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Optimal joint replenishment, delivery and inventory management policies for perishable products

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ABSTRACT

In this paper we analyze the optimal joint decisions of when, how and how much to replenish customers with products of varying ages. We discuss the main features of the problem arising in the joint replenishment and delivery of perishable products, and we model them under general assumptions. We then solve the problem by means of an exact branch-and-cut algorithm, and we test its performance on a set of randomly generated instances. Our algorithm is capable of computing optimal solutions for instances with up to 30 customers, three periods, and a maximum age of two periods for the perishable product. For the unsolved instances the optimality gap is always small, less than 1.5% on average for instances with up to 50 customers. We also implement and compare two suboptimal selling priority policies with an optimized policy: always sell the oldest available items first to avoid spoilage, and always sell the fresher items first to increase revenue.

1. Introduction

Inventory control constitutes an important logistics operation, especially when products have a limited shelf life. Keeping the right inventory levels guarantees that the demand is satisfied without incurring unnecessary holding or spoilage costs. Several inventory control models are available [3], many of which include a specific treatment of perishable products [30].

Problems related to the management of perishable products' inventories arise in several areas. Applications of inventory control of perishable products include blood management and distribution [5,9,17,18,20,25,26,33], as well as the handling of radioactive and chemical materials [1,11,37], of food such as dairy products, fruits and vegetables [4,12,29,31,35,36], and of fashion apparel [28]. Several inventory management models have been specifically derived for perishable items, such as the periodic review with minimum and maximum order quantity of Haijema [15], and the periodic review with service level considerations of Minner and Transchel [24]. Reviews of the main models and algorithms in this area can be found in Nahmias [30] and in Karaesmen et al. [19]. A unified analytical approach to the management of supply chain networks for time-sensitive products is provided in Nagurney et al. [27].

Efficient delivery planning can provide further savings in logistics operations. The optimization of vehicle routes is one of the most developed fields in operations research [21]. The integration of inventory control and vehicle routing yields a complex optimization problem called inventory-routing whose aim is to minimize the overall costs related to vehicle routes and inventory control. Recent overviews of the inventory-routing problem (IRP) are those of Andersson et al. [2] and of Coelho et al. [8].

The joint inventory management and distribution of perishable products, which is the topic of this paper, gives rise to the perishable inventory-routing problem (PIRP). Nagurney and Masoumi [25] and Nagurney et al. [26] studied the distribution and relocation of human blood in a stochastic demand context, considering the perishability and waste of blood related to age and to the limited capacity of blood banks. Hemmelmayr et al. [16] studied the case of blood inventory control with predetermined fixed routes and stochastic demand. The problem was solved heuristically by integer programming and variable neighborhood search. Gumasta et al. [14] incorporated transportation issues in an inventory control model restricted to two customers only. Custódio and Oliveira [10] proposed a strategical heuristic analysis of the distribution and inventory control of several frozen groceries with stochastic demand. Mercer and Tao [23] studied the weekly food distribution problem of a supermarket chain, without

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considering product age. A theoretical paper developing a column generation approach was presented by Le et al. [22] to provide solutions to a PIRP. The optimality gap was typically below 10% for instances with eight customers and five periods under the assumptions of fixed shelf life and flat value throughout the life of the product.

This paper makes several scientific contributions. We first classify and discuss the main assumptions underlying the management of perishable products. We then formulate the PIRP as a mixed integer linear program (MILP) for the most general case, and we also model it to handle the cases where retailers always sell older items first, and where they sell fresher items first. We devise an exact branch-and-cut algorithm for the solution of the various models. To the best of our knowledge, this is the first time an IRP is modeled and solved exactly under general assumptions in the context of perishable products management. Our models do not require any assumption on the shape of the product revenue and inventory cost functions. We also establish some relationships between the PIRP and the multi-product IRP recently studied by the authors [7].

The remainder of the paper is organized as follows. In Section 2 we provide a formal description of the PIRP. In Section 3 we present our MILP model and its two variants just described, including new valid inequalities. This is followed by a description of the branch-and-cut algorithm in Section 4. Computational experiments are presented in Section 5. Section 6 concludes the paper.

2. Problem description

The joint replenishment and inventory problem for perishable products is concerned with the combined optimization of delivery routes and inventory control for products having a transient shelf life. Here, we consider a three-echelon supply chain in which suppliers deliver products to retailers who then sell products to the end-customers. These products typically have an expiry date, after which they are no longer fit for consumption. This is the case not only of some law-regulated products such as food and drugs, but also of a wide variety of unregulated products whose quality, appearance or commercial appeal diminishes over time, such as flowers, cosmetics, paint, electronic products or fashion items. In this section we discuss four main assumptions underlying the treatment of these kinds of products, and we explain how we incorporate them in our model. Specifically, we discuss the types of product perishability in Section 2.1, the assumptions governing the inventory holding costs of these products in Section 2.2, their revenue as a function of age in Section 2.3, and the management of items of different ages held in inventory in Section 2.4.

2.1. Types of product perishability

There exist two main types of perishable products according to how they decay [30]. The first type includes products whose value does not change until a certain date, and then goes down to zero almost immediately. This is the case of products whose utility eventually ceases to be valued by the customers, such as calendars, year books, electronics or maps, which quickly become obsolescent after a given date or when a new generation of products enters the market. However, this is more a case of obsolescence than perishability. Even though these items may still be in perfect condition, they are simply no longer useful. Within the same category, we find products with an expiry date, such as drugs, yogurt and bottled milk. These products can be consumed whether they are top fresh or a few days old, but after their expiry date, they are usually deemed unfit for consumption. The second type includes products whose quality or perceived value decays gradually over time. Typical examples are fruits, vegetables and flowers. The models introduced in Section 3 can handle both types of products without any ad hoc modification. Raafat [34] describes a stochastic model in which the deterioration is a function of the on-hand inventory level. Our model does not work under the assumption of a random lifetime.

2.2. The impact of item age on inventory holding costs

As a rule, the unit inventory holding cost changes with respect to the age and value of a product. This general assumption holds, for instance, for insurance costs which are value related. All the variable costs related to the age of the product can be modeled through a single parameter, called the unit inventory holding cost, which depends on the age of the item. In some contexts, all items yield the same holding cost, regardless of their age. Products with a short shelf life usually fit in this category. In this case, the holding cost, which encompasses all other variable costs, can be captured by a unique input parameter independent of the value and age of the product, which is the case in most applications.

2.3. Revenue of the item according to its age

A parameter that greatly affects the profit yielded by products of different ages is their perceived value by consumers. Brand new items usually have a higher selling price, which decreases over time according to some function. In this paper we do not make any specific assumption regarding the shape of this function. Rather, we assume that the selling price is known in advance for each product age. Note that the function describing the relation between price and age can be non-linear, non-continuous or even non-convex, but it can still be accommodated by our model, as will be shown in Section 3.

2.4. Inventory management policies

The final assumption we discuss relates to the management of items of different ages held in inventory. It is up to the retailer to decide which items to offer to customers, which will influence the associated revenue. In such a context, three different selling priority policies can be envisaged. The first one consists of applying a *fresh first* (FF) policy by which the retailer always sells the fresher items first. This policy ensures a longer shelf life and increases utility for the customers but, at the same time, yields a higher spoilage rate. The second policy is the reverse. Under an *old first* (OF) policy, older items are sold first, which generates less spoilage, but also less revenue. The third policy, which we introduce in our model, is more flexible and general, and encompasses these two extremes. The *optimized priority* (OP) policy lets the model determine which items to sell at any given time period in order to maximize profit. This means that depending on the parameter settings, one may prefer to spoil some items and sell fresher ones because they generate higher revenues.

Although they are similar, FF and OF policies are different from the traditional FIFO and LIFO policies common in inventory management. Under a FIFO policy, the first product delivered will be the first to be sold. This coincides with an OF policy only if deliveries from the supplier to the retailer is always of fresh items. However, when the supplier delivers products of different ages in different periods, the sequence of deliveries does not necessarily coincide with the ages of the products in inventory. To illustrate, consider the case where the supplier delivers new items on day one, and three-day old items on day two. Then, on day three, different solutions will be obtained under the OF and the FIFO policies. Indeed, under the FIFO policy, the newer items (delivered on day one) will be sold first, but the older items (delivered on day two) will be selected under an OF policy.

