



# Optimal allocation of distributed generation and energy storage system in microgrids

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**Abstract:** This study presents a new approach for optimal allocation of distributed generation (DG) and energy storage system (ESS) in microgrids (MGs). The practical optimal allocation problems have non-smooth cost functions with equality and inequality constraints that make the problem of finding the global optimum difficult using any mathematical approaches. A dynamic capacity adjustment algorithm is incorporated in the matrix real-coded genetic algorithm (MRCGA) framework to deal with the non-smooth cost functions. The proposed cost function takes into consideration operation cost minimisation as well as investment cost minimisation at the same time for the MG. Moreover, an energy storage equality constraint is applied to manage the state of charge of EES in MGs. The MRCGA is used to minimise the cost function of the system while constraining it to meet the customer demand and security of the system. For each studied case, sets of optimal capacities and economic operation strategies of ESS and DG sources are determined. The computational simulation results are presented to verify the effectiveness of the proposed method.

## Nomenclature

$A_i^{DG}$	power capacity of $i$ th DG unit, kW	$C_j^{OM}$	operating and maintenance cost constant of $j$ th ESS, \$/kWh
$A_j^{ESS}$	power capacity of $j$ th ESS, kW	$C_{Fi}$	daily fuel cost of $i$ th generator unit, \$
$C_C^{ESS}$	cost of ESS charge from hour $t_0$ to hour $t_1$ , \$	$C_{Ei}^{DG}$	daily emission cost of $i$ th generator unit, \$
$C_D^{ESS}$	revenue of ESS discharge from hour $t_1$ to hour $t_2$ , \$	$C_{Ini}^{DG}$	daily investment cost of $i$ th DG unit, \$
$C_G^{ESS}$	profit of ESS from hour $t_0$ to hour $t_2$ , \$	$C_{Inj}^{ESS}$	daily investment cost of $j$ th ESS, \$
$C_{inh}^{ESS}$	hourly investment cost of ESS, \$	$C_{OMi}$	daily operating and maintenance cost of $i$ th generator unit, \$
$C_{OMh}^{ESS}$	operating and maintenance cost constant of ESS, \$/kWh	$C_{OMj}^{ESS}$	daily operating and maintenance cost of $j$ th ESS, \$
$C_{Ai}^{DG}$	price of $i$ th DG unit, \$/kW	$C_G^{MG}$	daily profit of energy exchanged with the grid, \$
$C_{Aj}^{ESS}$	price of ESS charge/discharge inverter, \$/kW	$C_{Si}$	daily startup cost of $i$ th generator unit, \$
$C_{Qj}^{ESS}$	price of ESS battery, \$/kWh	$C_{in}^{MG}$	daily scheduling cost of the microgrid, \$
$C_{FCi}$	daily fuel cost of $i$ th fuel cell, \$	$C_{op}^{MG}$	daily investment cost of the microgrid, \$
$C_{fci}$	fuel price of fuel cell, \$/kWh	$C_{tc}^{MG}$	daily total cost of the microgrid, \$
$C_{MTi}$	daily fuel cost of $i$ th microturbine, \$	$D_L(t)$	load demand at hour $t$
$C_{mti}$	fuel price of microturbine, \$/kWh	$L$	total number of DG units
$C_{ik}^E$	emission factor of $i$ th generating unit and $k$ th emission type, \$/kWh	$M$	total number of ESSs
$C_i^f(t)$	fuel price of $i$ th generator unit at hour $t$ , \$/kWh	$n$	the depreciation period in years
$C_g(t)$	power price of open power market at hour $t$ , \$/kWh	$N_i$	the number of startup
$C_i^{OM}$	operating and maintenance cost constant of $i$ th generator unit, \$/kWh	$P_{DGi}(t)$	active power output of the $i$ th generator at hour $t$ , kW
		$P_{ESSj}(t)$	active power output of the $j$ th ESS at hour $t$ , kW
		$P_g(t)$	active power exchanged with the grid at hour $t$ , kW
		$P_i(t)$	power output of $i$ th unit at hour $t$ , kW
		$P_i^{min}$	power lower limit of $i$ th unit, kW

$P_i^{\max}$	power upper limit of $i$ th unit, kW
$P_r^{\text{ESS}}$	power capacity of ESS, kW
$Q_s(t)$	aggregated capacity of ESS at hour $t$ , kWh
$Q_{\min}$	minimum capacity of ESS, kWh
$Q_{\max}$	maximum capacity of ESS, kWh
$Q_j^{\text{ESS}}$	energy capacity of $j$ th ESS, kWh
$r$	annual interest rate
$\Delta t$	scheduling interval (1 h in this paper)
$T$	time of the daily scheduling
$T_{\text{DG}i}$	yearly operating days of $i$ th DG unit
$T_{\text{ESS}j}$	yearly operating days of $j$ th ESS
$T_{\text{off}, i}$	time to which the unit is switched off
$T_{ci}$	cooling time constant
$W_{\text{hourly}}^{\text{ESS}}$	hourly discharged energy of ESS, kWh
$\eta_c^{\text{ESS}}$	charging efficiency of ESS
$\eta_d^{\text{ESS}}$	discharging efficiency of ESS
$\sigma_i$	hot startup cost of $i$ th generator unit, \$
$\delta_i$	cold startup cost of $i$ th generator unit, \$

## 1 Introduction

The needs to reduce pollutant gas emissions and the increasing energy consumption have led to an increase in installation capacity of renewable energy sources and energy storage system (ESS) [1–4]. Nowadays, electrical and energy engineering have to face a new scenario in which small distributed generation (DG) sources and dispersed energy-storage devices have to be integrated together into the electrical grid [5]. The new electrical grid, also named smart grid (SG), will deliver electricity from suppliers to consumers using digital technology, thus reducing cost and increasing reliability and transparency [6–8]. As the impact of geography, climate, weather and other external factors, the output energy of renewable energy sources is intermittent and unpredictable [9, 10], which will cause the complexity of energy exchange between the DG sources, ESSs and load. Furthermore, the user can purchase electricity from the grid and can also sell surplus energy of the own DG sources to the grid, which will increase the complexity of energy exchange between the distributed sources, ESSs and load in the microgrid (MG) and the main grid [11–13].

