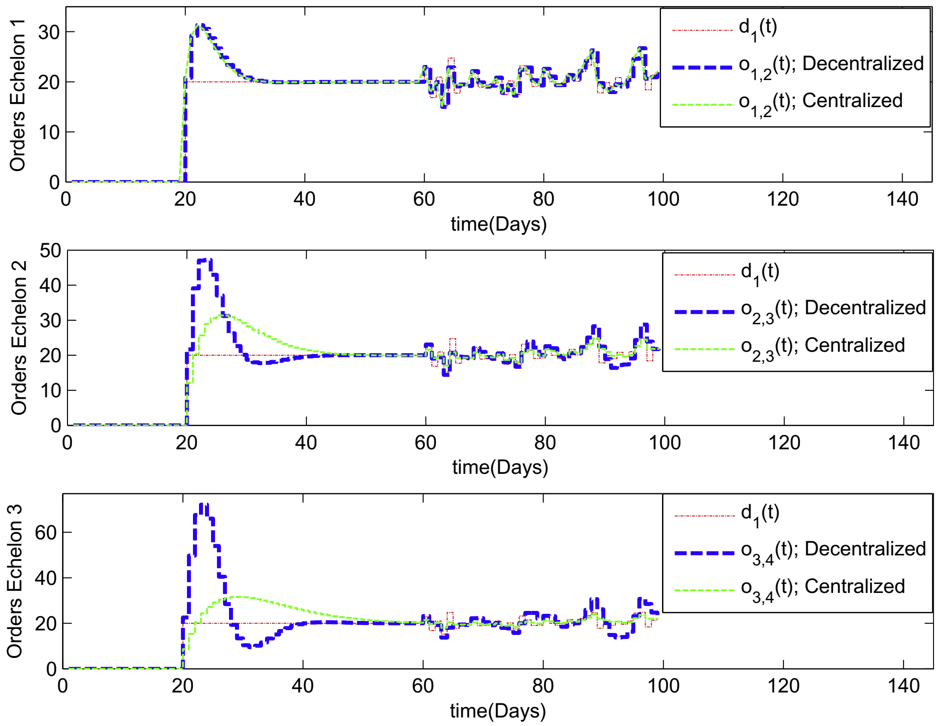

Fig. 8. Orders responses to changes in the demand assuming no changes in the inventory target.

Table 2
Ratio of the variance of the order rate to the variance of the demand rate (BW).

Strategy	Decentralized $\lambda_1^d = \lambda_2^d = \lambda_3^d = 0.695$	Centralized $\lambda_{1,1}^d = 0.695;\ \lambda_{2,1}^d = 0.84;\ \lambda_{3,1}^d = 0.89$
$\frac{Var(o_{1,2}(t))}{Var(d_1(t))}$	1.07	1.07
$\frac{Var(o_{2,3}(t))}{Var(d_1(t))}$	1.62	0.46
$\frac{Var(o_{3,4}(t))}{Var(d_1(t))}$	3.22	0.32

is to apply a significant detuning echelon by echelon mitigating progressively the bullwhip effect. In this case, an analytical trade-off is no possible. Since in this paper the main point is to compare the decentralized and centralized strategies, this analysis is proposed for future works.

On the other hand, Fig. 7 also shows that the centralized controller $Q^d(z)$ with the λ_{ij}^d values given in Fig. 4 provides a fast recuperation of inventory without overshoot in the first echelon. The second and third echelons present sluggish recuperation of inventory, that is due to the detuning for bullwhip mitigation. Fig. 8 shows that with the centralized control design, the overshots in the orders in response to a step change in the demand is slower than for the decentralized strategy. Moreover, this strategy presents a better bullwhip effect mitigation echelon by echelon in comparison with the decentralized strategy. Therefore, a trade-off between a fast response to the demand and bullwhip effect avoidance in the entire supply chain is available by the centralized control strategy.

