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A path-planning algorithm for parallel automatic parking

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***Abstract*—Path-planning is a key issue of automatic parking assist system due to the non-holonomic constraints. A shortest path algorithm for the parallel parking problem in a certain condition is proposed and proved. A feasible path-planning approach is presented to meet the requirement that the parking space is narrow and needs park iteratively by improving the shortest path. Considering several possibilities of collision with obstacles in the parking process, the parking region where the cars can park with no collision based on the proposed algorithm is designed. The proposed algorithm is verified combined with vehicle dynamics constraints under the limitation of the steering angle rotating speed in high-precision vehicle dynamics simulation software veDYNA. The presented simulation results clearly show that the proposed algorithm provides practical solutions for automatic parallel parking problems.**

Keywords-parallel automatic parking; shortest path-planning algorithm; iterative path-planning algorithm; collision-free path

I. INTRODUCTION

With the increase of traffic pressure, the parking space is reducing which makes the parking more and more difficult and contributes to the rapid development of automatic parking technology. Many researchers have proposed the approach of generating paths of cars with non-holonomic constraints to park. Laumond proposed a two-step approach which is applied to plan a feasible path using the clothoid curve [1,2]. The drawback is that it is difficult to approximate holonomic path by smooth non-holonomic path which may causes more operations in the parking process. Paromtchik [3,4] studied on motion generation with trigonometric function that plans a continuous and iterative path. But the parking space should be larger than other method to avoid moving forward and backward too many times. Some approaches were designed for real-time feedback path-following control such as fuzzy logic [5,6]. The approach means obtaining the information in real-time which is not easy to feedback synchronously, so collision avoidance is difficult for such planners.

In this paper, after synthetically considering the problems such as path optimization, narrow space parking and path with no collision, we propose one kind of shortest collision-free path in a certain condition in order to approach the car to the goal following a stable trajectory while avoiding the obstacles. And an iterative path-planning algorithm is presented to park in the narrow space which is not big enough to operate parking in one time. And the feasibility of the algorithm is verified with the dynamics constraints in high-precision vehicle dynamics simulation software.

II. VEHICLE KINEMATIC MODEL

It is assumed that the vehicle moves with non-sliding method in the parking process because of the low speed. The model of front wheel drive car is illustrated in Fig.1. In the reference coordinate system, r is the midpoint of the rear wheel, f is the midpoint of the front wheel, $x=x(t)$ and $y=y(t)$ are the coordinates of r, $\theta=\theta(t)$ is the course angle of the car with respect to global coordinate system, $\varphi=\varphi(t)$ is the steering angle, $v=v(t)$ is the velocity of f, l is the wheel base, R is curvature radius of r.

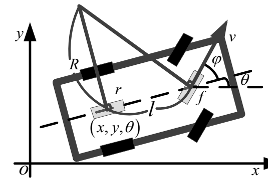

Figure 1. Kinematic model of front wheel drive car.

The kinematic model of car can be written as follows

$$\begin{cases} \dot{x} = v\cos\varphi\cos\theta \\ \dot{y} = v\cos\varphi\sin\theta \\ \dot{\theta} = \left(v/l\right)\sin\varphi \end{cases} . \tag{1}$$

The relation between R and φ is given by

$$\tan\varphi = l/R . \tag{2}$$

The parking environment can be constructed with the information from sensors as shown in Fig.2. In which, *abcd* represent the four corners of car, while *ABCD* represent the four corners of parking space, let the length of rear and front overhang be L_b and L_f, L and H are length and width of car, and the length and width of parking space which dominate the difficulty of parking are defined as L_s and H_s.

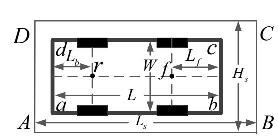

Figure 2 Description of car and parking space.

III. SHORTEST PATH OF PARALLEL PARKING

The working procedure of path-planner is partitioned into two parts: firstly, the system decides whether the space is big

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enough to park by analyzing the information from the sensors; secondly, the planner generates a feasible collision-free parking path with the consideration of choosing the appropriate start and end position if the space contents the parking requirement.