The effective model is required to match the total power production to the demand in an optimal way [14, 15]. This concept is pertinent in the framework of SGs through the combined use of an additional communication network within an intelligent EMS and local controllers. This scheme is a step between current grid requirements and future SGs. Many researches have been done to study the optimal energy management of the DG sources and ESSs in MGs considering the different aspects. In [16], a central unit commitment (UC) and economic dispatch model is extended with models for three large-scale energy storage technologies: pumped hydro accumulation storage (PAC), underground PAC and compressed air energy storage. In [17], a distributed intelligent energy management system is presented to minimise the MG operation cost by managing the charging and discharging rates of the battery. In [18], Berkeley Lab's Distributed Energy Resources Customer Adoption Model is presented to evaluate combined heat and power opportunities since it selects the optimal combination of distributed energy resources (DER) and storage investment options, fully taking their interdependence into account. In [19], an optimal participation strategy is presented for the wind electric generators that

employ an energy storage device for participating in the day-ahead UC process. In [20], the cooperative control strategy of microsources and the ESS during islanded operation is presented and evaluated by a simulation and experiment. In [21], a linear mixed integer UC problem is formulated in a general algebraic modelling system environment to simulate the impacts of different wind profiles on fuel saving benefits, startup costs and wind power curtailments.

As previously described, the effect of the MG energy management is strongly dependent on the scheduling and configuration optimisation of DG units and the ESSs. However, fewer researchers found that a conflict of interest exists regarding the optimal allocation and optimal short-term scheduling during the day. On the one hand, power companies want to minimise the investment of the installed DG and ESS to meet the increasing load requirements. On the other hand, power companies also want to maximise their profits by storing energy as much as possible at the lowest possible price and selling energy at the highest possible price. Both objectives should be weighted and considered in a smart algorithm, but fewer mathematical formulations are proposed.

In our previous study [22], we presented a methodology for the optimal allocation and economic analysis of ESS in MGs. By discretising the state of charge (SOC) of battery, the best charge/discharge trajectory of battery was calculated in order to maximise the net present value. However, we assume that the capacity of DG units in the MG is fixed, and only discussed the optimal allocation of the ESS in [22]. In essence, our approach in [22] could face difficulty when the capacity of DG units changed. Therefore a dynamic capacity adjustment algorithm is incorporated in the matrix real-coded genetic algorithm (MRCGA) framework in order to provide the solutions satisfying the coordinate optimisation of capacity configuration and economic dispatch. It aims to find a more comprehensive optimal configuration and economic dispatch of DG and ESS in MGs. This paper also tries to find the relationship between the operation cost and the investment cost of the MG. The forecasting simulation is used to deal with the effects of uncertainties of stochastic solar power generation, electricity market price and load values on technical and economic issues of the optimisation problem. ESS will follow the same initial SOC everyday. Typical results will be obtained based on the case study of a chosen day in this paper.

The rest of this paper is arranged as follows: Section 2 describes the various factors that should be considered when analysing the allocation and operation of MGs. In order to evaluate the economic performance of MGs, the ESS and DG investment cost, operating and maintenance costs, fuel costs, startup costs and energy purchase costs are given in Section 3, including the objective function and constraints. Section 4 gives the characteristics of the MRCGA. Results and discussion are given for a low-voltage dc MG in Section 5, illustrating the optimal allocation methods and economic operation strategies of MGs. Conclusions are given in Section 6.

## 2 System description of MGs

A MG potentially consists of a large number of energy resources of different types, investment costs, operating costs etc. It is necessary to limit the number and the type of the DG sources and ESSs. The configuration of the MG and the available DGs are described as follows.

### 2.1 Smart energy management system (SEMS) of the MG

Typically the objective of power dispatch is the minimisation of the production cost of all units, emission reduction and sustainability objectives can also be considered within the optimisation. The period of the UC is 24 h in general [23], and the scheduling units include the generating units, ESSs and interruptible loads.

This work proposes a new approach for the coordination between the short-term scheduling and configuration optimisation of DG units and the ESSs based on SEMS [24]. Fig. 1 presents the inputs and outputs of SEMS. The inputs of the SEMS include: the forecasted generating upper limits of the renewable energy sources, the forecasted load, the pricing values of energy market, the units production costs etc. In each elementary time interval, based on the input data listed above, the scheduler elaborates a set of control signals that are sent to the DG units and the ESSs. In this paper, the period of UC is 24 h and the elementary time interval is 1 h. The time-series forecasts of the renewable energy sources and loads are carried out using the neural networks modules. The SEMS calculates the active power set points during the following day in order to optimise different technical, economical and environmental objectives.

### 2.2 Photovoltaic (PV) array

A PV module is composed of PV cells in series, and multiple modules in series-parallel connection can form the PV array. The output characteristics of the PV module differ in different operating conditions, and the solar irradiation  $I_{SI}$  and ambient temperature  $T_c$  are the key factors that affect the output characteristics of PV module. The output power of the PV array can be predicted using forecasting model  $f_{forecast}()$ .

$$P_{pv} = f_{forecast}(I_{SI}, T_c) \tag{1}$$

### 2.3 Fuel cell (FC)

The FCs consume hydrogen and oxygen to produce electrical energy and can operate as long as fuel is being supplied. The fuel cost of FC is calculated as follows