There are several measures of the bullwhip effect proposed in the literature (Dejonckheere et al., 2003). The most common measure is the ratio of the variance of the order rate to the variance of the demand rate, i.e. $BW = \frac{Var(Orders(t))}{Var(Demand(t))}$, where BW values less than or equal to one means a total mitigation of bullwhip effect. Therefore, in Table 2, BW is calculated for the entire supply chain under the decentralized and centralized control strategies. The results confirm that the decentralized control strategy is effective in the first echelon but deteriorates dramatically echelon by echelon while in the centralized control strategy, the control of bullwhip effect is more effective echelon by echelon.

6. Discussion and conclusions

The proposed scheme allows us to tackle the two problems of inventory target tracking and demand rejection with two controllers separately. That is an advantage to other replenishment inventory policies based on control theory proposed in the literature (Hoberg et al., 2007; Jaksic and Rusjan, 2008; Balan et al., 2009; Dejonckheere et al., 2003). However, optimal tuning of these controllers produce aggressive orders that are unacceptable for factory managers. Then, two trade-off must be taken into account in the design of these controllers for inventory management in the supply chain, (Inventory target tracking vs. aggressive orders mitigation for the $\mathbf{Q}^t(z)$ design) and (Demand rejection vs. bullwhip effect mitigation for the $\mathbf{Q}^d(z)$ design). Previous works have analyzed these issues in a particular echelon. In this paper, these two control issues have been analyzed in the entire supply chain, under the decentralized and centralized control approaches. Since the interest of this work is to perform a comparative analysis between the decentralized and centralized control strategies in closed loop, forecasting demand which are feedforward schemes used to adjust the inventory target are excluded of the analysis. The analysis of appropriate schemes to adjust dynamically the inventory target are proposed for future works.

6.1. Inventory target tracking vs. aggressive orders mitigation for the $\mathbf{Q}^t(z)$ design

In the decentralized control approach based on multi-degree-of-freedom controller, the performance to inventory target tracking is optimal in the first echelon. That is, there is no overshoot in the inventory level when is subjected to inventory target changes. However, for the rest of the echelons, there are significant overshoots in the inventory response to an inventory target change which lead to a large spike in factory orders that is unacceptable for factory managers.

Since in the centralized control approach the controller for the entire supply chain is designed at once, the controller performance to inventory target tracking holds for all echelons. Therefore, the detuning of optimal controller to smooth the aggressive orders must be stronger for the decentralized control approach than for a centralized control approach.

6.2. Demand rejection vs. bullwhip effect mitigation for the $\mathbf{Q}^d(z)$ design

Respect to the trade-off between the demand rejection and the bullwhip effect mitigation in the $\mathbf{Q}^d(z)$ design, the tuning proposed by (Lin et al., 2004) for a single echelon is generalized in this work for the entire supply chain. Again, in the decentralized control approach this tuning works well in the first echelon. However, in the second echelon and onwards, the performance to bullwhip effect mitigation under this tuning is deteriorated. Since, in the centralized control approach the tuning for bullwhip effect mitigation is applied in each component of the Γ matrix, the tuning for bullwhip effect mitigation works successfully in the entire supply chain.

As is mentioned in the introduction, a decentralized control approach is more intuitive and easier to implement. Moreover, it is more suitable for supply chains where its elements belong to different companies and do not share information. Nevertheless, its performance to inventory target tracking and demand rejection and bullwhip effect mitigation of this controller can be deteriorated significantly in supply chains with several echelons and interactions.

On the other hand, when all or most of the supply chain elements belong to the same company or share internal information the centralized control approach is the more suitable. The IMC control scheme simplifies the design controllers for the multivariable system allowing to generalize the design guidelines for an echelon of the supply chain to multiple echelons in a decentralized and centralized control ways. Simulations shown in Section 5 evidence the improvement in the performance to inventory target tracking disturbance rejection and bullwhip effect mitigation provided by a centralized control implementation with information sharing. Therefore, the information sharing and the centralization of the inventory control are recommended in all cases where it is possible.

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