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Path Following for Marine Surface Vessels with Rudder and Roll Constraints: an MPC Approach

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Abstract— **The problem of path following for marine surface vessels using the rudder control is addressed in this paper. The need to enforce the roll constraints and the fact that the rudder actuation is limited in both amplitude and rate make the model predictive control (MPC) approach a natural choice. The MPC design is based on a linearized model for computational and implementation considerations, while the evaluation of the performance of MPC controller is performed on a nonlinear 4 degree of freedom surface vessel model. The simulation results are presented to verify the effectiveness of the resulting controller and a simulation based tuning process for the controller is also presented. Furthermore, the performance of the path following MPC control in wave fields is evaluated using an integrated maneuvering and seakeeping model, and the simulation confirms its robustness.**

I. INTRODUCTION

Controlling of marine surface vessels to follow a prescribed path or track a given trajectory has been a representative control problem for marine applications and has attracted considerable attention from the control community [1]–[9]. One challenge for path following of marine surface vessels stems from the fact that the system is often underactuated. Conventional ships are usually equipped with one or two main propellers for forward speed control, and rudders for course keeping of the ship. For ship maneuvering problems, such as path following and trajectory tracking, where we seek control for all three degrees of freedom (surge, sway and yaw), the two controls can not influence all three variables independently, thereby leading to under-actuated control problems. Recent development [2], [4]–[6], [8] in nonlinear control and control of under-actuated systems has offered new tools and promising solutions to deal with all 3-DoF using two independent controls.

Another challenge in the path following of marine surface vessels is the inherent physical limitations in the control inputs, namely the rudder saturation and rudder rate limit. More recently, given that the roll motion produces the highest acceleration and is considered as the principal villain for the sailor seasickness and cargo damage [10], enforcing roll constraints while maneuvering in seaways becomes an important design consideration in surface vessel control. While typical nonlinear control methodologies such as those pursued in [1]–[9] do not take these input and output constraints explicitly into account in the design process, the constraint enforcement is often achieved through numerical simulations and trial-and-error tuning of the controller parameters. Few other control methodologies, such as the model predictive control (MPC) [11], [12] and reference governor [13], have a clear advantage in addressing input and state constraints explicitly. [14] considers rudder saturation in its MPC controller for tracking control of marine surface vessels and [15] achieves the roll reduction for the heading control problem using an MPC approach. For the path following control problem considered in this paper where both the cross-tracking error and heading error are controlled by the rudder angle as an under-actuated problem and rudder limitation and roll constraints need to be enforced simultaneously, MPC applications have not been found in the open literature, to the best knowledge of the authors.

MPC, also known as the receding horizon control (RHC), is a control technique which embeds optimization within feedback to deal with systems subject to constraints on inputs and states [11], [12]. Over the last few decades, MPC has proven to be successful for a wide range of applications including chemical, food processing, automotive and aerospace systems [11]. Using an explicit model and the current state as the initial state to predict the future response of a plant, it determines the control action by solving a finite horizon open-loop optimal control problem on-line at each sampling interval. Furthermore, because of its natural appeal to multi-variable systems, MPC can handle underacuated problem gracefully by combining all the objectives into a single objective function.

This paper presents an MPC design of the path following problem for an integrated model of the surface vessel dynamics and 2-DoF path following kinematics. Our focus is on satisfying all the inputs and state constraints while achieving satisfactory path following performance. A 3-DoF simplified linear container model is adopted in the controller design and a corresponding 4-DoF nonlinear container model is used in simulations in order to study interactions between the path following maneuvering control and seakeeping roll dynamics. The path following performance of the proposed MPC controller and its sensitivity to the major controller parameters, such as the sampling time, predictive horizon and weighting matrices in the cost-function, are analyzed by numerical simulations. Finally, the effectiveness of the MPC path following controller in the wave field is studied by simulation on a numerical test-bed which combines both ship dynamics and wave impacts on vessels.