We plan our path under the following assumption:

- The orientation of the car in start and end positions are parallel. In order to maintain this parallel relationship, the path in the first and stage of parking should be circle.
- From (1), the nonholonomic constraint should be $\dot{y}-\dot{x}\tan\theta=0$ in this model, in which the $\theta(t)$ is continuous. So derivative of the path (i.e. dy/dx) is continuous which is necessary if the path is feasible.
- The information of parking bay and the coordinates of the start and end position are already given.

According to [7], Reeds proposed several optimal paths, and proved the shortest feasible is combined by circles and lines, in this paper we choose the CSC (Circle-Straight line-Circle) type as parking path, as shown in Fig.3.

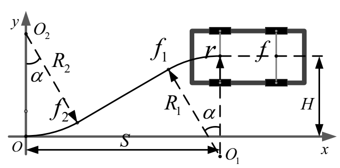


Figure 3 CSC path.

Where O_1 and O_2 are the circle centers of the first and final stage, R_1 and R_2 are the radius of the circles O_1 and O_2, f_1 and f2 are tangency points between lines and circles O_1 and O_2. α is the angle of arc path, O the origin of the coordinate is ideal end position, S and H are the horizontal and vertical coordinates of start position. From Fig.3 the coordinates of O_1 and O_2 are $(x_1,y_1)=(S,H-R_1)$ and $(x_2,y_2)=(0,R_2)$, it is assumed in this model that the tangent equation is

$$kx-y+m=0. \tag{3}$$

The path can be described as the equations

$$\begin{cases} r\rightarrow f_1:(x-S)^2+\left[y-(H-R_{\min})\right]^2=R_{\min}^2 \\ f_1\rightarrow f_2:kx-y+m=0 \\ f_2\rightarrow O:x^2+(y-R_{\min})^2=R_{\min}^2 \end{cases}. \tag{4}$$

Where the values of k, m and the coordinates of f_1, f_2 are as follows

$$\begin{cases} k=\dfrac{S(H-2R_{\min})+\sqrt{4R_{\min}^2(S^2+H^2)-16R_{\min}^3H}}{S^2-4R_{\min}^2} \\ m=R_{\min}\left(1-\sqrt{1+k^2}\right) \\ (x_{f_1},y_{f_1})=\left(S-\dfrac{k}{\sqrt{1+k^2}}R_{\min},H-\left(1-\dfrac{1}{\sqrt{1+k^2}}\right)R_{\min}\right) \\ (x_{f_2},y_{f_2})=\left(\dfrac{k}{\sqrt{1+k^2}}R_{\min},\left(1-\dfrac{1}{\sqrt{1+k^2}}\right)R_{\min}\right) \end{cases}$$

Let the total length of the path be S_0, the length of the line path be S_1, and equations can be obtained from Fig.3 and (4).

$$\begin{cases} (R_1+R_2)\sin\alpha+S_1\cos\alpha=S \\ (R_1+R_2)(1-\cos\alpha)+S_1\sin\alpha=H \\ S_0=S_1+(R_1+R_2)\alpha \end{cases} \tag{5}$$

Where

$$\begin{cases} S_1=\dfrac{S-(R_1+R_2)\sin\alpha}{\cos\alpha}=\dfrac{H-(R_1+R_2)(1-\cos\alpha)}{\sin\alpha} \\ S_0=\left(\dfrac{H\cos\alpha-S\sin\alpha}{\cos\alpha-1}\right)\left(\alpha-\dfrac{1-\cos\alpha}{\sin\alpha}\right)+\dfrac{H}{\sin\alpha} \end{cases}. \tag{6}$$

Let $f(\alpha)$ be the function of the sum of R_1 and R_2 with respect to α.

$$f(\alpha)=R_1+R_2=\frac{H\cos\alpha-S\sin\alpha}{\cos\alpha-1}. \tag{7}$$

From (5), $\cos\alpha=(S^2-H^2)/(S^2+H^2)$ is obtained when $S_1=0$ (i.e. two circle tangent), from which admissible range of α is $\alpha\in\left(0,\arccos\left((S^2-H^2)/(S^2+H^2)\right)\right)$.

