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# Mathematical Modeling of Particle Deposition in Hydraulic Settler

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**Abstract**

The present analysis is based upon the numerical solution of the full Navier-Stokes equations for laminar viscous flow. The convection-diffusion model is used for the determination of the flow particle concentration and the formation of typical sedimentation zones. The results are compared with available experimental data.

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*Keywords:* Navier-Stokes equations; particle deposition; recirculation zone; finite-difference method.

## 1. Introduction

One of the most important problem in hydrotechical construction is to remove the bottom and suspended sediments from water flows on its way to energy, irrigation and water conduits. Removal of the bottom sediment can be accomplished by appropriate design of the intake without building special structures. To precipitate suspended solids settlings a construction of expensive tanks in which the particles settle to the bottom under the gravity effect is required. The studying of the hydrodynamics of flow is necessary to create more favorable conditions contributing to the most rapid and efficient deposition of sediment. Physical modeling of flows in such designs is rather a sophisticated problem. Therefore, the mathematical simulation of flows in the settler taking into account the deposition of particles is an actual task.

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**Nomenclature**

| | |
|---|---|
| $V_x$, $V_y$ | axial and transverse velocity components |
| $\psi$ | stream function |
| $\Omega$ | vorticity |
| $Q$ | flow rate |
| $\nu$ | kinematic viscosity |
| $\rho$ | density |
| Re | Reynolds number |
| $t$ | time |
| $c$ | particle concentration |
| St | Stokes number |
| Pr | Prandtl number |
| Sc | Schmidt number |
| $g$ | gravity acceleration |

One of the simplest designs to be applied for this purpose is shown in Fig. 1 and has the following principle. From the intake channel the water enters the chamber of the settler, bypassing the sill representing a sudden deepening with a height equal to $h$. A developed recirculation zone is formed in the chamber of the settler. The bottom flow in the chamber transports the settled sediment particles toward the entry sill, in the lowest part of which the flushing slot is located. Through it sediments are discharged into the downstream facilities.

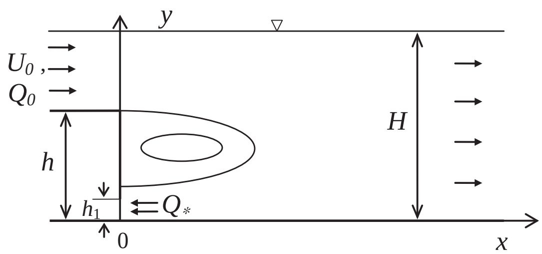


Fig. 1. Schematic of the problem.

## 2. Formulation of the problem and numerical procedure

As a mathematical model of the flow in the chamber settler, let’s consider the problem of the motion of an incompressible viscous fluid moving through the sill of high $h$, on the bottom of which there is a runoff with a given flow rate $Q_*$. Let’s introduce a Cartesian coordinate system $0xy$ with the center at the base of the sill. The system of Navier-Stokes equations describing the laminar flow of a viscous incompressible fluid by means of variables stream function $\psi$ and vorticity $\Omega$ can be written as follows:

$$\frac{\partial \Omega}{\partial t} + \frac{\partial}{\partial x}(V_x \Omega) + \frac{\partial}{\partial y}(V_y \Omega) = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2}\right), \tag{1}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\Omega, \tag{2}$$

$$V_x = \frac{\partial \psi}{\partial y}, \qquad V_y = -\frac{\partial \psi}{\partial x}, \qquad \Omega = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}. \tag{3}$$

The boundary of the free surface is assumed to be a straight line $y = y_k = \text{const}$ parallel to the axis $x$. The flow is considered in the rectangular region $G$ ($0 \le x \le x_k$, $0 \le y \le y_k$). The value $y_k = H/h$ characterizes the flow depth in the settling chamber (where $y_k^{-1}$ is the sill height), and $y_1 = h_1/h$ is the size of the flushing hole.

