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A fuzzy reliability assessment of basic events of fault trees through qualitative data processing

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Abstract

Probabilistic approaches are common in the analysis of reliability of complex engineering systems. However, they require quantitative historical failure data for determining reliability characteristics. In many real-world areas, such as e.g., nuclear engineering, quantitative historical failure data are unavailable or become inadequate and only qualitative data such as expert opinions, which are described in linguistic terms, can be collected and then used to assess system reliability. Moreover, experts are more comfortable justifying event failure likelihood using linguistic terms, which capture uncertainties rather than by expressing judgments in a quantitative manner. New techniques are therefore needed that will help construct models of reliability of complex engineering system without being confined to quantitative historical failure data. The objective of this study is to develop a fuzzy reliability algorithm to effectively generate basic event failure probabilities without reliance on quantitative historical failure data through qualitative data processing. The originality of this study comes with an introduction of linguistic values articulated in terms of component failure possibilities in order to qualitatively assess basic event failure possibilities treated as inputs of the proposed model and generate basic event failure probabilities as its outputs. To demonstrate the feasibility and effectiveness of the proposed algorithm, actual basic event failure probabilities collected from nuclear power plant operating experiences are compared with the failure probabilities generated by the algorithm. The results demonstrate that the proposed fuzzy reliability algorithm arises as a suitable alternative for the probabilistic reliability approach when quantitative historical failure data are unavailable.

Keywords: Engineering system reliability; Fuzzy reliability algorithm; Failure probability; Qualitative data processing; Nuclear safety assessment

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1. Introduction

Safety is a major issue in complex and safety critical engineering systems such as nuclear engineering systems. The problems of safety assessment have been studied by many researchers using various methods. In reliability theory, it is commonly assumed that components of a complex engineering system are described by precise probability distributions describing their reliability characteristics. However, this might not be the case in some real-world applications. If a system whose reliability has to be assessed is new, there will not be sufficient statistical data to estimate reliabilities of its component. Data used to determine these reliabilities may also arise from various sources. Therefore, the assumption of precise failure probability distributions of system component might be arguable. These difficulties emphasize the need for new techniques, which could effectively determine basic event failure probabilities without the need to resort to quantitative historical failure data.

Fuzzy set theory was first introduced as a useful tool to complement conventional reliability theories in [36]. Since then, there have been a number of approaches where the technology of fuzzy sets was used to evaluate system reliability. For example, in Bing et al. [3], a fuzzy linear regression method is combined with a finite element method to evaluate the reliability of mechanical structures. In this approach, a membership function of a triangular fuzzy number is used to express the structure stress. In order to overcome the limitation of the traditional failure mode, effects and criticality analysis (FMECA), a fuzzy rule-based approach has been implemented in [5,13,52]. Furthermore, Zio et al. [56] developed a fuzzy expert system for human reliability analysis to elicitate factors influencing conditional human error for two dependence successive operator actions in a nuclear power plant accident. In Karimi and Hüllermeier [24], fuzzy set theory has been used to complement probability theory to assess the risk of natural disaster when statistical data and/or physical knowledge are insufficient for probabilistic analysis. Meanwhile, Ding and Lisnianski [11] developed a fuzzy universal generating function in which fuzzy numbers used to represent the state probability and fuzzy composition operators were introduced to assess the reliability of a multi-state system. Moreover, Pandey and Tyagi [38] proposed a profust reliability to evaluate degradable systems and a fuzzy numbers-based method to assess system failure rate parameters.

We can also encounter some natural language-based system assessment methods which are used when quantitative data is unavailable or inadequate to invoke probabilistic reliability models [6–8,18,20,29]. In addition, real-world case studies indicate that experts are more comfortable justifying event failure likelihoods using natural languages/linguistic terms such as '*low failure possibility*', '*medium failure possibility*', and '*high failure possibility*' to represent component failure likelihood rather than quantitative judgment [12,33]. For example, it is common for experts to say that 'there is a *low possibility* that the component *A fails*' rather than the *probability of failure* of the component *A* is '1.5E−3'. These terms can be quantified with the use of membership functions of the corresponding fuzzy sets [7,53].

Fuzzy sets have also been incorporated in the fault tree analysis for assessing the safety of nuclear power plants. Fuzzy probabilities have been used to represent basic event failures for assessing the occurrence probability of a typical emergency core cooling system [34]. A fuzzy uncertainty importance measure has been proposed to quantify the source of uncertainty in the fault tree analysis completed for the reactor protective system (WASH-1400) [45]. The importance measure has been used to identify critical components in the fault tree analysis for the auxiliary feedwater system of Angra-I Westinghouse nuclear power plant [15] and for the containment cooling system of a typical four-loop pressurized water reactor [17]. However, those existing applications cannot assess basic event failure probabilities without historical failure data.

The motivation of this study is how to obtain basic event failure probabilities when basic events do not have probability distributions of their lifetime to failures. Therefore, we develop a fuzzy reliability algorithm to assess basic event failure probabilities through qualitative linguistic value processing without the need to engage basic event failure probability distribution. There are several aspects of originality of this study: (1) a use of qualitative linguistic values in terms of failure likelihoods to assess basic event failure possibilities and (2) an integration of the proposed fuzzy reliability algorithm into the quantification of the top event failure probability of the fault tree analysis. The proposed algorithm of fuzzy reliability exhibits two advantages: (1) an ability to assess basic event failure probabilities of new engineering systems using expert opinions articulated in linguistic terms; (2) an ability to capture factors of subjectivity and imprecision of expert linguistic assessments of the description of the basic event failure probabilities. To demonstrate the feasibility of the proposed algorithm, nuclear event failure probabilities generated by the algorithm are compared with the reliability data taken from the actual nuclear power plant operating experiences.

The paper is organized as follows. Section 2 briefly outlines some research carried out in the area of system reliability involving fuzzy sets. Problem statement along with a general flow of processing is given in Section 3. Section 4 presents the proposed fuzzy reliability algorithm in detail. A validation of the algorithm is described in Section 5. Section 6 is focused on the verification of the algorithm. Finally, conclusions and further research directions are offered in Section 7.

2. Preliminary studies

Generally, system reliability is assessed in a probabilistic manner by using quantitative historical failure data. However, if the event is absent (not recorded) or, we are provided with inadequate (too few data to draw sound statistical inference), improper (poor record keeping), and inaccurate data, in the modeling of system reliability, we resort ourselves to expert opinions [6,7,12,20]. Expert opinions have also been successfully implemented in risk analysis [26,32]. These opinions are commonly expressed linguistically and the words used in the description form a term set of linguistic values.