In order to illustrate the FF and the OF policies, first consider the case of bottled milk having a limited shelf life. A retailer holds in inventory one unit of old milk having a remaining shelf life of one day, and one unit of one-day old milk still good for several days. The unit revenue is $2. If the retailer applies an FF policy, he sells his one-day old milk today, making $2 of revenue. Tomorrow, the remaining bottle will be spoiled and he will make no revenue. The total revenue under the FF policy is then $2. If, on the other hand, he applies an OF policy, he sells his old bottle today, and the newer one tomorrow, making a total revenue of $4, or twice the revenue achieved under the FF policy.

Now consider the case of flowers, whose value declines quickly from one day to the next. A one-day old bouquet of flowers generates a revenue of $10, whereas a two-day old bouquet yields only $4. Under an FF policy, she will sell the one-day old flowers today, and nothing tomorrow, making a total revenue of $10. Under an OF policy, the retailer will sell the older flowers today and the other ones tomorrow, achieving a smaller revenue of $8.

Note that in these two examples, the OP policy coincides with either the OF or the FF policy. However, this is not always the case, namely when the revenue function is not monotonic with respect to the age of the product. Consider for example the case of bananas, which start their shelf life as green products, not yet ripe for consumption, then turn yellow when they reach their peak value, and finally become brown close to their spoilage date. Suppose there are two hands of bananas of each color in inventory. Let the revenue be $1.50 for a hand of green bananas, $2 for a yellow hand, and $0.50 for a brown hand. Note how the green product is valued higher than the brown one, because it will mature over time and will eventually become yellow. For a daily demand of one hand over two periods, the FF policy yields a revenue of $3, the OF policy yields only $1, but an OP policy consisting of selling yellow bananas each day yields an optimal revenue of $4. If the inventory contains only green and yellow bananas, then the OF and OP policies coincide; similarly, if only yellow and brown bananas are considered, then the FF and OP policies coincide.

Thus, the choice of which of the FF or OF policy to apply depends on the trade-off between the inventory level and the revenue functions of the product under consideration. The advantage of the OP policy is that it does not impose any constraints on the age of the items to sell and is able to generate the most general and profitable solutions.

We implement all three policies and we analyze their trade-offs in the context of profit maximization.

3. Mathematical formulations

We now formally describe the mathematical formulation of PIRP under the assumptions just presented for a single product and under the three inventory management policies just described. The case of several products is conceptually similar, but requires an additional index [7]. We assume that the routing cost matrix is symmetric. Thus, we define the problem on an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{0,\dots,n\}$ is the vertex set and $\mathcal{E}=\{(i,j): i,j\in\mathcal{V}, i<j\}$ is the edge set. Vertex 0 represents the supplier and the remaining vertices $\mathcal{V}'=\mathcal{V}\backslash\{0\}$ correspond to n customers. A routing cost c_{ij} is associated with edge $(i,j)\in\mathcal{E}$.

Because of the general assumptions presented in Section 2, we consider that both the supplier and customers are fully aware of the number of items in inventory according to their age. This is important because the sales revenue and inventory holding costs are affected by the age of the product. The supplier has the choice to deliver fresh or aged product items, and each case yields different holding costs. Each customer has a maximum inventory holding capacity C_i, which cannot be exceeded in any period of the planning horizon of length p. At each time period $t\in\mathcal{T}=\{1,\dots,p\}$, the supplier receives or produces a fresh quantity r^t of the perishable product. We assume that the supplier has sufficient inventory to meet the demand of its customers during the planning horizon, all the demand has to be satisfied. At the beginning of the planning horizon the decision maker knows the current inventory level of the product at each age held by the supplier and by the customers, and receives information on the demand d_i^t of each customer i for each time period t. Note again that, as discussed in the previous section, the demand can be equally satisfied by fresh or aged products, which will in turn affect the revenue.

As is typically the case in the IRP literature [8], we assume that the quantity r^t made available at the supplier in period t can be used for deliveries to customers in the same period, and the delivery amount received by customer i in period t can be used to meet the demand in that period. A set $\mathcal{K}=\{1,\dots,K\}$ of vehicles are available. We denote by Q_k the capacity of vehicle k. Each vehicle can perform at most one route per time period, visiting a subset of customers, starting and ending at the supplier's location. Also, as in other IRP papers, we do not allow split deliveries, i.e., customers receive at most one vehicle visit per period.

The perishable product under consideration becomes spoiled after s periods, i.e., the age of the product belongs to a discrete set $\mathcal{S}=\{0,\dots,s\}$. The product is valued according to its age, and the decision maker is aware of the selling revenue u_g of one unit of product of age g. Likewise, the inventory holding cost h_i^g in location $i\in\mathcal{V}$ is a function of the age g of the product. This general representation allows for flat or variable revenues, and for flat or variable holding costs depending on the age and value of the product, thus covering all situations described in Section 2.

The inventory level I_i^t held by customer i in period t comprises items of different ages. We break down this variable into $I_i^t=\sum_{g\in\mathcal{S}}I_i^{gt}$, where I_i^{gt} represents the quantity of product of age h in inventory at customer i in period t. Likewise, we decompose the demand d_i^t into $\sum_{g\in\mathcal{S}}d_i^{gt}$.

The aim of the problem is to simultaneously construct vehicle routes for each period and to determine delivery quantities of products of different ages for each period and each customer, in order to maximize the total profit, equal to the sales revenue, minus the routing and inventory holding costs. This problem is extremely difficult to solve since it encompasses several NP-hard problems such as the vehicle routing problem [21] and a number of variants of the classical IRP [8].

Our MILP model works with routing variables x_{ij}^{kt} equal to the number of times edge (i,j) is used on the route of vehicle k in period t. We also use binary variables y_i^{kt} equal to one if and only if node i is visited by vehicle k in period t. Formally, variables $I_i^t=\sum_{g\in\mathcal{S}}I_i^{gt}$ denote the inventory level at vertex $i\in\mathcal{V}$ at the end of period $t\in\mathcal{T}$, and d_i^{gt} denotes the quantity of product of age g used to satisfy the demand of customer i in period t, and we denote by q_i^{gkt} the quantity of product of age g delivered by vehicle k to customer i in period t. The problem can then be formulated under an OP policy as follows:

$$\text{(PIRP) maximize} \sum_{g\in\mathcal{S}}\sum_{t\in\mathcal{T}}u_i^g d_i^{gt}-\sum_{i\in\mathcal{V}}\sum_{g\in\mathcal{S}}\sum_{t\in\mathcal{T}}h_i^g I_i^{gt}-\sum_{(i,j)\in\mathcal{E}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}c_{ij}x_{ij}^{kt}, \tag{1}$$

subject to

$$I_0^{gt}=I_0^{g-1,t-1}-\sum_{i\in\mathcal{V}'}\sum_{k\in\mathcal{K}}q_i^{gkt}, \quad g\in\mathcal{S}\backslash\{0\},\ t\in\mathcal{T} \tag{2}$$

$$I_0^{0t}=r^t, \quad t\in\mathcal{T} \tag{3}$$

$$I_i^{gt}=I_i^{g-1,t-1}+\sum_{k\in\mathcal{K}}q_i^{gkt}-d_i^{gt}, \quad i\in\mathcal{V}',\ g\in\mathcal{S}\backslash\{0\},\ t\in\mathcal{T} \tag{4}$$

$$I_i^{0t} = \sum_{k \in \mathcal{K}} q_i^{0kt} - d_i^{0t}, \quad i \in \mathcal{V}', \ t \in \mathcal{T} \tag{5}$$

$$\sum_{g \in \mathcal{S}} I_i^{gt} \le C_i, \quad i \in \mathcal{V}', \ t \in \mathcal{T} \tag{6}$$

$$d_i^t = \sum_{g \in \mathcal{S}} d_i^{gt}, \quad i \in \mathcal{V}', \ t \in \mathcal{T} \tag{7}$$

$$\sum_{g \in \mathcal{S}} \sum_{k \in \mathcal{K}} q_i^{gkt} \le C_i - \sum_{g \in \mathcal{S}} I_i^{g,t-1}, \quad i \in \mathcal{V}', \ t \in \mathcal{T} \tag{8}$$

