Electrical Power and Energy Systems 87 (2017) 89–98

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

New reactive power flow tracing and loss allocation algorithms for power grids using matrix calculation

Ali Enshaee [a,*], Parisa Enshaee [b]

[a] Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan 8415683111, Iran
[b] Electrical Engineering Department, College of Engineering, Payame Noor University, Isfahan 81395671, Iran

ARTICLE INFO

Article history:
Received 10 August 2016
Received in revised form 24 October 2016
Accepted 25 November 2016

Keywords:
Loss allocation
Power flow tracing
Reactive power
Transmission system

ABSTRACT

A novel simple method is suggested in this paper to evaluate the contributions of the sources (including the generators and branches' charging capacitances) or the loads to the branches' reactive flows and losses separately as well as to calculate the sources' shares in providing the loads' reactive powers. In the method, the study system is first converted to the system, each branch of which only has reactive loss, using a new technique for modeling the generating branches based on the AC load flow results. The properties of two new matrices (i.e. injection-bus and absorption-bus matrices), which are constituted for the obtained system, are then used to derive three other matrices. These matrices, which express reactive power productions of the sources in terms of reactive power consumptions of the demands (viz. the loads and branches' losses) and vice versa, contain the intended contributory factors. Three-bus system is applied to demonstrate the computing process of the method whereas several IEEE systems are used to show its capability to implement on the transmission systems with arbitrary topologies and sizes. Some advantages of the method compared to the earlier methods are also illustrated.

1. Introduction

With the trend toward the deregulation of the electric power industry, the allocation of the transmission service cost to the users in an equitable and reasonable manner becomes much more significant. To ensure fair calculation of the price of all power wheeling transactions in this new competitive environment, it should be needed to know how each market participant utilizes the transmission network. This information can be determined only when the paths of delivering the powers from the sources to the loads and their amounts are assessed by power flow tracing. However, it is very difficult to answer the question 'what fractions of the given branch flow and loss are attributed to a particular source or load?' in a nonlinear power system. Thus, one complicated issue that has attracted many researchers is finding the widely accepted solution for power flow tracing problem.

Since the direction of the reactive power flow incurred by the selected source through the specified branch may not be the same as the direction of the active power flow, power flow tracing should be accomplished separately for active and reactive power [1]. Because the main commodity traded in the present day electricity markets is active power, a large part of the technical literature focus on active power flow tracing and the methods developed in some of them are said to be applied straightforwardly to trace the reactive power flows. In general, this statement is not correct because the branches always waste active power, whereas they can be considered to be both producer and consumer of reactive power due to their capacitive and inductive behaviors [2]. Owing to these facts and regarding the vital role of the reactive power in maintaining voltage at all buses of the system within limits and improving active energy transfer capability, there are the outstanding papers which emphasize on finding the shares of various sources in the reactive power flow and loss of each branch to identify that each source feeds which loads and how much. The methods presented in these papers can be broadly classified into three main groups:

(1) This kind of approaches assumes that the power flow coming into each bus contributes to all power flows leaving that bus the same as the proportion of its amount to the total inflow powers [1–13]. Although this assumption is intuitively logical, but its correctness can never be theoretically proved. This makes the validity of the application of these approaches to be doubtful.

* Corresponding author.
E-mail addresses: a.enshaee@ec.iut.ac.ir (A. Enshaee), p.enshaee@gmail.com (P. Enshaee).

http://dx.doi.org/10.1016/j.ijepes.2016.11.010
0142-0615/© 2016 Elsevier Ltd. All rights reserved.

(2) Since these methods use the admittance (Y) or impedance (Z) matrix and the load flow equations directly to derive the sources and loads' participations in the transmission costs and losses [14–29], they pay more attention to the actual characteristics of the system and thus have a great advantage over the previous group of methods. In some of these methods [14–17], the basic circuit relations are combined with the game theory for the division of the branch power flow and loss among the generators and loads. To implement this type of methods, the storage of a huge amount of data and a considerable computational burden are often required. Therefore, they are inappropriate for real-time applications. Moreover, Refs. [14,15] obtain non-zero values for each generator's contributions to the reactive power flows of all branches. This issue does not seem reasonable, because the reactive power cannot actually flow to long distances from its producer. Refs. [16,17] suppose that the generating and demand buses have the same responsibility in the use of every branch and consequently half of the total costs or losses of the transmission system should be allocated to each class of buses. The lack of a physical and economic justification for this allocation ratio causes its specification to be arbitrary, which is not satisfactory for market participants. Refs. [18–22] also do not consider the generator as a supplier of reactive losses, notwithstanding they employ no predefined sharing ratio to split the reactive loss of each branch between the generators and loads simultaneously. In addition, since the effect of the branches' charging capacitances on providing the reactive loads and losses is not discussed in Refs. [17–20,23–25]; the reactive power produced by the charging capacitance of each branch is considered as a part of its reactive loss in Refs. [16,21,22,14,15,26,27] represent every branch by the series impedance and ignore the shunt admittances from its equivalent circuit model; these references do not take into account the charging capacitance as a reactive power source. Even though the reactive power production of the charging capacitance is accounted in Refs. [28,29] by integrating it with the end bus injections according to the π-equivalent circuit of the branch, but this modification may lead to a significant change in the system's topological features, which can influence the generators' contributions to some loads and branches' flows. Furthermore, negative loss allocation to a number of generators or loads is seen in Refs. [14–16,21,22,26]. This situation, which results in negative cost assignment, can be interpreted as cross subsidy and thus it is not allowable from an economics point of view.

(3) To enhance the computation time of evaluating the amount of the reactive powers transmitted from generators to loads, Refs. [30–32] apply artificial intelligence. In these references, the generators' shares in reactive loads obtained by the procedure belonging to the other two groups of methods, are used for training the neural network or support vector machine. However, the proper performance of these techniques is highly dependent on the training process, which must be repeated for every change in the system topology, and fine-tuning of their various parameters, which should be done by the heuristic optimization algorithms. In addition, the methods developed by these references in such a complicated manner cannot specify the contribution of each reactive source or load to the branch power flow and loss.

In this paper, two novel algorithms are proposed to determine the share of each source or load in the reactive loss of each branch. These shares are used for specifying the contributions of the sources or loads to the reactive powers at both ends of each branch. One of these algorithms can also calculate the participation of each source in the reactive power consumption of each load. To consider the charging capacitance of each branch as an independent reactive power source and take reactive losses into account directly in the course of the algorithms, the new model, which is developed based on AC load flow solution, is applied for representing the branches. Therefore, neither embedding reactive power productions of the charging capacitances into the nodal powers nor adding more virtual buses and branches to make the system lossless is required in the proposed method. Instead of the proportional sharing assumption, the method employs algebraic and physical meaningful equations to obtain non-negative quantities for the sources and loads responsibilities in reactive losses; thus, not only the existence of cross-subsidies is avoided, but also the relative locations of the sources and loads within the system are reflected in the allocation result.

The proposed technique for acquiring a proper model of a branch is introduced in the next section. Afterwards, the above-mentioned algorithms are described in detail. Subsequently, the procedure of implementing the method is illustrated using a simple example system, and its capability to trace reactive power flows in any system configuration is shown by applying it to different standard test systems. Additionally, the method is compared with the previous ones to demonstrate its merits. Finally, conclusions are discussed.

2. Proposed model for representation of system branches

In the proposed method, a branch connects two buses of the system; thus, each line or transformer is considered as a branch. In addition, a source produces reactive power in the system; hence, a generator or branch can be regarded as a source. It is to be noted that in the circuit model of each branch, the charging capacitance does not exist, but there is the inductance; so, some of branches play the role of a source, while all branches have reactive loss. Therefore, if the number of the system branches is N_L and the number of the branches, which also generate reactive power, is N_C, it can be written: $N_C \leqslant N_L$.

