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# A Novel Approach to Simultaneous Robust Design of Product Parameters and Tolerances Using Quality Loss and Multivariate ANOVA Concepts

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**Design and development of high quality products are of utmost importance to any production plant. Product design consists of parameter design and tolerance design, which affect the product performances and the manufacturing costs, respectively. Most products involve more than one quality feature. Design and development of such products raise multi-response surface problems in which it is necessary to determine the optimal values of parameters and the tolerances for all responses simultaneously. In this research, an approach for simultaneous robust parameter and tolerance design is proposed to deal with multi-response problems. The proposed method employs quality loss concept and one-way multivariate analysis of variance. Two simulation studies are performed to validate the applicability of the proposed method. Research findings show that the proposed method performs better in quality improvement as well as in cost reduction than the existing methods. The variances of the responses are also lower than those of the other methods, that is, the proposed method results in a more robust approach to product design. Copyright © 2016 John Wiley & Sons, Ltd.**

**Keywords:** Robust Design; Product Design; Quality Loss; Multivariate Anova; Multi-response Optimization

## 1. Introduction

In today's highly competitive market, quality and proper functioning of the product according to the customers' needs are essential for the businesses to survive. Hence, the products with high level of quality and acceptable performance gain customer satisfaction through their useful life. Taguchi [1] has introduced three stages for developing a product or a process including system design, parameter design, and tolerance design. In system design, scientific and engineering principles are employed to determine the structure and the arrangement of the system. The parameter design procedure is assigning specific values to the controllable variables to meet the customer requirements. In the tolerance design stage, adequate tolerances for the parameters are determined. Conventionally, each of the three stages with specific inputs and outputs is conducted independently. All these stages are interrelated, and each one cannot guarantee the success of the product by itself. The use of concurrent engineering may yield a proper relationship among the three stages and thus cause a significant decrease in introduction and development period of the product. One of the most important features of the concurrent engineering is to consider manufacturing conditions in the early stage of the product design. Although product quality and process quality are dependent, it is difficult to find a quantitative relationship between them [2]. The knowledge about manufacturing conditions solely may not result in high quality products. The manufacturing conditions and related constraints should be considered in the early stages of the product design. Therefore, customer satisfaction is gained when a product includes the following features:

1. A proper performance in all conditions to satisfy the customers' requirements. In other words, it should be robust to unpredictable conditions.
2. Adequate tolerances which are designed to minimize the quality loss and the manufacturing costs.
3. Commercially available in the market in the least possible amount of time.

Robust design as a useful tool in the product and process design may be applied to achieve these goals. By conducting the stages of parameter and tolerance design simultaneously, better solutions may be found for a robust product. In Section 1, the robust

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parameter design, tolerance design, and simultaneous parameter and tolerance design are reviewed. The proposed method is introduced and discussed in Section 2. In Section 3, two applied examples are provided to demonstrate and validate the proposed method. Finally, the conclusions are drawn in Section 4.

### 1.1. Robust parameter design

The product designers attend to minimize the effects of uncertain conditions on product performances. Therefore, robust design is applicable in many different areas. 'Robust parameter design is an engineering methodology intended as a cost-effective approach for the quality of products and processes'[3]. In robust design, the quality of the product increases by decreasing the variation. Robust parameter design is the selection of the levels of the controllable variables to minimize the effects of nuisance variables. Robinson *et al.* [3], Montgomery [4], and Hasenkamp *et al.* [5] have carried out a comprehensive literature review, which can be referred to for further studies. Problems in engineering design involve the optimization of the product performance for multiple quality features. 'Multi-response optimization typically employs the loss or utility functions of individual responses into a multivariate function to evaluate the set of responses created by a particular set of design factors'[5]. Multi-response optimizations have been discussed in [6] and [7]. In the area of robust design where relations between responses and the controllable variables are not known, response surface methodology (RSM) is an effective tool. Two major approaches in the RSM are single response surface and dual response surface. For more details on the RSM and its approaches, Anderson-cook *et al* [8] and Khuri and Mukhopadhyay [9] can be referred to.

### 1.2. Tolerance design

Tolerance design is one of the most important stages in the product design and development [10]. From the designer's point of view, tolerances affect the final performance of the product. On the other hand, from the producers' perspective, tolerances affect the processes and the machine selection, the jigs, the fixtures, and especially the production costs [2]. Statistical methods and mathematical programming are used to design tolerances. A thorough study on the statistical methods of tolerance design may be found in Nigam and Turner [11]. Moskowitz *et al.* [12] proposed a model to allocate tolerances, considering second-order loss functions related to the tolerances and tolerance costs. Lee and Tang [13] considered the assembly constraints in the tolerance design problem. In addition to assembly constraint, process selection and process precision are also considered in the model, and genetic algorithm (GA) is employed to solve the model as indicated in [14] and [15]. Having considered quality loss costs and tolerance costs, Muthu *et al.* [16] used particle swarm optimization algorithm for optimization of the tolerance design problem. Sivakumar *et al.* [17] formulated the tolerance design problem in a multi-objective framework whose objectives are to minimize the quality loss and the tolerance costs. Tsai [18] proposed a method to solve the tolerance design problem using the orthogonal arrays.

### 1.3. Simultaneous parameter and tolerance design

In the product design and development procedure, the tolerance design is usually conducted after parameter design. This approach cannot always guarantee a cost-effective procedure to achieve acceptable quality features [19]. To address this, Li and Wu [20] proposed a method for simultaneous parameter and tolerance design. By means of a case study in chemical engineering, they showed that when the parameter design and the tolerance design are conducted simultaneously, the decrease in variability would be lower than or, at least, the same as the time when they were carried out independently. In order to design parameters and tolerances simultaneously, Jeang and Lue [21] employed computer simulations to estimate costs associated with the tolerances of the quality characteristics. These two researches paved the way for simultaneous parameter and tolerance design with regard to costs associated with the tolerances. Using loss function associated with the tolerances and the orthogonal arrays, simultaneous parameter and tolerance design is conducted in Jeang and Chang [19]. Jeang [22] suggested an approach to integrate the product and the process design via optimization of process mean and process tolerance. Through this integration, the optimal process parameters and process tolerances for specified design targets and product tolerances were achieved. Asymmetric cost function is applicable to the model by Jeang [23]. In order to estimate the relationship between the tolerances and the costs, Jeang *et al.* [24] employed the response surface methodology. Tsai [25] proposed a metaheuristic method to design parameter and tolerance simultaneously. Jeang [26] proposed a model, with assumption of process shifting, to simultaneously determine initial setting of process parameters, process tolerances, and resetting cycle, which is the time needed to set back process parameters to their initial settings. Process-capability limits, functionality requirements, and conforming rate are also considered in the model. Jeang [27] proposed an integrated method to optimize production lot size and process features, which are process parameters and process tolerances. Assuming non-constant variance of residuals or non-normal responses, the quality loss function associated with the parameters and tolerances is developed by generalized linear models in [28]; in addition, the model is solved using GA. Jeang and Lin [29] carried out a concurrent product and process parameter optimization for cost reduction and quality improvement. They considered process mean, process tolerance, and product tolerance as key controllable variables. Jeang [30] proposed a model to determine parameters of product and process simultaneously. Jeang [31] conducted simultaneous product and process design considering process mean, process tolerance, and product specifications. He also assumed that production costs and quality losses are random variables with known probability contributions. There is, however, a little work on designing the parameters and the tolerances of the products simultaneously. In this research, simultaneous robust design of product parameters and tolerances is suggested by means of integrating a modified quality loss function and the one-way multivariate analysis of variance (MANOVA). The proposed method is introduced in the next section.