$$q_i^{gkt} \le C_i y_i^{kt}, \quad i \in \mathcal{V}', \ g \in \mathcal{S}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{9}$$

$$\sum_{i \in \mathcal{V}'} \sum_{g \in \mathcal{S}} q_i^{gkt} \le Q_k y_0^{kt}, \quad k \in \mathcal{K}, \ t \in \mathcal{T} \tag{10}$$

$$\sum_{j \in \mathcal{V}, i<j} x_{ij}^{kt} + \sum_{j \in \mathcal{V}, j<i} x_{ji}^{kt} = 2y_i^{kt}, \quad i \in \mathcal{V}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{11}$$

$$\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{S}, i<j} x_{ij}^{kt} \le \sum_{i \in \mathcal{S}} y_i^{kt} - y_m^{kt}, \quad \mathcal{S} \subseteq \mathcal{V}', \ k \in \mathcal{K}, \ t \in \mathcal{T}, \ m \in \mathcal{S} \tag{12}$$

$$\sum_{k \in \mathcal{K}} y_i^{kt} \le 1, \quad i \in \mathcal{V}', \ t \in \mathcal{T} \tag{13}$$

$$I_i^{gt}, d_i^{gt}, q_i^{gkt} \ge 0, \quad i \in \mathcal{V}', \ g \in \mathcal{S}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{14}$$

$$x_{0i}^{kt} \in \{0,1,2\}, \ i \in \mathcal{V}', \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{15}$$

$$x_{ij}^{kt} \in \{0,1\}, \quad (i,j) \in \mathcal{E}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{16}$$

$$y_i^{kt} \in \{0,1\}, \quad i \in \mathcal{V}, \ k \in \mathcal{K}, \ t \in \mathcal{T}. \tag{17}$$

The objective function (1) maximizes the total sales revenue, minus inventory and routing costs. Constraints (2) define the inventory conservation conditions for the supplier, aging the product by one unit in each period. Constraints (3) ensure that the supplier always produces or receives top fresh products. Constraints (4) and (5) define inventory conservation and aging of the items for the customers. Constraints (6) impose a maximal inventory capacity at each customer. Constraints (7) state that the demand of each customer in each period is the sum of product quantities of different ages. Note that by design, any product whose age g is higher than s is spoiled, e.g., it no longer appears in the inventory nor it can be used to satisfy the demand. Constraints (8) and (9) link the quantities delivered to the routing variables. In particular, they only allow a vehicle to deliver products to a customer if a vehicle has been assigned to it. Constraints (10) ensure that the vehicle capacities are respected. Constraints (11) and (12) are degree constraints and subtour elimination constraints, respectively. Inequalities (13) ensure that at most one vehicle visits each customer in each period, thus forbidding split deliveries. Constraints (14)–(17) enforce integrality and non-negativity conditions on the variables.

This model can be strengthened through the inclusion of the following families of valid inequalities [6]:

$$x_{0i}^{kt} \le 2y_i^{kt}, \quad i \in \mathcal{V}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{18}$$

$$x_{ij}^{kt} \le y_i^{kt}, \quad i,j \in \mathcal{V}, \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{19}$$

$$y_i^{kt} \le y_0^{kt}, \quad i \in \mathcal{V}', \ k \in \mathcal{K}, \ t \in \mathcal{T} \tag{20}$$

$$y_0^{kt} \le y_0^{k-1,t}, \quad k \in \mathcal{K} \backslash \{1\}, \ t \in \mathcal{T} \tag{21}$$

$$y_i^{kt} \le \sum_{j<i} y_j^{k-1,t}, \quad i \in \mathcal{V}, \ k \in \mathcal{K} \backslash \{1\}, \ t \in \mathcal{T}. \tag{22}$$

Constraints (18) and (19) enforce the condition that if the supplier is the immediate successor of a customer in the route of vehicle k in period t, then i must be visited by the same vehicle. A similar reasoning is applied to customer j in inequalities (19). Constraints (20) ensure that the supplier is visited if any customer i is visited by vehicle k in period t.

When the vehicle fleet is homogeneous, one can break some of the vehicle symmetry by means of constraints (21), thus ensuring that vehicle k cannot leave the depot if vehicle $k-1$ is not used. This symmetry breaking rule is then extended to the customer vertices by constraints (22) which state that if customer i is assigned to vehicle k in period t, then vehicle $k-1$ must serve a customer with an index smaller than i in the same period.

We also introduce additional cuts in order to strengthen this formulation. If the sum of the demands over $[t_1, t_2]$ is at least equal to the maximum possible inventory held, then there must be at least one visit to this customer in the interval $[t_1, t_2]$. This constraint can be strengthened by considering that if the quantity needed to satisfy future demands is larger than the maximum inventory capacity, then several visits are needed. Since the maximum delivery size is the minimum between the holding capacity and the maximum vehicle capacity, one can round up the right-hand side of (23). Making the numerator tighter by considering the actual inventory instead of the maximum possible inventory yields inequalities (24), which cannot be rounded up because they would then become non-linear due to the presence of the $I_i^{t_1}$ variable in their right-hand side

$$\sum_{k \in \mathcal{K}} \sum_{t'=t_1}^{t_2} y_i^{kt'} \ge \left\lceil \frac{\sum_{t'=t_1}^{t_2} d_i^{t'} - C_i}{\min\{\max_k\{Q_k\}, C_i\}} \right\rceil, \quad i \in \mathcal{V}', \ t_1, t_2 \in \mathcal{T}, \ t_2 \ge t_1 \tag{23}$$

$$\sum_{k \in \mathcal{K}} \sum_{t'=t_1}^{t_2} y_i^{kt'} \ge \frac{\sum_{t'=t_1}^{t_2} d_i^{t'} - I_i^{t_1}}{\min\{\max_k\{Q_k\}, C_i\}}, \quad i \in \mathcal{V}', \ t_1, t_2 \in \mathcal{T}, \ t_2 \ge t_1. \tag{24}$$

A different version of the same inequalities can be written as follows. It is related to whether the inventory hold at each period is sufficient to fulfill future demands. In particular, if the inventory held in period t_1 by customer i is not sufficient to fulfill future demands, then a visit to this customer must take place in the interval $[t_1, t_2]$. This condition can be enforced by the following set of inequalities:

$$\sum_{k \in \mathcal{K}} \sum_{t'=t_1}^{t_2} y_i^{kt'} \ge \frac{\sum_{t'=t_1}^{t_2} d_i^{t'} - I_i^{t_1}}{\sum_{t'=t_1}^{t_2} d_i^{t'}}, \quad i \in \mathcal{V}', \ t_1, t_2 \in \mathcal{T}, \ t_2 \ge t_1. \tag{25}$$

Even if these inequalities are redundant for our model, they are useful in helping CPLEX generate new cuts.

It is relevant to note that this model distinguishes items of different ages through the use of index g. The variables have a meaning similar to those of the multi-product IRP [6]. In the case of a single perishable product, the model works as if products of different ages are different from each other (through their index) and have different profits, but contrary to what happens in the multi-product case, any of these products can be used to satisfy the same demand. Another particularity of this model is that at each period, an item transforms itself into another one through the process of aging. Thus, our problem shares some features of the multi-product problem [6], but it is structurally different from it.

3.1. Modeling an FF policy

We now show how the formulation just described can be used to solve the problem under an FF policy under which the retailer sells fresher items first. We add extra variables and constraints to the PIRP formulation in order to restrict the choice of products age to be sold.