Linguistic values are introduced in the fuzzy reliability approach to characterize phenomena that are too complex or ill-defined. The advantage of using linguistic values in engineering system safety analysis is that they can intuitively and easily express expert opinions that cannot be adequately represented in a numeric way [21,28,29,31].

The granularity of the set of linguistic values that are commonly used in engineering system safety depends on the number of linguistic terms; commonly this number varies from four to nine. The granularity level is decided upon by experts in the field and in line with the situation of the case of the interest. For example, in offshore engineering systems, five to seven linguistic values are used for antecedents and four linguistic values are used for consequences in the fuzzy rules [27,42,53]. Meanwhile, Guimaraes and Lapa use five linguistic values to estimate the safety level of the containment cooling system of a nuclear power plant [16].

Linguistic values present in human reasoning, can be formalized as membership functions of fuzzy sets [20,55]. The selection of a certain type of membership function depends on the nature of the problem at hand [30]. Previous studies indicate that trapezoidal and triangular fuzzy numbers (membership functions) form a sound practical alternative to reflect uncertainties, inaccuracy and fuzziness of human justifications involving in linguistic values [12,20,44,51,55]. Furthermore, Onisawa [35] has proposed a logarithmic function to fit the very small error possibility, which is expressed by a fuzzy subset of the unit interval [0, 1], to the nature of human judgment. This function considers the proportionality of human sensation to the logarithmic value of a physical quantity.

3. Problem statement

Probabilistic safety assessment (PSA) by fault tree analysis (FTA) has been considered as an important tool to assess the safety level of nuclear power plants (NPP). In this safety assessment, nuclear safety analysts must have confidence in the input data to gain confidence in the results. On the basis of this consideration, it is recommended to use plant specific data, which can be taken from operator logs and maintenance logs. Since the estimation of failure probabilities of rare events with high consequences is the focus of the NPP PSA, it is often very difficult to obtain component failure data, which are specific to that NPP. It is inevitable to obtain component failure data from other sources such as data from other NPPs or nuclear industries other than NPPs or non-nuclear experiences. However, these data sources carry uncertainties such as imprecision, ambiguity, and/or vagueness.

Since nuclear and other complex engineering systems do not come with historical failure data, expert opinions are often used to determine basic event failure likelihoods. Therefore, it is necessary to capture the subjectivity and imprecision of component failure probabilities in FTA. Hence, we propose a fuzzy reliability algorithm combining fuzzification and defuzzification modules to assess basic event failure probabilities through qualitative data processing. The objective of the fuzzification module is to convert basic event qualitative data expressed in terms of failure possibilities, which are subjectively assessed by experts, into the operational format of fuzzy numbers. The objective of the defuzzification module is to transform fuzzy numbers into a single scalar quantity to be used to generate basic event failure probabilities as the outputs of the algorithm.

The proposed fuzzy reliability algorithm developed to generate basic event failure probabilities from qualitative data consists of five functional modules, which are briefly described in this section. In the sequel, the details of the modules and the quantification steps of the algorithm are given in Section 4. An overall architecture of the quantification process is shown in Fig. 1.

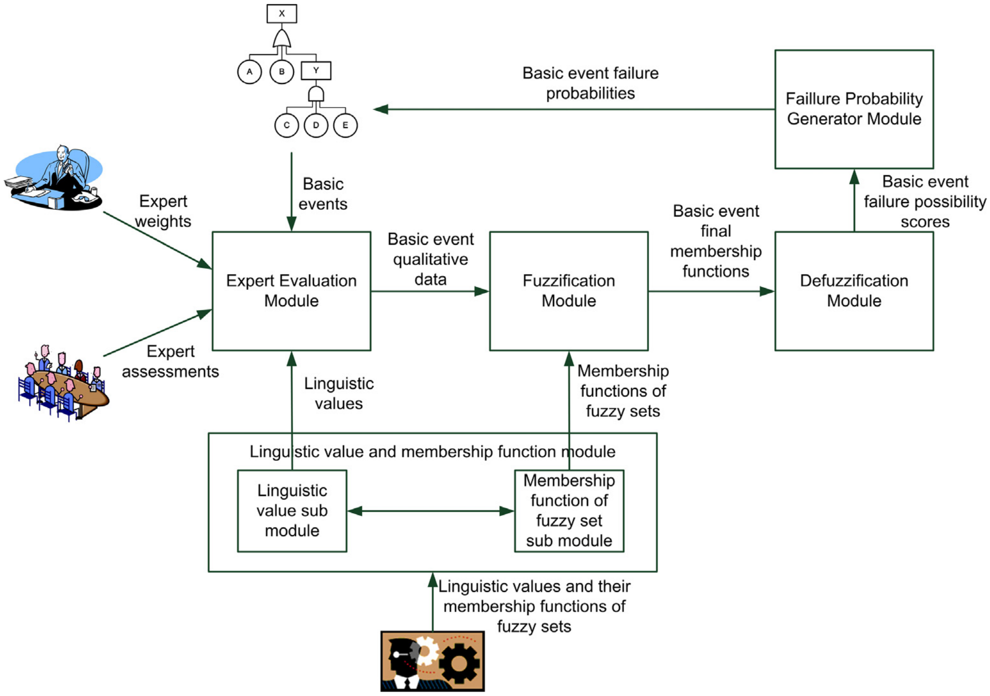

Fig. 1. Structure of the quantification process of the proposed fuzzy reliability algorithm.