$$C_{FCi} = \sum_{t=1}^T C_{fci}(t)P_{fci}(t)(\Delta t) \tag{2}$$

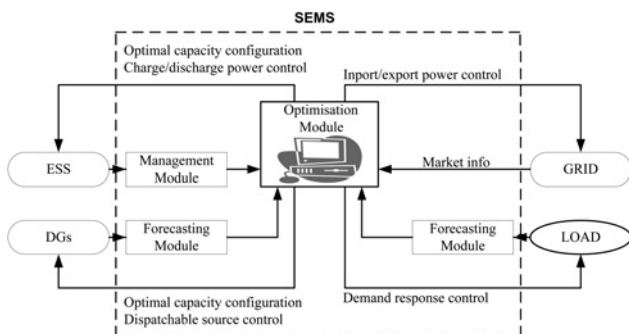


Fig. 1 Schematic of the inputs and outputs of SEMS

### 2.4 Microturbine (MT)

MT models are similar to those of FCs. The MT fuel cost is as follows

$$C_{MTi} = \sum_{t=1}^T C_{mti}(t)P_{mti}(t)(\Delta t) \tag{3}$$

### 2.5 ESS model

Batteries are one of the most cost-effective ESSs available, with energy stored electrochemically. A battery storage system is made up of a set of small-power battery modules connected in series and parallel to achieve a desired electrical characteristic. In MGs, the ESSs are ‘charged’ when the supply from the DG sources exceeds the load demand. They deliver the absorbed energy, or ‘discharge’ when the supply of the MG is insufficient. In order to obtain the highest profit of energy prices differences between light-load and peak-load periods, energy storage charge/discharge operation must be scheduled such as, to store low-price energy during light-load periods and then to deliver it during peak-load ones. Benefits can be made only if ESS efficiency is greater than the ratio (off-peak energy price/peak energy price).

When the ESS is charging ( $P_{ESSj}(t_1) < 0$ ) from hour  $t_0$  to hour  $t_1$

$$Q_s(t_1) = Q_s(t_0) - \eta_c^{ESS} P_{ESSj}(t_1) \Delta t \tag{4}$$

$$C_C^{ESS} = C_g(t_1) |P_{ESSj}(t_1)| \Delta t + C_{inh}^{ESS} + C_{OMh}^{ESS} P_r^{ESS} \Delta t \tag{5}$$

and when the ESS is discharging ( $P_{ESSj}(t_2) > 0$ ) from hour  $t_1$  to hour  $t_2$

$$Q_s(t_2) = Q_s(t_1) - P_{ESSj}(t_2) \Delta t / \eta_d^{ESS} \tag{6}$$

$$C_D^{ESS} = C_g(t_2) P_{ESSj}(t_2) \Delta t - C_{inh}^{ESS} - C_{OMh}^{ESS} P_r^{ESS} \Delta t \tag{7}$$

Assume  $Q_s(t_0) = Q_s(t_2)$

$$P_{ESSj}(t_2) = \eta_c^{ESS} \eta_d^{ESS} |P_{ESSj}(t_1)| \tag{8}$$

$$\begin{aligned} C_G^{ESS} &= C_D^{ESS} - C_C^{ESS} \\ &= C_g(t_2) P_{ESSj}(t_2) \Delta t - C_{inh}^{ESS} - C_{OMh}^{ESS} P_r^{ESS} \Delta t \\ &\quad - \left( C_g(t_1) |P_{ESSj}(t_1)| \Delta t + C_{inh}^{ESS} + C_{OMh}^{ESS} P_r^{ESS} \Delta t \right) \\ &= \left( \eta_c^{ESS} \eta_d^{ESS} C_g(t_2) - C_g(t_1) \right) |P_{ESSj}(t_1)| \Delta t \\ &\quad - 2C_{inh}^{ESS} - 2C_{OMh}^{ESS} P_r^{ESS} \Delta t \end{aligned} \tag{9}$$

In this paper, the charging efficiency and the discharging efficiency are assumed to be 0.9 and 0.9, respectively.

## 3 Optimisation problem

The UC problem is a frequently visited area of research, where the operational schedule of generators is optimised, usually with the aim of minimising the cost or maximising the profit from meeting electricity demand. This paper deals

with the optimal configuration of DG and ESS in MGs by formulating it in the SEMS. The SEMS will consider high level system design where broad parameters such as types, power/energy capacity of the DG and ESS are optimised at the same time as their operational schedule is optimised. This results in a more optimum system than the simple UC problem because it gives an indication of what DG and ESS are best suited to a MG demand profile and energy price combination, rather than only indicating how best to operate them after they have been purchased/installed. The modelling of SEMS can be represented as follows.

### 3.1 Optimisation model

1. Load demand. Load curves of the whole MG for a typical weekday are shown in Fig. 2.
2. Data about locally available energy resources. These include solar irradiation data as depicted in Fig. 3a, temperature as in Fig. 3b.
3. Investment cost. The daily investment cost for DG unit has been calculated from

$$C_{Ini}^{DG} = \left( \frac{r(1+r)^n}{(1+r)^n - 1} \right) \left( \frac{C_{Ai}^{DG} A_i^{DG}}{T_{DGi}} \right) \quad (10)$$

The daily investment cost of ESS will be expressed as

$$C_{Inj}^{ESS} = \left( \frac{r(1+r)^n}{(1+r)^n - 1} \right) \left( \frac{C_{Aj}^{ESS} A_j^{ESS} + C_{Qj}^{ESS} Q_j^{ESS}}{T_{ESSj}} \right) \quad (11)$$

The daily investment cost of MG will be expressed as

$$C_{in}^{MG} = \sum_{i=1}^L C_{Ini}^{DG} + \sum_{j=1}^M C_{Inj}^{ESS} \quad (12)$$

4. FC. FC for PV can be taken as zero. For fuel powered generators, it is assumed that fuel prices of fuel powered generators are different according to the operation date. The