We implement this idea as follows. We first introduce new binary variables L_i^{gt} equal to one if and only if products of age g can be used to satisfy the demand of customer i in period t. The first set of new constraints restricts the use of variables d_i^{gt}, i.e., the use of products of age g to satisfy the demand of customer i in period t, only to those products allowed by the respective L_i^{gt} variables, that is

$$d_i^{gt} \le C_i L_i^{gt}, \quad i \in \mathcal{V}', \ g \in \mathcal{S}, \ t \in \mathcal{T}. \tag{26}$$

We also order the new variables in increasing order of age index. The following set of constraints allows selling products of age $g+1$ only if products of age g have been used to satisfy the demand of customer i in period t:

$$L_i^{gt} \ge L_i^{g+1,t}, \quad i \in \mathcal{V}', \ g \in \mathcal{S} \backslash \{s\}, \ t \in \mathcal{T}. \tag{27}$$

We then impose the following constraints to disallow the use of older products if there exists enough inventory of fresher products. The use of products of age $g+1$ is allowed if and only if the total inventory of products of ages $g, g-1, \ldots, 0$ is insufficient to satisfy the demand of customer i in period t. This can be enforced through the following constraints:

$$C_i(1 - L_i^{g+1,t}) \ge \sum_{j=0}^{g} I_i^{jt} + \sum_{j=0}^{g} \sum_{k \in \mathcal{K}} q_i^{jkt} - d_i^t + 1, \quad i \in \mathcal{V}', \ g \in \mathcal{S} \backslash \{s\}, \ t \in \mathcal{T}. \tag{28}$$

3.2. Modeling an OF policy

It is straightforward to model the OF policy from the constraints developed for the FF case. This policy can be enforced by considering the same L_i^{gt} variables and the following three sets of constraints:

$$d_i^{gt} \le C_i L_i^{gt}, \quad i \in \mathcal{V}', \ g \in \mathcal{S}, \ t \in \mathcal{T}. \tag{29}$$

Constraints (29) only allow the use of inventory of age g to satisfy the demand if its associated L_i^{gt} variable is set to one. Then, we also rank the L_i^{gt} variables in increasing order of age index. The following set of constraints allow selling products of age $g-1$ only if products of age g are being used to satisfy the demand of customer i in period t:

$$L_i^{g-1,t} \le L_i^{g,t}, \quad i \in \mathcal{V}', \ g \in \mathcal{S} \backslash \{0\}, \ t \in \mathcal{T}. \tag{30}$$

Finally, we force some of the L variables to take value zero by adding the following constraints to the model. If the total inventory available of ages $\{g, g+1, \ldots, s\}$ is sufficient to satisfy the demand of customer i in period t, then the right-hand side of inequalities (31) is positive, which in turn guarantees that $L_i^{g-1,t}$ will take value zero

$$C_i(1 - L_i^{g-1,t}) \ge \sum_{j=g}^{s} I_i^{jt} + \sum_{j=g}^{s} \sum_{k \in \mathcal{K}} q_i^{jkt} - d_i^t + 1, \quad i \in \mathcal{V}', \ g \in \mathcal{S} \backslash \{0\}, \ t \in \mathcal{T}. \tag{31}$$

3.3. Extending the model to consider waste and salvage revenues

We note that the revenue function for aged products can easily consider a salvage value for wasted products. This can be achieved by including a revenue for a product when it is no longer fit for consumption, and by artificially extending its shelf life in the model. This has the disadvantage, however, that the demand for an old (wasted) product is shared with the demand for products that are still good. Salvage costs and an account for wasted products can be modeled as follows.

Let w_i^t be the quantity of wasted products at customer i in period t. It is easily observed that

$$w_i^t = I_i^{s,t-1}, \quad i \in \mathcal{V}', \ t \in \mathcal{T} \backslash \{1\}. \tag{32}$$

If γ is the salvage cost per unit, the total salvage cost is then

$$\sum_{i \in \mathcal{V}'} \sum_{t \in \mathcal{T} \backslash \{1\}} \gamma w_i^t, \tag{33}$$

which can be added to the objective function (1).

4. Branch-and-cut algorithm

For instances of very small sizes, the model presented in Section 3 can be fully described and all constraints and variables generated. It can then be solved by feeding it directly into an integer linear programming solver. However, for instances of realistic sizes, the number of subtour elimination constraints (12) is too large to allow full enumeration and these must be dynamically generated throughout the search process. The exact algorithm we present is then a classical branch-and-cut scheme in which subtour elimination constraints are only generated and incorporated into the program whenever they are found to be violated. It works as follows. At a generic node of the search tree, a linear program containing a subset of the subtour elimination constraints is solved, a search for violated inequalities is performed, and some of these are added to the current program which is then reoptimized. This process is reiterated until a feasible or dominated solution is reached, or until there are no more cuts to be added. At this point, branching on a fractional variable occurs. We provide a sketch of the branch-and-bound-and-cut scheme in Algorithm 1.

Algorithm 1. Branch-and-cut algorithm

At the root node of the search tree, generate and insert all valid inequalities into the program.
$z^* \leftarrow \infty$.
Termination check:
if there are no more nodes to evaluate **then**
  Stop with the incumbent and optimal solution of cost z^*.
else
  Select one node from the branch-and-bound tree.
endif
Subproblem solution: solve the LP relaxation of the node and let z be its cost.
if the current solution is feasible **then**
  if $z \le z^*$ **then**
    Go to termination check.
  else
    $z^* \leftarrow z$.
    Update the incumbent solution.
    Prune nodes with lower bound lower than or equal to z^*.
    Go to termination check.
  endif
endif
Cut generation:
if the solution of the current LP relaxation violates any cuts **then**
  Identify connected components as in Padberg and Rinaldi [32].
  Determine whether the component containing the supplier is weakly connected as in Gendreau et al. [13].
  Add violated subtour elimination constraints (12).
  Go to subproblem solution.
endif
Branching: branch on one of the fractional variables.
Go to the termination check.

5. Computational experiments

In order to evaluate the proposed algorithm, we have coded it in C++ and used IBM Concert Technology and CPLEX 12.5 running in parallel with two threads. All computations were executed on a machine equipped with an Intel Xeon™ processor running at 2.66 GHz with 24 GB of RAM, with the Scientific Linux 6.1 operating system.

5.1. Instances generation

We have created randomly generated instances to assess the performance of our algorithm on a wide range of situations. We have generated a total of 60 different instances which vary in terms of the number of customers, periods, vehicles and maximum age of the product. Our testbed is composed of instances generated with the following parameters:

- Number of customers n: 10, 20, 30, 40, 50.
- Number of periods H: 3 for up to $n=50$; 6 for up to $n=40$; and 10 for up to $n=30$.
- Number of vehicles K: 1 for $n=10$; 2 for $n=20$ and 30; 3 for $n=40$ and 50.
- Maximum age of the products s: 2 for $H=3$; 3 for $H=6$; 5 for $H=10$.
- Demand d_i^t: randomly selected from the interval [30, 210].
- Position (x, y) of the supplier and customers: randomly selected from the interval [0, 1000].
- Customers inventory capacity C_i: $R \times \max_t\{d_i^t\}$, where R is randomly selected from the set {2, 3}.
- Initial inventory I_i^0 of fresh products: equal to $C_i - d_i^1$.
- Revenue u_i^g: equal to $R_1 - (R_1 - R_2)g/s$, where R_1 and R_2 are randomly selected from the intervals [10, 20] and [4, 7], respectively.
- Inventory holding cost h_i^g: equal to $(R_1 + gR_2/(1+g))/100$, where R_1 and R_2 are randomly selected from the intervals [0, 100] and [0, 70], respectively.
- Vehicle capacities Q_k: equal to $\lfloor 1.25 \sum_{i \in \mathcal{V}'} \sum_{t \in \mathcal{T}} d_i^t / (HK) \rfloor$.

For each combination of the n, s, K and H parameters we have generated five instances, yielding a total of 60 instances.

In what follows we provide average statistics over five instances per combination. Detailed results are presented in Appendix A. These results along with the instances are also available for download from http://www.leandro-coelho.com.

5.2. Solutions for an OP policy

We provide in Table 1 average computational results for these instances under the OP policy. We have allowed the algorithm to run for a maximum of 2 h. When the time limit is reached, we report the best available lower and upper bound (solution value) and the optimality gap. We report the instance sizes as (n-s-K-H), where n is the number of customers, s is the maximum age of the product, K is the number of vehicles, and H is the length of the planning horizon. The next columns report the average best solution value obtained, the average best bound, the average

Table 1
Summary of the computational results for the PIRP under the OP policy.