1) *Linguistic value and membership function module*: This module defines the terms of linguistic values used to represent basic event failure possibilities and their corresponding mathematical representation. The inputs for this module come from safety analysts, who understand the systems, as well as qualitative data. It consists of two sub-modules, i.e., *linguistic value sub-module* and *membership function of fuzzy set sub-module*. The output of the linguistic value sub-module is a set of qualitative linguistic values (H), to express basic event failure possibilities. Basic event failure possibilities could be graded based on the type of the components or the likelihood of failure occurrences. Based on the component types, for example, *very low failure possibility* can be used to represent components, which are rigid and very unlikely to be failure even once. Meanwhile, the term *very high failure possibility* can be used to represent components, which have many moving parts and are near certain to fail several times. Based on the likelihood of failure occurrences, for example, *very low failure possibility* can be used to represent components whose predicted failure probabilities could be less than 10^{-8}. Meanwhile, *very high failure possibility* can be used to represent components whose predicted failure probabilities could be greater than 10^{-3}. This grading will, of course, be different for different application. For instance, 10^{-3} could be defined as *high failure possibility* for nuclear accidents but as *low failure possibility* for motorcycle accidents. Therefore, safety analysts have to define this failure possibility grading based on the system problems at hands, the extent of the gathered information and the expert knowledge. Moreover, Yu and Park [54] stated that defining the failure possibility distribution is a matter of subjective opinion. This set of qualitative linguistic values (H) will be used by experts in the expert evaluation module to subjectively assess basic event failure likelihoods.

Meanwhile, the outputs of the membership function of fuzzy set sub-module are membership functions to represent each member of H. These fuzzy sets represent qualitative basic event failure possibilities defined in the [0, 1] universe of discourse. This means that the closer the fuzzy probabilities are to 0, the less likely the basic events are to fail. On the other hand, the closer the fuzzy probabilities are to 1, the more likely the basic events are to fail. Meanwhile, the horizontal axis represents the failure probability of basic events, which is also defined between 0 and 1. This means that the closer the fuzzy numbers are to the point of origin, the lower the basic event failure probabilities are. On the other hand, the farther the fuzzy numbers are from the point of origin, the higher the basic event failure probabilities are. These phenomena can easily be understood from Table 1 and Fig. 2 below.

It is also important to note that membership function used in this module can have different form for different engineering systems. To assign values for those failure possibility membership functions, safety analysts may choose

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Table 1
Nuclear event failure likelihood values.

Nuclear event failure possibilities	Failure probabilities
Very Low (*VL*)	<1.0E−8
Low (*L*)	1.0E−8–1.0E−7
Reasonably Low (*RL*)	1.0E−7–1.0E−6
Moderate (*M*)	1.0E−6–1.0E−5
Reasonably High (*RH*)	1.0E−5–1.0E−4
High (*H*)	1.0E−4–1.0E−3
Very High (*VH*)	>1.0E−3

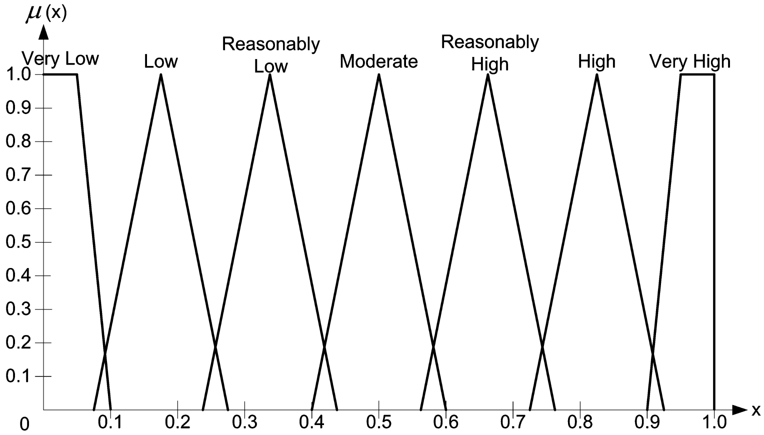

Fig. 2. Graphical representation of the nuclear event membership functions.

a technique coming from the six straightforward methods described by Ross [43], i.e. intuition, inference, rank ordering, neural networks, genetic algorithms, and inductive reasoning. The membership functions developed in this module will then be used in the fuzzification module to generate basic event final membership functions.

$$H = \{very\ low, low, moderate, \ldots, very\ high\} \tag{1}$$

$$U = \left\{very\ low(u), low(u), moderate(u), \ldots, very\ high(u)\right\} \tag{2}$$

As noted earlier, there are m linguistic terms, say *very low*, *low*, *moderate*, ..., *very high* where each of them is described by fuzzy sets and the corresponding membership functions, say *very low*(u), *low*(u), *moderate*(u), ..., and *very high*(u).

2) *Expert evaluation module*: This evaluation module generates a set of qualitative data representing basic event failure possibilities. Inputs to this module are a set of basic events from the system fault tree under evaluation, a set of experts to subjectively evaluate basic event failure and a set of basic event subjective assessment coming from the experts. An expert is a person who is familiar with the system, understands the system working environment, and has considerable training in and knowledge of the system operation. Cooke et al. [9] recommended three indicators to choose experts, i.e. the number of scientific publications, recommendations from a wide range of experts, and experiences with previous similar studies. By scoring each criteria and sum-up the total score, the experts whose expertise are more relevant to the study what it is intended for will be properly selected.

In real-world applications, the experts may have different levels of expertise, background and working experience. Hence, they may demonstrate different perceptions about the same events and subjectively provide different assessment. To reflect their differences of assessment, different justification weights from 0 to 1 may be assigned to every expert. Cooke and Goossens [10] have formulated two key performance-based indicators to weight experts, i.e. calibration and informativeness. This technique needs 'seed variables' whose values have been known but at the time of assessment the experts do not know those values. Using calibration questions, the probabilities of experts to correctly answer the questions can be drawn. The seed variables and the calibration questions must be as closely as possible to

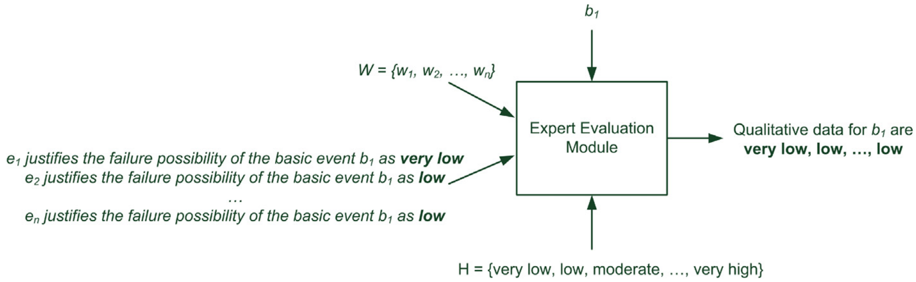

Fig. 3. Description of links presented by (3)–(7).

the problems that the study is intended to solve [26]. This technique has also been implemented in Tuomisto et al. [47] to weight experts on air pollution epidemiology and can also be implemented in this module to weight experts.

$$B = \{b_1, b_2, \ldots, b_l\} \quad \text{and} \quad B \in FT \tag{3}$$

$$E = \{e_1, e_2, \ldots, e_n\} \tag{4}$$

$$W = \left\{ w_1, w_2, \ldots, w_n;\ 0 \leqslant w_i \leqslant 1 \text{ and } \sum_{i=1}^{n} w_i = 1 \right\} \tag{5}$$

There are l basic events in the system fault tree FT, say $b_1, b_2, \ldots, b_l$ which are subjectively evaluated by n experts, say $e_1, e_2, \ldots, e_n$ which have justification weights of say, $w_1, w_2, \ldots, w_n$ where each weight is defined in space [0, 1] and the total weight must be 1.