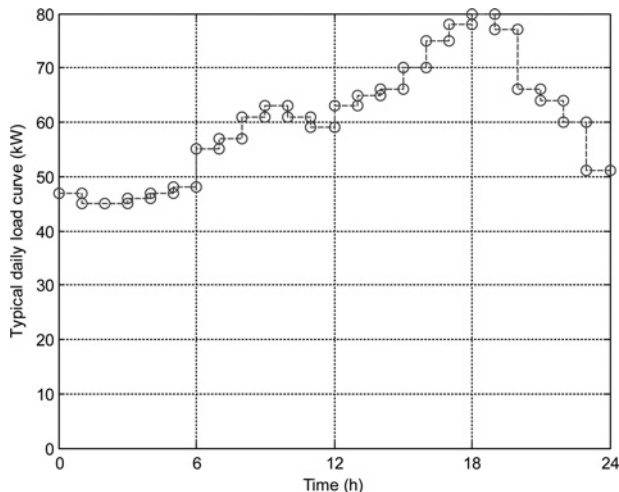


Fig. 2 Typical load curve of the MG

FC is expressed as

$$C_{Fi} = \sum_{t=1}^T C_i^f(t) P_i(t) \Delta t \quad (13)$$

5. Daily purchased and sold power tariffs. In order to obtain maximum profit, the MG can be controlled to reduce the operating cost by storing low-price energy during light-load periods and then delivering it to load or selling it to grid during peak-load ones. The benefit under this operation can be expressed as

$$C_G^{MG} = \sum_{t=1}^T C_g(t) P_g(t) \Delta t \quad (14)$$

6. Startup cost. Startup cost is considered only for fuel-consuming units. For the MT, the fuel cost for the startup period at full capacity and half efficiency has been taken into account to calculate the startup cost. For the FC, the startup cost considered is defined as a function of two main parts: the hot startup cost and the cold startup cost.

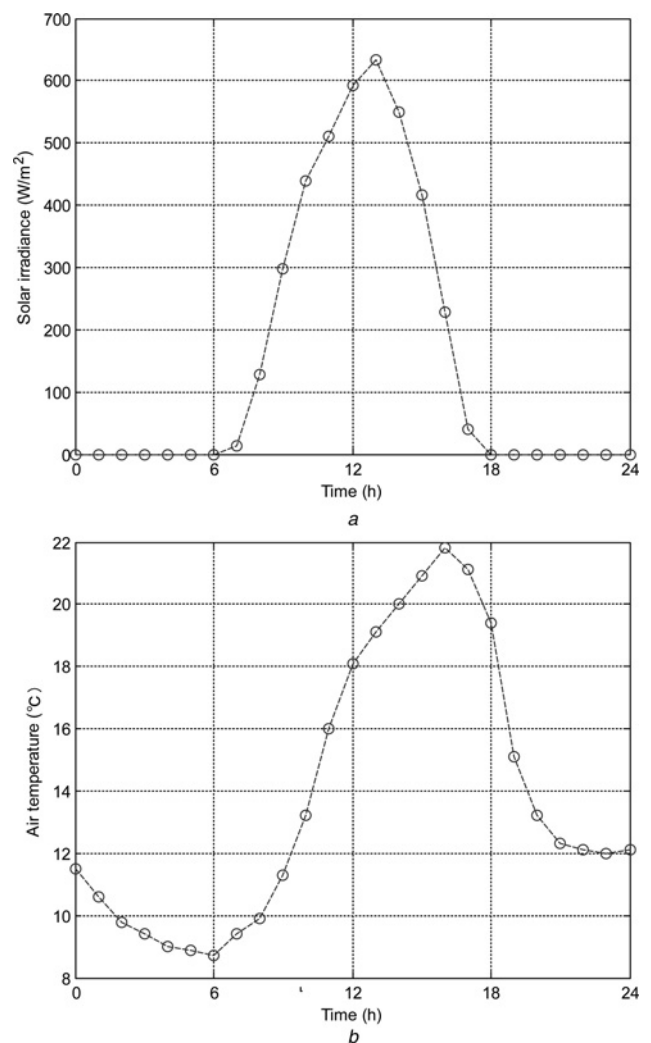


Fig. 3 Data about locally available energy resources

- a Input solar irradiation data used in the model
- b Input temperature data used in the model

**Table 1** Externality costs and emission factors for NO<sub>x</sub>, SO<sub>2</sub>, CO and CO<sub>2</sub> [25, 26]

Emission type	Penalty for pollutant emission, \$/kg	Emission factors for FC, g/kWh	Emission factors for MT, g/kWh
NO <sub>x</sub>	1.250	0.023	0.6188
SO <sub>2</sub>	0.875	0	0.000928
CO	0.145	0.0544	0.1702
CO <sub>2</sub>	0.004125	635.04	184.0829

The startup cost is expressed as

$$C_{Si} = \left[ \sigma_i + \delta_i \left( 1 - e^{(-T_{off,i})/(T_{ci})} \right) \right] * N_i \quad (15)$$

7. Pollutant emission costs (Table 1). The pollutant emission costs of DG sources is expressed as

$$C_{Ei} = \sum_{t=1}^T \sum_{k=1}^K C_{ik}^E P_i(t) \Delta t \quad (16)$$

8. Operating and maintenance costs. The operating and maintenance cost of DG sources may be specified in \$/kW. The operating and maintenance cost of DG combination is expressed as

$$C_{OMi} = \sum_{t=1}^T C_i^{OM} A_i^{DG} \Delta t \quad (17)$$

The operating and maintenance cost of ESS is expressed as

$$C_{OMj}^{ESS} = \sum_{t=1}^T C_j^{OM} A_j^{ESS} \Delta t \quad (18)$$

9. Objective function. The scheduling problem is to determine the supply quantities of various energy sources and the operating status of different energy supply equipment and devices over the scheduling horizon such that the total cost of the MG is minimised. It is formulated as the following optimisation problem

$$C_{tc}^{MG} = C_{in}^{MG} + C_{op}^{MG} \quad (19)$$

$$C_{op}^{MG} = \sum_{i=1}^L (C_{Fi} + C_{OMi} + C_{Si} + C_{Ei}) + \sum_{j=1}^M C_{OMj}^{ESS} - C_G^{MG} \quad (20)$$

The proposed cost function takes into consideration operation cost  $C_{op}^{MG}$  minimisation as well as investment cost  $C_{in}^{MG}$  minimisation at the same time for the MG and the objective function to minimise the total cost  $C_{tc}^{MG}$ .

### 3.2 Optimisation constraints

1. Power balance constraints. To satisfy the active power balance in the MG, an equality constraint is

$$\sum_{i=1}^L P_{DG_i}(t) + \sum_{j=1}^M P_{ESS_j}(t) + P_g(t) = D_L(t) \quad (21)$$

2. Unit generation output limits

$$P_i^{\min} \leq P_i(t) \leq P_i^{\max} \quad (22)$$

The units ( $P_1(t)$ ,  $P_2(t)$ , ...,  $P_i(t)$ ) in this paper include DG units, ESSs and the grid. For the PV system, the predicted output of the forecasting simulation is used as the power limit of PV system.