The boundary conditions for the system (1)-(3) are set as follows. An axial flow with flow rate $Q_0$ and average velocity $U_0$ is assigned at the entry to settling chamber for $x = 0$, $1 \le y \le y_k$. For $x = 0$, $0 \le y \le y_1$ we have a runoff with flow rate $Q_1 = Q_*/Q_0$ and a parabolic velocity distribution. On rigid surfaces, when $0 \le x \le x_k$, $y = 0$ and $x = 0$, $y_1 \le y \le 1$ no-slip conditions are assigned. Conditions of shear stresses are determined as equal to zero and are placed on the surface of the fluid for $0 \le x \le x_k$, $y = y_k$. Soft boundary conditions are imposed in the outlet section $x = x_k$, $0 \le y \le y_k$. The set of boundary conditions can be written as:

$$\psi = \psi_0(y), \quad \frac{\partial \psi}{\partial x} = 0, \quad x = 0, \quad 1 \le y \le y_k, \tag{4}$$

$$\psi = \psi_1(y), \quad \frac{\partial \psi}{\partial x} = 0, \quad x = 0, \quad 0 \le y \le y_1, \tag{5}$$

$$\psi = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad x = 0, \quad y_1 \le y \le 1, \tag{6}$$

$$\psi = \psi_1(0) = \text{const}, \quad \frac{\partial \psi}{\partial y} = 0, \quad y = 0, \quad 0 \le x \le x_k, \tag{7}$$

$$\psi = \psi_0(y_k) = \text{const}, \quad \Omega = 0, \quad y = y_k, \quad 0 \le x \le x_k, \tag{8}$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \Omega}{\partial x} = 0, \quad x = x_k, \quad 0 \le y \le y_k, \tag{9}$$

where $$\psi_0(y) = y - 1, \quad \psi_1(y) = \frac{3}{2y_1} Q_1 \left[ 4\left( \frac{y^2}{2y_1} - \frac{y^3}{3y_1^2} \right) - \frac{2}{3} y_1 \right] \tag{10}$$

To solve numerically the boundary value problem (1)-(3), (4)-(9) the finite difference method was used. The incomplete reduction method was applied for solving the Poisson equation (2). The transport equation (1) was solved using the implicit block iteration method. The diffuse terms were approximated by means of central differences. For approximating the convective terms we used a modified Leonard scheme with quadratic upstream terms of third-order accuracy. This method has been successfully applied in [1] to calculate flows of various types.

## 3. Flow fields calculations results

Flow calculations in the settling chamber determined by the boundary problem (1)-(9) was carried out for values $\mathrm{Re} = 100$; 150; 200; 250; $y_1 = 0.1$, $y_k = 1.7$; 1.8; 1.9; 2; 2.1. The typical streamline patterns for the calculated flows are presented in Fig. 2,a-c.

Experimental model of the settler had the following parameters. The total length of the tray was 5050 mm, the length of the supply channel was 1200 mm, the length of settling chamber was 2800 mm, flow rate was $Q_0 = 3 \div 15$ l/sec, the Reynolds number was $\mathrm{Re} \sim 10^4 - 10^5$.

The results of the experiments can be compared with obtained numerical solutions based on the concept of the effective parameters of the flow. For this purpose in accordance with the Prandtl's hypothesis let's introduce the value of the length of the mixing path $\ell$, which is associated with turbulent viscosity $\nu_t$ as follows:

$$\ell^2 = \nu_t (\partial V_x / \partial y)^2 \tag{11}$$

The effective viscosity of the flow $\nu_{\mathrm{eff}}$ is determined by the sum $\nu_{\mathrm{eff}} = \nu + \nu_t$ and $\nu_t >> \nu$. Taking the known value $\ell \approx 0.11$ for the flows of this type, and estimating the value $\partial V_x / \partial y$ from the experimental data [2], we find that the effective Reynolds number is $\mathrm{Re}_{\mathrm{eff}} = U_0 h / \nu \approx 140 - 150$.

