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Coupled axisymmetric vibration of nonlocal fluid-filled closed spherical membrane shell

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Abstract In this paper, the axisymmetric vibration of a fluid-filled spherical membrane shell is studied based on nonlocal elasticity theory. The membrane shell is considered elastic, homogeneous and isotropic. The shell model is reformulated using the nonlocal differential constitutive relations of Eringen. The membrane shell is completely filled with an inviscid fluid. The motion of the fluid is governed by the wave equation. Nonlocal governing equations of motion for the fluid-filled spherical membrane shell are derived. Along the contact surface between the membrane and the fluid, the compatibility requirement is applied and Legendre polynomials, associated Legendre polynomials and spherical Bessel functions are used to obtain the natural frequencies of the fluid-filled spherical membrane shells. The frequencies for both empty and fluid-filled spherical membrane shell are evaluated, and their comparisons are performed to confirm the validity and accuracy of the proposed method. An excellent agreement is found between the present and previous ones available in the literature. The variations of the natural frequencies with the small-scale parameter, density ratio, wave speed ratio and Poisson's ratio are also examined. It is observed that the frequencies are affected when the size effect is taken into consideration.

1 Introduction

Free vibration of a spherical shell is one of the basic elastodynamic problems. Most spherical shell systems operate in complex environments that are coupled with an inner or outer fluid. It is well known that the dynamic behavior of a fluid-filled spherical shell will be different from uncoupled ones. Moreover, the dynamics of the fluid-filled spherical shell is of technological importance in some modern industrial, biomedical, biological and many other applications [1]. Therefore, the dynamic behavior of the spherical shells filled with fluid is of substantial practical interest and has been widely investigated. Rayleigh [2] solved the problem of axisymmetric vibrations of a fluid in a rigid spherical shell. The solution for vibrations of the fluid-filled spherical membrane appears in [3]. Frequency equations and mode shapes have been obtained analytically for the axisymmetric, extensional and nontorsional vibrations of the fluid-filled elastic spherical shells and rigid prolate spheroidal shells [4]. Motivated by the fact that the human head can be represented as a shell filled with fluid, Engin and Liu [5] considered the free vibrations of a thin homogeneous spherical shell containing an inviscid irrotational fluid. Advani and Lee [6] investigated the vibration of the fluid-filled shell using higher-order shell theory including transverse shear and rotational inertia. Guarino and Elger [7] have looked at the frequency spectra of a fluid-filled sphere, both with and without a central solid sphere, in order to explore the use of auscultatory percussion as a clinical diagnostic tool. The general nonaxisymmetric free vibration of a spherically isotropic

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elastic spherical shell filled with a compressible fluid medium has been investigated by Chen and Ding [8]. Exact frequency equations have been presented for a piezoceramic spherical shell submerged in a compressible fluid medium [9]. The general solutions for the spherical-symmetric steady-state response problem of the fluid-filled piezoelectric spherical shell have been presented in Ref. [10]. Young [11] studied the free vibration of spheres composed of inviscid compressible liquid cores surrounded by spherical layers of linear elastic, homogeneous and isotropic materials. In another work, the dynamical behavior of a gas-filled neo-Hookean spherical shell surrounded by a Newtonian fluid has been studied for the special case of spherically symmetric motions [12]. Recently, axisymmetric vibrations of a hollow piezoelectric sphere submerged in a compressible viscous fluid medium have been studied [13]. More recently, axisymmetric vibrations of a viscous fluid-filled piezoelectric sphere, with radial polarization, submerged in a compressible viscous fluid medium have been investigated by Hu et al. [1].

In recent years, size-dependent theories of continuum mechanics have attracted attention because of the necessity of modeling and analysis of very small-sized mechanical structures and devices in the rapid developments of micro-/nanotechnologies. One of the well-known models is the nonlocal elasticity theory [14–16]. It seems that this theory could potentially play a useful role in analysis related to nanotechnology applications. Therefore, several researchers have applied the nonlocal elasticity theory for the mechanical analysis of micro- and nanostructures in more recent years [17–25]. However, most of these studies have focused on beam-like, plate-like and cylindrical shell-like structures.