This paper is organized as follows: in Section II, the 4-DoF container model and the corresponding simplified 3-DoF linear model are presented along with the Serret-Frenet

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formulation to facilitate the path following control design. In Section III, the MPC algorithm is developed to address the path following problem with rudder and roll constraints. The simulation results in both calm water and wave fields are presented in Section IV together with some discussions on the controller parameter tuning, followed by the conclusions in Section V.

II. PATH FOLLOWING ERROR DYNAMICS AND MARINE SURFACE VESSEL MODEL

A. Path Following Error Dynamics

In the open literature, the path following problem for marine surface vessels has been addressed with two different approaches: one is to treat it as a tracking control problem [3], [5], the other is to simplify the tracking control problem into the regulation problem by adopting proper path following error dynamics [1], [8], [16], [17]. For the latter approach, the Serret-Frenet frame [18] is often adopted to derive the error dynamics.

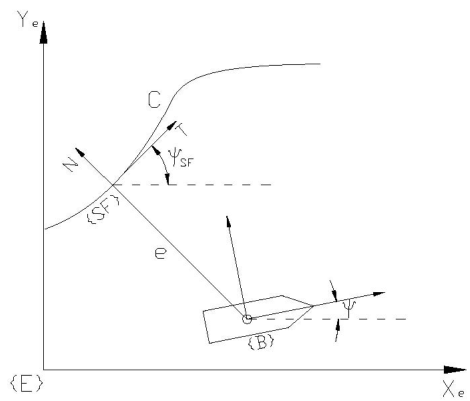

Fig. 1. Illustration of the coordinations in the earth frame (inertial frame) {E}, the ship body-fixed frame {B} and the Serret-Frenet frame {SF}.

Fig. 1 shows the Serret-Frenet frame used for path following control. The origin of the frame {SF} is located at the closest point on the path curve C from the origin of the frame {B}. The error dynamics based on the Serret-Frenet equations are introduced in [17], given as:

$$\begin{aligned}\dot{\bar{\psi}} &= \dot{\psi} - \dot{\psi}_{SF} \\ &= \frac{\kappa}{1-e\kappa}(usin\bar{\psi} - vcos\bar{\psi}) + r, \end{aligned} \quad (1)$$

$$\dot{e} = usin\bar{\psi} + vcos\bar{\psi}, \quad (2)$$

where e, defined as the distance between the origins of {SF} and {B}, and $\bar{\psi} := \psi - \psi_{SF}$, are referred to as the cross-track error and heading error respectively, u, v, r are the surge, sway and yaw velocity respectively. ψ is the heading angle of the vessel and ψ_{SF} is the path tangential direction as shown in Fig. 1 [17], κ is the curvature of the given path. The control objective of the path following problem is to drive e and $\bar{\psi}$ to zero.

For most path following problems for surface vessels in the open sea, the path is a straight line or a way-point path, which consists of piecewise straight lines. In these cases, the curvature κ is zero, therefore the heading error dynamics (1) could be simplified as:

$$\dot{\bar{\psi}} = r. \quad (3)$$

B. Marine Surface Vessel Model

Marine surface vessels have 6 degrees of freedom, which are described in Figure 2 [19], [20]. Two frames have been adopted for modeling. One is the inertial frame fixed on the earth; and the other is the body-fixed frame on the ship body.

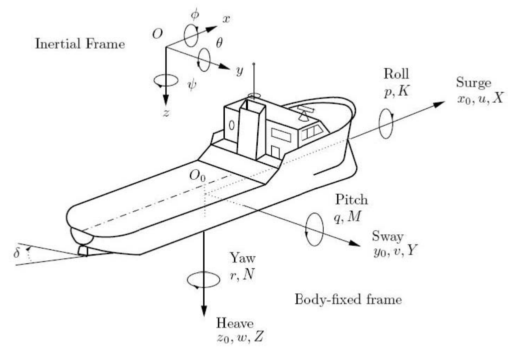

Fig. 2. Reference frames and variables for ship motion description [20]. Figure adapted from [19].