To trace the reactive power productions of the branches by the proposed algorithms, the generated reactive power of each branch, which is computable after executing the AC load flow, is first modeled as a source on the independent fictitious bus. This bus is then connected to the end buses of the original branch by two fictitious branches. The reactive loss of each fictitious branch is assumed to be equal to one half of the reactive loss of the original branch. In other words, each generating branch is replaced with a bus and two branches; thus, the system obtained by this action has $(N_L + N_C)$ branches, none of which neither produces reactive power nor has bidirectional reactive power flows. Furthermore, if the main system has N_B buses, the obtained system will have $(N_B + N_C)$ buses. Fig. 1 shows the values of reactive powers come from the fictitious bus, which is denoted by N in a dashed-circle. In this figure, $\boldsymbol{Q_C}$ is the N_C-dimension vector whose elements are formed by reactive power productions of the generating branches of the main system, and the N_L elements of the $\boldsymbol{Q_{Loss}}$ vector are equal to the values of reactive losses of the main system branches.

3. Tracing the reactive power produced by sources

The first step of the proposed algorithm is the formation of the injection-bus matrix for the modified system considering the values of reactive power flows of its branches. This matrix is defined as:

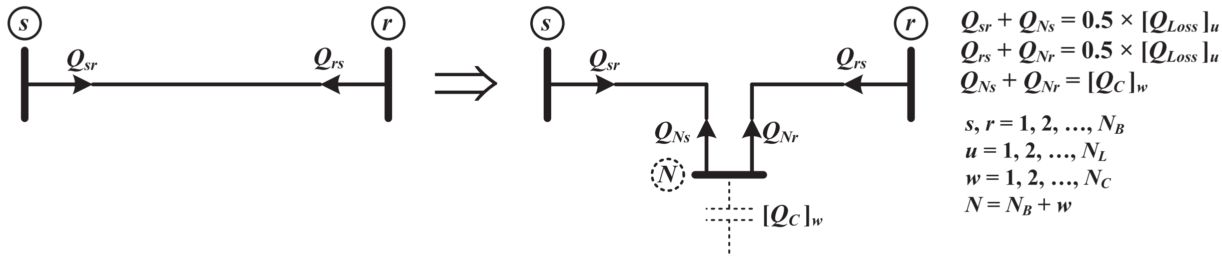

Fig. 1. Representation of equivalent model for generating branch.

$$[\boldsymbol{A_1}]_{ij} = \begin{cases} Q_{ij} & \text{for } i \neq j \text{ and } Q_{ij} < 0 \\ [\boldsymbol{Q_S}]_i - \sum_{\substack{\alpha=1, \alpha\neq i \\ Q_{i\alpha}<0}}^{N_B+N_C} Q_{i\alpha} & \text{for } i = j \\ 0 & \text{otherwise} \end{cases} \quad (1)$$
$$\text{for } i, j = 1, 2, \ldots, (N_B + N_C)$$

where $\boldsymbol{Q_S}$ is the $(N_B + N_C)$-dimension vector. If the values of reactive power productions of the main system sources constitute the elements of the N_B rows of the $\boldsymbol{Q_G}$ vector, then $\boldsymbol{Q_S}$ will have the following structure:

$$\boldsymbol{Q_S} = \begin{bmatrix} \boldsymbol{Q_G} \\ \boldsymbol{Q_C} \end{bmatrix} \quad (2)$$

Eq. (1) indicates that the element of the i^{th} row and the j^{th} column of $\boldsymbol{A_1}$ is equal to the negative of the reactive power injected to the i^{th} bus by the branch connecting the buses i and j. The element located at the main diagonal of this matrix is equal to the sum of the reactive powers injected to its corresponding bus.

The following properties, which will be used to show that $\boldsymbol{A_1}$ is certain to be invertible, should be enumerated for $\boldsymbol{A_1}$ considering its definition.

(1) The sum of the elements of the i^{th} row of $\boldsymbol{A_1}$ is equal to the reactive power produced by the source on the i^{th} bus. If $\boldsymbol{E}$ is a unit vector of size $(N_B + N_C)$, this property can be expressed as:

$$\boldsymbol{A_1 E} = \boldsymbol{Q_S} \quad (3)$$

(2) The sum of the elements of the j^{th} column of $\boldsymbol{A_1}$ is equal to the reactive power consumed by the load on the j^{th} bus plus the sum of reactive losses of all branches to which bus j sends reactive power. If $\boldsymbol{Q_{Load}}$ is the N_B-dimension vector whose elements are formed by the values of reactive power consumptions of the main system loads, $\boldsymbol{O}$ is a zero vector of size N_C, and the values of reactive losses of the modified system branches constitute the elements of the $(N_L + N_C)$ rows of the $\boldsymbol{Q_L}$ vector, then this property can be written as:

$$\boldsymbol{A_1^T E} = \boldsymbol{Q_D} + \boldsymbol{B_1 Q_L} \quad (4)$$

where superscript T denotes the transpose of the matrix and

$$\boldsymbol{Q_D} = \begin{bmatrix} \boldsymbol{Q_{Load}} \\ \boldsymbol{O} \end{bmatrix} \quad (5)$$

$$[\boldsymbol{B_1}]_{il} = \begin{cases} 1 & \text{if } Q_{Branch_l} = Q_{ik} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$
$$\text{for } i, k = 1, 2, \ldots, (N_B + N_C) \quad l = 1, 2, \ldots, (N_L + N_C)$$

Eq. (6) shows that if bus i is the sending-end bus of branch l, the element of the i^{th} row and the l^{th} column of $\boldsymbol{B_1}$ is equal to one.

(3) The main diagonal elements of $\boldsymbol{A_1}$ are always positive because there is at least one reactive power injection to each bus either by the source on it or the branches incident to it. From these properties, it can be concluded that $\boldsymbol{A_1}$ is invertible. Using Eqs. (3) and (4), $\boldsymbol{E}$ can be calculated as:

$$\boldsymbol{E} = \boldsymbol{A_1^{-1} Q_S} \quad (7)$$

$$\boldsymbol{E} = (\boldsymbol{A_1^T})^{-1}(\boldsymbol{Q_D} + \boldsymbol{B_1 Q_L}) \quad (8)$$

These equations are applied to find the share of each source in the reactive power consumed by each load or wasted through each branch.

Suppose that the diagonal matrix, the main diagonal entries of which are formed by the elements of $\boldsymbol{Q_S}$, is represented as diag ($\boldsymbol{Q_S}$). Hence, the following relation can be written:

$$\boldsymbol{Q_S} = \text{diag}(\boldsymbol{Q_S})\boldsymbol{E} = \text{diag}(\boldsymbol{Q_S})(\boldsymbol{A_1^T})^{-1}(\boldsymbol{Q_D} + \boldsymbol{B_1 Q_L}) \quad (9)$$

If $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ are defined as follows:

$$\begin{aligned} \boldsymbol{S_1} &= \text{diag}(\boldsymbol{Q_S})(\boldsymbol{A_1^T})^{-1} \\ \boldsymbol{S_2} &= \boldsymbol{S_1 B_1} \end{aligned} \quad (10)$$

then the following equation can be used to specify how much of the reactive power consumption of load r is provided by the source on bus i.

$$Q_{Source_i \to Load_r} = [\boldsymbol{S_1}]_{ir}[\boldsymbol{Q_D}]_r \quad (11)$$
$$\text{for } i = 1, 2, \ldots, (N_B + N_C) \quad r = 1, 2, \ldots, N_B$$

The contribution of the source on bus i to the reactive loss of branch l can also be determined using the following equation:

$$Q_{Source_i \to Loss_l} = [\boldsymbol{S_2}]_{il}[\boldsymbol{Q_L}]_l \quad (12)$$
$$\text{for } i = 1, 2, \ldots, (N_B + N_C) \quad l = 1, 2, \ldots, (N_L + N_C)$$

It is obvious that the share of each source in reactive loss of the main system branch, which is replaced by two fictitious branches in the modified system, is equal to the sum of its shares in reactive losses of those branches. Moreover, in the modified system, the source on the i^{th} bus supplies $[\boldsymbol{S_2}]_{il} \times 100$ percent of reactive power flow of the l^{th} branch; thus, with attention to correspondence between the branches of the main and modified systems, the sources' contributions to reactive powers at both ends of each branch of the main system can easily be identified using Eq. (12).