In this range there are $f'(\alpha)>0$, $S'(\alpha)>0$, from which the S_0 increases with the sum of two radius increasing.

It is assumed that $\max\varphi$ is the maximum of φ, from (2), the $R_{\min}$ (minimum of R) is $R_{\min}=l/\tan(\max\varphi)$, so the shortest path is generated when $R_1=R_2=R_{\min}$.

Let $(S,H)=(7,2.5)$, the shortest path is compared with other path with bigger curvature radius in Fig.4.

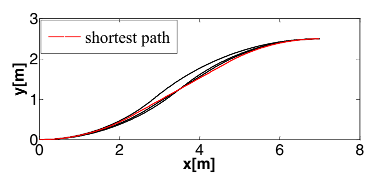


Figure 4 Comparison between shortest path and other paths.

Let S=7, H ranges from 2 to 3 by 0.1, the shortest paths are illustrated in Fig.5.

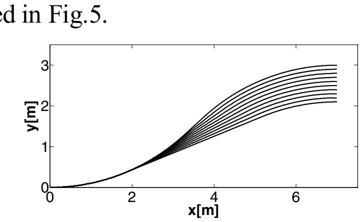


Figure 5 Shortest paths when S=7, H∈(2,3).

The parking space is defined as narrow space when the car fails to park by one operation. For narrow parking space, drivers may move forward and backward several times to park, the shortest parking path is improved to adjust the narrow space circumstance to a certain extent. We describe the two operations parking approach for simplicity of the iterative algorithm for example.

Let L_i be the ideal length of parking space which is longer than the real L_s by d_1. In Fig.6 the parking maneuver of two operations is illustrated under the following process. In Fig.6 *ABCD* is real space, *A'BCD'* is ideal space. Let the car execute the parking path as ideal space in real space, the car should stop at f_4 in the path from f_2 to O to avoid collision with *AD*. Then by turning the steering wheel to the other side that makes the car move with radius of $R_{\min}$ from f_4 to f_5. Then change the steering angle to the other side again to make the car move from f_5 to f_6 with the same radius. In which, the horizontal distance between f_4 and O is d_1, f_4 and f_5 have the same vertical coordinates. The vertical coordinates of f_5 and f_6 are 0, which means the body is straighten in f_6. And there should have admissible range as follows

$$3d_1 \le L_s - L. \tag{8}$$

From Fig.6, the coordinates of f_4、f_5、f_6 are

$$\begin{cases} f_4 : (0, R_{\min} - \sqrt{R_{\min}^2 - d_1^2}) \\ f_5 : (2d_1, R_{\min} - \sqrt{R_{\min}^2 - d_1^2}) \\ f_6 : (3d_1, 0) \end{cases}$$

The path $f_4 \to f_5$, $f_5 \to f_6$ are

$$\begin{cases} f_4 \to f_5 : (x - d_1)^2 + \left(y - \left(R_{\min} - 2\sqrt{R_{\min}^2 - d_1^2}\right)\right)^2 = R_{\min}^2 \\ f_5 \to f_6 : (x - 3d_1)^2 + (y - R_{\min})^2 = R_{\min}^2 \end{cases}. \tag{9}$$

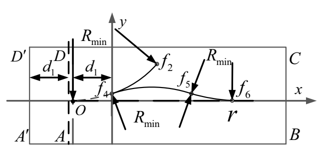


Figure 6 Parking maneuver of two operations.

When the range of $L_s - L$ changes to $2d_1 < L_s - L < 3d_1$ (i.e. the car stop in the path from f_5 to f_6 to avoid collision with *BC*), the car can iteratively park with the same maneuver above.