The magnitudes describing the position and orientation of the ship are usually expressed in the inertial frame, and the coordinates are noted: $[x,y,z]^T$ and $[\psi,\phi,\theta]^T$ respectively. The forces $[X,Y,Z]^T$, moments $[K,M,N]^T$, linear velocities $[u,v,w]^T$, and angular velocities $[p,q,r]^T$ are usually expressed in the body-fixed frame. δ is the rudder angle. See Figure 2.

For maneuvering of surface vessels, normally 3 DoFs are discussed, namely the surge, sway and yaw. In some cases, the surge is decoupled and 2 DoFs are left. In this paper, in order to address the path following problem with roll constraints, a 4-DoF model is needed, including 3-DoF discussed in traditional maneuvering [21] and an additional DoF focusing on seakeeping characteristics, namely the roll (p). A mathematical model for a single-screw high-speed container ship (often referred to as S175 in the marine engineering community) in surge, sway, roll and yaw has been presented in [21]. This 4-DoF dynamical ship model is highly nonlinear with 10 states and 2 control inputs: $X = [u,v,r,p,x,y,\psi,\phi,n,\delta]^T$ and $U = [n_c,\delta_c]^T$. u, v, r and p are the surge velocity, sway velocity, yaw rate and roll rate with respect to the ship-fixed frame respectively, the corresponding displacements with respect to the inertial frame are denoted as x, y, ψ and ϕ. Other two states are the propeller shaft speed n and the rudder angle δ. The inputs to the model are the commanded propeller speed n_c and rudder angle δ_c respectively. The actuator input saturation and rate limits are also incorporated in this model. The 4-DoF nonlinear model [21] is one of the most comprehensive ship models available in open literature. It captures the fundamental characteristics of the ship dynamics and covers a wide range of operating conditions.

However, using the 10th order ship model for MPC implementation is computationally prohibitive, due to the complexity associated with the nonlinear constrained optimization. In our work, the 10th order nonlinear model is used as a virtual ship for simulation and performance evaluation. For control design, the following reduced order linear model is used to facilitate the model-based design approach:

$$\dot{v} = a_{11}v + a_{12}r + a_{13}p + a_{14}\phi + b_1\delta, \tag{4}$$

$$\dot{r} = a_{21}v + a_{22}r + a_{23}p + a_{24}\phi + b_2\delta, \tag{5}$$

$$\dot{\psi} = r, \tag{6}$$

$$\dot{p} = a_{31}v + a_{32}r + a_{33}p + a_{34}\phi + b_3\delta, \tag{7}$$

$$\dot{\phi} = p, \tag{8}$$

where a_{11}, a_{12}, a_{21}, a_{22}, a_{31}, a_{32}, a_{33}, a_{34}, b_1, b_2 and b_3 are constant parameters. The surge speed is assumed to be constant and the surge dynamics are neglected.

The performance of the control system designed using the reduced order model will be presented to justify the utility of the reduced order model when the same controller is applied to the full order model.

III. MPC Formulation for Path Following Control

This section presents the formulation of the MPC controller for the path following problem of marine surface vessels. For notational convenience, we rewrite the ship dynamics together with path following error dynamics into the matrix form:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\delta, \tag{9}$$

where

$$\bar{x} = [e, \bar{\psi}, v, r, p, \phi]^T, \tag{10}$$

$$\bar{A} = \begin{bmatrix} 0 & u & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & 0 & a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ b_1 \\ b_2 \\ b_3 \\ 0 \end{bmatrix}, \tag{11}$$