Considering Eqs. (9)–(12), it can be observed that the sum of the shares of each source in reactive power consumptions of the loads and reactive losses of the branches is equal to the value of the reactive power produced by that source. Furthermore, it can be proven that the sum of the sources' contributions to each reactive demand is equal to the value of the reactive power consumed/wasted by that demand. To do this, it must be shown that the sum of the elements of each column of $\boldsymbol{S_1}$ or $\boldsymbol{S_2}$ is equal to one. According to Eqs. (7) and (10), it can be written:

$$E^T S_1 = E^T(\mathrm{diag}(Q_S)(A_1^T)^{-1}) = [A_1^{-1}\mathrm{diag}(Q_S)E]^T = [A_1^{-1}Q_S]^T = E^T \tag{13}$$

$$E^T S_2 = E^T(S_1 B_1) = E^T B_1 = (E')^T \tag{14}$$

where E' is a unit vector of size ($N_L + N_C$). Hence, it can be concluded that the sum of the sources' shares in the reactive power at each end bus of every branch equals 100 in terms of percentage.

4. Tracing the reactive power consumed by loads

For this purpose, a relation should be found for Q_D in terms of Q_S and Q_L. This relation can simply be acquired using the properties of the absorption-bus matrix. This matrix must be formed for the modified system as follows:

$$[A_2]_{ik} = \begin{cases} -Q_{ik} & \text{for } i \neq j \text{ and } Q_{ik} > 0 \\ [Q_D]_i + \sum\limits_{\substack{\beta=1,\beta\neq i \\ Q_{i\beta}>0}}^{N_B+N_C} Q_{i\beta} & \text{for } i = k \\ 0 & \text{otherwise} \end{cases} \tag{15}$$
$$\text{for } i,k = 1,2,\ldots,(N_B+N_C)$$

This definition indicates that the element of the i^{th} row and the k^{th} column of A_2 is equal to the negative of the reactive power extracted from bus i by the branch between buses i-k. The element located at the main diagonal of this matrix is equal to the sum of the reactive powers absorbed from its corresponding bus, both by the load on that bus and the branches incident to it. In fact, Eq. (15) is the dual of Eq. (1). Thus, the dual of the second property of A_1 is expected to be established for A_2, i.e., the sum of the elements of the k^{th} column of A_2 is equal to the reactive power produced by the source on bus k minus the sum of reactive losses of all branches from which the k^{th} bus receives reactive power. The mathematical expression of this property is:

$$A_2^T E = Q_S - B_2 Q_L \tag{16}$$

where B_2 is defined as the dual of the definition presented for B_1, i.e., if bus i is the receiving-end bus of branch m, the element of the i^{th} row and the m^{th} column of this matrix is equal to one; in other words:

$$[B_2]_{im} = \begin{cases} 1 & \text{if } Q_{Branch_m} = Q_{ij} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$
$$\text{for } i,j = 1,2,\ldots,(N_B+N_C) \quad m = 1,2,\ldots,(N_L+N_C)$$

Moreover, A_2 is invertible like A_1; hence, the following can be obtained from Eq. (16):

$$E = (A_2^T)^{-1}(Q_S - B_2 Q_L) \tag{18}$$

Consequently, the relation mentioned at the beginning of this section can be expressed as:

$$Q_D = \mathrm{diag}(Q_D)\left(A_2^T\right)^{-1}(Q_S - B_2 Q_L) \tag{19}$$

Now, if D is defined as:

$$D = \mathrm{diag}(Q_D)(A_2^T)^{-1} B_2 \tag{20}$$

then the following equation can be employed to determine the contributions of the loads to the reactive loss of each branch of the modified system:

$$Q_{Load_r \leftarrow Loss_m} = [D]_{rm}[Q_L]_m \tag{21}$$
$$\text{for } r = 1,2,\ldots,N_B \quad m = 1,2,\ldots,(N_L+N_C)$$

Besides, in this system, $[D]_{rm} \times 100$ percent of reactive power flow of branch m is allocated to the load on bus r. Therefore, the contribution of each load to the reactive power flow and loss of each branch of the main system can easily be computed using the above-mentioned equation and regarding correspondence between the branches of the main and modified systems.

The last point, which should be cited here, is that the sum of the loads' participations in the reactive power flow or loss of each branch is equal to the value of the reactive power flow or loss of that branch. To show this, it must be proven that the elements of each column of D is equal to one. Due to the fact that A_2 has the dual of the first property of A_1, i.e. $A_2 E = Q_D$, and using Eq. (20), it can be written:

$$\begin{aligned} E^T D &= E^T(\mathrm{diag}(Q_D)(A_2^T)^{-1} B_2) = [B_2^T A_2^{-1}\mathrm{diag}(Q_D)E]^T \\ &= [B_2^T A_2^{-1} Q_D]^T = [B_2^T E]^T = (E')^T \end{aligned} \tag{22}$$

5. Implementation of proposed method

5.1. Flowchart of proposed method

To perform reactive power tracing on any system using the proposed method, the values of reactive power flows and loss of each branch as well as the value of reactive power generated by its charging capacitance must be calculated first. These data can be obtained, if the parameters in the π model of the branches and the tap ratios of the phase shifting transformers are available and the buses' voltages are determined form a converged AC load flow calculation or by on-line measurement units. Moreover, the net reactive power production or consumption at each bus can be evaluated. Once all the mentioned data are provided, a new system will be constructed according to the explanation presented in Section 2, which allows the reactive load and loss allocation to the charging capacitances of the study system branches to be possible. By using the vectors and matrices, which all are formed for the new system and discussed in Sections 3 and 4, the reactive power flows from the producers to the consumers and vice versa can be traced. For better understanding, the procedure of the proposed method is shown in Fig. 2.

5.2. Test results of a simple system

To test the validity of the proposed method, it was implemented on a simple 3-bus system. Fig. 3 presents the values of reactive power productions and consumptions of the buses as well as the values of reactive power flows of the branches in the single-line diagram of this system. The value of the reactive power generated/lost by each branch of the main system along with the data required to achieve the modified system is also given in Table 1. Fig. 4 shows the single-line diagram of the modified system which has 6 (=$N_B + N_C$ = 3 + 3) buses and 6 (=$N_L + N_C$ = 3 + 3) branches. For this system, Q_S, Q_D and Q_L can be written as:

$$\begin{aligned} Q_S &= [99.44 \quad 0 \quad 42.53 \quad 20.47 \quad 16.28 \quad 20.26]^T \\ Q_D &= [0 \quad 100 \quad 0 \quad 0 \quad 0 \quad 0]^T \\ Q_L &= [24.700 \quad 24.700 \quad 4.435 \quad 4.435 \quad 20.355 \quad 20.355]^T \end{aligned} \tag{23}$$