 

## 2. Proposed method

In this section, the proposed method of simultaneous robust parameter and tolerance design is discussed. The proposed method has two characteristics: (i) modification of the quality loss functions for the parameters and the tolerances and (ii) employment of MANOVA to allocate tolerances.

Products are identified by their quality features (QFs). In cases where the quality feature is composed of more other quality features, it is usually broken down into its components to construct a product breakdown structure (PBS). In this situation, more than one level of break down is permitted. The components in the last level of the PBS are considered to be controllable variables, which are the output of particular processes. To illustrate, the PBS of a rectangular metal sheet from [33] is shown in Figure 1. The diameter of the sheet is the intended QF, which is considered to be a function of its length and width. The length and width of the sheet are being cut on two different machines.

Therefore, in general, a relationship between the QF and the controllable variables may be represented as follows:

$$y = f(x_1, x_2, \ldots, x_m) \tag{1}$$

Where $y$ is a quality feature, $x_i$ represents the $i$th controllable variable for $i = 1, 2,\ldots, m$, and $f$ is the transfer function. In Eq. (1), only one level of breakdown is considered. Equation (1) may be represented for all QFs in a product. In addition to different characteristics, there are other differences between the product design and the process design. In the product design, the design factors are controllable deterministic variables. Therefore, random variations are only transferred to the responses through the external nuisance factors. In the process design, controllable factors may be considered as nuisance factors. This can transfer random and systematic variations to the responses. 'In robust process design, design factors may be controllable only in the sense that their long-term means may be determined'[32]. In product design for each QF, the related target and tolerance values are allocated. The target value is determined in order to meet the customers' requirements. Because of some nuisance and uncontrollable factors in the production stage, the variation is transferred to the QFs. Thus, for each QF, tolerance limits are assigned that are compatible with manufacturing. Consequently, the effect of the allocated values of the tolerances in the product design stage may be realized in the manufacturing phase. Considering the mentioned facts, quality loss of a product is broken down into two terms: (i) the quality loss associated with the product parameters and (ii) the quality loss associated with the product tolerances. In the product design stage, there may be a bias in the QF when controllable variables are designed improperly. The quality loss associated with the product parameters is expressed as follows:

$$L_1(y) = K_1(y - T)^2 = K_1(f(x_1, x_2, \ldots, x_m) - T)^2 \tag{2}$$

*where* $L_1(y)$ is the quality loss associated with product parameters, T is the target value of the QF, and $K_1$ is the cost coefficient of the quality loss. Equation (2) is only used for nominal-the-best (NTB) scenarios. Many industrial applications also deal with both the smaller-the-better (STB) and the larger-the-better (LTB) scenarios. Therefore, the first term of the quality loss for all of the three common scenarios, STB, NTB, and LTB is modified and expressed as follows:

$$L_1(y) = K_1\left(\frac{f^{max} - f(x_1, x_2, \ldots, x_m)}{f^{max} - f^{min}} - 1\right)^2 \tag{3}$$

$$L_1(y) = \begin{cases} K_1\left(\dfrac{f(x_1, x_2, \ldots, x_m) - f^{min}}{T - f^{min}} - 1\right)^2 & f^{min} \le f(x_1, x_2, \ldots, x_m) \le T \\ K_1\left(\dfrac{f^{max} - f(x_1, x_2, \ldots, x_m)}{f^{max} - T} - 1\right)^2 & T \le f(x_1, x_2, \ldots, x_m) \le f^{max} \end{cases} \tag{4}$$

$$L_1(y) = K_1\left(\frac{f(x_1, x_2, \ldots, x_m) - f^{min}}{f^{max} - f^{min}} - 1\right)^2 \tag{5}$$

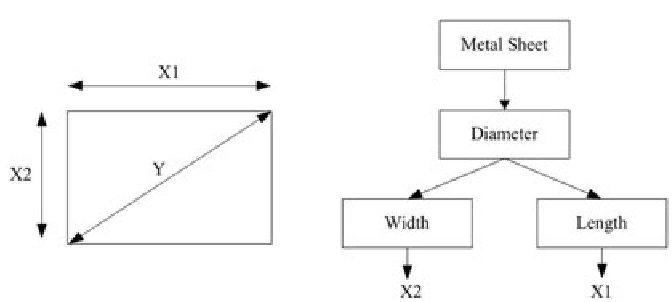


**Figure 1.** Product breakdown structure

*where* $f^{min}$ and $f^{max}$ are the minimum and maximum values of the transfer function, respectively. Equations (3), (4), and (5) are the quality loss functions associated with STB, NTB, and LTB scenarios, respectively. As shown in Eq. (1), QF is defined as a function of controllable variables. In product design, the value of these controllable variables must be determined to minimize the quality loss functions.

In order to evaluate the allocated tolerances of the product, the tolerances of the controllable variables should be considered in the production phase because tolerances are used to control dispersion. Because there are nuisance variables in the production phase, only adequacy of the tolerances must be assessed via the variation transmitted to the product during manufacturing; this variation is calculated in terms of the tolerances. To address this, *Var*(*y*) is used as a measure of dispersion to evaluate the allocated tolerances. Equation (6) expresses the quality loss function associated with the product tolerances:

$$L_2(y) = K_2 Var(y) \tag{6}$$

*where* $K_2$ is the cost coefficient of the quality loss associated with dispersion. A second-order Taylor series approximation expansion of the transfer function around $X_0 = (t_1, t_2, \ldots, t_i, \ldots, t_m)$ is given by Eq. (7). $X_0$ is the vector of nominal values allocated to the parameters in the parameter design stage.

$$y = f(x_1, x_2, \ldots, x_m) \cong f(X_0) + \sum \frac{\partial f}{\partial x_i}|_{t_i}(x_i - t_i) \tag{7}$$

Then the following approximations are calculated:

$$Var(y) = \sum \left(\frac{\partial f}{\partial x_i}|_{t_i}\right)^2 \sigma_{x_i}^2 \tag{8}$$

$$Var(x_i) = \left(\frac{\delta_i}{3C_{pm}}\right)^2 \tag{9}$$

*where* $C_{pm}$ is the process-capability index. $\sigma_{x_i}^2$ and $\delta_i$ are the variance and the tolerance associated with the *i*th controllable variable, respectively. Substitution of Eq. (9) into Eq. (8) results in the variation of the QF as follows:

$$Var(y) = \sum \left(\frac{\partial f}{\partial x_i}|_{t_i}\right)^2 \left(\frac{\delta_i}{3C_{pm}}\right)^2 \tag{10}$$

Therefore, the variation transmitted to the product in the manufacturing phase is defined in terms of the tolerances [33]. Equation (10) shows that when the designed values of the parameters change, it results in changes in the variation coefficients. Therefore, the parameters affect the tolerances. Thus, the relation between the product design stage and the manufacturing phase must be modeled. In addition, the process capability index is meant to reflect the process conditions early in the product design stage. When there is more than one QF to deal with, the quality loss function has to be determined for each response variable independently.