In various modern biomedical and biological applications, some components such as micro-/nanosized spherical shells, which are used as targeted drug delivery systems [26], biological cells, which are hollow spherical membranes filled with liquid [27], and spherical viruses can be modeled as a fluid-filled spherical membrane structure. For this purpose, the objective of the present study is to include the effect of small scale on the fluid–structure interactions by investigating the axisymmetric vibrations of the fluid-filled spherical membrane shell. Governing equations of the spherical membrane shell are reformulated using the nonlocal differential constitutive relations pioneered by Eringen [15,16]. It is assumed that the shell is completely filled with a compressible and inviscid fluid. Therefore, the motion of the fluid is governed by the wave equation. The governing equations of the membrane shell and inner compressible nonviscous fluid are coupled through the interface continuity conditions. The interface conditions for a shell joined to an acoustic medium are (i) the normal pressure load on the shell must be equal to the boundary pressure of the fluid and (ii) the normal velocity of the shell surface has to be equal to the normal velocity component of the fluid boundary. Using Legendre polynomials, associated Legendre polynomials and spherical Bessel functions, the coupled axisymmetric vibration of the fluid-filled spherical membrane shell with considering the small-scale effect is obtained in the form of a frequency equation. To validate the accuracy of solutions, the results are compared with those found in the literature. Furthermore, numerical results in the form of frequency spectra are presented for different material and small-scale parameters.

2 Formulation of the problem

2.1 Review of nonlocal elasticity theory

For nonlocal linear elastic solids, the equations of motion have the form [15,16]

$$t_{ij,j} + f_i = \rho \ddot{u}_i, \tag{1}$$

where ρ and f_i are the mass density and the body or applied force density, respectively. u_i is the displacement vector, and t_{ij} is the stress tensor of the nonlocal elasticity, which is defined by

$$t_{ij}(x) = \int_V \alpha(\left|x' - x\right|, \chi)\sigma_{ij}(x')dV(x'), \tag{2}$$

where x is a reference point in the body and $\alpha(\left|x' - x\right|, \chi)$ is the nonlocal modulus or attenuation function whose arguments are the Euclidean distance $\left|x' - x\right|$ and material constant $\chi = e_0 a/l$. e_0 is a nonlocal scaling parameter that has been assumed as a constant appropriate to each material, a and l are the internal and external characteristic lengths, respectively. σ_{ij} is the local stress tensor in classical elasticity theory at any point x' in the body, which satisfies the constitutive relations

$$\sigma_{ij} = C_{ijkl} e_{kl}, \tag{3}$$

in which C_{ijkl} are the elastic modulus components and e_{kl} is the strain tensor. The nonlocal modulus is found by matching the curves of plane waves with those due to atomic lattice dynamics. Various different forms of $\alpha(\left|x'-x\right|,\chi)$ and their properties have been discussed in detail by Eringen [15,28]. When $\alpha(|x|)$ takes on a Green's function of a linear differential operator L, that is

$$L\alpha\left(\left|x'-x\right|\right)=\delta\left(\left|x'-x\right|\right), \tag{4}$$

the nonlocal constitutive relation (2) is reduced to the differential equation

$$Lt_{ij}=\sigma_{ij}, \tag{5}$$

and the integro-partial differential equation (1) is correspondingly reduced to the partial differential equation

$$\sigma_{ij,j}+L(f_i-\rho\ddot{u}_i)=0. \tag{6}$$

By matching the dispersion curves with lattice models, Eringen [15,16] proposed a nonlocal model with the linear differential operator L defined by

$$L=1-(e_0a)^2\nabla^2, \tag{7}$$

where ∇^2 is the Laplace operator. Therefore, according to Eqs. (3), (5) and (7), the constitutive relations may be simplified to

$$\left(1-(e_0a)^2\nabla^2\right)t_{ij}=C_{ijkl}e_{kl}. \tag{8}$$

It should be noted that Eq. (8) has been widely adopted for tackling various problems of linear elasticity and micro-/nanostructural mechanics.