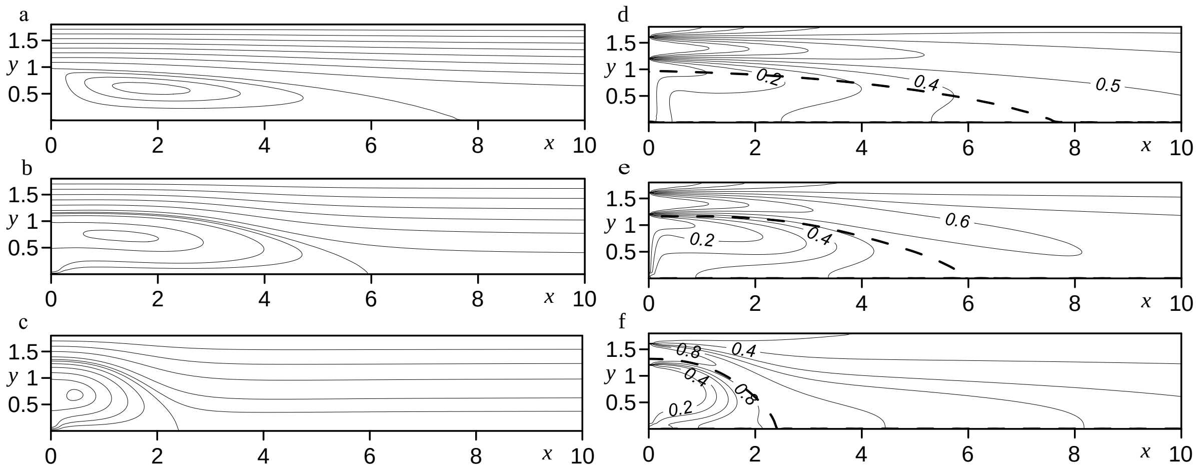


Fig. 2. Streamlines (a-c) and concentration isolines (d-f) for $\mathrm{Re}=150$ ; $y_k=1.8$ ; $Q_1=0;\,0.2;\,0.4$ ; $\mathrm{Fr}=0.01$ ; $\mathrm{St}=5\cdot 10^{-5}$ .

A comparison of the experimental data with the calculation results is given in Fig. 3,a. It can be seen that there is a good agreement between theory and experiment.

In general, through the choice of appropriate and effective Reynolds number it's possible to describe properly the large-scale vortex structures (such as the recirculation zones).

## 4. Convection-diffusion model

In the settling chamber among all forces acting on a moving particle, the most significant are the force of gravity and the force of viscous resistance (Stokes force). The equations of motion of the particles under the influence of these forces in the dimensionless form are:

$$\frac{dV_{xs}}{dt}=\frac{1}{2\,\mathrm{St}}(V_x-V_{xs})\ , \tag{12}$$

$$\frac{dV_{ys}}{dt}=-\frac{1}{\mathrm{Fr}}+\frac{1}{2\,\mathrm{St}}(V_y-V_{ys})\ , \tag{13}$$

$$\mathrm{Fr}=\frac{U_0^2}{g\,h},\qquad \mathrm{St}=\frac{\rho_s r_s^2 U_0}{9\nu\rho\,h}\ ,$$

where St is the Stokes number, Fr is Froude number and the subscript $s$ denotes the particles. For $\mathrm{St}<<1$ from (12) it follows that the axial particle velocities $V_{xs}$ coincide with the corresponding velocity of the basic flow $(V_{xs}=V_x)$ .

The velocity of the particles in the transverse direction will be determined from the equation (13). In the stationary case $(\partial/\partial t=0)$ we have

$$V_{ys}\frac{\partial V_{ys}}{\partial r}=-\frac{1}{\mathrm{Fr}}+\frac{1}{2\,\mathrm{St}}(V_y-V_{ys})\ . \tag{14}$$

Let's consider the process of particle transfer for the flows calculated in paragraph 3. The numerical investigations of two-phase flow with rigid particles are based upon the convection-diffusion model [3]. We assume that the influence of the particle motion on the basic flow is negligible. The flow field is defined as a sum of the particle velocity and the liquid phase velocity. In this case the equation of volume global continuity transforms to