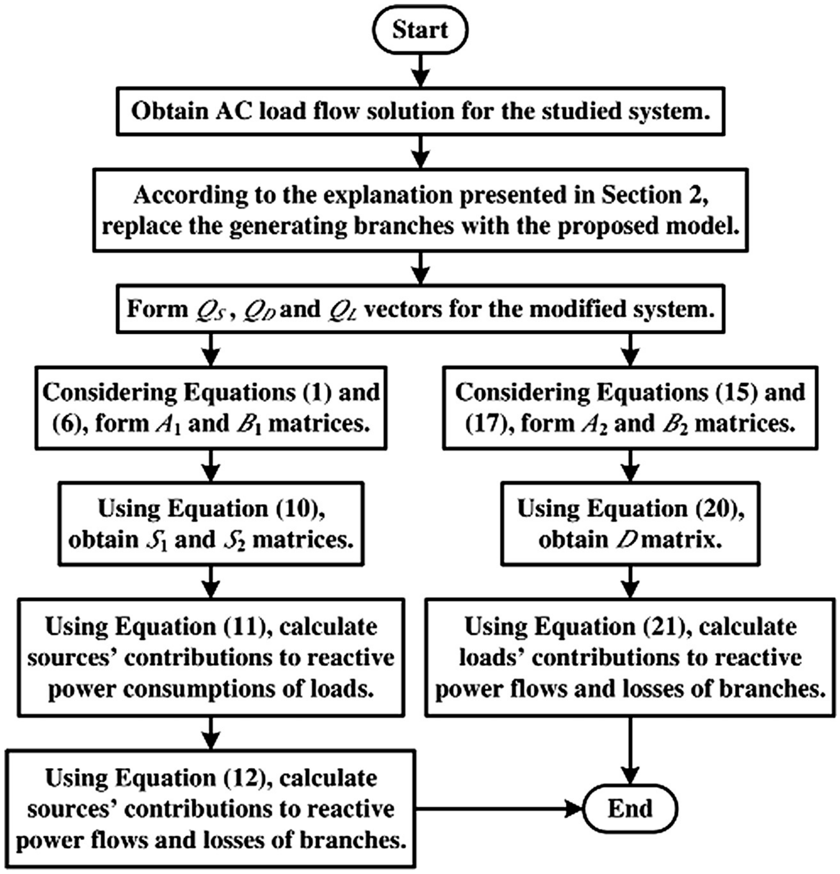

Fig. 2. Flowchart of the proposed method for reactive power tracing.

$\boldsymbol{A_1}$, $\boldsymbol{B_1}$, $\boldsymbol{A_2}$ and $\boldsymbol{B_2}$ will take the forms:

$$\boldsymbol{A_1}=\begin{bmatrix} 99.44 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & -54.93 & 0 & -45.07 \\ 0 & 0 & 65.52 & 0 & -22.99 & 0 \\ -59.16 & 0 & 0 & 79.63 & 0 & 0 \\ -11.145 & 0 & 0 & 0 & 27.425 & 0 \\ 0 & 0 & -45.165 & 0 & 0 & 65.425 \end{bmatrix} \quad \boldsymbol{B_1}=\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
$$\boldsymbol{A_2}=\begin{bmatrix} 99.44 & 0 & 0 & -83.86 & -15.58 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 65.52 & 0 & 0 & -65.52 \\ 0 & -79.63 & 0 & 79.63 & 0 & 0 \\ 0 & 0 & -27.425 & 0 & 27.425 & 0 \\ 0 & -65.425 & 0 & 0 & 0 & 65.425 \end{bmatrix} \quad \boldsymbol{B_2}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Using these vectors and matrices, $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ can be calculated as:

$$\boldsymbol{S_1}=\begin{bmatrix} 1 & 0.4524 & 0.1426 & 0.7429 & 0.4064 & 0.0984 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2020 & 0.6491 & 0 & 0 & 0.4481 \\ 0 & 0.1412 & 0 & 0.2571 & 0 & 0 \\ 0 & 0.0648 & 0.2083 & 0 & 0.5936 & 0.1438 \\ 0 & 0.1396 & 0 & 0 & 0 & 0.3097 \end{bmatrix}$$
$$\boldsymbol{S_2}=\begin{bmatrix} 1 & 0.7429 & 1 & 0.4064 & 0.0984 & 0.1426 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4481 & 0.6491 \\ 0 & 0.2571 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5936 & 0.1438 & 0.2083 \\ 0 & 0 & 0 & 0 & 0.3097 & 0 \end{bmatrix} \tag{25}$$

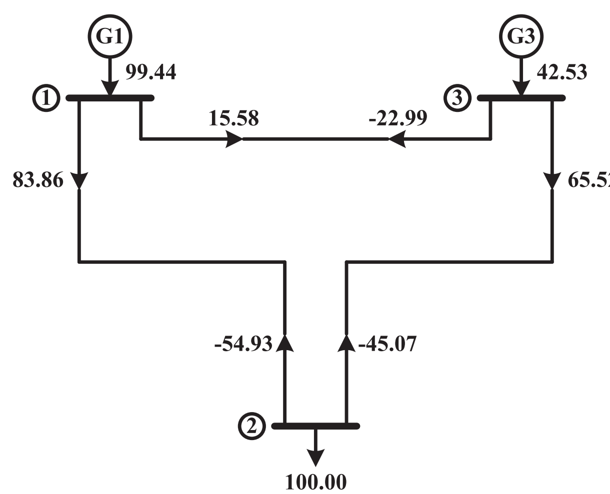

Fig. 3. One-line diagram of the 3-bus system with reactive power flows (MVAR).

$\boldsymbol{D}$ can also be computed as:

$$\boldsymbol{D}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

Table 1
Reactive power flow and loss for branches of the main and modified 3-bus system.

Main system							Modified system					
Branch no.	From bus s	To bus r	Q_{sr} (MVAR)	Q_{rs} (MVAR)	$[\boldsymbol{Q_C}]_w$ (MVAR)	$[\boldsymbol{Q_{Loss}}]_u$ (MVAR)	Branch no.	From bus i	To bus j	Q_{ij} (MVAR)	Q_{ji} (MVAR)	$[\boldsymbol{Q_L}]_m$ (MVAR)
1	1	2	83.86	−54.93	20.47	49.40	1	1	4	83.860	−59.160	24.700
							2	2	4	−54.930	79.630	24.700
2	1	3	15.58	−22.99	16.28	8.87	3	1	5	15.580	−11.145	4.435
							4	3	5	−22.990	27.425	4.435
3	2	3	−45.07	65.52	20.26	40.71	5	2	6	−45.070	65.425	20.355
							6	3	6	65.520	−45.165	20.355
Sum					57.01	98.98					57.010	98.980

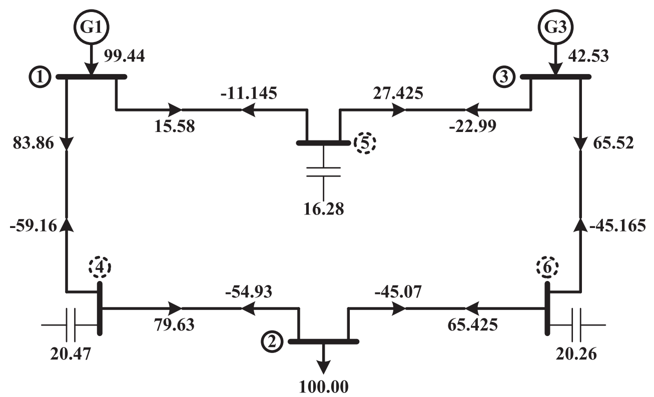

Fig. 4. One-line diagram of the modified 3-bus system with reactive power flows (MVAR).

Tables 2 and 3 present the results of the application of the $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$ matrices for determining the sources' shares. Table 2 includes the sources' contributions to reactive power flows of the main system branches. Table 3 shows the contributions of the sources to the reactive load and losses. From Eqs. (21) and (26), it is easy to understand that the proposed method allocates all reactive power flows and losses of the branches to load 2, which is the sole load of the system. Hence, the load shares are not presented in the form of Tables 2 and 3.