As mentioned earlier, tolerances affect the product performance. Conventionally, tolerances are allocated based on the designers' prior experiences. On the contrary, the proposed method allocates tolerances in a way that the product performance does not change significantly when a controllable variable varies within its tolerance limits. Assuming *p* correlated responses, one-way MANOVA is employed to allocate tolerances. Wilk's lambda test statistic is applied to construct the MANOVA table. Wilk's lambda is defined as follows:

$$\Lambda^* = \frac{|\boldsymbol{B}|}{|\boldsymbol{B} + \boldsymbol{W}|} \tag{11}$$

*where* $\boldsymbol{B}$, $\boldsymbol{W}$, and $\Lambda^*$ are treatment sum of squares and cross products, residual sum of squares and cross products, and Wilk's lambda test statistic, respectively [34]. $\boldsymbol{B}$ and $\boldsymbol{W}$ are calculated using Eqs (12) and (13):

$$\boldsymbol{B} = \sum_{l=1}^{g} n(\overline{\boldsymbol{y}_l} - \overline{\boldsymbol{y}})'(\overline{\boldsymbol{y}_l} - \overline{\boldsymbol{y}}) \tag{12}$$

$$\boldsymbol{W} = \sum_{l=1}^{g}\sum_{j=1}^{n}\left(\boldsymbol{y}_{lj} - \overline{\boldsymbol{y}}\right)'\left(\boldsymbol{y}_{lj} - \overline{\boldsymbol{y}}\right) \tag{13}$$

*where* $\overline{\boldsymbol{y}_l}$ and $\overline{\boldsymbol{y}}$ are the mean of the *l*th treatment and the overall mean, respectively. *g* is the number of the treatments. In the proposed method, three treatments are considered when the tolerance effects on the response variables are evaluated. Assuming that controllable variables are independent, each controllable variable is assessed separately. In order to assess the effects of the assigned tolerances on the responses, three levels, $t_i - \delta_i, t_i, t_i + \delta_i$, are considered for the *i*th controllable variable. A design of experiment with one factor in three levels is employed to construct the MANOVA tests; *n* observations are generated in each treatment using a multivariate normal distribution. Each observation is a p-vector of QF values, which are of interest. For the *lth* treatment associated with the *ith* controllable variable, observations are generated using normal distribution with mean vector $\boldsymbol{\mu}_{l(i)}$ and covariance matrix $\boldsymbol{\Sigma}$. For $l = 1, 2,$ *and* 3, $\boldsymbol{\mu}_{l(i)}$ is defined as follows:

$$\boldsymbol{\mu}_{1(i)} = \left[f_1(t_1, t_2, \ldots, t_i - \delta_i, \ldots, t_m), \ldots, f_p(t_1, t_2, \ldots, t_i - \delta_i, \ldots, t_m)\right] \tag{14}$$

$$\boldsymbol{\mu}_{2(i)} = \left[f_1(t_1, t_2, \ldots, t_i, \ldots, t_m), \ldots, f_p(t_1, t_2, \ldots, t_i, \ldots, t_m)\right] \tag{15}$$

$$\boldsymbol{\mu}_{3(i)} = \left[f_1(t_1, t_2, \ldots, t_i + \delta_i, \ldots, t_m), \ldots, f_p(t_1, t_2, \ldots, t_i + \delta_i, \ldots, t_m)\right] \tag{16}$$

Notations are as defined earlier. Table I shows the treatments and the distribution of the generated observations under each treatment. Now, covariance matrix, $\boldsymbol{\Sigma}$, must be defined. Considering the $k$th QF where $k = 1, 2, \ldots, p$, the $k$th QF requires a variation as follows:

$$\sigma_{kR}^2 = \left[(1/3)\left(\frac{y_{kmax} - y_{kmin}}{2}\right)\right]^2 \tag{17}$$

In Eq. (17), $y_{kmin}$ and $y_{kmax}$ are the maximum and the minimum values of customer requirements for the $k$th QF, respectively. $\sigma_{kR}^2$ is the variation associated with the $k$th QF imposed by the customer [26]. Therefore, the diagonal elements of the covariance matrix are calculated using Eq. (17). For the off diagonal elements, the covariance among all the QFs must be computed using:

$$\begin{aligned}\sigma_{kh} &= cov(Y_k, Y_h) = E(Y_k.Y_h) - E(Y_k)E(Y_h)\\ &= E[f_k(x_1, x_2, \ldots, x_i, \ldots, x_m).f_h(x_1, x_2, \ldots, x_i, \ldots, x_m)]\\ &\quad - E[f_k(x_1, x_2, \ldots, x_i, \ldots, x_m)Ef_h(x_1, x_2, \ldots, x_i, \ldots, x_m)]\\ for \quad & k, h = 1, 2, \ldots, p \ , \quad k \neq h\end{aligned} \tag{18}$$

*where* $E(.)$ is the expected value. Because controllable variables are assumed to be the outputs of particular processes, then the controllable variables are independently distributed as $N\left(\mu_i, \left(\frac{\delta_i}{3C_{pm}}\right)^2\right)$. It is also assumed that all processes perform on their targets, that is, $E(X_i) = \mu_i = t_i$. So, the moments of the normal distribution may be employed to simplify Eq. (18). As a result, the covariance matrix, $\boldsymbol{\Sigma}$, is now completely defined.