2.2 Spherical membrane shell equations based on nonlocal elastic model

Spherical shells may vibrate in both axisymmetric and nonaxisymmetric modes. The axisymmetric modes are independent of the circumferential coordinate (θ), whereas the nonaxisymmetric modes depend upon both meridional (φ) and circumferential coordinates. The nonaxisymmetric modes are degenerate, meaning that the nonaxisymmetric frequencies are identical to corresponding axisymmetric modes [29]. Hence, the axisymmetric modes are studied here. Considering the axisymmetric torsionless motion of the spherical membrane shell of median radius R and denoting the meridional (φ-direction) and radial (r-direction) displacements at the median radius, as u and w, respectively, it can be shown that the strain–displacement relations are written as

$$t_{\varphi\varphi}-\left(\frac{e_0a}{R}\right)^2\left(\frac{\partial^2 t_{\varphi\varphi}}{\partial\varphi^2}+\cot\varphi\frac{\partial t_{\varphi\varphi}}{\partial\varphi}\right)=\frac{E}{1-\nu^2}\left(e_{\varphi\varphi}+\nu e_{\theta\theta}\right), \tag{9}$$

$$t_{\theta\theta}-\left(\frac{e_0a}{R}\right)^2\left(\frac{\partial^2 t_{\theta\theta}}{\partial\varphi^2}+\cot\varphi\frac{\partial t_{\theta\theta}}{\partial\varphi}\right)=\frac{E}{1-\nu^2}\left(e_{\theta\theta}+\nu e_{\varphi\varphi}\right), \tag{10}$$

where E is the elastic modulus of the spherical shell and ν denotes Poisson's ratio. Moreover, the strains are expressed as

$$\begin{aligned} e_{\varphi\varphi}&=\frac{1}{R}\left(\frac{\partial u}{\partial\varphi}+w\right),\\ e_{\theta\theta}&=\frac{1}{R\sin\varphi}\left(u\cos\varphi+w\sin\varphi\right). \end{aligned} \tag{11}$$

In the nonlocal elastic shell model, the stress resultants are defined based on the stress components in Eqs. (9) and (10), and thus can be expressed as follows by referencing the kinematic relations:

$$\begin{aligned} N_{\varphi\varphi}&=\int_{-\frac{h}{2}}^{\frac{h}{2}} t_{\varphi\varphi}\left(1+\tfrac{z}{R}\right)\mathrm{d}z,\\ \text{i.e. } N_{\varphi\varphi}-\left(\tfrac{e_0a}{R}\right)^2\left(\tfrac{\partial^2 N_{\varphi\varphi}}{\partial\varphi^2}+\cot\varphi\tfrac{\partial N_{\varphi\varphi}}{\partial\varphi}\right)&=\tfrac{Eh}{R(1-\nu^2)}\left[\tfrac{\partial u}{\partial\varphi}+w+\nu\left(u\cot\varphi+w\right)\right], \end{aligned} \tag{12}$$

$$N_{\theta\theta} = \int_{-\frac{h}{2}}^{\frac{h}{2}} t_{\theta\theta}\mathrm{d}z,$$
$$\text{i.e. } N_{\theta\theta} - \left(\frac{e_0 a}{R}\right)^2 \left(\frac{\partial^2 N_{\theta\theta}}{\partial\varphi^2} + \cot\varphi \frac{\partial N_{\theta\theta}}{\partial\varphi}\right) = \frac{Eh}{R(1-\nu^2)}\left[u\cot\varphi + w + \nu\left(\frac{\partial u}{\partial\varphi} + w\right)\right], \tag{13}$$

where h is the wall thickness of the spherical shell. It should be noted that the principle of virtual work is independent of the constitutive relations. So this can be applied to derive the equilibrium equations of the nonlocal spherical shell. Using the principle of virtual displacements, following governing equations can be obtained:

$$\frac{\partial N_{\varphi\varphi}}{\partial\varphi} + (N_{\varphi\varphi} - N_{\theta\theta})\cot\varphi = R\rho_s h\frac{\partial^2 u}{\partial t^2}, \tag{14}$$

$$-(N_{\varphi\varphi} + N_{\theta\theta}) + Rq = R\rho_s h\frac{\partial^2 w}{\partial t^2}, \tag{15}$$

in which $\rho_s h$ is the mass density per unit lateral area of the spherical shell and q is normal pressure load. Substituting Eqs. (12) and (13) into (14) and (15), the equations of motion for the spherical shell in terms of meridional and radial displacements of the mean surface of the spherical shell, (u, w), are obtained as