It can be observed that all equalities, mentioned for the contributions of the sources and loads in Sections 3 and 4, are valid here. Establishment of these equalities confirms the correctness of the proposed method for reactive power tracing.

5.3. Test results of standard systems

To determine whether or not the proposed method can be implemented on every system, reactive power tracing for IEEE 14-, 24-, 30-, 39-, 57-, 118-, and 300-bus standard systems was carried out. The AC load flow results of these systems were obtained using MATPOWER [33]. To implement the proposed method on these systems, a program was written in MATLAB. Since the technique, which is used to write a program for implementation of each reactive power tracing method, has a lot of influence on time required for its calculation, the fair comparison between the computation times of different methods cannot be made. Consequently, to demonstrate the fast performance of the proposed

Table 2
Sources' contributions to branches' reactive power flows (MVAR).

		Branch 1		Branch 2		Branch 3	
		$Q_{1,2}$	$Q_{2,1}$	$Q_{1,3}$	$Q_{3,1}$	$Q_{2,3}$	$Q_{3,2}$
Generators	Bus 1	$[\boldsymbol{S_2}]_{1,1} \times Q_{1,2} = 83.86$	$[\boldsymbol{S_2}]_{1,2} \times Q_{2,1} = -40.8095$	$[\boldsymbol{S_2}]_{1,3} \times Q_{1,3} = 15.58$	$[\boldsymbol{S_2}]_{1,4} \times Q_{3,1} = -9.3427$	$[\boldsymbol{S_2}]_{1,5} \times Q_{2,3} = -4.4365$	$[\boldsymbol{S_2}]_{1,6} \times Q_{3,2} = 9.3427$
	Bus 3	$[\boldsymbol{S_2}]_{3,1} \times Q_{1,2} = 0$	$[\boldsymbol{S_2}]_{3,2} \times Q_{2,1} = 0$	$[\boldsymbol{S_2}]_{3,3} \times Q_{1,3} = 0$	$[\boldsymbol{S_2}]_{3,4} \times Q_{3,1} = 0$	$[\boldsymbol{S_2}]_{3,5} \times Q_{2,3} = -20.1961$	$[\boldsymbol{S_2}]_{3,6} \times Q_{3,2} = 42.53$
Charging capacitances	Branch 1	$[\boldsymbol{S_2}]_{4,1} \times Q_{1,2} = 0$	$[\boldsymbol{S_2}]_{4,2} \times Q_{2,1} = -14.1205$	$[\boldsymbol{S_2}]_{4,3} \times Q_{1,3} = 0$	$[\boldsymbol{S_2}]_{4,4} \times Q_{3,1} = 0$	$[\boldsymbol{S_2}]_{4,5} \times Q_{2,3} = 0$	$[\boldsymbol{S_2}]_{4,6} \times Q_{3,2} = 0$
	Branch 2	$[\boldsymbol{S_2}]_{5,1} \times Q_{1,2} = 0$	$[\boldsymbol{S_2}]_{5,2} \times Q_{2,1} = 0$	$[\boldsymbol{S_2}]_{5,3} \times Q_{1,3} = 0$	$[\boldsymbol{S_2}]_{5,4} \times Q_{3,1} = -13.6473$	$[\boldsymbol{S_2}]_{5,5} \times Q_{2,3} = -6.4807$	$[\boldsymbol{S_2}]_{5,6} \times Q_{3,2} = 13.6473$
	Branch 3	$[\boldsymbol{S_2}]_{6,1} \times Q_{1,2} = 0$	$[\boldsymbol{S_2}]_{6,2} \times Q_{2,1} = 0$	$[\boldsymbol{S_2}]_{6,3} \times Q_{1,3} = 0$	$[\boldsymbol{S_2}]_{6,4} \times Q_{3,1} = 0$	$[\boldsymbol{S_2}]_{6,5} \times Q_{2,3} = -13.9567$	$[\boldsymbol{S_2}]_{6,6} \times Q_{3,2} = 0$
Sum		83.86	−54.93	15.58	−22.99	−45.07	65.52

Table 3
Sources' contributions to reactive power consumptions of demands (MVAR).

Sources		Demands				Sum
		Loads	Losses			
		Bus 2	Branch 1	Branch 2	Branch 3	
Generators	Bus 1	$[\boldsymbol{S_1}]_{1,2} \times [\boldsymbol{Q_D}]_2 = 45.2460$	$[\boldsymbol{S_2}]_{1,1} \times [\boldsymbol{Q_L}]_1 + [\boldsymbol{S_2}]_{1,2} \times [\boldsymbol{Q_L}]_2 = 43.0505$	$[\boldsymbol{S_2}]_{1,3} \times [\boldsymbol{Q_L}]_3 + [\boldsymbol{S_2}]_{1,4} \times [\boldsymbol{Q_L}]_4 = 6.2373$	$[\boldsymbol{S_2}]_{1,5} \times [\boldsymbol{Q_L}]_5 + [\boldsymbol{S_2}]_{1,6} \times [\boldsymbol{Q_L}]_6 = 4.9062$	99.44
	Bus 3	$[\boldsymbol{S_1}]_{3,2} \times [\boldsymbol{Q_D}]_2 = 20.1961$	$[\boldsymbol{S_2}]_{3,1} \times [\boldsymbol{Q_L}]_1 + [\boldsymbol{S_2}]_{3,2} \times [\boldsymbol{Q_L}]_2 = 0$	$[\boldsymbol{S_2}]_{3,3} \times [\boldsymbol{Q_L}]_3 + [\boldsymbol{S_2}]_{3,4} \times [\boldsymbol{Q_L}]_4 = 0$	$[\boldsymbol{S_2}]_{3,5} \times [\boldsymbol{Q_L}]_5 + [\boldsymbol{S_2}]_{3,6} \times [\boldsymbol{Q_L}]_6 = 22.3339$	42.53
Charging capacitances	Branch 1	$[\boldsymbol{S_1}]_{4,2} \times [\boldsymbol{Q_D}]_2 = 14.1205$	$[\boldsymbol{S_2}]_{4,1} \times [\boldsymbol{Q_L}]_1 + [\boldsymbol{S_2}]_{4,2} \times [\boldsymbol{Q_L}]_2 = 6.3495$	$[\boldsymbol{S_2}]_{4,3} \times [\boldsymbol{Q_L}]_3 + [\boldsymbol{S_2}]_{4,4} \times [\boldsymbol{Q_L}]_4 = 0$	$[\boldsymbol{S_2}]_{4,5} \times [\boldsymbol{Q_L}]_5 + [\boldsymbol{S_2}]_{4,6} \times [\boldsymbol{Q_L}]_6 = 0$	20.47
	Branch 2	$[\boldsymbol{S_1}]_{5,2} \times [\boldsymbol{Q_D}]_2 = 6.4807$	$[\boldsymbol{S_2}]_{5,1} \times [\boldsymbol{Q_L}]_1 + [\boldsymbol{S_2}]_{5,2} \times [\boldsymbol{Q_L}]_2 = 0$	$[\boldsymbol{S_2}]_{5,3} \times [\boldsymbol{Q_L}]_3 + [\boldsymbol{S_2}]_{5,4} \times [\boldsymbol{Q_L}]_4 = 2.6327$	$[\boldsymbol{S_2}]_{5,5} \times [\boldsymbol{Q_L}]_5 + [\boldsymbol{S_2}]_{5,6} \times [\boldsymbol{Q_L}]_6 = 7.1666$	16.28
	Branch 3	$[\boldsymbol{S_1}]_{6,2} \times [\boldsymbol{Q_D}]_2 = 13.9567$	$[\boldsymbol{S_2}]_{6,1} \times [\boldsymbol{Q_L}]_1 + [\boldsymbol{S_2}]_{6,2} \times [\boldsymbol{Q_L}]_2 = 0$	$[\boldsymbol{S_2}]_{6,3} \times [\boldsymbol{Q_L}]_3 + [\boldsymbol{S_2}]_{6,4} \times [\boldsymbol{Q_L}]_4 = 0$	$[\boldsymbol{S_2}]_{6,5} \times [\boldsymbol{Q_L}]_5 + [\boldsymbol{S_2}]_{6,6} \times [\boldsymbol{Q_L}]_6 = 6.3033$	20.26
Sum		100	49.4	8.87	40.71	198.98

method, the average CPU time for executing AC load flow and reactive power tracing programs in each system after 20 runs on a PC with Intel Pentium 3.00-GHz processor and 4-GB of RAM is depicted in Fig. 5. This figure indicates that although the execution time increases when the system becomes large, but the proposed method is still fast enough. Small execution time for tracing reactive power flows in the large-scale system, i.e. 300-bus system, verifies the implementation feasibility of the proposed method in real power systems. In addition, easy access to the test results and their validation by examining the equalities pointed in the previous subsection show that the answer of the aforementioned question is positive. Fig. 6 summarizes the results of reactive