Instance size (n-s-K-H)	Best known solution value	Best known upper bound	Gap (%)	# solved	Time (s)
PIRP-10-2-1-3	31 529.90	31 529.90	0.00	5/5	0.4
PIRP-10-3-1-6	61 684.44	61 684.44	0.00	5/5	2.4
PIRP-10-5-1-10	81 094.96	81 094.96	0.00	5/5	210.2
PIRP-20-2-2-3	62 936.24	62 936.24	0.00	5/5	27.8
PIRP-20-3-2-6	126 736.20	128 894.40	1.75	0/5	7200.6
PIRP-20-5-2-10	180 919.00	186 553.20	3.30	0/5	7201.4
PIRP-30-2-2-3	97 580.90	97 580.90	0.00	5/5	322.0
PIRP-30-3-2-6	192 817.80	196 322.20	1.79	0/5	7201.0
PIRP-30-5-2-10	294 582.20	300 742.00	2.17	0/5	7201.4
PIRP-40-2-3-3	127 961.60	129 832.00	1.45	0/5	7201.4
PIRP-40-3-3-6	250 435.80	258 103.40	3.10	0/5	7201.2
PIRP-50-2-3-3	177 157.40	179 724.40	1.46	0/5	7201.8

Table 2
Summary of the computational results for the PIRP under an FF policy.

Instance size (n-s-K-H)	% decrease	Opt gap (%)	# solved	Time (s)
PIRP-10-2-1-3	0.00	0.00	5/5	0.6
PIRP-10-3-1-6	0.17	0.00	5/5	3.2
PIRP-10-5-1-10	0.51	0.64	3/5	3251.0
PIRP-20-2-2-3	0.01	0.00	5/5	50.6
PIRP-20-3-2-6	0.14	1.97	0/5	7200.4
PIRP-20-5-2-10	0.10	3.42	0/5	7202.4
PIRP-30-2-2-3	0.12	0.33	4/5	1526.0
PIRP-30-3-2-6	−0.01	1.83	0/5	7201.2
PIRP-30-5-2-10	0.35	2.45	0/5	7202.6
PIRP-40-2-3-3	0.14	1.62	0/5	7201.0
PIRP-40-3-3-6	0.25	3.38	0/5	7202.8
PIRP-50-2-3-3	0.35	1.89	0/5	7202.6

Table 3
Summary of the computational results for the PIRP under an OF policy.

Instance size (n-s-K-H)	% decrease	Opt gap (%)	# solved	Time (s)
PIRP-10-2-1-3	13.91	0.00	5/5	0.2
PIRP-10-3-1-6	14.99	0.00	5/5	35.8
PIRP-10-5-1-10	10.43	0.85	3/5	3638.6
PIRP-20-2-2-3	18.84	0.00	5/5	6.0
PIRP-20-3-2-6	11.94	2.10	1/5	6622.0
PIRP-20-5-2-10	8.64	4.91	0/5	7201.6
PIRP-30-2-2-3	18.09	0.00	5/5	40.4
PIRP-30-3-2-6	9.97	1.76	0/5	7201.6
PIRP-30-5-2-10	7.95	3.22	0/5	7202.0
PIRP-40-2-3-3	16.09	0.60	2/5	6249.4
PIRP-40-3-3-6	9.56	3.36	0/5	7202.0
PIRP-50-2-3-3	16.53	1.38	0/5	7202.8

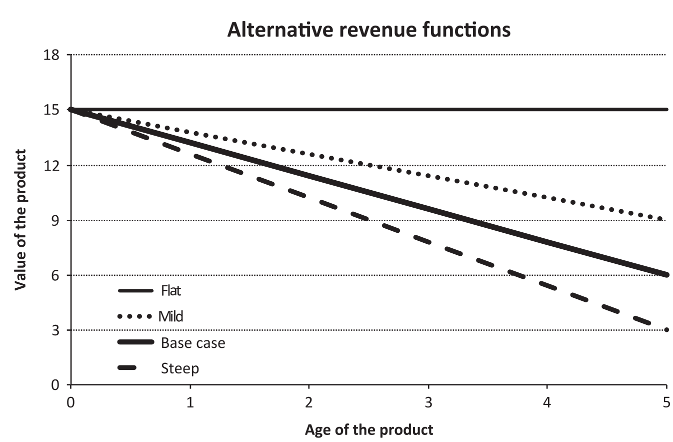

Fig. 1. Four alternative revenue functions.

Table 4
Percentage decrease in profit when using alternative revenue functions.

Instance size (n-s-K-H)	FF policy				OF policy			
	Base case	Mild	Steep	Flat	Base case	Mild	Steep	Flat
PIRP-10-2-1-3	0.00	0.00	0.00	0.00	13.91	21.37	10.76	0.01
PIRP-10-3-1-6	0.17	0.00	0.00	0.02	14.99	19.69	10.52	0.46
PIRP-10-5-1-10	0.51	0.00	0.00	0.35	10.43	16.27	9.19	1.59
PIRP-20-2-2-3	0.01	0.00	0.00	0.00	18.84	25.02	14.01	0.11
PIRP-20-3-2-6	0.14	−0.01	−0.11	0.12	11.94	15.83	8.59	0.80
PIRP-20-5-2-10	0.10	0.05	−0.03	0.40	8.64	11.22	7.44	1.58

optimality gap, the number of instances out of the five that were solved to optimality, and the average running time in seconds.

These results clearly indicate that the performance of the algorithm is directly related to the number n of customers and to the length H of the planning horizon. For the instances with shorter planning horizons ($H=3$), the algorithm is always able to find optimal solutions within a few seconds of computational time. This remains true even when the number of customers and vehicles increases, e.g., all five instances with 30 customers and three periods were solved to optimality, taking on average 5 min. Larger instances with up to 40 and 50 customers also with three periods were solved with a gap of less than 1.50% on average.

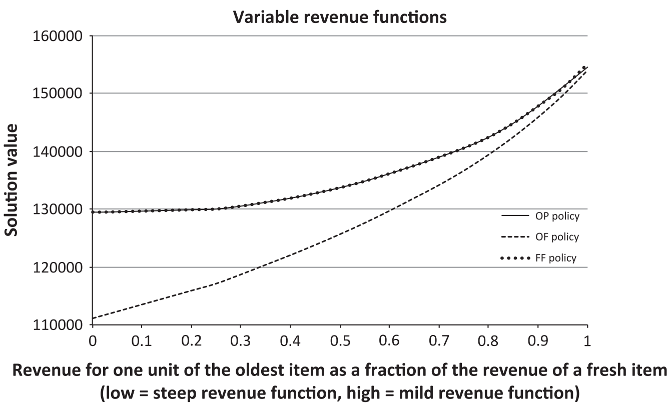

Fig. 2. Variable revenue functions. The horizontal axis indicate the revenue for one unit of the oldest item as a fraction of the revenue of a fresh item. Low values on the horizontal axis indicate a steep revenue function with respect to the age of the products. High values on the horizontal axis indicate a mild revenue function with respect to the age of the products.

5.3. Solutions for an FF and an OF policy

We also compare the solution cost of the optimized policy with respect to the age of the products sold with the cost of the alternative FF and OF policies. We first consider the FF policy which maximizes the revenue by always selling fresher items. This policy, on the other hand, leads to more spoilage, which in turn increases the need for more deliveries, thus increasing distributions costs. The results are shown in Table 2 as percentages representing the profit decrease of the FF policy with respect to the OP policy. We also report the optimality gap, the number of instances solved optimally, and the running time in seconds. We note that the difficulty of solving the PIRP under an FF policy is similar to that observed for the OP policy, and the profit is only slightly lower. Finally, we provide the same comparison with respect to the OF policy. The summary of the results is shown in Table 3.

As was the case of the FF policy, the difficulty of obtaining optimal and quasi-optimal solutions is not affected by the inclusion of the new binary variables and the new constraints. However, unlike the previous policy, the effect on cost of selling older items first, thus deriving lower revenues, has a major effect on the total profit observed, which decreases substantially over all instances.