3. The equality constraint of the ESS periodical behaviour (see (23))

### 4 Implementation of the optimisation model

When designing MGs, several goals could be set, including economic dispatch and optimal allocation. To achieve this, the MRCGA has been developed to find optimal mix and economic dispatch of available resources in MGs. The MRCGA algorithm consists of two important stages, namely economic dispatch and genetic operations, which are performed recursively till a feasible solution satisfying all the constraints is obtained. Performing economic dispatch prior to genetic operation would ensure the satisfaction of the equality constraint and then minimisation proceeds through genetic operation on the strings (schedules, in this case). A dynamic capacity adjustment algorithm is incorporated in the MRCGA framework in order to provide the solutions satisfying the uniform optimisation of capacity configuration and economic dispatch. In order to more efficiently calculate the objective function and the optimisation constraints, the real number matrix is applied. The optimisation process is expressed as follows:

1. Read data about locally available energy resources, fuel prices and electric prices of open market.
2. Run MRCGA to generate a real number matrix  $G_k$ .

$$G_k = \begin{bmatrix} C_{DG1,1} & C_{DG1,2} & \dots & C_{DG1,t} & \dots & C_{DG1,T} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ C_{DGL,1} & C_{DGL,2} & \dots & C_{DGL,t} & \dots & C_{DGL,T} \\ C_{ESS1,1} & C_{ESS1,2} & \dots & C_{ESS1,t} & \dots & C_{ESS1,T} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ C_{ESSM,1} & C_{ESSM,2} & \dots & C_{ESSM,t} & \dots & C_{ESSM,T} \\ C_{Grid,1} & C_{Grid,2} & \dots & C_{Grid,t} & \dots & C_{Grid,T} \end{bmatrix}$$

$$\begin{cases} Q_S(0) = Q_S(T) \text{ and } Q_{\min} \leq Q_S(t) \leq Q_{\max} \\ \frac{1}{\eta_d^{ESS}} \sum_{P_{ESS_j}(t) > 0, t=1, \dots, 24} P_{ESS_j}(t) + \eta_c^{ESS} \sum_{P_{ESS_j}(t) < 0, t=1, \dots, 24} P_{ESS_j}(t) + \sum_{P_{ESS_j}(t) = 0, t=1, \dots, 24} W_{hourly} = 0 \end{cases} \quad (23)$$

$$= \begin{bmatrix} C_{1,1} & C_{1,2} & \dots & C_{1,t} & \dots & C_{1,T} \\ C_{2,1} & C_{2,2} & \dots & C_{2,t} & \dots & C_{2,T} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ C_{i,1} & C_{i,2} & \dots & C_{i,t} & \dots & C_{i,T} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ C_{N,1} & C_{N,2} & \dots & C_{N,t} & \dots & C_{N,T} \end{bmatrix} \quad (24)$$

where  $G_k$  is the  $k$ th individual of genetic populations;  $C_{DGL,t}$  is the power output of the  $L$ th DG unit at time  $t$ ,  $C_{ESSM,t}$  is the power output of the  $M$ th ESS unit at time  $t$ ,  $C_{Grid,t}$  is the power output of the grid unit at time  $t$ ,  $C_{i,t}$  is the  $i$ th row,  $t$ th column element of coding matrix, which is the power output of the unit  $i$  at time  $t$ ,  $N=L+M+1$ .

3. Update the capacity configuration of DG and ESS when  $k=k+1$ .

$$A_i(k) = \max(C_{i,1}^k, C_{i,2}^k, \dots, C_{i,T}^k) \quad (25)$$

$$C_{lni}^{DG}(k) = \left( \frac{r(1+r)^n}{(1+r)^n - 1} \right) \left( \frac{C_{Ai}^{DG} A_i^{DG}(k)}{T_{DGi}} \right) \quad (26)$$

It is assumed that the ratio of charge/discharge current of the ESS is 0.1 C. Therefore the energy capacity of ESS can be calculated

$$\begin{cases} Q_j^{ESS}(k) = 10A_j^{ESS}(k) * \Delta t \\ A_j^{ESS}(k) = \max(|C_{j,1}^k|, |C_{j,2}^k|, \dots, |C_{j,T}^k|) \end{cases} \quad (27)$$

where  $C_{j,t}$  is the  $j$ th row,  $t$ th column element of coding matrix, which is the power output of the  $j$ th ESS at time  $t$ .

$$C_{lnj}^{ESS}(k) = \left( \frac{r(1+r)^n}{(1+r)^n - 1} \right) \times \left( \frac{C_{Aj}^{ESS} A_j^{ESS}(k) + C_{Qj}^{ESS} Q_j^{ESS}(k)}{T_{ESSj}} \right) \quad (28)$$

4. Adjust each variable to satisfy its constraints. The elements value of coding matrix is adjusted to satisfy unit generation output limits. The power capacity adjustment of DG is described as follows

$$\begin{cases} P_i^{\max} = A_i(k) \\ P_i^{\min} = \eta_i^{\min} A_i(k) \end{cases} \quad (29)$$

$$C_{i,t}^* = \begin{cases} C_{i,t}^k, & P_i^{\min} < C_{i,t}^k \leq P_i^{\max} \\ P_i^{\min}, & \lambda P_i^{\min} < C_{i,t}^k < P_i^{\min} \\ 0, & \text{else} \end{cases} \quad (30)$$

where  $C_{i,t}^k$  is the value before adjustment,  $C_{i,t}^*$  is the adjusted value,  $\eta_i^{\min}$  is the minimum portion of rated capacity that can output of unit  $i$ ,  $\lambda$  is the constant between 0 and 1, this paper is taken to 0.6.

The power and energy constraints of ESS are calculated according to formula (23).