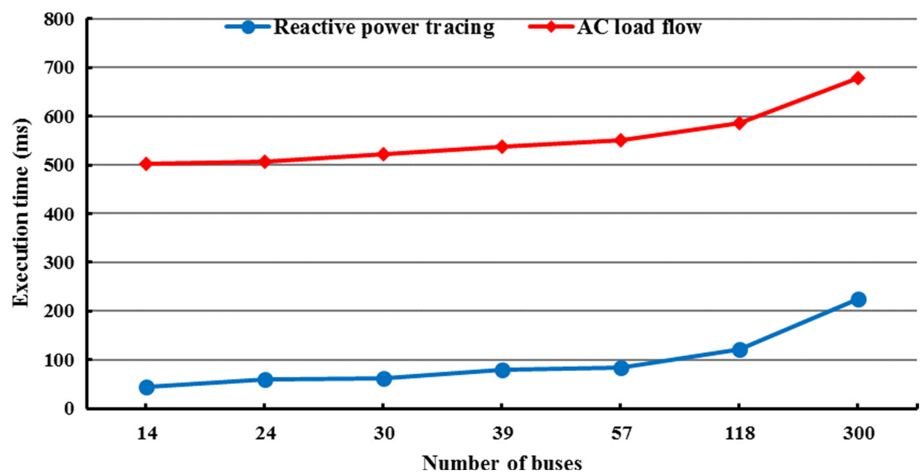

Fig. 5. Execution time of AC load flow and reactive power tracing programs in IEEE standard systems.

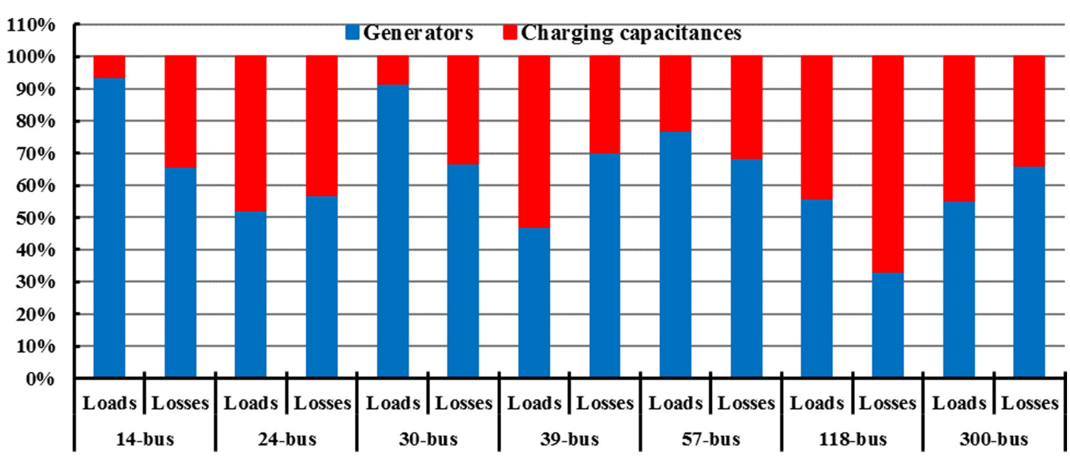

Fig. 6. Sources' contributions to total reactive loads and losses of IEEE standard systems.

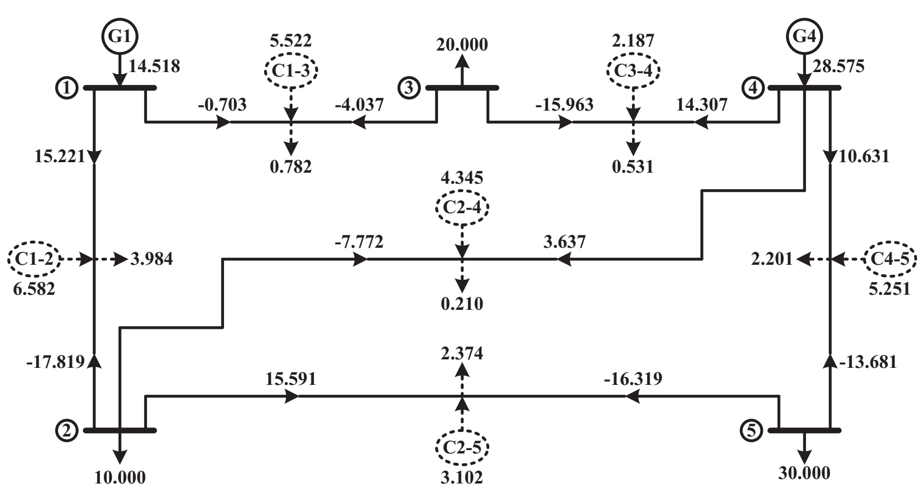

Fig. 7. One-line diagram of the 5-bus system with reactive power flows (MVAR).

power tracing to simplify its presentation. This figure specifies what percentage of the total reactive loads and losses of each system are supplied by its generators and branches' charging capacitances. As seen in Fig. 6, the proposed method allocates at least 30% of the total reactive losses to the charging capacitances of every system as well as greater than 40% of the total reactive loads of four systems, i.e. 24-, 39-, 118-, and 300-bus systems. Therefore, the share of the charging capacitances in providing reactive demands is not negligible at all.

5.4. Comparison of proposed method with other methods

Two systems, i.e. simple 5-bus and IEEE 9-bus, were selected for comparison. The single-line diagrams of these systems are shown in Figs. 7 and 8, respectively. In addition to the reactive power production and consumption at each bus and the reactive power flows at both ends of each branch, the reactive power generation and loss of each branch are indicated in these figures by dashed-arrowheads at the midpoint of that branch.