5.4. Solutions for alternative revenue functions

In order to assess the trade-off between the OP, FF and OF policies, we have changed how the product revenue varies linearly as a function of age. We have generated three variations. In the first *mild* scenario, the difference in cost between fresh and old products is reduced. In the second *steep* scenario, the difference is

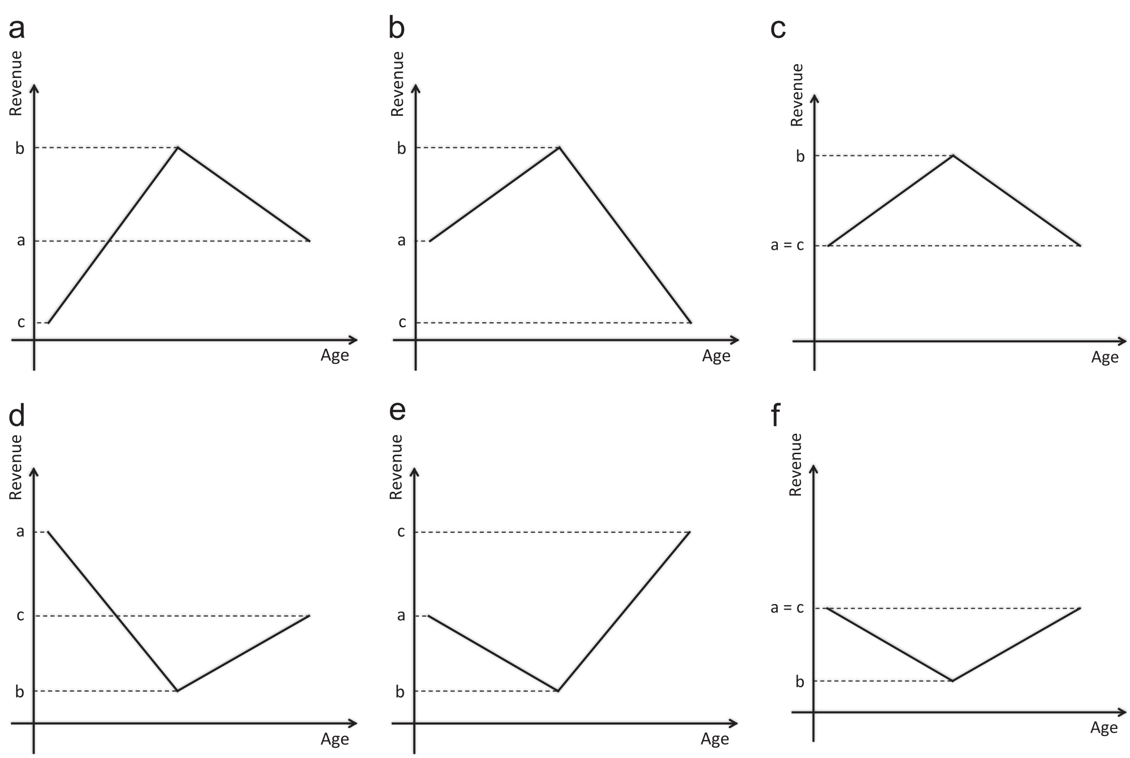

Fig. 3. Examples of the six different non-monotonic revenue functions tested. (a) Case with $a<c<b$. (b) Case with $c<a<b$. (c) Case with $a=c<b$. (d) Case with $a>c>b$. (e) Case with $c>a>b$. (f) Case with $a=c>b$.

Table 5
Average results for the OP, OF, and FF policies on instances with non-monotonic revenue functions.

Instance	OP policy				OF policy					FF policy				
	Best known solution value	Best known upper bound	Gap (%)	Time (s)	Best known solution value	Best known upper bound	Gap (%)	Time (s)	Increase over OP (%)	Best known solution value	Best known upper bound	Gap (%)	Time (s)	Increase over OP (%)
PIRP-A-20-5-2-6	150 784.2	154 088.6	2.19	7200	14 8894.0	151 786.6	1.94	7200	1.25	146 016.8	149 616.8	2.49	7200	3.16
PIRP-B-20-5-2-6	146 755.0	150 372.6	2.48	7200	143 425.6	146 530.4	2.18	7200	2.26	145 228.6	149 122.6	2.69	7201	1.04
PIRP-C-20-5-2-6	146 953.0	150 523.6	2.45	7200	144 423.0	147 789.4	2.35	7200	1.72	145 122.4	149 061.6	2.75	7200	1.24
PIRP-D-20-5-2-6	149 296.8	150 630.4	0.90	6318	138 932.4	140 970.6	1.51	7200	6.94	147 369.0	149 701.6	1.57	7200	1.29
PIRP-E-20-5-2-6	220 262.4	220 262.4	0.00	533	217 987.4	217 987.4	0.00	624	1.03	187 765.6	191 287.2	1.88	7200	14.75
PIRP-F-20-5-2-6	173 871.6	173 871.6	0.00	1312	168 254.6	169 828.0	0.94	6065	3.23	161 485.4	164 105.6	1.62	7200	7.12
Average	164 653.8	166 624.8	1.33	4961	160 319.5	162 482.0	1.49	5915	2.63	155 497.9	158 815.9	2.17	7200	5.56

increased. Finally, we have also created a *flat* scenario case in which the revenue of the product is constant as a function of age. These three scenarios and the base case are depicted in Fig. 1. The slopes of the linear functions in the increasing order are equal to -2.4, -1.8 and -1.2.

We have designed the following experiments in order to evaluate the impact of these changes in the trade-off between the different policies. We have selected all 30 instances containing 10 and 20 customers. Each instance was solved under the three policies and under the three alternative revenue functions. In Table 4 we report the percentage decrease in profit with respect to the optimized policy for each of the revenue functions considered.

To better understand how different revenues for products of different ages affect the trade-off between each of the three policies, we have conducted the following experiments. We have selected one instance (PIRP-10-5-1-10-1) and we have solved it using the three policies for several slopes of the revenue functions. Specifically, we have set the revenue of a fresh product to 20 and we have set the revenue of the oldest item ranging from zero to 20, in steps of one unit. We have then plotted the values of the objective functions of each one in the graph of Fig. 2.

These new sets of experiments confirm that on our data set the FF policy provides solution values that are almost identical to the OP policy. Note how the thin continuous line of the OP policy is only slightly higher than the dotted line of the FF policy, but visually indistinguishable from it. This implies that here the optimal policy tends to favor the sale of fresher products. The OF policy, on the other hand, provides solutions whose cost is greatly affected by the revenue value of older products. The difference between the policies is largest when these products are valued very low. When the revenue value for older products increases, so does increase the profit of applying an OF policy, and the difference between this policy and the other two tends to vanish.

Finally, we have also tested the performance of the three policies with respect to non-monotonic revenue functions. The shape of the non-monotonic instances is depicted in Fig. 3. There are three cases in which the revenue increases before decreasing, and three cases in which the revenue first decreases and then increases again. The six cases are illustrated in Fig. 3. We have created five instances of each type. Average results are presented in Table 5. We observe that for all cases, the OP policy outperforms both the FF and the OF policies.

6. Conclusions

We have introduced the joint replenishment and inventory control of perishable products. We have modeled the problem under general assumptions as a MILP, and we have solved it exactly by branch-and-cut. We have also introduced, modeled and solved exactly two variants of the problem defined by applying the OF and the FF selling priority policies, in which the retailer sells with higher priority older and fresher items, respectively. Our model remains linear even when the product revenue decreases in a non-linear or even in a non-convex fashion over time. It keeps track of the number of items of each age, and considers different holding costs for products of different ages. The model identifies products of different ages independently from each other, which is very similar to dealing with several products, as in a multi-product environment, but not identical since the state of the product changes over time. The model optimally determines which items to sell at each period based on the trade-off between cost and revenue. The algorithm can effectively compute optimal joint replenishment and delivery decisions for perishable products in an inventory-routing context for medium size instances. We have also shown that on our testbed, the profit changes drastically depending on the shape of the revenue of the product. On

Table 6
Detailed results of the computational experiments for the PIRP.