In the second operation, let

$$x_1 = L_1 - L - 2d_1. \tag{10}$$

From (10), in the third operation, let

$$x_2 = L_1 - L - 2(d_1 - x_1) = 3x_1 \tag{11}$$

In the *n* time parking operation, there is

$$x_{n-1} = 3^{n-2} x_1. \tag{12}$$

If $x_{n-2} < d_1 < x_{n-1}$, *n* times operation is needed in this situation.

When the range of $L_s - L$ changes to $L_s - L \le 2d_1$, the space is not big enough to park even by iterative algorithm.

IV. Planning Collision-free Region

The path above is planned under the following assumptions: the start and end positions are given. But the path-planner in parking assist system should also concern about the collision avoidance problem based on the planned path. So where the car starts and stops can plan the shortest path with no collision is studied in the following research with the consideration of several collision possibilities.

In the final stage of parking, the vehicle straightens the body through rotary process, *a* may have collision with *AB* as illustrated in Fig.7. The minimum distance between *ab* and *AB* is

$$d_2 = \sqrt{(R_{\min} + W/2)^2 + L_b^2} - R_{\min} - W/2 \tag{13}$$

In the initial stage of entering parking space, *b* may have collision with *C* as shown in Fig.8. The R_b is given as

$$R_b = \sqrt{(l + L_f)^2 + (R_{\min} + W/2)^2} \tag{14}$$

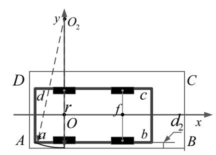


Figure 7 Collision circumstance 1.

The minimum length of parking space is

$$\min L_s = \sqrt{R_b^2 - (R_{\min} + W/2 + d_2 - H_s)^2} + L_b \tag{15}$$

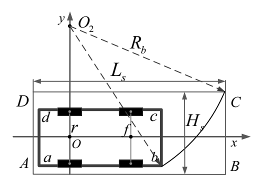


Figure 8 Collision circumstance 2.

Fig.9 shows that in the initial process of reversing, the body may have collision with *C* because of the narrow distance between *ab* and *CD*. The minimum distance between *ab* and *CD* is

$$d_3 = (R_{\min} - W/2) - \sqrt{(R_{\min} - W/2)^2 - (S - L_s + L_b)^2} \tag{16}$$

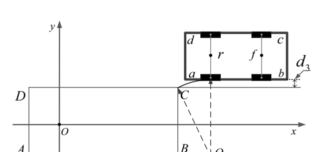


Figure 9 Collision circumstance 3.

In the initial process of reversing, c may have collision with lateral obstacle which is illustrated in Fig.10. The minimum distance between cd and lateral obstacle is

$$d_4 = R_b - (R_{\min} + W/2) \tag{17}$$

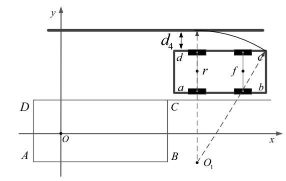


Figure 10 Collision circumstance 4.

To compute how to avoid the collision in the line path of reversing, the shortest path is compared with the connecting line of rO as shown in Fig.11.

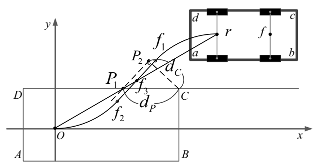


Figure 11 Collision circumstance 5.

Where rO and f_1f_2 intersect in f_3, and rO and CD intersect in P_1, P_1P_2 parallel to f_1f_2, CP_2 perpendicular to P_1P_2, the length of CP_2 is d_c, the length of CP_1 is d_p. If the d_c has the range $d_c > W/2$, and f_3 is beyond the CD, the body will have no collision with C in this circumstance.