The comparison begins with the simple 5-bus system. As observed in Fig. 7, the charging capacitance of the branch connecting buses 1–3 only contributes to the reactive power flow of this branch. The same can be seen for the branch between buses 3–4; the charging capacitance of this branch and generator 4 are the sole contributors to the reactive power going to bus 3 from bus 4. Using these findings, it can be asserted that all reactive power consumption of load 3 should be supplied by these three sources. Refs. [8,11,32] produce results that are incompatible with this reasonable expectation. Because these references change the system topology by moving the reactive power generated by the charging capacitance of each branch to the end buses of that branch and uniting it with nodal injections, their suggested methods do not acquire zero value for the contribution of generator 1 to the required reactive power of load 3; while this situation does not

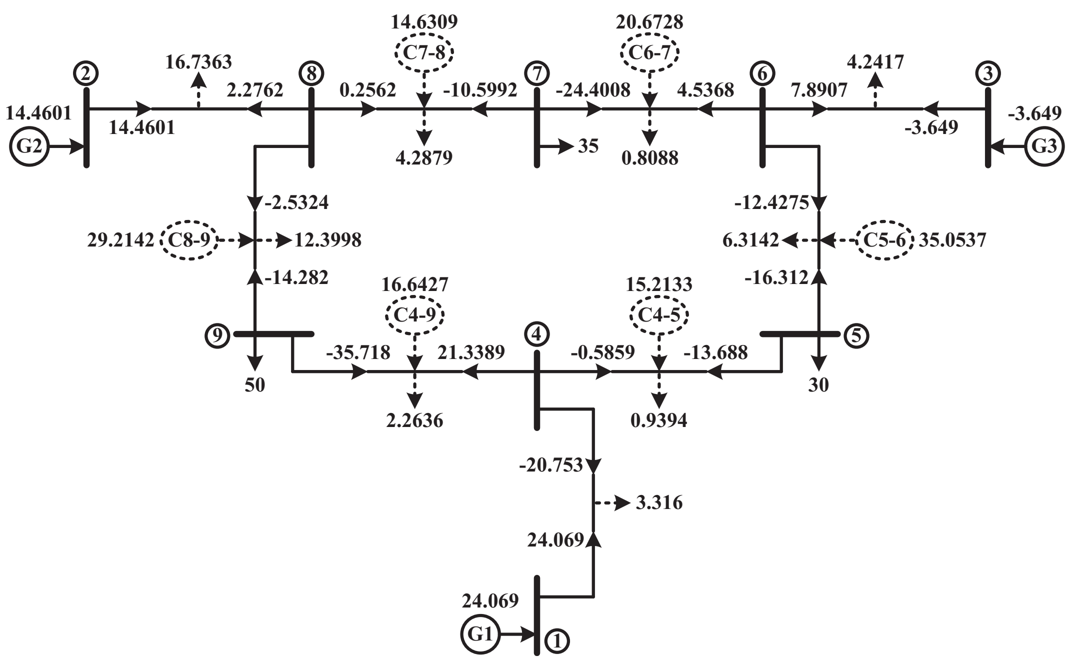

Fig. 8. One-line diagram of the IEEE 9-bus system with reactive power flows (MVAR).

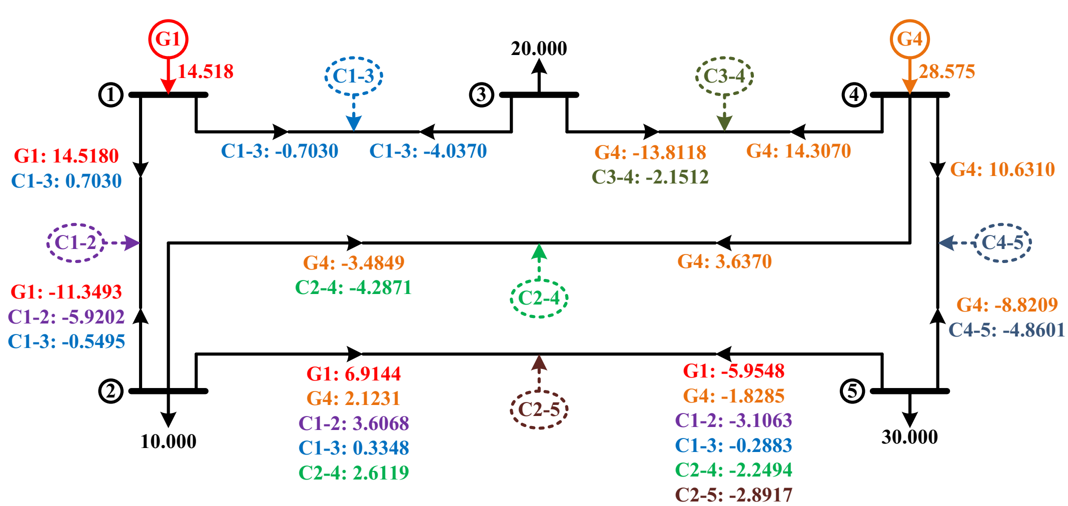

Fig. 9. Sources' contributions to reactive power flows of branches in the 5-bus system (MVAR).

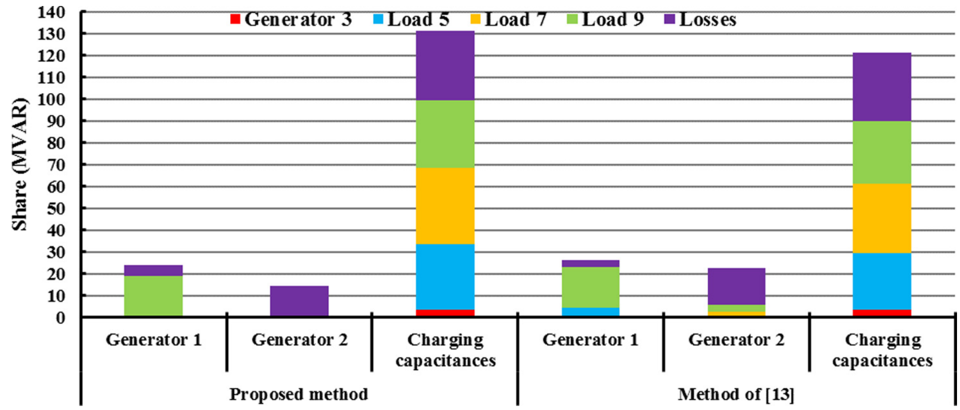

Fig. 10. Comparison of two methods for specifying sources' contributions to reactive power consumptions of demands in the IEEE 9-bus system.

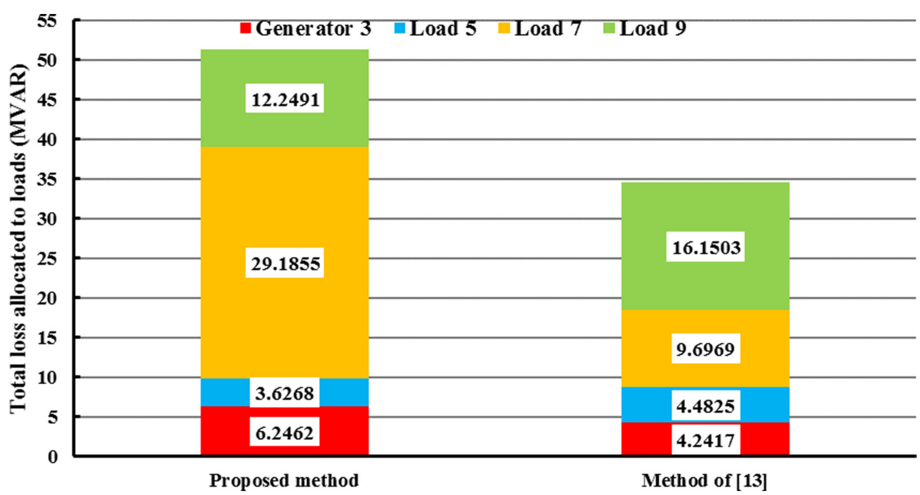

Fig. 11. Comparison of two methods for allocating total reactive losses to loads in the IEEE 9-bus system.

occur in the proposed method due to application of the new equivalent model for the generating branches. Fig. 9 shows the details of branch reactive power flow allocation to the sources using the proposed method. This figure clearly indicates that generator 4 and the charging capacitances of the branches connecting buses 1–3 and 3–4 (i.e. G4, C1-3 and C3-4, respectively) are only considered by the proposed method as the providers of load 3. Therefore, the proposed method is more consistent with the physical behavior of the system as compared to the methods presented in [8,11,32].