Given a specific sampling time T_s, the plant (9) is easily transformed into its discrete-time version:

$$\bar{x}_{k+1} = A\bar{x}_k + B\delta_k. \tag{12}$$

Then the MPC online optimization problem can be formulated as follows: at each time k, find the optimal control sequence $\{\delta_k^*, \delta_{k+1}^*, \cdots, \delta_{k+N-1}^*\}$ to minimize the following cost function (13):

$$J(U_k, \bar{x}_k) = \sum_{j=1}^{N} (\bar{x}_{k+j}^T Q \bar{x}_{k+j} + \delta_{k+j-1}^T R \delta_{k+j-1}), \tag{13}$$

subject to

$$-\delta_{max} \leq \delta_{k+j} \leq \delta_{max}, \ j = 0, 1, \cdots, N-1, \tag{14}$$

$$-\Delta\delta_{max} \leq \delta_{k+j} - \delta_{k+j-1} \leq \Delta\delta_{max}, \ j = 0, 1, \cdots, N-1, \tag{15}$$

$$-\bar{x}_{max} \leq \bar{x}_{k+j} \leq \bar{x}_{max}, \ j = 1, 2, \cdots, N, \tag{16}$$

where (14), (15) and (16) stand for rudder saturation, rudder rate limit and state limit respectively. Q and R are the corresponding weighting matrices and N is the predictive horizon. The control law is given by $\delta_k = \delta_k^*$.

Since the cost function (13) is quadratic in $\bar{x}$ and δ and all the constraints are linear, we can use quadratic programming (QP) to solve the optimization problem. In this paper, the optimization and simulation is performed in MATLAB. Through simulations, the design space of MPC, which consists of the sampling time T_s, predictive horizon N and weighting matrices Q and R, are explored for achieving different desired performance of the controlled system. Discussions on parameter tuning will be presented in Section IV.

IV. Simulation Results and Controller Parameter Tuning

The proposed control law is implemented and simulated on the full order nonlinear model. The actuator saturation and its rate limits ($|\delta| \leq 35$ deg and $|\dot{\delta}| \leq 5$ deg/sec) are incorporated in simulations, while different roll constraints are imposed to evaluate the effectiveness of the MPC and the trade-offs between tightening the roll constraint and achieving path following. Since only the relative penalty on $\bar{x}$ and δ will influence the performance, we choose Q and R to have the form of $Q = \{0.0001, c_1, 0, 0, 0, 0\}$, $R = c_2$, namely, the cost function is $J = \sum_{j=1}^{N}(0.0001 e_{k+j}{}^2 + c_1 \bar{\psi}_{k+j}^2 + c_2 \delta_{k+j-1}^2)$, with c_1, c_2 being positive constants.

A. Selecting of the Sampling Time

The general guideline for selecting the sampling rates for discrete dynamical system is about 4-10 samples per rise time [22], which is about 18 second for the roll dynamics of the container ship. Therefore, a rational choice of the sampling is between 1 to 4 seconds. For the MPC application, small sampling times provide more timely feedback but require more frequent optimization, a good trade-off between the path following performance specification and real-time implementation consideration can be achieved through the sensitivity analysis. The roll responses corresponding to different sampling times are summarized in Fig. 3. For each simulation, the predictive time window is set to 120 seconds (considering that the time constant for the maneuvering dynamics is around 20 seconds), which leads to different predictive step N for different sampling interval. From Fig. 3, we can see roll angle responses with 1 second and 2 second sampling period are almost indifferentiable, while the responses with 3 or 4 second sampling interval start to deviate. Fig. 3 shows that $T_s = 2$ sec is a good trade-off for the implementation of MPC controller for the container ship under consideration.

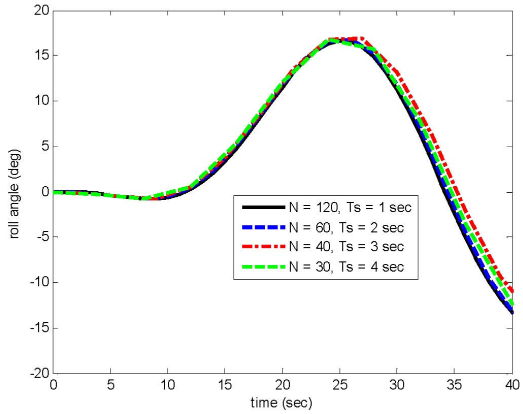

Fig. 3. Simulation results of the ship response with different sampling time.