In the basic event evaluation process, the experts $e_1, e_2, \ldots,$ and e_n subjectively justify the failure possibility of the basic event b_1, for example, as *very low*, *low*, …, and *low*, respectively as in (6).

$$Y = \big\{\{very\ low, low, \ldots, low\}, \{\ldots\}, \ldots, \{\ldots\}\big\} \tag{6}$$

Y is the set of the basic event subjective assessment coming from the experts. This module then generates the matrix of basic event qualitative data (Ql) in (7). For example, the qualitative data for the basic event b_1 are *very low*, *low*, …, and *low*.

$$Ql = \begin{bmatrix} very\ low & low & \cdots & low \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{7}$$

The links described by (3)–(7) are visualized in Fig. 3.

Since the objective of the proposed algorithm is to integrate basic event qualitative data into the quantitative phase of fault tree analysis, the proposed fuzzy reliability algorithm implements a fuzzification technique to convert qualitative linguistic values into their corresponding mathematical forms described by the membership functions of fuzzy sets and a defuzzification technique to convert those fuzzy quantities into their corresponding scalar quantities in the form of nuclear event failure possibility scores. Each technique is realized in a module as follows.

3) *Fuzzification module*: In this algorithm, basic event failures are subjectively assessed by experts using qualitative linguistic values in terms of failure possibilities as in (6). Since the purpose of the algorithm is to generate quantitative failure probabilities from qualitative failure possibilities, the objective of this fuzzification module is to quantify basic event qualitative data taken from the expert evaluation module into their corresponding quantitative data in the form of membership function of fuzzy numbers taken from the linguistic value and membership function module. This module, then, aggregates those n subjective quantitative data coming from n experts in (8) to generate a vector of final quantitative data in (9) to reach consensus for every basic event in (3).

$$Qn = \begin{bmatrix} very\ low(u) & low(u) & \cdots & low(u) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{8}$$

Qn shown in (8) is the corresponding quantitative data of the qualitative data Ql in (7), for example, the quantitative data for the basic event b_1 are *very low*(u), *low*(u), …, and *low*(u), where each of them is represented as a fuzzy set and described by the corresponding membership functions.

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$$M^B = \begin{bmatrix} b_1(u) \\ b_2(u) \\ \vdots \\ b_l(u) \end{bmatrix} \tag{9}$$

M^B is a vector of l basic event final quantitative data where each of them is aggregated from its n quantitative data subjectively evaluated by n experts in (8). For example, $b_1(u)$ is the final quantitative data for the basic event b_1, which is aggregated from its n quantitative data, *very low*(u), *low*(u), ..., *low*(u). This $b_1(u)$ is given in the form of a membership function of a fuzzy set.

The weighted averaging operator can be used to aggregate two or more values of different importance. It is a generalization of the arithmetic mean in the sense we assign different weights (importance) to every single value involved in the aggregation process. This technique has been implemented in multi-criteria and multi-expert decision making to aggregate the criteria given by the experts [14,28,46] and in system reliability analysis to aggregate the fuzzy justifications coming from experts [18]. Other aggregation operators, which consider the weight of individual value, can be implemented in this module as well.

4) *Defuzzification module*: The final quantitative data taken from the fuzzification module is still in the form of fuzzy numbers whereas the calculation of the actual reliability requires a single scalar quantity. Therefore, the output generated by the fuzzification module need to be transformed into a scalar quantity. Defuzzification is a process of synthesis the output of fuzzy systems, which incorporates the representations of imprecision and/or uncertainties, to be a single scalar quantity as opposed to a fuzzy set [25]. Apparently, there is no unique way to realize defuzzification and a method being selected is problem-oriented [21]. Therefore, safety analysts need to find the most suitable defuzzification technique for their area of investigation.

The objective of this defuzzification module is to generate a vector of l basic event failure possibility scores from the basic event final quantitative data taken from the fuzzification module.

$$R_s^B = \begin{bmatrix} R_s^{b_1} \\ R_s^{b_2} \\ \vdots \\ R_s^{b_l} \end{bmatrix} \tag{10}$$

R_s^B is a vector of l basis event failure possibility scores where each of them is a single scalar quantity, which is defuzzified from its final quantitative data. For example, $R_s^{b_1}$ is the failure possibility score for the basic event b_1, which is defuzzified from $b_1(u)$.

5) *Failure probability generator module*: This failure probability generator module generates a vector of basic event failure probabilities (R^{b_k}) from their corresponding quantitative failure possibilities taken from the defuzzification module.

$$R^B = \begin{bmatrix} R^{b_1} \\ R^{b_2} \\ \vdots \\ R^{b_l} \end{bmatrix} \tag{11}$$

R^B is a vector of l basis event failure probabilities where each of them is generated from its corresponding failure possibility score. For example, R^{b_1} is the failure probability for the basic event b_1, which is generated from $R_s^{b_1}$.

4. A fuzzy reliability algorithm

The proposed fuzzy reliability algorithm generates basic event failure probabilities from qualitative data, which are expressed in terms of failure possibilities using qualitative linguistic values. The inputs to the algorithm are the linguistic values, membership functions of fuzzy sets, basic events of the fault tree of the system under evaluation, experts and their justification weights, and expert subjective evaluation as in (1)–(6). The output of the algorithm is a set of failure probabilities representing the all l basic event failures as in (11).

The proposed fuzzy reliability algorithm consists of five steps and each step corresponds to a module presented in Fig. 1. In the sequel, we elaborate on their details.