In the proposed method, the initial values are allocated to the tolerances of the controllable variables in advance, and then the observations are generated using the previously mentioned equations. Performing MANOVA here requires modification of the test statistic in Eq. (11). Considering the conditions for the problem in hand, the modified test statistic is represented as follows:

$$F_0 = \frac{3(n-1)}{2} \times \frac{1 - \Lambda^*}{\Lambda^*} \tag{19}$$

Notations are as defined earlier. This modified test statistic follows an F distribution with 2 and $3(n-1)$ degrees of freedom [34]. When $F_0$ is smaller than or equal to $F_{(2,\ 3(n-1);\ \alpha)}$, then the *ith* controllable variable has no significant effect on the QFs under investigation as long as it changes within its tolerances. $F_{(2,\ 3(n-1);\ \alpha)}$ is the critical point of an F distribution with 2 and $3(n-1)$ degrees of freedom leaving $\alpha$ under its right tail. When MANOVA shows significant difference among the mean of the treatments at significance level $\alpha$, then the tolerance associated with *i*th controllable variable decreases. Otherwise, the *P*-value of the hypothesis is compared with a pre-specified value $\gamma$. The tolerance value associated with *ith* controllable variable increases as long as the *P*-value of the test is larger than $\gamma$. By experiment, $\gamma$ is set to be 0.05 to 0.10. This helps to allocate the tolerance as loose as possible without causing any significant change in the response variables as long as the controllable variable changes within its tolerance limits. Assuming that there are no significant interactions between the controllable variables, each controllable variable is assessed independently. This procedure, generating observations, performing MANOVA, and comparing the *P*-value of the test to $\alpha$ and $\gamma$, must be repeated until the *P*-value of the tests is larger than $\alpha$ but smaller than $\gamma$. The output of this procedure is the sets of assigned tolerances associated with each controllable variable. Figure 2 presents the block diagram of the proposed method. As shown in Figure 2, the steps of the proposed method for tolerance design are boxed in a red line. These steps illustrate the way that the one-way MANOVA concept is applied to tolerance design. The proposed method allocates tolerances through an iterative approach. As mentioned earlier, these steps must be followed for all controllable variables. The pseudo code for the tolerance design is available in Appendix 1.

As shown in Table I and Eqs (12) to (14), the only difference between the treatments is related to different levels of the *i*th controllable variable. Different levels of the *i*th controllable variable are constructed using its tolerances. Therefore, when MANOVA results in no significant difference among the treatments, it means that product performance does not change significantly when the controllable variable varies within its tolerance limits.

**Table I.** Treatments and generated observations

| Treatment 1 | Treatment 2 | Treatment 3 |
|---|---|---|
| $\mathbf{y}_{11}$ | $\mathbf{y}_{21}$ | $\mathbf{y}_{31}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $\mathbf{y}_{1u}$ | $\mathbf{y}_{2u}$ | $\mathbf{y}_{3u}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| $\mathbf{y}_{1n}$ | $\mathbf{y}_{2n}$ | $\mathbf{y}_{3n}$ |
| $\mathbf{y}_{1u} \sim N(\boldsymbol{\mu}_{1(i)}, \boldsymbol{\Sigma})$ | $\mathbf{y}_{2u} \sim N(\boldsymbol{\mu}_{2(i)}, \boldsymbol{\Sigma})$ | $\mathbf{y}_{3u} \sim N(\boldsymbol{\mu}_{3(i)}, \boldsymbol{\Sigma})$ |

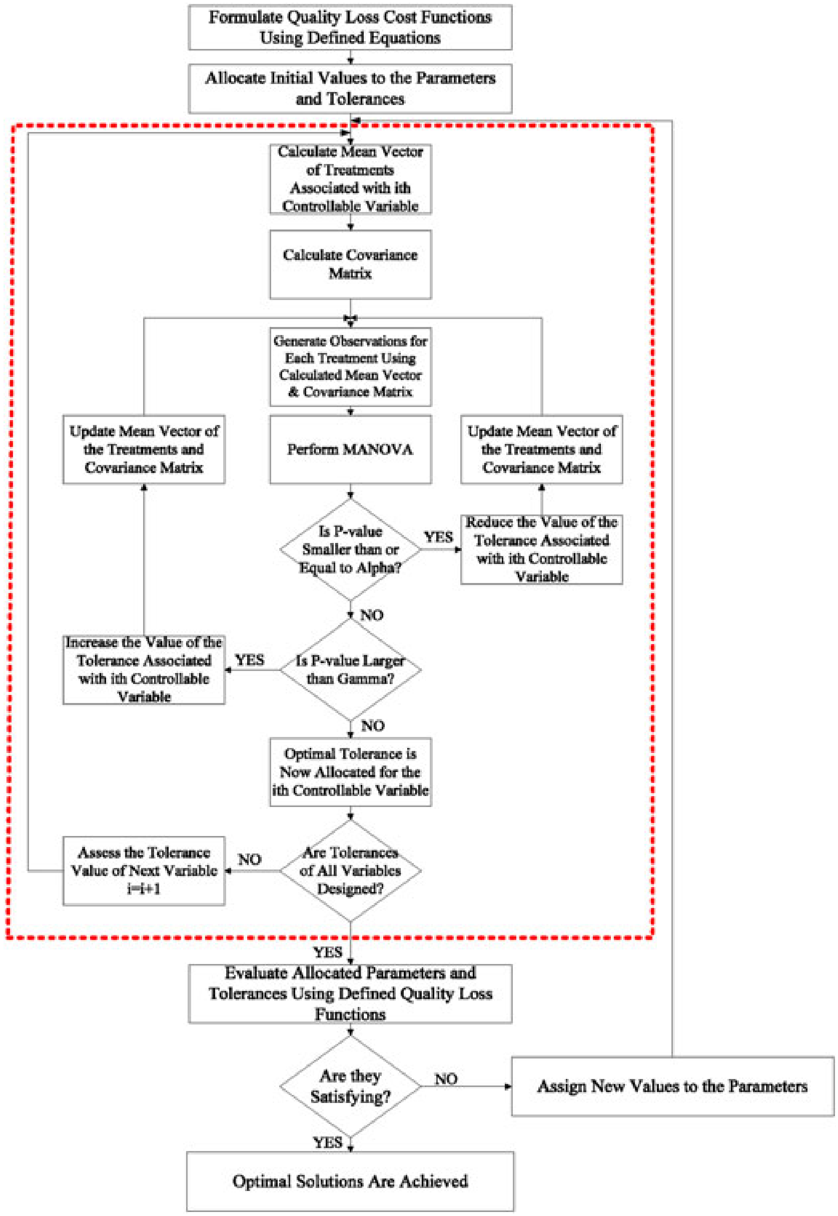


**Figure 2.** Block diagram of the proposed method

## 3. Applications

In this section, two case studies are presented to evaluate and to justify the proposed method. The first one is a wheel mounting assembly [19], and the second one is a design of a polymer [7,35]. Parameter design and tolerance design are provided in each case. The problems are formulated using the proposed method, and the formulations are explained in details later in solving process. For the proposed method, the sample sizes for the two cases are 30 and 25, respectively. The values of $\alpha$ and $\gamma$ are 0.05 and 0.07 for both cases, respectively. Furthermore, the initial value for all tolerances is 1. The multi-objective structure of the proposed method requires its integration with the non-dominated sorting genetic algorithm II introduced in [36]. Non-dominated sorting genetic algorithm II is a tool for finding the Pareto front for a multi-objective problem. Pareto front is a set of Pareto solutions, which, according to [7], are defined as follows:

'In a common multi-response problem, the optimization scheme for a vector function $\boldsymbol{f}(x)$ is of the form

$$\underset{x \epsilon S}{Min}\, \boldsymbol{f}(x) = \{f_1(x),\ f_2(x), \ldots,\quad f_k(x)\} \tag{20}$$

It is assumed that the control factors or decision variables $x \epsilon R^n$ belong to a nonempty compact feasible region $S \subset R^{n\prime}$. The decision variable $x^*$ is a Pareto solution to Eq. (20) if and only if there does not exist any $x \epsilon S$ such that $\boldsymbol{f}(x) \leq \boldsymbol{f}(x^*)$ and for at least one $i \epsilon$

 

$\{1, 2, \ldots, k\}$, $f_i(x) < f_i(x^*)$. Using the integrated method, the allocated values of the parameters and the tolerances are evaluated by means of the quality loss functions, which were defined earlier.

**Example 1: Wheel mounting assembly**

Figure 3 shows a wheel mounting assembly, which consists of components $x_1, x_2, x_3, x_4$, and $x_5$ [19].

Figure 4 shows the PBS of the wheel mounting assembly (Figure 4).

The components are linked with two interrelated dimension chains represented as follows:

$$y_1 = x_2 - x_4$$

$$y_2 = -x_1 - x_2 - x_3 + x_5$$

Parameters and tolerances for each component should be determined. The target values for both $y_1$ and $y_2$ are 0.14. The lower bound and the upper bound for each response variable are 0 and 0.3, respectively. Cost coefficients of the quality loss are $K_1 = 3000, K_2 = 4000$. Responses have the same dimensions, and both are NTB. Considering Eq. (2), the quality loss function associated with the parameters is calculated as follows:

$$L_1(y_1) = 3000(y_1 - 0.14)^2$$

$$L_1(y_2) = 3000(y_2 - 0.14)^2$$

Variation of the responses is calculated by considering Eqs (7) to (10) and $C_{pm} = 1$:

$$Var(y_1) = \frac{\delta_2^2}{9} + \frac{\delta_4^2}{9}$$

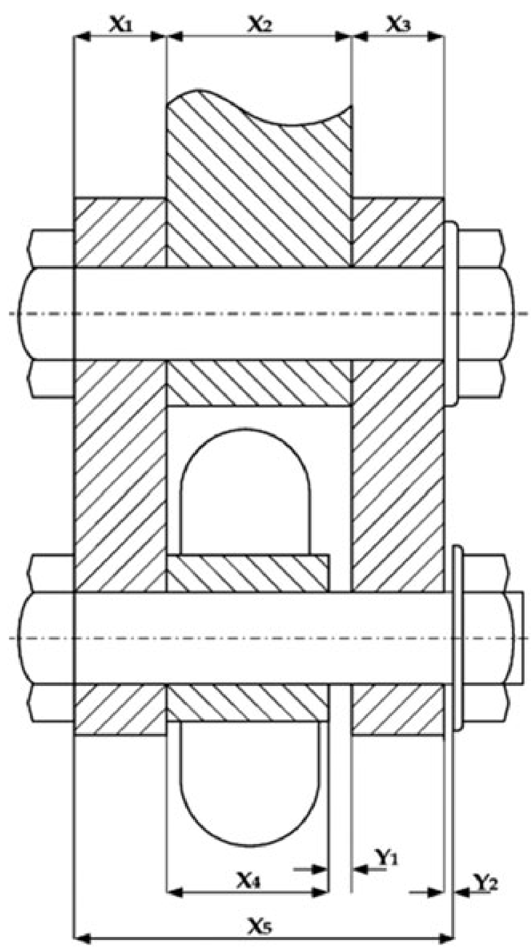


**Figure 3.** Wheel mounting assembly

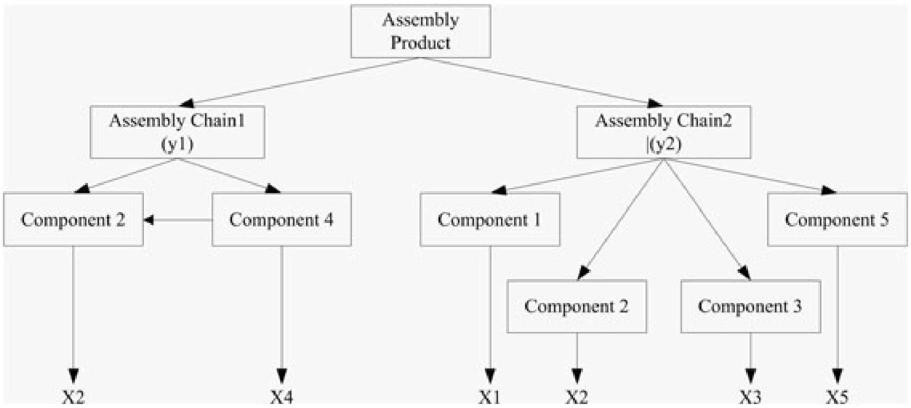


**Figure 4.** Product breakdown structure of wheel mounting assembly

$$Var(y_2) = \frac{\delta_1^2}{9} + \frac{\delta_2^2}{9} + \frac{\delta_3^2}{9} + \frac{\delta_5^2}{9}$$

As a result, the second term of the quality loss for each response is as follows:

$$L_2(y_1) = 4000Var(y_1) = 4000\left(\frac{\delta_2^2}{9} + \frac{\delta_4^2}{9}\right)$$

$$L_2(y_1) = 4000Var(y_2) = 4000\left(\frac{\delta_1^2}{9} + \frac{\delta_2^2}{9} + \frac{\delta_3^2}{9} + \frac{\delta_5^2}{9}\right)$$

The total quality loss function is defined as follows:

$$TQL = L_1(y_1) + L_1(y_2) + L_2(y_1) + L_2(y_2)$$

The tolerance cost function is as follows:

$$C_i(\delta_i) = a_i + b_i e^{-c_i \delta_i}$$

*where* $C_i(\delta_i)$ is the tolerance cost of the $i$th component and $a_i$, $b_i$, and $c_i$ are the coefficients that vary from one component to another. The values of coefficients are given in Table II.