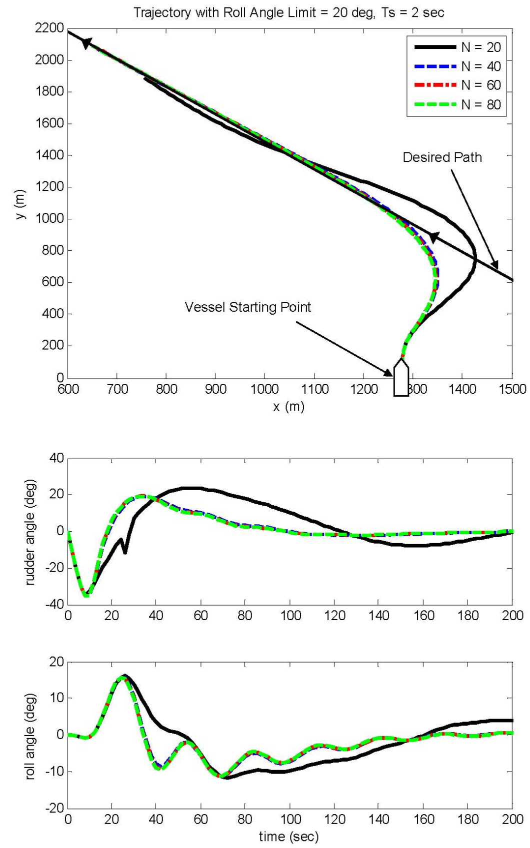

Fig. 4. Simulation results of the ship response with different predictive horizon.

B. Prediction Horizon

The length of the predictive horizon N is a basic tuning parameter for MPC controllers. Generally speaking, controller performance improves as N increases, at the expense of additional computation effort [11]. The effects of predictive horizon N on the path following performance are studied by simulations with results given in Fig. 4. It is clear from Fig. 4 that longer predictive horizon leads to faster path following and avoids over-steering but the benefits of extending the prediction horizon diminishes beyond $N = 40$. Given the heavy computational cost associated with long prediction horizon, we conclude that a value of 40-60 achieves a good trade-off for the predict horizon N, given 2 seconds as the sampling period.

Putting it all in the context of computational effort required for MPC implementation for a marine surface vessel path following control, the optimization problem with 2 second sampling interval and 60 step predictive horizon can be solved in about 0.6 second in simulations on a desktop computer with P4 2.4 CPU and 2G RAM. This moderate computational demand makes the MPC path following control promising for real-time implementation.

TABLE I

CONTROLLER GAINS FOR SIMULATION.

	G1	G2	G3	G4	G5
c_1	8	1.6	40	8	8
c_2	1	1	1	0.1	10

C. Effects of Weighting Matrices Q and R

The weighting matrices Q and R are used as the main tuning parameters to shape the closed-loop response for desired performance [12]. The numerical values of the different gains used for simulations are listed in Table. I. Investigating the performance sensitivity to the weighting matrix leads to useful insights that will be discussed in the sequel.

First, it is observed that the path following performance is primarily determined by the value of c_1 and is almost independent of the penalty of rudder c_2. This characteristic is revealed by inspecting the responses of Fig. 5 with weighting matrix selection G1, G4 and G5 on Fig. 5. Simulations are performed for many other combinations of c_1 and c_2, the same results are obtained (results are not presented due to space limit). To further confirm it, we performed the linear analysis of the closed-loop system with LQR controller, and it can be shown that the slowest eigenvalue, which dominates the cross-track error dynamics, are essentially un-affected by R matrix. The sensitivity of the path following performance to the parameter c_1 is shown in Fig. 5 when comparing the responses of G1, G2 and G3. In our simulation, the value of c_1 in the range of $[1.6, 40]$ yields reasonable path following performance, measured in the cross-tracking convergence speed.

Second, once c_1 is determined to achieve the desired path following performance, the parameter c_2 can be used to tune for different rudder response and roll response. Again consider responses corresponding to G1, G4 and G5, the difference in the rudder behavior reflects the impact of c_2.