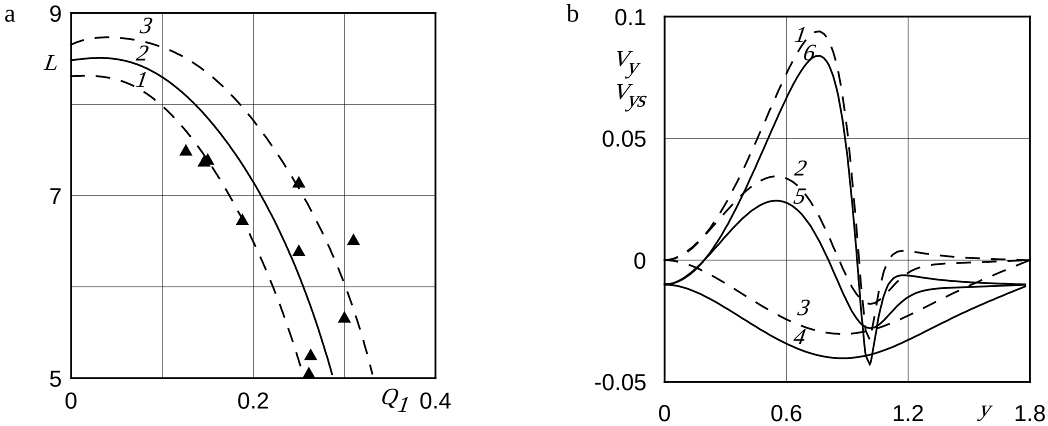


Fig. 3. Comparison of calculated length of the recirculation zone with the experiment (a) for Re=145, $y_k$ = 1.65, 1.7, 1.75 (curves *1-3*), distributions of the transverse velocity $V_y$ (b, curves *1-3*) and particle deposition flow rate $V_{ys}$ (b, curves *4-6*) for $\mathrm{Re}=150$, $Q_1=0$, $y_k=1.8$, $z$ = 0.234, 0.938, 5.

the diffusion equation for the particle concentration $c$:

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial x}(V_x\,c)+\frac{\partial}{\partial y}(V_{ys}\,c)=\frac{1}{\mathrm{Re}\,\mathrm{Sc}}\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right), \tag{15}$$

in which $\mathrm{Sc}=\nu/D$ is the Schmidt number, and $D$ is the diffusion coefficient. The distribution $V_{ys}$ in (15) is determined by solving the equation (14) with the condition $V_{ys}(y_k)=0$.

Assuming that the admixture of the particles is introduced into the basic flow in the initial cross-section $x=0$ with $y_{s1}\le y\le y_{s2}$, ($y_{s1}>1$, $y_{s2}<y_k$) we write down the boundary conditions for the equation (15) in the form:

$$x=0:\quad \frac{\partial c}{\partial x}=0,\quad 0\le y\le y_{s1},\quad y_{s2}\le y\le y_k;\quad c=1,\quad y_{s1}\le y\le y_{s2}\ , \tag{16}$$

$$x=x_k:\quad \frac{\partial c}{\partial x}=0,\quad 0\le y\le y_k\ , \tag{17}$$

$$y=0,\quad y=y_k:\quad \frac{\partial c}{\partial y}=0,\quad 0\le x\le x_k\ . \tag{18}$$

The boundary-value problem (15)-(18) was solved by the relaxation method on the basis of the finite-difference scheme used for the transport equations (1).

## 5. Calculation results for flow fields concentration

For the flows calculated above, the problem of the particle concentration distribution was solved numerically over the parameter ranges $\mathrm{Sc}=1-5$, $\mathrm{St}=10^{-4}-10^{-5}$. In Fig. 2,d-e we have reproduced the most characteristic results of the calculations in the form of concentration isolines. The dashed line in these figures shows the contour of the reverse flow zone.

In Fig. 3,b we have reproduced distributions of the transverse velocity $V_y$ and particle deposition flow rate $V_{ys}$. The difference in the distribution can be noticed only at a short distance $x=0.234$ (curves 1 and 6) away from the inlet cross-section. Further downstream the profile $V_y$ is different from the profile $V_{ys}$ at almost constant value corresponding to the dimensionless deposition rate in a fluid at rest $V_{ys}^{*}=-2\,\mathrm{St}/\mathrm{Fr}$.