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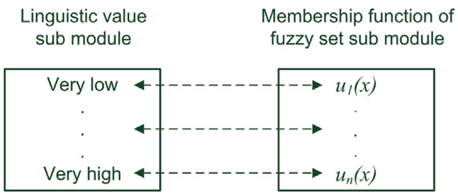

Fig. 4. The links between the linguistic value and the membership function of fuzzy sub-set sub-modules.

4.1. Linguistic value and membership function development

In this step, we develop a failure possibility distribution to be implemented in the linguistic value sub-module based on the likely failure occurrences and the membership functions of fuzzy sets to be implemented in the membership function of fuzzy set sub-module using the inductive reasoning approach. The failure possibility distribution is a set of m qualitative linguistic values used to scale basic event failure possibilities from the lowest rates to the highest rates as in (1). The membership functions to represent these qualitative linguistic values are in the form of triangular fuzzy numbers as in (12) to represent the ith linguistic value in (1).

$$u_i(x) = \begin{cases} u_i^L(x), & a \leqslant x \leqslant b \\ u_i^R(x), & b \leqslant x \leqslant c \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

This development process is realized in the linguistic value and membership function module shown in Fig. 1. The links between the linguistic value sub-module and the membership function of fuzzy sub-set sub-module are visualized in Fig. 4.

4.2. Basic event failure possibility evaluation

In this step, we collect a set of expert subjective evaluation about basic event failure possibilities as given in (6). Experts answer specific questions about basic event failure possibilities by choosing one failure possibility from m predefined failure possibilities in (1). Using (7), the matrix of basic event qualitative data is generated as a reply to questions assuming the form.

What is the failure possibility of the basic event b_i? Is it very low, low, …, or very high?

$$Ql = \begin{bmatrix} h_i^{e_1 b_1} & h_i^{e_2 b_1} & h_i^{e_3 b_1} & \cdots & h_i^{e_n b_1} \\ h_i^{e_1 b_2} & h_i^{e_2 b_2} & h_i^{e_3 b_2} & \cdots & h_i^{e_n b_2} \\ h_i^{e_1 b_3} & h_i^{e_2 b_3} & h_i^{e_3 b_3} & \cdots & h_i^{e_n b_3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_i^{e_1 b_l} & h_i^{e_2 b_l} & h_i^{e_3 b_l} & \cdots & h_i^{e_n b_l} \end{bmatrix} \tag{13}$$

where $h_i^{e_j b_k}$ is the ith failure possibility in H of the basic event b_k evaluated by the expert e_j. Meanwhile, i is the index of the failure possibility in (1), k is the index of the basic event in (3), and j is the index of the expert in (4). For example, the failure possibility of the basic event b_2 $(k = 2)$ is subjectively evaluated by the expert e_3 $(j = 3)$ as *very low* $(i = 1)$. Therefore, $h_i^{e_j b_k} = h_1^{e_3 b_2} = \textit{very low}$. This step is completed in the expert evaluation module as shown in Fig. 1.

4.3. Fuzzification process

This step takes the basic event qualitative data Ql from the expert evaluation module and the corresponding membership function $u_i(x)$ from the linguistic value and membership function module and then generates a matrix of quantitative data, which are basic event final membership function. Using (8)–(9), the matrix of basic event final quantitative data is expressed as follows.

$$Qn = \begin{bmatrix} u_i^{e_1 b_1} & u_i^{e_2 b_1} & u_i^{e_3 b_1} & \cdots & u_i^{e_n b_1} \\ u_i^{e_1 b_2} & u_i^{e_2 b_2} & u_i^{e_3 b_2} & \cdots & u_i^{e_n b_2} \\ u_i^{e_1 b_3} & u_i^{e_2 b_3} & u_i^{e_3 b_3} & \cdots & u_i^{e_n b_3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ u_i^{e_1 b_l} & u_i^{e_2 b_l} & u_i^{e_3 b_l} & \cdots & u_i^{e_n b_l} \end{bmatrix} \tag{14}$$

where $u_i^{e_j b_k}$ is the membership function of the basic event b_k to represent the failure possibility h_i justified by the expert e_j. For example, the failure possibility of the basic event b_2 is subjectively evaluated by the expert e_3 as *very low*. The linguistic value of *very low* is an h_1 in H, then $u_i^{e_j b_k} = u_1^{e_3 b_2} = u_{very\ low}(x)$, which comes from (2).

$$M^B = \begin{bmatrix} u^{b_1}(x) \\ u^{b_2}(x) \\ u^{b_3}(x) \\ \vdots \\ u^{b_l}(x) \end{bmatrix} = Qn \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} \tag{15}$$

where $u^{b_i}(x)$ is the final membership function for the ith basic event, which is aggregated from its n quantitative data, w_i is the weight for the ith expert, n is the number of expert, and l is the number of basic events.

We consider the weighted averaging operator in (15) as the most appropriate aggregation technique for this algorithm because it represents real situation in which experts may justify the same basic event with different failure possibilities. This step is performed in the fuzzification module in Fig. 1.

4.4. Defuzzification process

In this step, basic event failure possibility scores are generated from their final membership function taken from the fuzzification module. It is very important to choose a suitable defuzzification technique for a specific application. Among the diversity of the methods, we use the area defuzzification technique (*ADT*) to realize this algorithm for nuclear safety assessment. It is a suitable technique for defuzzifying the membership functions of fuzzy numbers into a failure possibility score for a nuclear safety assessment involving qualitative linguistic values [40,41]. More specifically, the method returns a numeric value computed as follows.

$$ADT = d\big(\mu_{\tilde{A}}(x)\big) = x_1 y_0 + \int_{x_2}^{d} \mu_{\tilde{A}}^{R}(x)\,dx \tag{16}$$

where y_0 is the centroid point of the real fuzzy number $\tilde{A}$ on the vertical axis, x_1 is the intersection point between the line y_0 and the left membership function $\mu_{\tilde{A}}^{L}(x)$ on the horizontal axis, and x_2 is the intersection point between the line y_0 and the right membership function $\mu_{\tilde{A}}^{R}(x)$ on the horizontal axis.

Using (10), the vector of basic event failure possibilities is generated as follows.

$$\begin{bmatrix} R_s^{b_1} \\ R_s^{b_2} \\ R_s^{b_3} \\ \vdots \\ R_s^{b_l} \end{bmatrix} = \begin{bmatrix} d(u^{b_1}(x)) \\ d(u^{b_2}(x)) \\ d(u^{b_3}(x)) \\ \vdots \\ d(u^{b_l}(x)) \end{bmatrix} \tag{17}$$

$R_s^{b_i}$ is a failure possibility score for the ith basic event, which is defuzzified from its final quantitative data $d(u^{b_i}(x))$, and l is the number of basic events. This step is completed in the defuzzification module in Fig. 1.