5. Select the fitness function. The fitness function is defined as the following form

$$f_{fit} = A / \left( C_{tc}^{MG} + \sum_{i=1}^N \delta_{pi} P_{Vi} \right) \quad (31)$$

where  $f_{fit}$  is the fitness function,  $P_{Vi}$  is the violation value of the constraints of unit  $i$ ,  $\delta_{pi}$  is the penalty factor for the violation value and  $A$  is the positive constant.

6. Repeat steps 2–5 until  $f_{fit}^{k-1}$  and  $f_{fit}^k$  converge to within a tolerable level. The capacity configuration of the DG/ESS and the constraints of economic dispatch in the MG will be updated according the genetic populations generated by MRCGA, including available DG/ESS equipment options and their associated costs (investment cost, startup cost, emission cost, operating and maintenance costs etc.), load profiles, energy tariff structures, fuel prices, the maximum/minimum storable energy of energy storage installations, the peak power and the equality constraint of the periodical behaviour.

7. Find the minimum total costs and index corresponding to the minimum total costs.

The flowchart in Fig. 4 illustrates the implementation of this iterative technique. The function options of MRCGA are expressed as follows:

1. Fitness scaling options. Select ‘Rank’ as the scaling function.
2. Selection options. Select ‘Remainder’ as the selection function.
3. Reproduction options. The elite count is set to 2 and the value of crossover fraction is 0.8.
4. Mutation options. Select ‘Uniform’ as the mutation function.

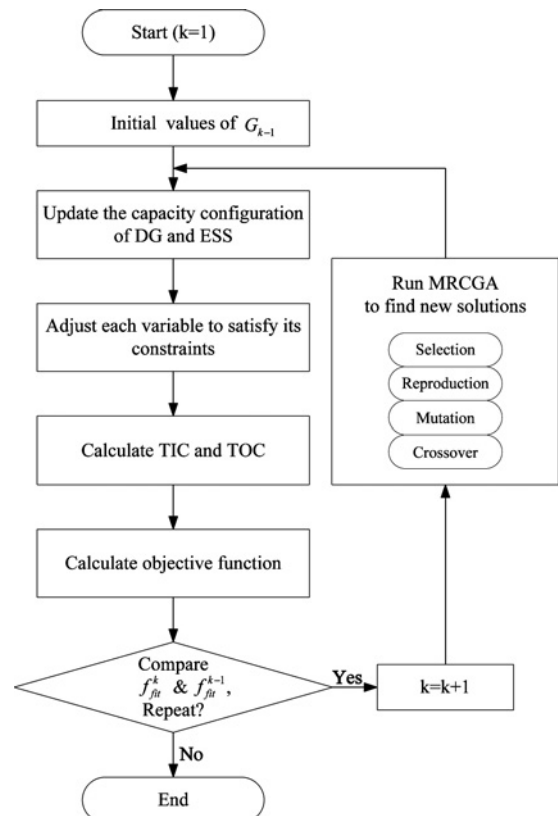


Fig. 4 Iterative method of MRCGA

5. Crossover options. Select ‘Two points’ as the crossover function.

In order to make the results of GA more accurate, the stopping criterion of GA is that there is no improvement in the best fitness value. Through repeated testing, it is found that the results of the iterative algorithm under the same conditions are basically similar.

### 5 Simulations and results analysis

As mentioned in the previous sections, the typical study case of low-voltage MG is shown in Fig. 1. The MG considered in this paper consists of a PV array, a MT, a FC and ESS. The inputs to the model of the PV were measured data. The ESSs is discharged to supply the load and at the same time the SOC is monitored and in the end of the simulation the battery should be charged to the initial state. Table 2 summarises the costs coefficients of the DG sources expressed in dollars. The hourly energy price of open market comes from [14]. The MG acts only on the daily market in this paper.

It is assumed that the consumer in actual MG system meets all of its electricity demand via utility purchases. The actual operating cost for the day is \$260. The MRCGA finds the optimal sets of power and energy capacities of the DG sources and ESS by minimisation of the operation cost in each internal loop. Simulations are done in eight cases in different operating environment and specific settings are presented in Table 3. In case 1, case 3, case 5 and case 7, the capacity configuration of the DG sources and ESS is fixed. Therefore the investment costs of the DG sources and ESS is fixed. In case 2, case 4, case 6 and case 8, the capacity configuration of the DG sources and ESS will be adjusted according to the MRCGA. The computation time of the iterative method is about 300 s.

Fig. 5a presents the optimal production diagrams of the MG with the preinstall DG sources in case 1, whereas the MG can absorb power from the electricity grid and cannot provide power into it. As the investment costs of DG sources are constant, the sum of other costs will decide which DG sources will operate for the next hour. The daily operating costs are \$219 in case 1. In case 2, the capacity configuration and power production of the MG are optimised uniformly,

whereas the MG can absorb power from the electricity grid but cannot provide power into it. Fig. 5b presents the optimal production diagrams of the MG with the preinstall DG sources in case 2. Note that FC systems were never selected by the optimisation model in this case, because their high investment cost means they are not competitive with the other technology options considered. The daily costs are \$143 in case 2. Compared with case 1, the daily total costs are reduced by 35% after dynamic capacity adjustment. Meanwhile, compared with the actual operating results, the daily total costs are reduced by 45%.

In case 3, the MG can absorb power from the electricity grid but cannot provide power into it. As ESS efficiency is less than the ratio (off-peak energy price/peak energy price), the power output of ESS is equal to 0. Therefore the optimal production diagrams of the MG with the preinstall DG sources and ESS are same with case 1. The daily costs are \$233 and the additional costs are the investment cost, operating and maintenance cost of ESS. In case 4, the capacity configuration and power production of the MG are optimised uniformly, whereas the MG can absorb power from the electricity grid but cannot provide power into it. Fig. 6 presents the optimal production diagrams of the MG in case 4. The daily total costs are \$145. Compared with case 3, the daily total costs are reduced by 38% after dynamic capacity adjustment.