Instance size (*n-s-K-H*)	OP policy				FF policy				OF policy			
	Best known solution value	Best known upper bound	Gap (%)	Time (s)	Best known solution value	Best known upper bound	Gap (%)	Time (s)	Best known solution value	Best known upper bound	Gap (%)	Time (s)
PIRP-10-2-1-3-1	28 863.4	28 863.4	0.00	1	28 863.4	28 863.4	0.00	1	26 221.0	26 221.0	0.00	1
PIRP-10-2-1-3-2	34 368.0	34 368.0	0.00	1	34 368.0	34 368.0	0.00	1	29 637.7	29 637.7	0.00	0
PIRP-10-2-1-3-3	27 895.7	27 895.7	0.00	0	27 895.7	27 895.7	0.00	1	23 062.5	23 062.5	0.00	0
PIRP-10-2-1-3-4	33 688.1	33 688.1	0.00	0	33 688.1	33 688.1	0.00	0	30 923.3	30 923.3	0.00	0
PIRP-10-2-1-3-5	32 834.3	32 834.3	0.00	0	32 834.3	32 834.3	0.00	0	25 874.1	25 874.1	0.00	0
PIRP-10-3-1-6-1	67 552.8	67 552.8	0.00	1	67 542.2	67 542.2	0.00	1	60 057.5	60 057.5	0.00	5
PIRP-10-3-1-6-2	53 367.7	53 367.7	0.00	1	53 330.5	53 330.5	0.00	2	43 119.8	43 119.8	0.00	1
PIRP-10-3-1-6-3	67 946.0	67 946.0	0.00	3	67 908.3	67 908.3	0.00	4	59 064.9	59 064.9	0.00	7
PIRP- 10-3-1-6-4	65 375.6	65 375.6	0.00	2	64 918.2	64 918.2	0.00	5	55 391.3	55 391.3	0.00	8
PIRP-10-3-1-6-5	54 180.1	54 180.1	0.00	5	54 176.5	54 176.5	0.00	4	44 555.6	44 555.6	0.00	158
PIRP-10-5-1-10-1	80 471.9	80 471.9	0.00	30	79 740.7	79 740.7	0.00	641	69 734.9	69 734.9	0.00	2000
PIRP-10-5-1-10-2	72 194.8	72 194.8	0.00	205	72 149.4	72 149.4	0.00	1018	66 685.6	67 738.8	1.57	7201
PIRP-10-5-1-10-3	101 043.0	101 043.0	0.00	427	100 508.0	102 792.0	2.27	7200	96 025.7	96 025.7	0.00	687
PIRP-10-5-1-10-4	82 829.4	82 829.4	0.00	28	82 336.3	82 336.3	0.00	196	73 296.0	73 296.0	0.00	1104
PIRP-10-5-1-10-5	68 935.7	68 935.7	0.00	361	68 657.0	69 301.6	0.93	7200	57 416.6	58 956.7	2.68	7201
PIRP-20-2-2-3-1	61 780.2	61 780.2	0.00	24	61 780.2	61 780.2	0.00	136	50 548.4	50 548.4	0.00	16
PIRP-20-2-2-3-2	75 757.3	75 757.3	0.00	1	75 753.0	75 753.0	0.00	2	64 271.6	64 271.6	0.00	2
PIRP-20-2-2-3-3	72 546.5	72 546.5	0.00	97	72 546.5	72 546.5	0.00	76	62 656.7	62 656.7	0.00	7
PIRP-20-2-2-3-4	52 850.8	52 850.8	0.00	14	52 842.2	52 842.2	0.00	36	42 008.2	42 008.2	0.00	4
PIRP-20-2-2-3-5	51 746.4	51 746.4	0.00	3	51 746.4	51 746.4	0.00	3	35 895.6	35 895.6	0.00	1
PIRP-20-3-2-6-1	110 437.0	112 517.0	1.88	7200	110 343.0	112 340.0	1.81	7200	87 287.3	90 277.8	3.42	7200
PIRP-20-3-2-6-2	133 377.0	136 382.0	2.25	7200	133 126.0	136 980.0	2.89	7201	117 342.0	120 768.0	2.91	7200
PIRP-20-3-2-6-3	106 120.0	108 651.0	2.38	7202	106 033.0	108 735.0	2.54	7200	91 104.1	94 204.4	3.40	7202
PIRP- 20-3-2-6-4	135 267.0	137 210.0	1.43	7200	134 850.0	137 238.0	1.77	7201	122 779.0	122 779.0	0.00	4306
PIRP-20-3-2-6-5	148 480.0	149 712.0	0.83	7201	148 395.0	149 662.0	0.85	7200	139 502.0	140 613.0	0.79	7202
PIRP-20-5-2-10-1	200 786.0	206 053.0	2.62	7202	200 646.0	206 351.0	2.84	7201	183 235.0	191 178.0	4.33	7202
PIRP-20-5-2-10-2	152 951.0	161 040.0	5.28	7201	153 008.0	160 525.0	4.91	7201	132 874.0	141 960.0	6.83	7202
PIRP-20-5-2-10-3	182 710.0	188 156.0	2.98	7200	182 683.0	188 459.0	3.16	7202	172 490.0	179 249.0	3.91	7201
PIRP-20-5-2-10-4	146 990.0	153 093.0	4.15	7202	146 316.0	153 154.0	4.67	7204	133 093.0	141 996.0	6.68	7201
PIRP-20-5-2-10-5	221 158.0	224 424.0	1.47	7202	221 003.0	224 387.0	1.53	7204	204 667.0	210 435.0	2.81	7202
PIRP-30-2-2-3-1	85 251.9	85 251.9	0.00	1101	84 740.0	86 178.2	1.69	7202	71 288.4	71 288.4	0.00	137
PIRP-30-2-2-3-2	94 711.4	94 711.4	0.00	114	94 633.3	94 633.3	0.00	118	75 580.3	75 580.3	0.00	13
PIRP-30-2-2-3-3	99 037.0	99 037.0	0.00	46	99 037.0	99 037.0	0.00	41	77 017.0	77 017.0	0.00	24
PIRP-30-2-2-3-4	113 737.0	113 737.0	0.00	12	113 737.0	113 737.0	0.00	80	91 090.1	91 090.1	0.00	13
PIRP-30-2-2-3-5	95 167.2	95 167.2	0.00	337	95 140.5	95 140.5	0.00	189	84 626.6	84 626.6	0.00	15
PIRP-30-3-2-6-1	190 666.0	196 082.0	2.84	7201	190 788.0	196 539.0	3.01	7201	176 515.0	181 188.0	2.64	7202
PIRP-30-3-2-6-2	195 358.0	196 565.0	0.61	7200	195 318.0	196 563.0	0.63	7201	177 796.0	179 130.0	0.75	7201
PIRP-30- 3-2-6-3	185 507.0	188 220.0	1.46	7200	185 153.0	188 306.0	1.70	7201	166 860.0	169 568.0	1.62	7202
PIRP-30-3-2-6-4	174 064.0	176 545.0	1.42	7201	174 029.0	176 540.0	1.44	7202	147 141.0	149 654.0	1.70	7201
PIRP-30-3-2-6-5	218 494.0	224 199.0	2.61	7203	218 991.0	224 188.0	2.37	7201	199 631.0	203 846.0	2.11	7202
PIRP-30-5-2-10-1	232 289.0	238 098.0	2.50	7201	230 669.0	237 852.0	3.11	7202	210 635.0	219 618.0	4.26	7201
PIRP-30-5-2-10-2	257 061.0	263 149.0	2.36	7201	256 029.0	262 717.0	2.61	7204	237 088.0	245 671.0	3.62	7202
PIRP-30-5-2-10-3	321 116.0	325 615.0	1.40	7202	320 413.0	326 010.0	1.74	7203	294 469.0	300 721.0	2.12	7202
PIRP-30-5-2-10-4	372 682.0	377 550.0	1.30	7201	372 132.0	377 642.0	1.48	7202	346 923.0	354 108.0	2.07	7203
PIRP-30-5-2-10-5	289 763.0	299 298.0	3.29	7202	288 474.0	298 017.0	3.30	7202	266 616.0	277 415.0	4.05	7202
PIRP-40-2-3-3-1	134 602.0	136 680.0	1.54	7200	133 995.0	136 662.0	1.99	7202	113 171.0	113 836.0	0.58	7200
PIRP-40-2-3-3-2	129 497.0	132 107.0	2.01	7205	129 618.0	132 099.0	1.91	7200	109 140.0	109 140.0	0.00	5353
PIRP-40-2-3-3-3	127 505.0	129 425.0	1.50	7202	127 438.0	129 524.0	1.63	7201	110 936.0	111 849.0	0.82	7201
PIRP-40-2-3-3-4	120 444.0	121 953.0	1.25	7200	120 255.0	121 919.0	1.38	7201	101 223.0	101 223.0	0.00	4292
PIRP-40-2-3-3-5	127 760.0	128 995.0	0.96	7200	127 601.0	129 128.0	1.19	7201	102 356.0	103 990.0	1.59	7201
PIRP-40-3-3-6-1	234 408.0	244 405.0	4.26	7201	233 405.0	244 450.0	4.73	7204	206 077.0	216 220.0	4.92	7202
PIRP- 40-3-3-6-2	216 410.0	223 526.0	3.28	7200	215 836.0	223 805.0	3.69	7203	194 909.0	201 572.0	3.41	7202