$$\frac{\partial^2 u}{\partial\varphi^2} + \frac{\partial u}{\partial\varphi}\cot\varphi - (\nu + \cot^2\varphi)u + (1+\nu)\frac{\partial w}{\partial\varphi}$$
$$= \frac{\rho_s(1-\nu^2)R^2}{E}\left[\frac{\partial^2 u}{\partial t^2} - \left(\frac{e_0 a}{R}\right)^2\left(\frac{\partial^4 u}{\partial t^2\partial\varphi^2} + \frac{\partial^3 u}{\partial t^2\partial\varphi}\cot\varphi - (1+\cot^2\varphi)\frac{\partial^2 u}{\partial t^2}\right)\right], \tag{16}$$

$$\frac{\partial u}{\partial\varphi} + u\cot\varphi + 2w = -\frac{\rho_s(1-\nu)R^2}{E}\left[\frac{\partial^2 w}{\partial t^2} - \left(\frac{e_0 a}{R}\right)^2\left(\frac{\partial^4 w}{\partial t^2\partial\varphi^2} + \frac{\partial^3 w}{\partial t^2\partial\varphi}\cot\varphi\right)\right]$$
$$+\frac{(1-\nu)R^2}{Eh}\left[q - \left(\frac{e_0 a}{R}\right)^2\left(\frac{\partial^2 q}{\partial\varphi^2} + \frac{\partial q}{\partial\varphi}\cot\varphi\right)\right]. \tag{17}$$

From relations (16) and (17), it is easily seen that the classical or local shell theory is recovered if the parameter $e_0 a$ is set to zero.

2.3 Fluid equation

The motion of an inviscid and irrotational fluid undergoing small oscillations is governed by the wave equation. In spherical coordinates, the wave equation can be written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\varphi}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial\Phi}{\partial\varphi}\right) = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}, \tag{18}$$

where Φ is the velocity potential and c is the speed of sound in the fluid. The velocity potential function is related to pressure through the relationship

$$P = -\rho_f\frac{\partial\Phi}{\partial t}, \tag{19}$$

where ρ_f is the density of the fluid.

2.4 Fluid–structure interaction

The interface conditions for a shell joined to a fluid are that the normal pressure load, q, on the shell has to be equal to the boundary pressure, P, of the fluid:

$$q(\varphi, t) = P(R, \varphi, t), \tag{20}$$

and the velocity potential and the radial displacement are interconnected through the kinematic boundary condition, that is the radial velocities of the shell and normal velocity component of the fluid boundary are equal:

$$\frac{\partial w}{\partial t}(\varphi, t) = \frac{\partial\Phi}{\partial r}(R, \varphi, t). \tag{21}$$

2.5 Frequency equation

To simplify the analysis, the following nondimensional quantities are defined:

$$\begin{aligned} &\psi = \frac{u}{R} \quad \xi = \frac{w}{R} \quad \tau = \frac{c_s t}{R} \quad c_s = \sqrt{\frac{E}{\rho_s(1-\nu^2)}} \quad s = \frac{c}{c_s} \\ &\Phi_1 = \frac{\Phi}{Rc_s} \quad f = \frac{\rho_f R}{\rho_s h} \quad r_1 = \frac{r}{R} \quad \Omega = \frac{\omega R}{c} \quad \mu = \frac{e_0 a}{R}. \end{aligned} \tag{22}$$

Using Eq. (20) and substituting from Eq. (22) into (16)–(18) and (21) yields:

$$\frac{\partial^2\psi}{\partial\varphi^2} + \frac{\partial\psi}{\partial\varphi}\cot\varphi - (\nu+\cot^2\varphi)\psi + (1+\nu)\frac{\partial\xi}{\partial\varphi} = \left[\frac{\partial^2\psi}{\partial\tau^2} - \mu^2\left(\begin{array}{l}\frac{\partial^4\psi}{\partial\tau^2\partial\varphi^2} + \frac{\partial^3\psi}{\partial\tau^2\partial\varphi}\cot\varphi \\ -(1+\cot^2\varphi)\frac{\partial^2\psi}{\partial\tau^2}\end{array}\right)\right], \tag{23}$$

$$\begin{aligned} (1+\nu)\left[\frac{\partial\psi}{\partial\varphi} + \psi\cot\varphi + 2\xi\right] = &-\left[\frac{\partial^2\xi}{\partial\tau^2} - \mu^2\left(\frac{\partial^4\xi}{\partial\tau^2\partial\varphi^2} + \frac{\partial^3\xi}{\partial\tau^2\partial\varphi}\cot\varphi\right)\right] \\ &-f\left[\frac{\partial\Phi_1}{\partial\tau}(1,\varphi,t) - \mu^2\left(\frac{\partial^3\Phi_1}{\partial\tau\partial\varphi^2}(1,\varphi,t) + \frac{\partial^2\Phi_1}{\partial\tau\partial\varphi}(1,\varphi,t)\cot\varphi\right)\right], \end{aligned} \tag{24}$$