The objectives are to minimize:

$$Z_1 = TQL = 3000(x_2 - x_4 - 0.14)^2 + 3000(-x_1 - x_2 - x_3 + x_5 - 0.14)^2 + 4000\left(\frac{\delta_2^2}{9} + \frac{\delta_4^2}{9}\right) + 4000\left(\frac{\delta_1^2}{9} + \frac{\delta_2^2}{9} + \frac{\delta_3^2}{9} + \frac{\delta_5^2}{9}\right)$$

$$Z_2 = C(\delta) = \sum_{i=1}^{5}\left(a_i + b_i e^{-c_i \delta_i}\right)$$

The variances are computed using

$$\sigma_{1R}^2 = \sigma_{2R}^2 = \left[(1/3)\left(\frac{0.3 - 0}{2}\right)\right]^2 = 0.0025$$

**Table II.** Tolerance cost function coefficients

| Variables | $a(i)$ | $b(i)$ | $c(i)$ |
|---|---|---|---|
| $x_1$ | 3.231 | 81.49 | 37.11 |
| $x_2$ | 6.498 | 40.77 | 43.4 |
| $x_3$ | 3.231 | 81.49 | 37.11 |
| $x_4$ | 0 | 16.48 | -15.21 |
| $x_5$ | 4.292 | 28.9 | 44.3 |

**Table III.** Some of the Pareto solutions of the proposed method

| Solutions | | | | | | |
|---|---|---|---|---|---|---|
| Parameters | Tolerances | $[y_1, y_2]$ | $[Var(y_1), Var(y_2)]$ | TQL | $C_M$ | TC |
| (5.0408, 8.3769, 4.1684, 8.2349, 17.5845) | (0.0816, 0.0794, 0.0831, 0.0264, 0.0719) | [0.1402,0.1404] | [0.00077,0.0028] | 14.2587 | 52.0474 | 66.3061 |
| (4.9688, 8.6670, 3.8364, 8.5208, 17.4729) | (0.0875, 0.0685, 0.0844, 0.0289, 0.0590) | [0.1462,0.1469] | [0.00061.0.0025] | 12.9136 | 53.7483 | 66.6619 |
| (4.9451, 8.5268, 4.0088, 8.3852, 17.4793) | (0.0816, 0.0915, 0.0872, 0.0246, 0.0789) | [0.1416,0.1402] | [0.00099,0.0032] | 16.8300 | 50.0036 | 66.8336 |

TQL: Total Quality Loss Costs; $C_M$: Tolerance Costs; TC: Total Costs.

**Table IV.** Optimal values of the implemented methods

| Methods | Variables | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|---|
| Jeang & Chang [19] | $t_i$ | 5.0006 | 8.4678 | 4.0058 | 8.3218 | 17.6145 |
| | $\delta_i$ | 0.0975 | 0.0935 | 0.1310 | 0.0262 | 0.0714 |
| Proposed method | $t_i$ | 4.8943 | 8.4870 | 4.0362 | 8.3454 | 17.4107 |
| | $\delta_i$ | 0.0972 | 0.0511 | 0.0858 | 0.0268 | 0.0796 |

The covariance between responses is computed using

$$\begin{aligned} E(Y_1.Y_2) &= E[(x_2 - x_4)(-x_1 - x_2 - x_3 + x_5)] \\ &= E\left(-x_2x_1 - {x_2}^2 - x_2x_3 + x_2x_5 + x_4x_1 + x_4x_2 + x_4x_3 - x_4x_5\right) \\ &= -t_1t_2 - \left(\frac{\delta_2^2}{9} + t_2^2\right) - t_2t_3 + t_2t_5 + t_4t_1 + t_4t_2 + t_4t_3 - t_4t_5 \end{aligned}$$

$$E(Y_1) = t_2 - t_4$$

$$E(Y_2) = -t_1 - t_2 - t_3 + t_5$$

$$\sigma_{12} = cov(Y_1, Y_2) = E(Y_1.Y_2) - E(Y_1)E(Y_2) = -\frac{\delta_2^2}{9}$$

Therefore, the covariance matrix is as follows:

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.0025 & -\frac{\delta_2^2}{9} \\ -\frac{\delta_2^2}{9} & 0.0025 \end{bmatrix}$$

The proposed method is applied to obtain the optimal values of the parameters and the tolerances. Some of the solutions of the Pareto front obtained from the proposed method are presented in Table III. The optimized results of the proposed method are compared with those of the method presented in Jeang and Chang [19]. Tables IV and V show the optimized results of the two methods.

The variances obtained using the proposed method and the Jeang and Chang [19] method are shown in Figure 5. For both responses, the proposed method generates smaller variances than Jeang and Chang [19].

Figure 6 presents total quality loss costs, tolerance costs, and total costs of the two methods. The total quality loss and the total costs for the proposed method are smaller than those of the Jeang and Chang [19], but the tolerance costs of the proposed method are larger.

**Table V.** Result comparison of the implemented methods

| | Jeang & Chang [19] | Proposed method |
|---|---|---|
| $[y_1, y_2]$ | [0.1459,0.1403] | [0.1417,0.1348] |
| $[Var(y_1), Var(y_2)]$ | [0.0010,0.0045] | [0.00036,0.0029] |
| TQL | 22.3155 | 13.0100 |
| $C_M$ | 46.5547 | 52.8938 |
| TC | 68.8701 | 65.9037 |

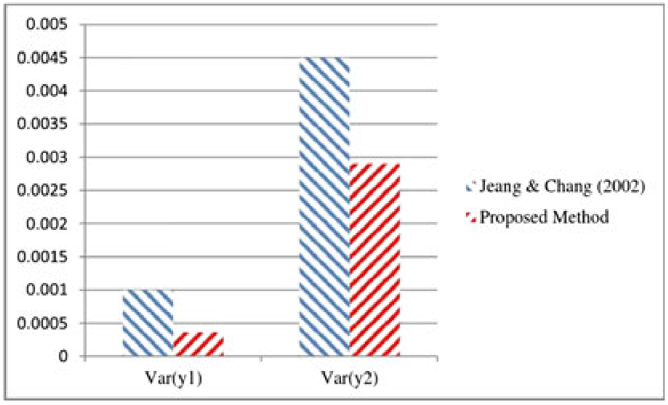


**Figure 5.** Comparison of the variances of the implemented methods

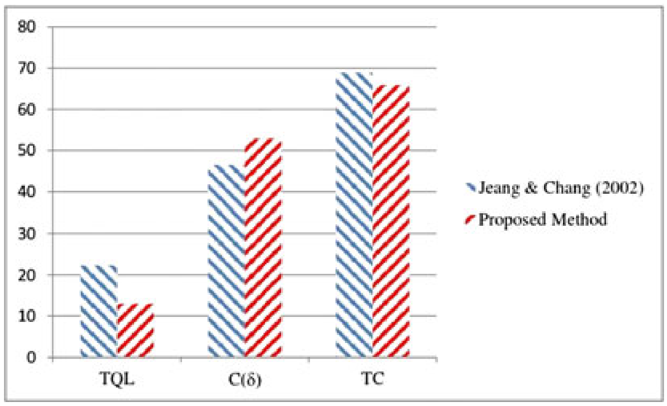


**Figure 6.** Result comparison of the implemented methods

### Example 2: Design of a polymer

The second example is from Ardakani and Wulff [7] and Myers *et al.* [35]. The controllable variables are the reaction time ($x_1$), the reaction temperature ($x_2$), and the amount of catalyst ($x_3$). Thus, the feasible region is cubical where $S = [-1.682, 1.682]^3 \subset R^3$. The responses are the percentage of conversion ($y_1$), and thermal activity ($y_2$) of a polymer. Usually, the relations among the responses and the controllable variables are unknown in such chemical processes. So, such relations can be estimated through RSM. In this case, for the data provided in Appendix 2, the estimated response function for the percentage of conversion ($y_1$ ) and the estimated response function for the thermal activity ($y_2$ ) are given as follows:

$$y_1(X) = 80.93 + 1.03x_1 + 4.10x_2 + 6.20x_3 - 1.63x_1^2 + 2.96x_2^2 - 5.18x_3^2 + 2.03x_1x_2 + 11.37x_1x_3 - 3.80x_2x_3$$
$$y_2(X) = 60.51 + 3.58x_1 + 2.23x_3$$

Figure 7 shows the PBS of the chemical product.

The objective is to maximize the percentage of conversion and to reach to the thermal activity response as close as possible to the target value of $T = 57.5$. Customer requirements indicate that the percentage of conversion has to be greater than 80 and lower than 100. Cost coefficients of the quality loss are $K_1 = K_2 = 2000$. Considering Eqs (2) through (5), the quality loss function associated with the parameters can be calculated as follows:

$$L_1(y_1) = K_1\left(\frac{y_1 - 5.7848}{116.2545 - 5.7848} - 1\right)^2$$

$$L_1(y_2) = \begin{cases} K_1\left(\dfrac{y_2 - f^{min}}{T - f^{min}} - 1\right)^2 & 50 \le f(x_1, x_2, \ldots, x_n) \le T \\ K_1\left(\dfrac{f^{max} - y_2}{f^{max} - T} - 1\right)^2 & T \le f(x_1, x_2, \ldots, x_n) \le 70 \end{cases}$$

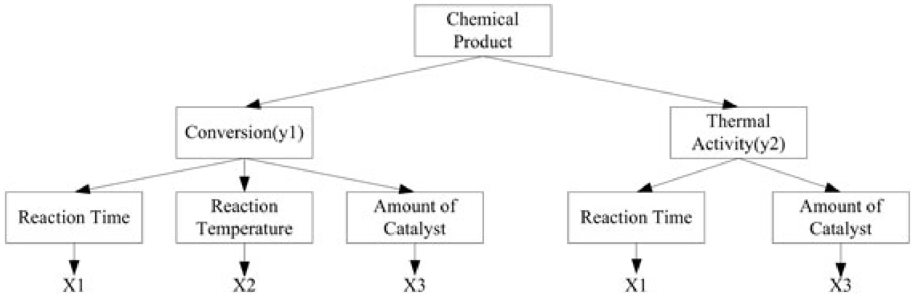


**Figure 7.** Product breakdown structure of the chemical product

Considering Eqs (7) through (10) and $C_{pm} = 1$, then the quality loss functions associated with the tolerances are as follows:

$$L_2(y_1) = 2000Var(y_1) = 2000\sum_{i=1}^{3}\left(\frac{\partial y_1}{\partial x_i}|_{t_i}\right)^2\left(\frac{\delta_i}{3}\right)^2$$

$$L_2(y_2) = 2000Var(y_2) = 2000\sum_{i=1}^{3}\left(\frac{\partial y_2}{\partial x_i}|_{t_i}\right)^2\left(\frac{\delta_i}{3}\right)^2$$

The tolerance cost function is the same as the one mentioned in the wheel mounting assembly example. The values of coefficients associated with this case are given in Table VI.

The objectives are to minimize:

$$Z_1 = TQL \quad Z_2 = C(\delta) = \sum_{i=1}^{3}\left(a_i + b_i e^{-c_i\delta_i}\right)$$

Similar to the example earlier, the covariance matrix of the response variables must be computed. Then using the covariance matrix, observations are generated. Some of the solutions of the Pareto front obtained from the proposed method are presented in Table VII.

**Table VI.** Tolerance cost function coefficients

| Variables | $a(i)$ | $b(i)$ | $c(i)$ |
|---|---|---|---|
| $x_1$ | 3.231 | 81.49 | 37.11 |
| $x_2$. | 6.498 | 40.77 | 43.4 |
| $x_3$ | 331 | 81.49 | 37.11 |

**Table VII.** Some of the Pareto solutions of the proposed method

| Solutions | | | | | | |
|---|---|---|---|---|---|---|
| Parameters | Tolerances | $[y_1, y_2]$ | $[Var(y_1), Var(y_2)]$ | TQL | $C_M$ | TC |
| (−0.4704,1.6784,0.5607) | (0.0613,0.014, 0.0783) | [94.4117,57.5756] | [0.0068,0.0087] | 109.3465 | 47.7678 | 157.1143 |
| (−0.4763,1.6787,−0.5656) | (0.0430,0.0104, 0.0768) | [94.4176,57.5434] | [0.0043.0.0059] | 98.4773 | 60.1428 | 158.6200 |
| (−0.4791.1.6782,−0.5683) | (0.0318,0.0114,0.0673) | [94.4100,57.5276] | [0.0044,0.0039] | 94.4100 | 69.5874 | 164.5535 |