This analysis leads to the following guidelines for parameter tuning of the proposed MPC path following controller: 1) Set $c_2 = 1$, and vary c_1 to achieve desired path following performance; 2) Fix c_1 as selected in 1), vary c_2 to tune for different rudder and roll responses.

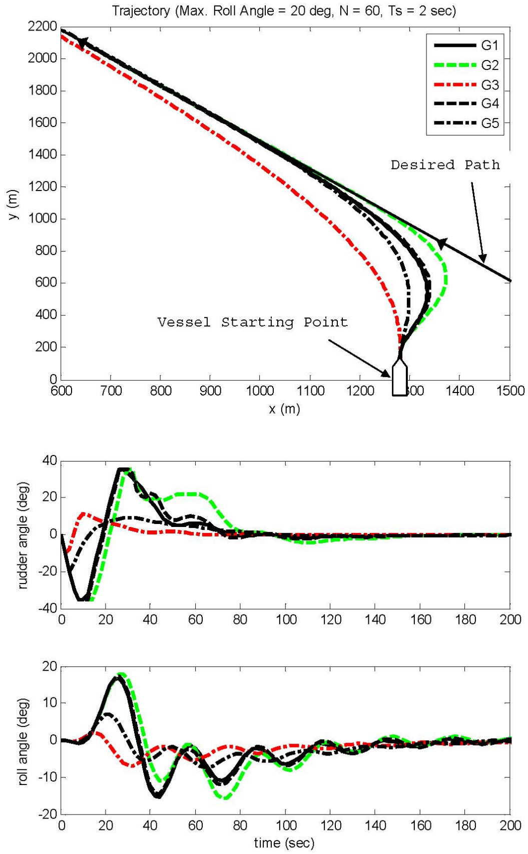

Fig. 5. Simulation results of the ship response with different weighting matrix.

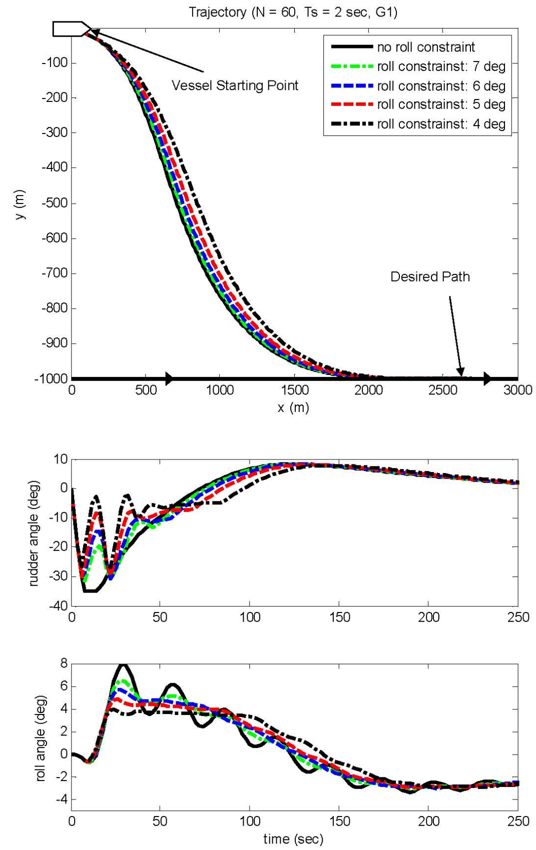

Fig. 6. Simulation results of the ship response with different roll constraints.

D. Enforcing Roll Constraints

Generally, a trade-off exists between the path following convergence speed and the roll minimization, namely, imposing roll constraints will deteriorate the path following performance of vessels. To understand this trade-off, simulations are performed with different roll constraints being imposed, and the results are summarized in Fig. 6. From Fig. 6, we can see that initially, the roll constraints slow down the heading changing speed because the large rudder action is not permissible due to the roll constraints. However, the final converge point of all scenario are very close because the MPC controller can compensate later by increasing the rudder angle. However, if the constraints on the roll is tightened further beyond 2 deg, the vessel will take very long time to converge to the path or even go into the infeasible region.