$$\frac{1}{r_1^2}\frac{\partial}{\partial r_1}\left(r_1^2\frac{\partial\Phi_1}{\partial r_1}\right) + \frac{1}{r_1^2\sin\varphi}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial\Phi_1}{\partial\varphi}\right) = \frac{1}{s^2}\frac{\partial^2\Phi_1}{\partial\tau^2}, \tag{25}$$

$$\frac{\partial\xi}{\partial\tau}(\varphi,\tau) = \frac{\partial\Phi_1}{\partial r_1}(1,\varphi,\tau). \tag{26}$$

Separating the wave equation (25) yields $\Phi_1(r_1,\varphi,\tau) = H(r_1)G(\varphi)\exp(i\Omega s\tau)$, with

$$\frac{1}{\sin\varphi}\frac{d}{\mathrm{d}\varphi}\left(\sin\varphi\frac{dG}{\mathrm{d}\varphi}\right) + n(n+1)G = 0, \tag{27}$$

$$\frac{1}{r_1^2}\frac{d}{dr_1}\left(r_1^2\frac{dH}{dr_1}\right) + \left[\Omega^2 - \frac{n(n+1)}{r_1^2}\right]H = 0, \tag{28}$$

where n is integer. Equation (27) is the self-adjoint from of Legendre's equation, and the spherical Bessel functions are the solution of Eq. (28). The requirement for boundedness and the linearity of (25) leads to its general solution:

$$\Phi_1(r_1,\varphi,\tau) = \sum_{n=0} C_n j_n(\Omega r_1)P_n(\cos\varphi)\exp(i\Omega s\tau), \tag{29}$$

when C_n are unknown coefficients, $j_n(\Omega r_1)$ are spherical Bessel functions of the first kind and $P_n(\cos\varphi)$ are Legendre polynomials of the first kind [3]. Now, let us consider the two partial differential equations (23) and (24). In order to reduce these equations to a pair of equivalent algebraic equations, assume the following expansions for ξ and ψ:

$$\xi(\varphi,\tau) = \sum_{n=0} A_n P_n(\cos\varphi)\exp(i\Omega s\tau), \tag{30}$$

$$\psi(\varphi,\tau) = \sum_{n=1} B_n P_n^1(\cos\varphi)\exp(i\Omega s\tau), \tag{31}$$

where A_n and B_n are coefficients and $P_n^1(\cos\varphi)$ are the associated Legendre polynomials of the first kind and first order. Substitution of equations (29) and (30) into the boundary condition (26) yields the coefficients C_n. These coefficients, for each integer value of n, are

$$C_n = \frac{is}{\frac{dj_n(\Omega)}{\mathrm{d}\Omega}}A_n. \tag{32}$$

Substituting Eqs. (29)–(32) into Eqs. (23) and (24) results in two sets of homogeneous algebraic equations for A_n and B_n. It should be noted that the higher-order derivatives of $P_n(\cos\varphi)$ and $P_n^1(\cos\varphi)$ have been eliminated by the recursive use of the differential equations that they satisfy:

$$\frac{d^2 P_n}{\mathrm{d}\varphi^2} + \cot\varphi \frac{dP_n}{\mathrm{d}\varphi} + n(n+1)P_n = 0, \tag{33}$$

$$\frac{d^2 P_n^1}{\mathrm{d}\varphi^2} + \cot\varphi \frac{dP_n^1}{\mathrm{d}\varphi} + (n(n+1) - (1+\cot^2\varphi))P_n^1 = 0. \tag{34}$$

For the nontrivial solution, the determinant of this set of equations must be zero. Hence, the frequency equation is obtained as below:
For $n = 0$

$$2(1+\nu) - \left(1 + f\frac{j_0(\Omega)}{\Omega\frac{dj_0(\Omega)}{\mathrm{d}\Omega}}\right)\Omega^2 s^2 = 0. \tag{35}$$