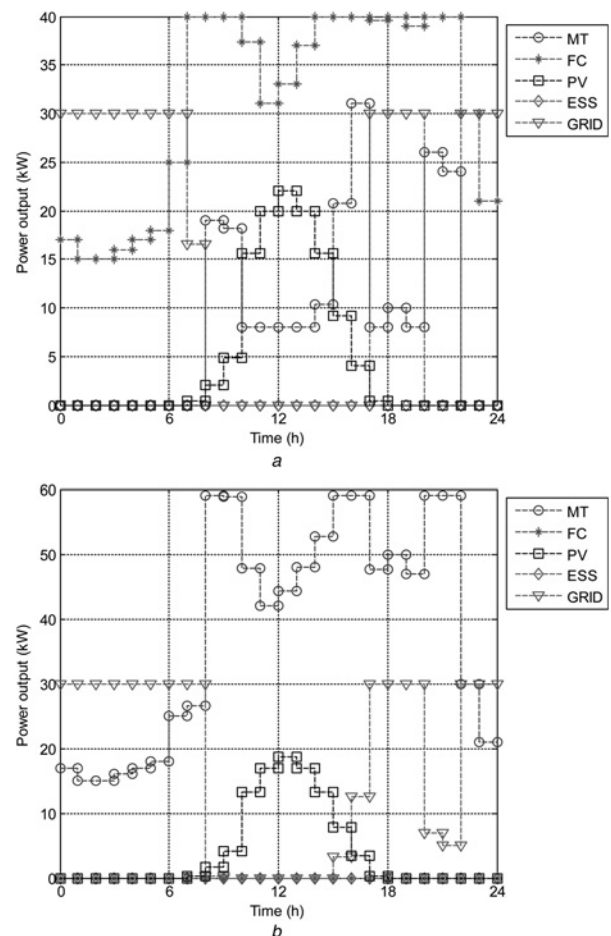
Fig. 7a presents the optimal production diagrams of the MG with the preinstall DG sources in case 5, whereas the MG can absorb/provide power from/into the electricity grid.

**Table 2** Costs of the DG sources

	MT	FC	PV
investment cost, \$/day	0.825	2.474	0.456
externality cost, \$/MWh	1.558	2.656	0
operating and maintenance cost, \$/kW	0.015	0.029	0.07
fuel cost, \$/kWh	0.056	0.036	0
startup cost, \$	0.115	0.205	0

**Table 3** Different operating environment of the MG

Type	Capacity configuration	ESS	Grid
case 1	fixed	without	unidirectional
case 2	changed	without	unidirectional
case 3	fixed	with	unidirectional
case 4	changed	with	unidirectional
case 5	fixed	without	bidirectional
case 6	changed	without	bidirectional
case 7	fixed	with	bidirectional
case 8	changed	with	bidirectional



**Fig. 5** Optimal production diagrams of the MG in cases 1 and 2

a Optimal produces diagrams of the MG in case 1

b Optimal produces diagrams of the MG in case 2

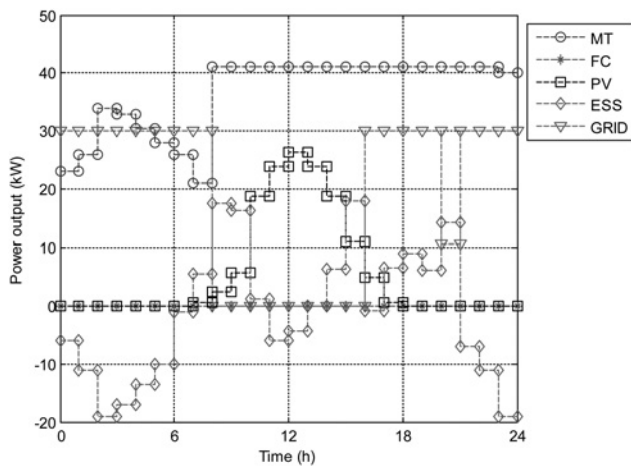


Fig. 6 Optimal produces diagrams of the MG in case 4

In case 6, the capacity configuration and power production of the MG are optimised uniformly, whereas the MG can absorb/provide power from/into the electricity grid. Fig. 7b presents the optimal production diagrams of the MG in case 6. Compared with case 5, the daily costs are reduced by 42% after dynamic capacity adjustment.

Fig. 8a presents the optimal production diagrams of the MG with the preinstall DG sources and ESS, whereas the

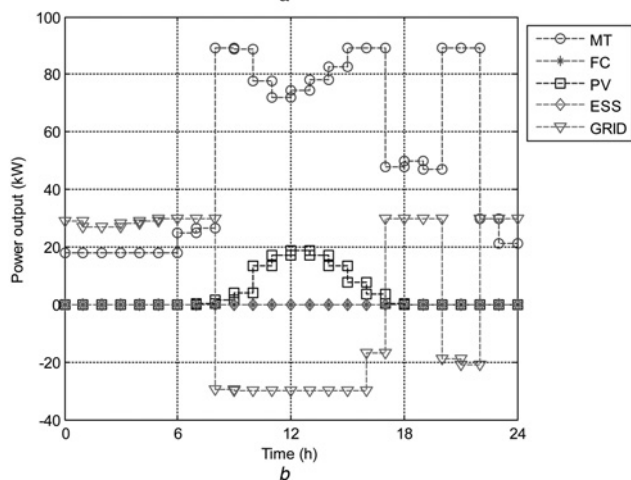
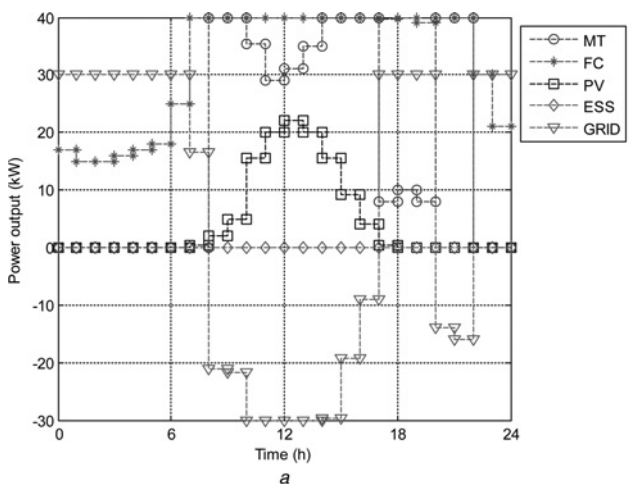


Fig. 7 Optimal production diagrams of the MG in cases 5 and 6

a Optimal produces diagrams of the MG in case 5  
b Optimal produces diagrams of the MG in case 6

MG can absorb/provide power from/into the electricity grid. In case 8, the capacity configuration and power production of the MG are optimised uniformly, whereas the MG can absorb/provide power from/into the electricity grid. Fig. 8b presents the optimal production diagrams of the MG in case 8. Compared with case 7, the daily costs are reduced by 45% after dynamic capacity adjustment.