4.5. Basic event failure probability generation

This step generates basic event failure probabilities from their corresponding failure possibility scores taken from the defuzzification module. By using the logarithmic function proposed by Onisawa [35] as expressed in (18) and integrating with (11), the set of basic event failure probabilities can be described as (19).

$$R_s^{b_i} = \frac{1}{1 + [K \times \log(\frac{1}{R^{b_i}})]^3} \tag{18}$$

$R_s^{b_i}$ is the failure possibility score and R^{b_i} is the failure probability of the basic event b_i. K is a constant representing the safety criterion, which equals to 0.435 [33,35,37].

$$R^{b_i} = \begin{cases} \frac{1}{10^z}, & R_s^{b_i} \neq 0 \\ 0, & R_s^{b_i} = 0 \end{cases} \tag{19}$$

R^{b_i} is a failure probability for the ith basic event and $z = [\frac{1-R_s^{b_i}}{R_s^{b_i}}]^{1/3} \times 2.301$. This process is completed by the failure probability generator module in Fig. 1.

The logarithmic function in (18) reflects a fact that human error may still occur even though the error probability used to derive the error possibility very small. From the reliability analysis point of view, the failure possibility and the error possibility are regarded to be the same entity.

5. Algorithm verification

In this section, we describe how the proposed fuzzy reliability algorithm is validated. To investigate the feasibility of the proposed algorithm, the actual failure probabilities taken from the US Combustion Engineering reactor protection system (CERPS) during the period 1984 through 1998 operating experience, which are well documented in Wierman et al. [50], are compared to the failure probabilities generated by the algorithm. Many authors in the past have used this data source to validate their experimental studies.

Bondavalli and Filippini [4] used this data source to validate their proposed stochastic Petri net to assess the availability and performance of the safety function of the reactor protection system. In the study by Bartha et al. [1], this data was used to validate their proposed periodic and outage testing methodology of the reactor protection systems in the Paks Nuclear Power Plant. Meanwhile, Kang and Han [23] used this data source to calculate alpha parameters to make the common cause failure event failure probabilities suitable for the emergency diesel generator for Ulchin Unit 3. Bickel [2] used this data set to evaluate the risk implications of the core protection calculator system failure in the reactor protection system.

Component failure probabilities in Wierman et al. [50] are presented in three different values, i.e. *best estimate*, *lower bound*, and *upper bound* reliability values. The best estimate reliability value is the recommended reliability data to be used in the FTA. Meanwhile, the upper and the lower bound reliability values represent a range of reliability data estimation. To verify the feasibility and the applicability of the proposed algorithm, the basic event failure probabilities generated by the proposed algorithm have to be between the upper and the lower bound reliability values and as close as possible to the best estimate reliability value. If the results show that the generated failure probabilities are beyond the range of the reliability data estimation, the membership functions to represent the qualitative data need to be explored in more detail.

6. An illustrative case study

This section describes the data sets used to verify the proposed algorithm and quantify the algorithm performance as well as carry out result analysis to verify the feasibility of the proposed algorithm.

Table 2
The basic event failure probabilities of the CERPS fault tree.

Basic events	Failure description	Known failure probability		
		Lower bound	Best estimate	Upper bound
b_1	Trip breaker local hardware faults	4.3E−6	1.8E−5	4.5E−5
b_2	Shunt trip device local faults	6.3E−6	1.5E−4	5.5E−4
b_3	Under-voltage coil device local faults	1.4E−4	1.1E−3	3.5E−3
b_4	Channel trip unit (bi-stable) fails to trip at its set point	3.4E−5	5.0E−4	1.8E−3
$\mathbf{b_5}$	**Channel analog core protection calculator fails to send a signal to the trip unit**	**1.6E−3**	**7.6E−3**	**2.0E−2**
b_6	Channel digit protection calculator fails to send a signal to the trip unit	6.5E−4	2.7E−3	6.8E−3
b_7	Channel reactor vessel pressure sensor/transmitter fails to detect a high pressure and sends a signal to the trip unit	1.1E−5	1.1E−4	3.5E−4
b_8	Channel reactor vessel temperature/transmitter (cold or hot leg) fails to detect a low level and sends a signal to the trip unit	4.2E−4	8.4E−4	1.5E−3
b_9	Manual scram switch fails to operate upon demand	4.1E−5	1.3E−4	2.8E−4
b_{10}	Control rod (or associated control rod drive) fails to insert fully into core upon demand	3.4E−7	1.7E−5	6.4E−5
b_{11}	Channel logic relay fails to de-energize upon demand	2.2E−5	2.6E−4	8.8E−4
b_{12}	CCF 2 of 8 trip breaker local hardware faults	1.9E−7	1.0E−6	2.7E−6
b_{13}	CCF 2 of 4 trip breaker local hardware faults	8.0E−8	7.1E−7	2.2E−6
b_{14}	CCF 2 of 8 shunt trip device local faults	3.9E−7	1.1E−6	4.0E−5
b_{15}	CCF 2 of 4 shunt trip device local faults	2.5E−7	8.7E−6	3.3E−5
b_{16}	CCF 2 of 8 under-voltage coil device local faults	5.1E−6	5.4E−5	1.8E−4
b_{17}	CCF 2 of 4 under-voltage coil device local faults	2.3E−6	3.7E−5	1.3E−4
$\mathbf{b_{18}}$	**CCF specific 2 of 3 bi-stables associated with either a pressure (P) or temperature (T) signal (T&M)**	**1.1E−6**	**2.6E−5**	**9.5E−5**
b_{19}	CCF specific 3 of 4 bi-stables associated with either a pressure (P) or temperature (T) signal	1.4E−7	7.2E−6	2.8E−5
b_{20}	CCF specific 4 of 6 bi-stables (T&M)	3.7E−8	1.7E−6	6.6E−6
b_{21}	CCF specific 6 of 8 bi-stables	7.1E−9	7.7E−7	2.9E−6
b_{22}	CCF 2 of 3 analog core protection calculators (T&M)	4.9E−5	3.8E−4	1.2E−3
b_{23}	CCF 3 of 4 analog core protection calculators	1.3E−5	1.7E−4	5.6E−4
b_{24}	CCF 2 of 3 digital core protection calculators (T&M)	2.3E−5	1.4E−4	3.8E−4
b_{25}	CCF 3 of 4 digital core protection calculators	6.3E−6	5.7E−5	1.8E−4
b_{26}	CCF 2 of 3 pressure sensor/ transmitters (T&M)	3.0E−7	5.0E−6	1.8E−5
b_{27}	CCF 3 of 4 pressure sensor/ transmitters	4.0E−8	1.5E−6	5.8E−6
b_{28}	CCF 2 of 3 temperature sensor/ transmitters (T&M)	8.0E−6	3.7E−5	9.8E−5
b_{29}	CCF 3 of 4 temperature sensor/ transmitters	7.5E−7	1.0E−5	3.5E−5
b_{30}	CCF specific 2 of 4 manual trip switches	7.4E−7	5.0E−6	1.5E−5
b_{31}	CCF specific 2 of 4 trip breaker shunt trip device power	2.3E−7	2.5E−6	8.3E−6
b_{32}	CCF 50% (18 of 36) or more CRD/rods fail to insert	7.5E−10	3.6E−8	1.4E−7
b_{33}	CCF specific 6 of 12 logic relays (T&M)	4.8E−9	1.6E−7	6.0E−7
b_{34}	CCF specific 12 of 24 logic relays	5.3E−10	4.3E−8	1.7E−7
b_{35}	CCF 3 of 3 logic relays (T&M)	4.8E−9	4.7E−7	1.8E−6
b_{36}	CCF 6 of 6 logic relays	8.2E−10	2.0E−7	7.2E−7
b_{37}	CCF 2 of 4 trip relays	5.7E−7	4.8E−6	1.5E−5