For $n \geq 1$

$$\begin{aligned}
&\Omega^4 s^4 (1+\mu^2 n(n+1))^2 \left(1 + f\frac{j_n(\Omega)}{\Omega\frac{dj_n(\Omega)}{\mathrm{d}\Omega}}\right) \\
&- \Omega^2 s^2 (1+\mu^2 n(n+1)) \left\{2(1+\nu) - \left(1 + f\frac{j_n(\Omega)}{\Omega\frac{dj_n(\Omega)}{\mathrm{d}\Omega}}\right)[(1-\upsilon) - n(n+1)]\right\} \\
&- \left\{2(1+\nu)[(1-\upsilon) - n(n+1) + (1+\nu)^2 n(n+1)]\right\} = 0.
\end{aligned} \tag{36}$$

It should be noted that by setting $f = 0$ in (36), the frequency equation of an empty closed spherical membrane shell is obtained. For $f = 0$, it was shown that (35) and (36) can be written as polynomial equations, so there exists a finite number of roots for each mode. However, for a membrane shell containing compressible and inviscid fluid, the situation is quite different. Since the frequency equations (35) and (36) contain spherical Bessel's functions of the first kind that involve circular transcendental functions, there exists an infinite number of roots for each mode.

In general, the frequency equations (35) and (36), being nonlinear equations involving transcendental functions, cannot be easily solved analytically. However, the roots of frequency equations can easily be solved numerically. In the next Section, the numerical results are given to present a better understanding of the coupled axisymmetric vibration of the fluid-filled spherical membrane shell.

3 Numerical results and discussion

The reliability of the present formulation for spherical membrane shells is checked by extensive comparison with the existing results in the literature. First, let us discuss the case of $f = 0$ corresponds to the absence of the fluid. Setting $f = 0$, $\mu = 0$ and defining a new nondimensional frequency $\bar{\Omega} = \Omega s$ in (36) yields the local frequency equation of the empty membrane shell. Table 1 compares the present results for an empty local spherical membrane shell and those obtained earlier by Nayfeh and Arafat [30]. Excellent agreement between the present solutions and those given in Ref. [30] is achieved.

In addition, in an attempt to demonstrate the relevance of the present elastic shell model for the coupled vibration of the fluid-filled spherical membrane shell, the frequency spectrum predicted by the present model

Table 1 Comparison of frequency parameters $\bar{\Omega} = \Omega s$ for spherical membrane shell when $\nu = 0.3$

Source of results	Mode sequence number						
	1	2	3	4	5	6	7
Present							
Lower branch	0	0.70093	0.82995	0.88057	0.90544	0.91948	0.92818
Upper branch	1.97484	2.72188	3.63472	4.59615	5.57496	6.56160	7.55238
Nayfeh and Arafat [30]							
Lower branch	0	0.70096	0.83006	0.88093	0.90630	0.92124	0.93139
Upper branch	1.97484	2.72190	3.63474	4.59617	5.57499	6.56163	7.55242

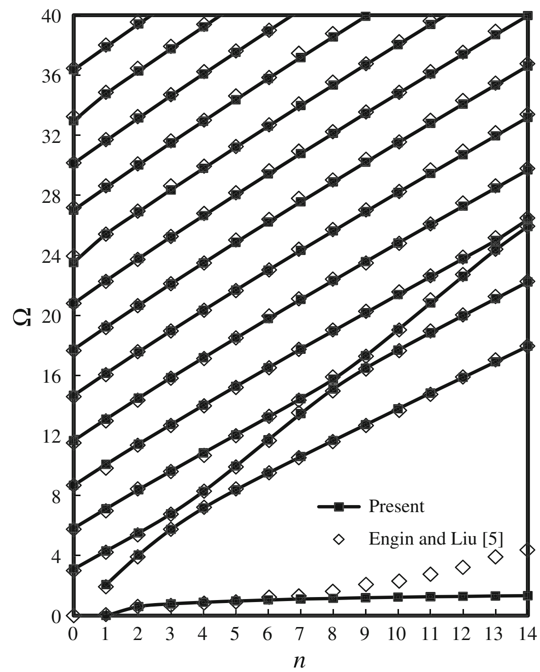

Fig. 1 The comparison between the results of present model and those given by Engin and Liu [5] for frequency spectrum of water-filled spherical bone shell