We examine the daily ESS schedule in Fig. 8b. In this figure, positive corresponds to hours when ESS is discharged, whereas negative corresponds to charging hours of the ESS. According to the hourly load profile with two peak hours, the power grid

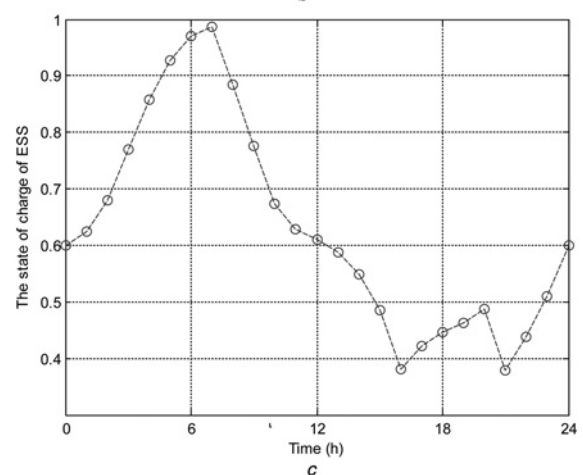
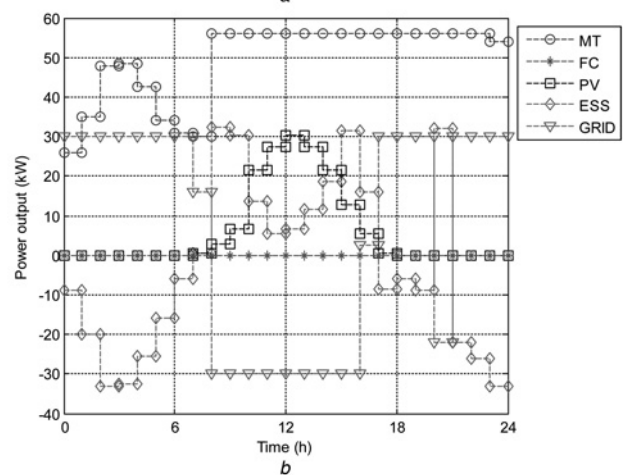
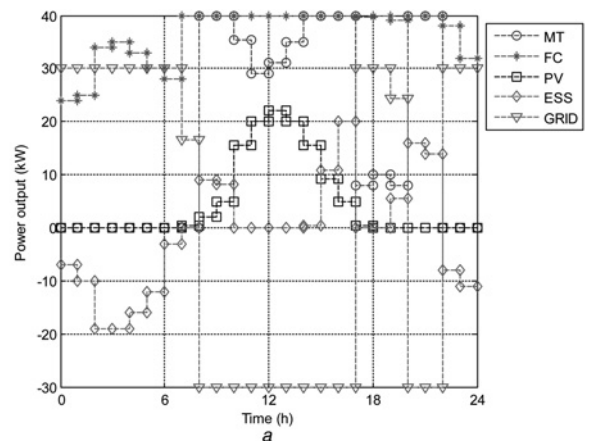


Fig. 8 Optimal production diagrams of the MG in cases 7 and 8

a Optimal produces diagrams of the MG in case 7  
b Optimal produces diagrams of the MG in case 8  
c SOC of ESS



**Table 4** Capacity configuration and total costs of the MG

Type	MT	FC	PV	ESS	Grid	Total costs
case 1 (Fig. 5a)	40	40	40	0	0–30	219
case 2 (Fig. 5b)	59	0	34	0	0–30	143
case 3 (Fig. 5a)	40	40	40	20	0–30	233
case 4 (Fig. 6)	41	0	48	19	0–30	145
case 5 (Fig. 7a)	40	40	40	0	–30–30	144
case 6 (Fig. 7b)	89	0	34	0	–30–30	84
case 7 (Fig. 8a)	40	40	40	20	–30–30	149
case 8 (Fig. 8b)	56	0	55	33	–30–30	82

and some DG sources will charge the ESS at midnight and afternoon hours, respectively, when hourly loads are relatively low. Then, during the peak load at noon, the DG sources and ESS will both supply the local load directly instead of charging the ESS. This operational scheme provides a lower cost for supplying the hourly load as compared with that of charging the ESS at noon hours when the prices of electricity purchased are high. The SOC of ESS is shown in Fig. 8c.

The average optimum capacities of the considered generation components are presented in Table 4. From the results in Table 4, it can be seen that installing DG and ESS with optimal size could reduce total costs of the MG.

## 6 Conclusion

This paper presents a new approach for optimal allocation of DG and ESS in MGs. Compared with the past-proposed capacity configuration methods, the advantage of the method presented in this paper is taking into account the dynamic capacity adjustment of DG and ESS, which is suitable for the coordinate optimisation between capacity configuration and short-term scheduling of MG systems. An interesting feature of the proposed approach is that the dynamic capacity adjustment algorithm is incorporated in the MRCGA framework; therefore the capacity adjustment in the loop iteration is directly achieved by means of energy capacity coefficients that can be obtained from the results of a single power flow calculation. The energy storage equality constraint of ESS is also resolved by fitness function. The strategies for handling constraints are devised while preserving the dynamic process of the MRCGA. The computational simulation results show that optimal allocation of DG and ESS can be achieved according to the MRCGA, and total costs during the system operational lifetime period are minimised. It can be reasonably expected that the methods proposed in the present work may provide useful indications on optimal allocation for MG system compatible with a large penetration of renewable distributed resources.

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