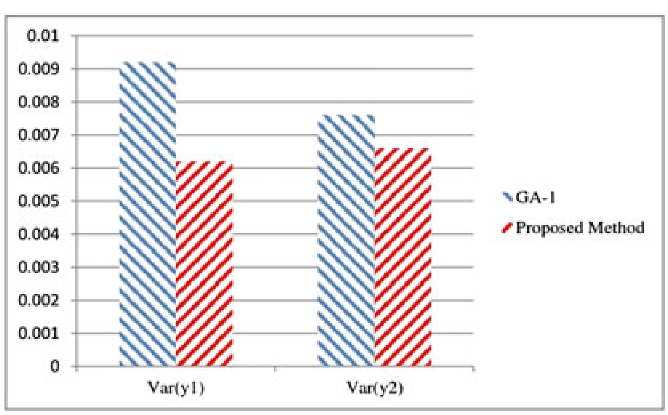


**Figure 8.** Comparison of the variances of the implemented methods

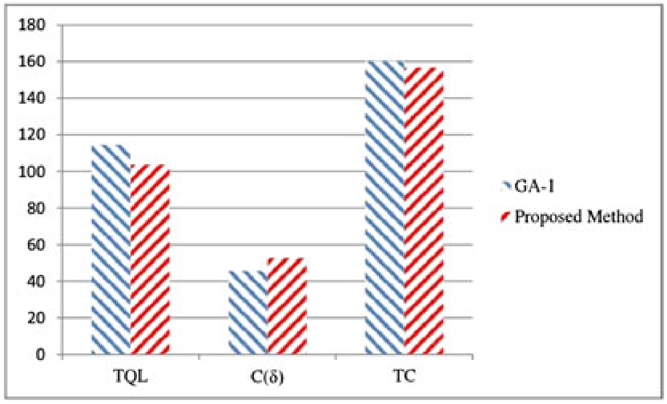


**Figure 9.** Result comparison of the implemented methods

**Table VIII.** Optimal values of the implemented methods

| Methods | Variables | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|---|
| GA-1 | $t_i$ | -0.5351 | 1.6527 | -0.4942 |
| | $\delta_i$ | 0.0584 | 0.0195 | 0.0703 |
| Proposed | $t_i$ | -0.4719 | 1.6798 | -0.5591 |
| | $\delta_i$ | 0.0537 | 0.0140 | 0.0678 |

GA, genetic algorithm.

**Table IX.** Result comparison of the implemented methods

| | GA-1 | Proposed method |
|---|---|---|
| $[y_1, y_2]$ | [94.0224,57.4922] | [94.4312,57.5737] |
| $[Var(y_1), Var(y_2)]$ | [0.0092,0.0076] | [0.0062,0.0066] |
| TQL | 114.6131 | 103.7943 |
| $C_M$ | 45.7789 | 52.8204 |
| TC | 160.3920 | 156.6147 |

For the purpose of comparison, a second approach called GA-1 is applied to this example. The quality loss functions of this approach are the same as the proposed method, but they are optimized using Optimization Toolbox of MATLAB 2010Ra. Results of the optimization procedures are given in Tables VIII and IX.

Results of the two approaches are presented in Figures 8 and 9. Figure 8 shows that the proposed method results in a lower variation of the responses than the GA-1.

Figure 9 presents total quality loss costs, tolerance costs, and total costs of the two approaches. Total quality loss and total costs for the proposed method are lower than those of GA-1, but the summation of tolerance costs for the proposed method is higher. The proposed method is coded using MATLAB 2010Ra Software and is run on a Laptop with four processors each 2.53 GHz and 4.00 GB of RAM.

## 4. Discussions and conclusions

A novel approach to simultaneous robust parameter and tolerance design for multi-response problems is proposed in this research. The proposed method takes into account the conditions of the manufacturing phase early in the product design stage to evaluate effects of the tolerances on the cost and quality. It employs the quality loss concept to formulate quality loss associated with the parameters and the tolerances of a product. One-way MANOVA is also used to allocate the optimal tolerances. The applicability and effectiveness of the proposed method are demonstrated through two case studies. The first case has linear responses, while the second one consists of a linear and a nonlinear response. In the first example, Mount Wheel Assembly, the proposed model yields both lower quality loss costs and lower total costs than those of the Jeang and Chang [19]. In this example, the proposed model sets the response variables closer to their target values in comparison to those of Jeang and Chang [19]. In the second example, Design of a Polymer, the variation for the thermal activity of the polymer for the proposed method is lower than that of the GA-1. Moreover, the variation for the conversion percentage for the proposed method is smaller. It is also shown that the proposed method allocates tolerances more properly than when we use traditional methods. Moreover, the proposed method reduces the costs of the quality loss as well as the total costs. In both cases, it is shown that the proposed method provides a high quality and low cost approach to the simultaneous parameter and tolerance design, that is, the proposed method achieves higher quality with lower cost. Because the proposed method achieves lower variation for the responses than the other methods, the proposed method is more robust in the product design. The proposed method is flexible, so it may be applied to all three common scenarios, STB, NTB, and LTB. Multi-objective formulation of the problem generates more solutions, which may help the user to make appropriate decisions. For further research, considering reliability concept in the product design can be intended.

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## Appendix A

Pseudo code of the tolerance design procedure
**Input**: Initial tolerances, $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, $\alpha$, $\gamma$, sample size
**Repeat** for all variables $(x_1, x_2, x_3, \ldots, x_m)$ /*Controllable variables*/
    **Repeat** for $x_i$
        Data = Mnormrnd ($\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$); /* Generate observations from a Multivariate Normal Distribution*/
        *P*-value = Manova (Data); /*Perform MANOVA*/
        **If** *P*-value $\leq \alpha$ **Then** Reduce the Tolerance
        $\boldsymbol{\mu}$ = g ($\boldsymbol{\mu}$); /* Update mu */
        $\boldsymbol{\Sigma}$ = g ($\boldsymbol{\Sigma}$)/* update the covariance matrix*/
        **Else if** *P*-value $> \gamma$, **Then** increase the tolerance
        $\boldsymbol{\mu}$ = g ($\boldsymbol{\mu}$); /* Update mu */
        $\boldsymbol{\Sigma}$ = g ($\boldsymbol{\Sigma}$)/* update the covariance matrix*/
    **Until** *P*-value $> \alpha$ value $<= \gamma$ /*Stopping criteria*/
    **Go to** $x_{i+1}$; /* Tolerance design for next variable*/
**Until** Stopping criteria; /*e.g. $i = $ m*/
**Output**: Optimal tolerances

## Appendix B

Experimental data of the chemical product example are presented in Table X [35].

**Table X.** Experimental data of the chemical product example

| $x_1$ (time) | $x_2$ (temperature) | $x_3$ (catalyst) | $y_1$ (conversion) | $y_2$ (thermal activity) |
|---|---|---|---|---|
| -1 | -1 | -1 | 74 | 53.2 |
| 1 | -1 | -1 | 51 | 62.9 |
| -1 | 1 | -1 | 88 | 53.4 |
| 1 | 1 | -1 | 70 | 62.6 |
| -1 | -1 | 1 | 71 | 57.3 |
| 1 | -1 | 1 | 90 | 67.9 |
| -1 | 1 | 1 | 66 | 59.8 |
| 1 | 1 | 1 | 97 | 67.8 |
| -1.1682 | 0 | 0 | 76 | 59.1 |
| 1.1682 | 0 | 0 | 79 | 65.9 |
| 0 | -1.1682 | 0 | 85 | 60.0 |
| 0 | 1.1682 | 0 | 97 | 60.7 |
| 0 | 0 | -1.1682 | 55 | 57.4 |
| 0 | 0 | 1.1682 | 81 | 63.2 |
| 0 | 0 | 0 | 81 | 59.2 |
| 0 | 0 | 0 | 75 | 60.4 |
| 0 | 0 | 0 | 76 | 59.1 |
| 0 | 0 | 0 | 83 | 60.6 |
| 0 | 0 | 0 | 80 | 60.8 |
| 0 | 0 | 0 | 91 | 58.9 |

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