E. MPC Controller Evaluation in the Wave Field

The proposed MPC path following controller, developed based on the reduced order linear ship model in calm water, is implemented and simulated with the full order original model incorporating the wave effects to evaluate the performance. The wave-induced disturbances can be represented by the 1st-order and 2nd-order effects: namely the wave excitation load and wave drift load respectively [23]. These two loads are calculated separately and their combined effect serves as the total wave loads acting on the vessels. Fig. 7 shows the block diagram of the overall model. The wave load program calculates the wave induced forces and moments based on the wave field information (sea state, dominant wave direction) and the ship states (position, heading and speed). The ship maneuvering model is driven by the wave forces and moments, together with the control input (rudder angle calculated from the current ship states). The details about the numerical test-bed are reported in [24].

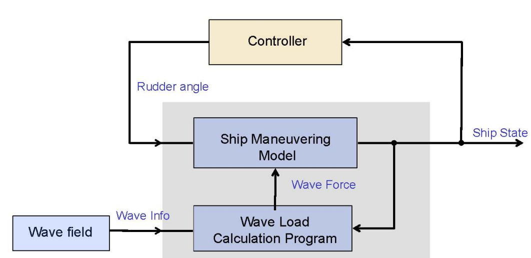

Fig. 7. Block Diagram of the Simulation Model.

The simulation results of the MPC path following controller in the wave field without roll constraints and with 20 deg roll constraints are summarized in Fig. 8. The significant wave height in the simulations is 3.25 m, which corresponds to sea state 5. From Fig. 8, we can see that the MPC controller achieves path following while satisfying the roll constraints while the convergence speed is slowed down due to the wave effects. If we further tighten the roll constraints, the vessel probably goes to the infeasible region. We are exploring the wave dynamics to improve the feasibility of the controller.

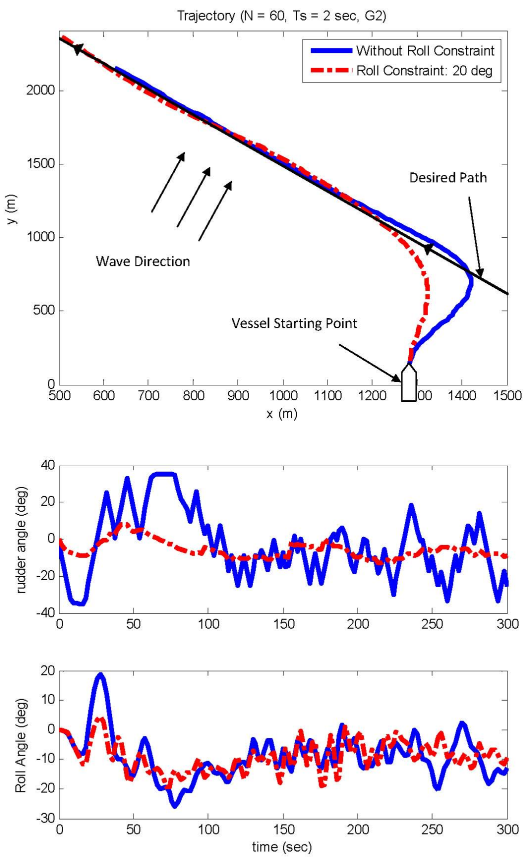

Fig. 8. Simulation results of the ship response in the wave field.

V. Conclusions

In this paper, an MPC approach addressing the path following of marine surface vessels with input and state constraints is presented. The detailed MPC controller formulation is described and the simulation results show that the MPC controller can achieve the path following of marine surface vessels while satisfying the pre-scribed input and state constraints. Furthermore, the proposed controller is shown to be robust in the wave field. The sensitivity analysis of the performance to the sampling time, predictive horizon and weighting matrices are also performed, which leads to the guidelines in the MPC parameter tuning. The sampling rate (2 seconds) and the prediction horizon ($N \simeq 50$) determined from simulations provide evidence that the real-time implementation of MPC controller in path following of marine surface vessels is feasible with moderate computational resource.

VI. Acknowledgement

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