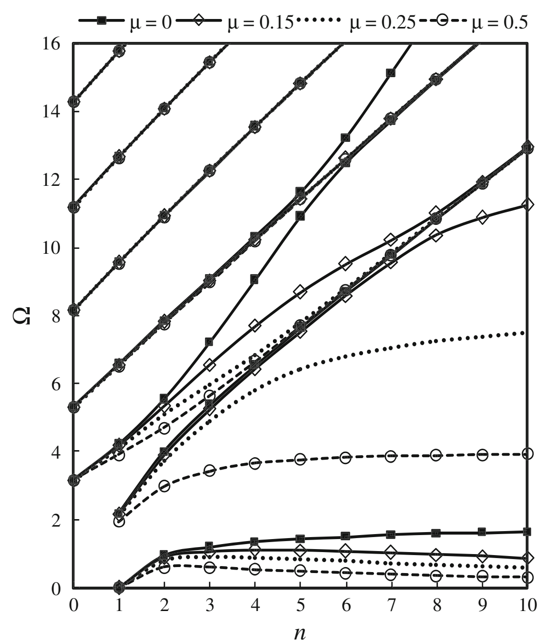

Fig. 2 Comparisons of frequency spectrum with different values of the small-scale parameter

(35) and (36) with local condition, $\mu = 0$, for a fluid-filled spherical membrane shell with $\nu = 0.3$ and f equal to 9.38 (bone-water) is plotted in Fig. 1 with comparison to the results given by Engin and Liu [5] based on the classical elastic model. It is to be noted that this and all other spectra that are plotted in this paper are discrete, that is only those points corresponding to integer values of the n are physically meaningful. It can be seen that the predicted numerical results by the present model are in reasonable agreement with the results reported in literature.

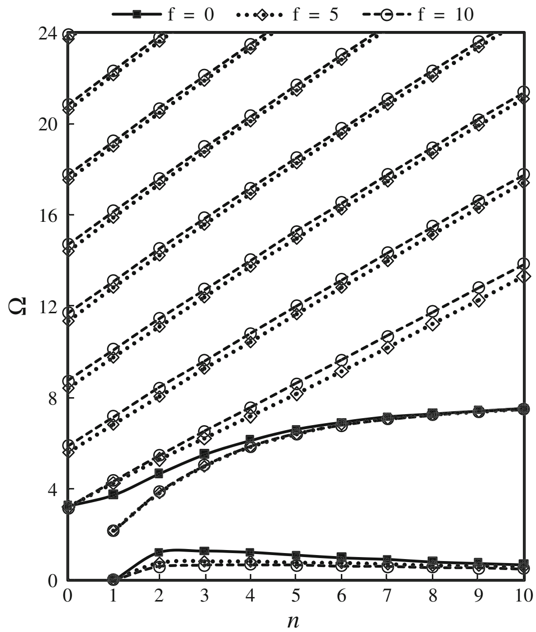

Fig. 3 Comparisons of frequency spectrum with different values of density ratio f

The effect of the dimensionless small-scale parameter, μ, on the frequencies of the fluid-filled spherical membrane shell given by the present model for $s = 0.5$, $\nu = 0.3$ and $f = 3$ as a function of the mode number is shown in Fig. 2. Our results from Fig. 2 show that the frequencies are not very sensitive to changes of the small-scale parameter for higher frequency branches ($\Omega > 10$). For lower branches, all frequencies decrease with increasing small-scale parameter, and the rate of decrease of the frequencies are not the same for all branches. Actually, the nonlocal theory introduces a more flexible model as the nanostructures can be viewed as atoms linked by elastic springs while the local model assumes spring constants to take on an infinite value. Consequently, the frequency reduction in the nonlocal model is physically justifiable.

To elucidate the effect of density ratio, the frequency spectrum is plotted for $s = 0.5$, $\nu = 0.3$ and $\mu = 0.25$ in Fig. 3. It is observed that there are an upper branch and a lower branch of the frequency spectrum in absence of the fluid. In addition, the coupled frequencies of the two lower branches decrease as the density ratio increases due to the added mass effect. On the contrary, the situation is reverse for higher branches. Moreover, it can be seen from Fig. 3 that the frequencies of the second lower branch are not very sensitive to changes of the density ratio for higher mode numbers ($n > 7$).