The IEEE 9-bus system was used to compare the proposed method with the method described in [13] by De. The results related to the determination of the sources' contributions to the reactive power demands by these two methods are shown in Fig. 10. The results of De's method do not appear to be justifiable, especially for generators 1 and 2. According to Fig. 8, it is rational that only the charging capacitances of the branches between buses 4–5 and 5–6 supply load 5, and the contribution of generator 1 to the reactive power consumption of this load equals zero. However, unlike the proposed method, De's method does not conform to this fact. Furthermore, it can be seen that all reactive power produced by generator 2 is wasted through the branch connecting buses 2–8. Nonetheless, contrary to the proposed method, by De's method not only generator 2 has contribution to the reactive power consumption of loads 7 and 9, but the sum of all allocated terms to this generator is also greater than its reactive power output. Hence, the correctness of this method is somewhat doubtful.

Fig. 11 presents the reactive losses assigned to each load by the proposed method and De's method. It should be noted that generator 3 absorbs reactive power in the study case; so, it is regarded as a load. The figure demonstrates that the sum of the loads' contributions specified by De's method is far below the value of the total reactive losses (i.e. 51.3076 MVAR); whereas the proposed method keeps a balance between the total reactive losses and the sum of the loads' shares.

6. Conclusion

This paper presented two new algorithms to trace the reactive powers that come from the sources or go to the loads. These algorithms used three matrices, which were acquired based on the properties of the so-called bus power matrices, to allocate a fraction of the reactive power flow and loss of each branch to each source or load as well as to assess the reactive power contributed by each source to meet each load. The desirable characteristics satisfied by the proposed method are:

- It requires only one solved AC load flow.
- It recognizes the branches' charging capacitances and generators as the sources.

- It is straightforwardly implementable and fast enough for online application in real transmission systems.
- It assigns the reactive loads and losses to the sources without any problems of negative allocation and over-allocation.
- The sum of the allocated losses to the loads, which all are non-negative, is the same as the total reactive losses of the system.
- Different modeling of the generating branches makes the tracing results achieved by the method to be coherent with intuitive expectations.
- It has no need to add virtual buses and branches for considering the branches' reactive losses.

References

[1] De M, Goswami SK. A direct and simplified approach to power-flow tracing and loss allocation using graph theory. Electr Power Compon Syst 2010;38(3):241–59.
[2] Augugliaro A, Dusonchet L, Favuzza S, Ippolito MG, Sanseverino ER. A model for reactive power tracing by addition of fictitious nodal injections. Electr Power Syst Res 2012;83(1):196–202.
[3] Bialek J. Tracing the flow of electricity. IEE Proc-Gener Transmiss Distrib 1996;143(4):313–20.
[4] Kirschen D, Allan R, Strbac G. Contributions of individual generators to loads and flows. IEEE Trans Power Syst 1997;12(1):52–60.
[5] Dai Y, Liu XD, Ni YX, Wen FS, Han ZX, Shen CM, et al. A cost allocation method for reactive power service based on power flow tracing. Electr Power Syst Res 2003;64(1):59–65.
[6] Pantos M, Gubina F. Ex-ante transmission-service pricing via power-flow tracing. Int J Electr Power Energy Syst 2004;26(7):509–18.
[7] Gang D, Dong ZY, Bai W, Wang XF. Power flow based monetary flow method for electricity transmission and wheeling pricing. Electr Power Syst Res 2005;74(2):293–305.
[8] Pantos M, Verbic G, Gubina F. Modified topological generation and load distribution factors. IEEE Trans Power Syst 2005;20(4):1998–2005.
[9] Su CT, Liaw JH, Li CM. Power-flow tracing and wheeling costing considering complex power and convection lines. IEE Proc-Gener Transmiss Distrib 2006;153(1):1–10.
[10] Su CT, Liaw JH. Complex power flow tracing considering convection lines using nominal-T model. Int J Electr Power Energy Syst 2007;29(1):28–35.
[11] Abdelkader SM. Allocating transmission loss to loads and generators through complex power flow tracing. IET Gener Transm Distrib 2007;1(4):584–95.
[12] Abdelkader SM. Transmission loss allocation through complex power flow tracing. IEEE Trans Power Syst 2007;22(4):2240–8.
[13] De M, Goswami SK. Reactive power cost allocation by power tracing based method. Energy Convers Manage 2012;64:43–51.
[14] Peng JC, Jiang H, Song YH. A weakly conditioned imputation of an impedance-branch dissipation power. IEEE Trans Power Syst 2007;22(4):2124–33.
[15] Peng JC, Zeng YG, Jiang H. Resolution of line-transferred power in grids yielded by circuit-laws' symmetry under deductive reasoning of Shapley theorem. IET Gener Transm Distrib 2012;6(7):627–35.
[16] Molina YP, Prada RB, Saavedra OR. Complex losses allocation to generators and loads based on circuit theory and Aumann-Shapley method. IEEE Trans Power Syst 2010;25(4):1928–36.
[17] Molina YP, Saavedra OR, Amarís H. Transmission network cost allocation based on circuit theory and the Aumann-Shapley method. IEEE Trans Power Syst 2013;28(4):4568–77.
[18] Molina YP, Saavedra OR, Portugal C. Allocation of transmission network cost using modified Z_{bus} matrix. Int J Electr Power Energy Syst 2014;63:323–30.
[19] Lo KL, Alturki YA. Towards reactive power markets Part 2: differentiated market reactive power requirements. IET Gener Transm Distrib 2008;2(4):516–29.
[20] Alturki YA, Lo KL. Real and reactive power loss allocation in pool-based electricity markets. Int J Electr Power Energy Syst 2010;32(4):262–70.
[21] Elmitwally A, Eladl A, Abdelkader S. Efficient algorithm for transmission system energy loss allocation considering multilateral contracts and load variation. IET Gener Transm Distrib 2015;9(16):2653–63.
[22] Elmitwally A, Eladl A, Abdelkader S. Algorithm for transmission loss allocation in market-based power systems with wind generation. J Energy Eng 2016.
[23] Parida SK, Singh SN, Srivastava SC. Reactive power cost allocation by using a value-based approach. IET Gener Transm Distrib 2009;3(9):872–84.
[24] De M, Goswami SK. Voltage support cost allocation. Energy Convers Manage 2011;52(2):1184–91.
[25] Araújo MM, Saavedra OR, Prada RB. Allocation of reactive power support based on a voltage source model. Int J Electr Power Energy Syst 2011;33(3):439–44.
[26] Wu ZQ. Determination of the branch power and loss caused by node injection power. IEE Proc-Gener Transmiss Distrib 2003;150(6):722–6.
[27] Teng JH. Power flow and loss allocation for deregulated transmission systems. Int J Electr Power Energy Syst 2005;27(4):327–33.
[28] Thukaram D, Vyjayanthi C. Relative electrical distance concept for evaluation of network reactive power and loss contributions in a deregulated system. IET Gener Transm Distrib 2009;3(11):1000–19.
[29] De M, Goswami SK. Reactive support allocation using improved Y-bus matrix method. IET Gener Transm Distrib 2011;5(4):448–60.
[30] Mustafa MW, Khalid SN, Shareef H, Khairuddin A. Reactive power transfer allocation method with the application of artificial neural network. IET Gener Transm Distrib 2008;2(3):402–13.
[31] Sulaiman MH, Mustafa MW, Shareef H, Khalid SNA. An application of artificial bee colony algorithm with least squares support vector machine for real and reactive power tracing in deregulated power system. Int J Electr Power Energy Syst 2012;37(1):67–77.
[32] Mustafa MW, Sulaiman MH, Shareef H, Khalid SNA. Reactive power tracing in pool-based power system utilising the hybrid genetic algorithm and least squares support vector machine. IET Gener Transm Distrib 2012;6(2):133–41.
[33] MATPOWER Version 5.1. <http://www.pserc.cornell.edu/matpower/>.