PIRP-40-3-3-6-3	262 138.0	268 344.0	2.36	7201	260 320.0	268 184.0	3.02	7203	239 755.0	246 071.0	2.63	7201
PIRP-40-3-3-6-4	257 099.0	265 133.0	3.12	7202	257 786.0	265 014.0	2.80	7201	230 334.0	237 589.0	3.14	7203
PIRP-40-3-3-6-5	282 124.0	289 109.0	2.47	7202	281 599.0	289 187.0	2.69	7203	261 346.0	268 376.0	2.68	7202
PIRP-50-2-3-3-1	171 373.0	174 079.0	1.57	7201	170 394.0	174 429.0	2.36	7201	140 191.0	142 451.0	1.61	7203
PIRP-50-2-3-3-2	188 028.0	189 710.0	0.87	7202	187 998.0	1897 18.0	0.91	7203	160 553.0	162 094.0	0.95	7202
PIRP-50-2-3-3-3	182 640.0	184 651.0	1.10	7202	182 079.0	184 624.0	1.39	7204	156 807.0	158 018.0	0.77	7202
PIRP-50-2-3-3-4	166 408.0	170 039.0	2.18	7202	166 458.0	170 129.0	2.20	7203	134 964.0	137 487.0	1.86	7204
PIRP-50-2-3-3-5	177 338.0	180 143.0	1.58	7202	175 681.0	180 209.0	2.57	7202	146 842.0	149 333.0	1.69	7203

monotonically decreasing revenue functions, the value of the solution obtained under the OP is reduced when an OF policy is applied, but the decrease is only marginal under an FF policy. Extensive computational experiments carried out on randomly generated instances support these conclusions.

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Appendix A. Detailed computational results for the OP, FF and OF policies

We present in Table 6 the detailed computational results for all instances under the OP, the FF and the OF policies.

References

[1] Aelion V, Castells F, Veroutis A. Life cycle inventory analysis of chemical processes. Environ Prog 2006;14(3):193–200.
[2] Andersson H, Hoff A, Christiansen M, Hasle G, Løkketangen A. Industrial aspects and literature survey: combined inventory management and routing. Comput Oper Res 2010;37(9):1515–36.
[3] Axsäter S. Inventory control. In: International series in operations research & management science, vol. 90. New York: Springer; 2006.
[4] Blackburn J, Scudder G. Supply chain strategies for perishable products: the case of fresh produce. Prod Oper Manag 2009;18(2):129–37.
[5] Brodheim E, Derman C, Prastacos GP. On the evaluation of a class of inventory policies for perishable products such as blood. Manag Sci 1975;21(11):1320–5.
[6] Coelho LC, Laporte G. The exact solution of several classes of inventory-routing problems. Comput Oper Res 2013;40(2):558–65.
[7] Coelho LC, Laporte G. A branch-and-cut algorithm for the multi-product multi-vehicle inventory-routing problem. Int J Prod Res 2013;51(23–24):7156–69.
[8] Coelho LC, Cordeau J-F, Laporte G. Thirty years of inventory-routing. Transp Sci 2014;48(1):1–19.
[9] Cumming PD, Kendall KE, Pegels CC, Seagle JP, Shubsda JF. A collections planning model for regional blood suppliers: description and validation. Manag Sci 1976;22(9):962–71.
[10] Custódio A, Oliveira R. Redesigning distribution operations: a case study on integrating inventory management and vehicle routes design. Int J Logist 2006;9(2):169–87.
[11] Emmons H. A replenishment model for radioactive nuclide generators. Manag Sci 1968;14(5):263–74.
[12] Farahani P, Grunow M, Günther H-O. Integrated production and distribution planning for perishable food products. Flex Serv Manuf J 2012;24(1):28–51.
[13] Gendreau M, Laporte G, Semet F. The covering tour problem. Oper Res 1997;45(4):568–76.
[14] Gumasta K, Chan FTS, Tiwari MK. An incorporated inventory transport system with two types of customers for multiple perishable goods. Int J Prod Econ 2012;139(2):678–86.
[15] Haijema R. A new class of stock-level dependent ordering policies for perishables with a short maximum shelf life. Int J Prod Econ 2013;143(2):434–9.
[16] Hemmelmayr V, Doerner KF, Hartl RF, Savelsbergh MWP. Vendor managed inventory for environments with stochastic product usage. Eur J Oper Res 2010;202(3):686–95.
[17] Jagannathan R, Sen T. Storing crossmatched blood: a perishable inventory model with prior allocation. Manag Sci 1991;37(3):251–66.
[18] Jennings JB. Blood bank inventory control. Manag Sci 1973;19(6):637–45.
[19] Karaesmen IZ, Scheller-Wolf A, Deniz B. Managing perishable and aging inventories. In: Kempf KG, Keskinocak P, Uzsoy R, editors. Planning production and inventories in the extended enterprise. International series in operations research & management science, vol. 151. New York: Springer; 2011. p. 393–436.
[20] Kendall KE, Lee SM. Formulating blood rotation policies with multiple objectives. Manag Sci 1980;26(11):1145–57.
[21] Laporte G. Fifty years of vehicle routing. Transp Sci 2009;43(4):408–16.
[22] Le T, Diabat A, Richard J-P, Yih Y. A column generation-based heuristic algorithm for an inventory routing problem with perishable goods. Optim Lett 2013;7(7):1481–502.
[23] Mercer A, Tao X. Alternative inventory and distribution policies of a food manufacturer. J Oper Res Soc 1996;47(6):755–65.

[24] Minner S, Transchel S. Periodic review inventory-control for perishable products under service-level constraints. OR Spectr 2010;32(4):979–96.
[25] Nagurney A, Masoumi AH. Supply chain network design of a sustainable blood banking system. In: Boone T, Jayaraman V, Ganeshan R, editors. Sustainable supply chains international series in operations research & management science, vol. 174. New York: Springer; 2012. p. 49–72.
[26] Nagurney A, Masoumi AH, Yu M. Supply chain network operations management of a blood banking system with cost and risk minimization. Comput Manag Sci 2012;9(2):205–31.
[27] Nagurney A, Yu M, Masoumi AH, Nagurney LS. Networks against time—supply chain analytics for perishable products. New York: Springer; 2013.
[28] Nagurney A, Yu M, Masoumi AH, Nagurney LS. Fast fashion apparel supply chains. In: Networks against time—supply chain analytics for perishable products. New York: Springer; 2013. p. 117–39.
[29] Nagurney A, Yu M, Masoumi AH, Nagurney LS. Food supply chains. In: Networks against time—supply chain analytics for perishable products. New York: Springer; 2013. p. 65–88.
[30] Nahmias S. Perishable inventory systems. International series in operations research & management science, vol. 160. New York: Springer; 2011.
[31] Osvald A, Stirn LZ. A vehicle routing algorithm for the distribution of fresh vegetables and similar perishable food. J Food Eng 2008;85(2):285–95.
[32] Padberg MW, Rinaldi G. A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems. SIAM Rev 1991;33(1):60–100.
[33] Pegels CC, Jelmert AE. An evaluation of blood-inventory policies: a Markov chain application. Oper Res 1970;18(6):1087–98.
[34] Raafat F. Survey of literature on continuously deteriorating inventory models. J Oper Res Soc 1991;42(1):27–37.
[35] Singh SP, Chonhenchob V, Singh J. Life cycle inventory and analysis of re-usable plastic containers and display-ready corrugated containers used for packaging fresh fruits and vegetables. Packag Technol Sci 2006;19(5):279–93.
[36] van Donselaar K, van Woensel T, Broekmeulen RACM, Fransoo J. Inventory control of perishables in supermarkets. Int J Prod Econ 2006;104(2):462–72.
[37] Yang X. Choosing transportation alternatives for highly perishable goods: a case study on nuclear medicine [Ph.D. thesis]. Massachusetts Institute of Technology; 2006.