The minimum length of d_p is

$$\min d_P = (W\sqrt{1+k^2})/2k \tag{18}$$

From Fig.11, x_p is horizontal coordinate of P_1 as follows

$$x_p = S(H_s - d_2 - W/2)/H + L_b \tag{19}$$

L_s (L_i in iterative process) should meet the requirement is

$$L_s \geq (W\sqrt{1+k^2})/2k + S(H_s - d_2 - W/2)/H + L_b \tag{20}$$

In the iterative parking process, the b may have collision with AB at point f_5, as shown in Fig.12. In which $\beta = \arcsin(d_1/R_{\min})$. The d_2 should be replaced by d_5 which is the minimum distance between ab and AB in this situation.

$$d_5 = R_b \cos(\arcsin((l + L_f)/R_b) - \beta) - (R_{\min} + W/2) \tag{21}$$

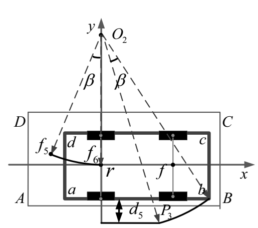


Figure 12 Collision circumstance 6.

Let safe clearance distance δ be 0.2m to every obstacle. Take the Audi A4 as the experimental car.

Let L_s=7m, H_s=2.4m, and the goal position has 1.2m and 0.2m to AB and AD. The start region is shown in Fig.13.

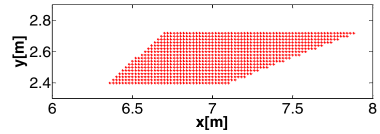
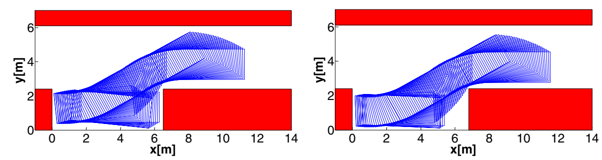


Figure 13 Start region.

Let L_i=7m, (S,H) be (7,2.5), simulation of the iterative parking process when the sizes of parking spaces (L_s,H_s) are (6.5,2.4) ,(6.8,2.4) are illustrated in Fig.14.



(a) (6.5,2.4) (b) (6.8,2.4)

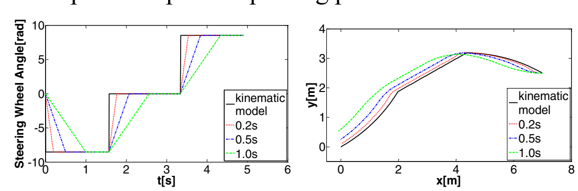
Figure 14 Iterative parking process.

V. Model Verification

As the shortest path model has two points of tangency, where the steering wheel should have saltation, which is impossible to follow in practical car with the consideration of vehicle dynamics constraints if the car does not stop at these points. The algorithm is verified in veDYNA under the assumption that the car follows a smooth and continuous path with no stop.

The reference coordinate is transferred to midpoint of the front wheel to solve the problem related to vehicle dynamics. In setting part of veDYNA, the car's parameters are set as Audi A4. Let v be 5km/s, and set the time that the steering angle changes from 0 to maxφ need to be 0.2s, 0.5s, 1.0s separately. Compare the simulation results in veDYNA with kinematic model of the shortest path. Let the (S,H) be (7,2.5), the comparison is shown in Fig.15.

From Fig.15, though the accuracy of path following is getting worse with the speed of rotating the steering wheel decreasing, it still has capability to follow the kinematic model path to a certain extent. The algorithm is verified to realize a practical parallel parking path.



(a) Steering wheel angle (b) Path of midpoint of the front wheel

Figure 15 Simulation results in veDYNA and comparison.

VI. Conclusion

One kind of shortest path-planning algorithm of non-holonomic vehicle parallel auto-parking in a certain condition has been proposed. A feasible model with iterative parking operation in narrow space is developed by improving the shortest path-planning algorithm. Several collision circumstances are considered to plan the start and end position region which makes the vehicle move with no collision. And the model is verified in vehicle dynamics model by high-precision vehicle dynamics simulation software with the consideration of speed of rotating the steering wheel. The results show the effectiveness of the proposed parallel parking algorithm in practical environment.

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