6.1. Basic event data sets

A reactor protection system is one of many safety systems in commercial reactors that comprises numerous electronic and mechanical components to produce an automatic or manual rapid shutdown when the reactor experiences disturbed conditions and requires a trip to stop the nuclear reaction. Basic events used here are taken from the CERPS fault tree presented in Wierman et al. [50].

We can see from Table 2 that there are 37 basic events to be assessed. To illustrate how the proposed algorithm generates the basic event failure probabilities of the CERPS fault tree, we choose two basic events from Table 2, i.e. b_5 and b_{18}. The failure probabilities for all other basic events generated by the algorithm are shown in Appendix A.

Table 3
The results of the experimentations to find parameters for $\mu_{VL}(x)$ and $\mu_{VH}(x)$.

Experimentation goals	Membership functions	Generated failure probabilities
Finding a membership function representing the *very low failure possibility*	(0.00, 0.04, 0.08)	6.36E−13
	(0.00, 0.03, 0.05, 0.08)	1.30E−12
Finding a membership function representing the *very high failure possibility*	(0.92, 0.96, 1.00)	1.03E−03
	(0.92, 0.95, 0.97, 1.00)	1.87E−03

6.2. Basic event assessment process by the fuzzy reliability algorithm

For illustration purposes, let us assume that the higher management level assign seven experts with the same level of expertise about the Combustion Engineering reactor protection system to evaluate those basic events shown in Table 2. Therefore, we assign to all the seven experts the same justification weight of 1/7.

6.2.1. Linguistic value and membership function development

Based on the likely failure occurrences and the range of nuclear event failure data collected from nuclear power plant operating experiences (10^{-13}–10^{-2}) [22,39,49,50], seven qualitative linguistic values are defined to assess basic event failure possibilities as in (20).

$$\begin{aligned} H &= \{VL, L, RL, M, RH, H, VH\} \\ &= \{\textit{very low}, \textit{low}, \textit{reasonably low}, \textit{moderate}, \textit{reasonably high}, \textit{high}, \textit{very high}\} \end{aligned} \tag{20}$$

Nuclear events with '*very low failure possibilities* (VL)' mean that the failure probabilities of these events are predicted to be less than 10^{-8} and very unlikely to become failures. Nuclear events with '*very high failure possibilities* (VH)' mean the failure probabilities of these events are predicted to be greater than 10^{-3} and are near certain to become failures. Events with '*low*' (L), '*reasonably low*' (RL), '*moderate*' (M), '*reasonably high*' (RH), and '*high*' (H) *failure possibilities* are up-graded from '*very low*' to '*very high*' *failure possibilities* and their failure likelihood values are shown in Table 1 Section 3.

Since previous researches confirm that trapezoidal and triangular fuzzy numbers form a sound practical alternative to reflect uncertainties, inaccuracy and fuzziness of human justifications involving in linguistic values [12,51] and have smooth transitions from one linguistic term to another term [48], these types of membership functions are considered in this study to represent nuclear event failure possibilities in (20). In addition, based on the fact that the real nuclear event reliability data are mostly less than 10^{-2} and could be of order 10^{-5} to 10^{-13} [22,39,49,50], the algorithm has to be able to generate basic event failure probabilities, which are between 10^{-2} and 10^{-13}. This rule can be defined in a fuzzy rule as follows.

$$\text{If } \tilde{A} = \{\tilde{A}_i \mid i = 1, 2, \ldots, n\} \text{ and } R = \left\{R_i = f(A_i) \,\middle|\, i = 1, 2, \ldots, n\right\} \text{ then } 10\text{E}{-13} \leqslant R_i \leqslant 10\text{E}{-2}$$

where $\tilde{A}_i$ is a normal fuzzy number and R_i is a failure probability generated by the algorithm from the normal fuzzy number $\tilde{A}_i$ [41].

In this study, the inductive reasoning is used to develop the membership values in (21)–(27) to represent those basic event failure possibilities in H. In the experimentation, firstly, we tried to find which membership function could be used to generate higher failure probability range by comparing the failure probabilities generated by those two membership functions. In this first experimentation, we also tried to find the left most and the right most membership functions of each fuzzy numbers, which could generate nuclear event failure probabilities within the range of the real nuclear event failure probabilities. The results, which are shown in Table 3, confirm that the triangular membership function can generate bigger failure probability range than the trapezoidal membership function can do. The triangular membership functions also can produce smaller failure probabilities than that produced by trapezoidal membership functions. These experimentation results justify that nuclear event failure possibilities should be mathematically represented by the membership functions of triangular fuzzy numbers.