The influence of Poisson's ratio on the coupled natural frequencies of the fluid-filled spherical membrane shell is shown in Fig. 4. Numerical results in this figure have been calculated for the following particular values of different parameters: $s = 0.25$, $f = 3$ and $\mu = 0.1$. For a given mode number, the coupled frequencies of two lower branches decrease as Poisson's ratio increases. Moreover, the frequencies of third lower branch are not sensitive to Poisson's ratio. It is also observed that the frequencies of the higher branches increase with increasing Poisson's ratio.

The effect of the wave speed ratio, s, is studied as the final numerical example. Figure 5 shows the variation of the coupled frequencies with two different values of wave speed ratio $s = 0.25$ and 0.5, $f = 2.5$, $\nu = 0.3$ and $\mu = 0$ as function of mode number. It is observed that by increasing the wave speed ratio, frequencies have a decreasing trend. Moreover, the frequency spectrum is distorted when the value of the wave speed ratio is changed.

4 Conclusion

The influence of small scale on the coupled axisymmetric vibration of the fluid-filled spherical membrane shell was studied in this paper. In spite of some achievement in vibration analysis of the fluid-filled spherical membrane shell, to the authors' knowledge, there has been no attempt to tackle the problem described in the present investigation. Developing the nonlocal elastic shell model is the main contribution of the present paper.

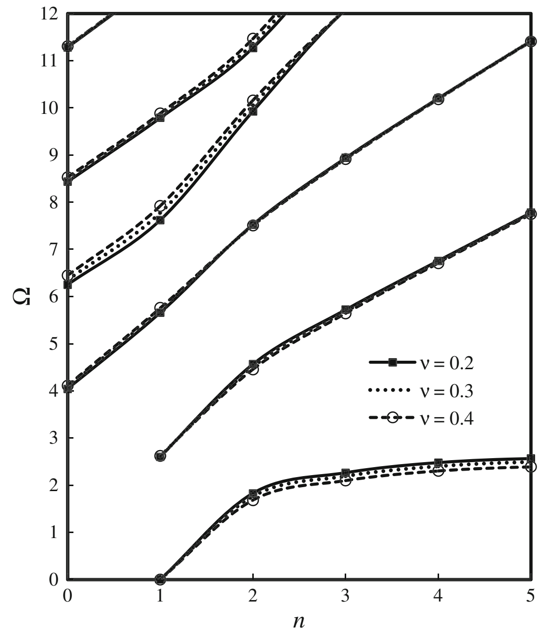

Fig. 4 Frequency spectrum for various values of Poisson's ratio with $s = 0.25$, $f = 3$ and $\mu = 0.1$

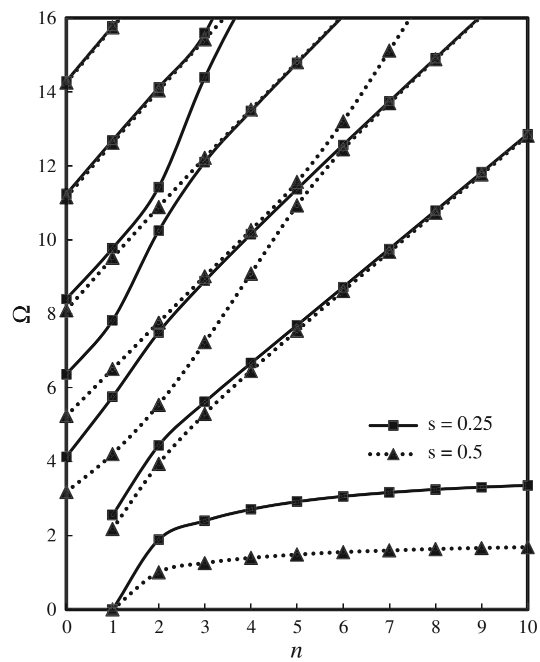

Fig. 5 Frequency spectrum for two different values of wave speed ratio $s = 0.25$ and 0.5, $f = 2.5$, $\nu = 0.3$ and $\mu = 0$

The validity of the obtained results was successfully verified through comparison with data available in the literature. The main results of the present work are summarized as follows.

(1) For a membrane shell containing the compressible and inviscid fluid, there exist an infinite number of frequencies for each mode.
(2) For lower frequency branches, all frequencies decrease with increasing small-scale parameter, and the rate of decrement is not identical for all branches.
(3) The coupled frequencies of two lower branches decrease as the density ratio increases due to the added mass effect.

Finally, it is hoped that the results proposed in this investigation would be helpful for the design of the micro-/nanosized spherical shells containing fluid.

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