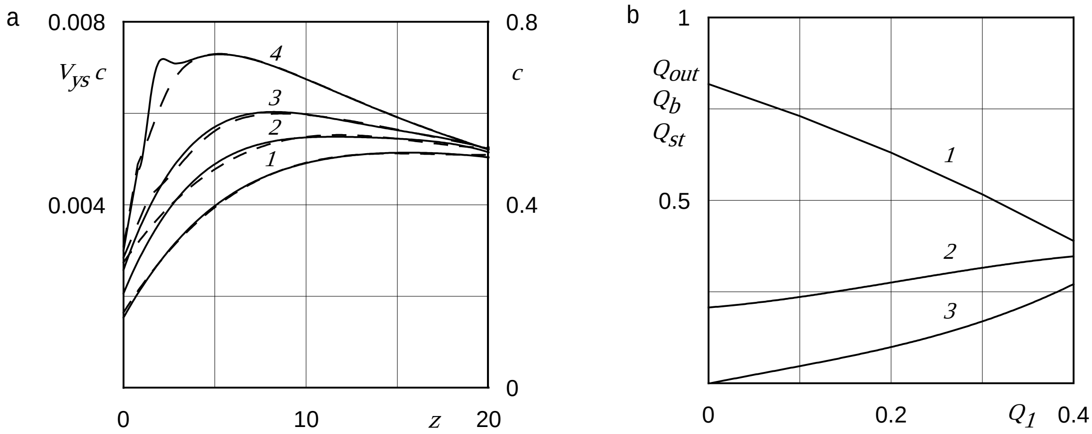


Fig. 4. Concentration $c$ (dashed lines) and flux concentration $V_{ys}c$ (solid lines) for $y=0$, $\mathrm{Re}=150$, $y_k=1.8$, $Q_1=0$, 0.1, 0.2, 0.3 (a, curves 1-4); dependencies of the particle flow rates $Q_{out}$, $Q_b$, $Q_{st}$ (b, curves 1-3) on the runoff $Q_1$ for $\mathrm{Re}=150$, $y_k=1.8$, $\mathrm{Fr}=0.01$, $\mathrm{St}=5\cdot10^{-5}$.

The nature of the particle deposition on the bottom of the settler is illustrated by the distribution of concentration $c$ and flux concentration $V_{ys}c$ for $y=0$ (Fig. 4,a). Using these dependencies we can determine the location of maximum deposition of particles.

For a more detailed view of the particle deposition process in the settling chamber let's introduce the following integral characteristics: the particle flow rates $Q_{out}$, $Q_b$ and $Q_{st}$ through the tube exit cross-section $(x=x_k)$, at the bottom of the settler $(y=0)$ and in the flushing hole $(x=0, 0\le y\le y_1)$ respectively:

$$Q_{out}=\int_0^{y_k}(V_x c)\big|_{x=x_k}dy,\quad Q_b=-\int_0^{x_k}(V_{ys}c)\big|_{y=0}dx,\quad Q_{st}=-\int_0^{y_1}(V_x c)\big|_{x=0}dy\ .$$

Their sum $Q_{out}+Q_b+Q_{st}$ divided by the flow rate of particles $Q_{in}$ in the initial cross-section

$$Q_{in}=\int_{y_{s_1}}^{y_{s_2}}(V_x c)\big|_{x=0}dy\ .$$

in accordance with the mass conservation condition must be equal to unity. This property was used to check the calculation accuracy. The dependencies $Q_{out}$, $Q_b$, $Q_{st}$, divided by the $Q_{in}$, from the flow rate value $Q_1$, are presented for $\mathrm{Re}=150$, $y_k=1.8$ in Fig. 4,b. They also show that an increase of the flow rate in the flushing hole leads to a more intensive deposition of particles on the bottom of the settling chamber, whereas the particle flow rate $Q_{st}$ directly through the flushing hole increases slightly.

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