Those two triangular fuzzy numbers in Table 3 are then used to represent nuclear events with the *very low failure possibility*, i.e. $\mu_{VL}(x)$, and the *very high failure possibility*, i.e. $\mu_{VH}(x)$, as given in (21) and (27), respectively.

Table 4
The results of the experimentations to find parameters for $\mu_M(x)$.

Experimentation goals	Membership functions	Generated failure probabilities
Finding a membership function representing the *moderate failure possibility*	(0.35, 0.50, 0.65)	6.39E−05
	(0.40, 0.50, 0.60)	7.91E−05
	(0.45, 0.50, 0.55)	9.65E−05

The membership parameters for other five failure possibilities are then generated by segmenting the area between the two membership functions in Table 3, i.e. $\mu_{VL}(x)$ and $\mu_{VH}(x)$. To find the parameters of the membership functions for *moderate failure possibility*, i.e. $\mu_M(x)$, we segmented the area between $\mu_{VL}(x)$ and $\mu_{VH}(x)$ into two areas by choosing the center of the Cartesian plane, which is 0.50, as its core. Then, we varied the pair of its left and right supports to find the parameters that could generate the lowest failure probabilities for the *moderate failure possibility*. We chose the lowest failure probabilities because nuclear event failure probabilities are mostly very small. The results of this experimentation are shown in Table 4.

From Table 4, we then chose the triangular membership function of (0.35, 0.50, 0.65) to mathematically represent nuclear events with *moderate failure possibilities*, i.e. $\mu_M(x)$, as in (24).

To find membership parameters for *reasonably high failure possibility*, i.e. $\mu_{RH}(x)$, and *high failure possibility*, i.e. $\mu_H(x)$, we follow the rule saying that fuzzy sub-sets, which are distributed in the Cartesian plane, overlap [43]. Based on this specific character, since the right support for the $\mu_M(x)$ is 0.65, then we chose 0.63 as the left support for $\mu_{RH}(x)$. We also use symmetrical membership functions to mathematically represent nuclear event failure possibilities. Therefore, the right support for the $\mu_{RH}(x)$ is 0.83. Hence, the triangular membership function of (0.63, 0.73, 0.83) is used to represent nuclear events with *reasonably high failure possibilities*, i.e. $\mu_{RH}(x)$, as in (25). Meanwhile, since the left support for the $\mu_{VH}(x)$ is 0.92, then we chose 0.93 as the right support and 0.81 as the left support for the $\mu_H(x)$. Hence, the triangular membership function of (0.81, 0.87, 0.93) is used to represent nuclear events with *high failure possibilities*, i.e. $\mu_H(x)$, as in (26).

Using the same segmentation procedures, we finally chose those membership functions of triangular fuzzy numbers shown below and graphically shown in Fig. 2 Section 3, to describe nuclear event qualitative failure possibilities defined in (20).

$$u_{VL}(x) = \begin{cases} \frac{x}{0.04}, & 0.00 \leqslant x \leqslant 0.04 \\ \frac{0.08-x}{0.04}, & 0.04 \leqslant x \leqslant 0.08 \\ 0, & x \geqslant 0.08 \end{cases} \tag{21}$$

$$u_L(x) = \begin{cases} \frac{x-0.07}{0.06}, & 0.07 \leqslant x \leqslant 0.13 \\ \frac{0.19-x}{0.06}, & 0.13 \leqslant x \leqslant 0.19 \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

$$u_{RL}(x) = \begin{cases} \frac{x-0.17}{0.10}, & 0.17 \leqslant x \leqslant 0.27 \\ \frac{0.37-x}{0.10}, & 0.27 \leqslant x \leqslant 0.37 \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

$$u_M(x) = \begin{cases} \frac{x-0.35}{0.15}, & 0.35 \leqslant x \leqslant 0.50 \\ \frac{0.65-x}{0.15}, & 0.50 \leqslant x \leqslant 0.65 \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

$$u_{RH}(x) = \begin{cases} \frac{x-0.63}{0.10}, & 0.63 \leqslant x \leqslant 0.73 \\ \frac{0.83-x}{0.10}, & 0.73 \leqslant x \leqslant 0.83 \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

$$u_H(x) = \begin{cases} \frac{x-0.81}{0.06}, & 0.81 \leqslant x \leqslant 0.87 \\ \frac{0.93-x}{0.06}, & 0.87 \leqslant x \leqslant 0.93 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

$$u_{VH}(x)=\begin{cases}\frac{x-0.92}{0.04}, & 0.92\leqslant x\leqslant 0.96\\ \frac{1.00-x}{0.04}, & 0.96\leqslant x\leqslant 1.00\\ 0, & x\leqslant 0.92\end{cases} \tag{27}$$

6.2.2. Basic event failure possibility evaluation

There are four inputs in this process. One of the inputs is the failure possibility distribution H taken from the linguistic value and membership function module as in (20). The other three inputs are a set of seven expert weights (W), a set of 37 basic events of the CERPS fault tree (B) and a matrix of expert subjective evaluation (Y) as shown below.

$$W=\{w_i \mid i=1,2,3,\ldots,7 \text{ and } w_i=1/7\} \tag{28}$$

$$B=\{b_i \mid i=1,2,\ldots,37 \text{ and } b_i\in FT(CERPS)\} \tag{29}$$

$$Y=\begin{bmatrix} M & RL & M & RL & M & RL & RL\\ RH & M & M & M & RH & M & RH\\ VH & VH & VH & VH & VH & VH & VH\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ RL & RL & RL & RL & RL & RL & RL\end{bmatrix} \tag{30}$$

Table 5 represents the matrix Y in the tabular form to easily understand how each expert evaluates basic event failure possibilities. Those values in Table 5 could be obtained by requesting that each expert completes a questionnaire about basic event failure possibilities. The questions there could assume the following format.

What is the failure possibility of the basic event b_i? Is it VL, L, RL, M, RH, H, or VH?

Those justification results in Table 5 are just of illustrative character of experts to obtain the closest matching failure probabilities to the known best estimate values.

The output of this process is generated using (13). For example, the qualitative data for basic events b_5 and b_{18} are shown in (31). The qualitative data for other basic events in B are generated by the same processes.

$$Q_l=\begin{bmatrix} VH & VH & VH & VH & VH & VH & VH\\ M & RL & RL & M & RL & M & M\end{bmatrix} \tag{31}$$

6.2.3. Fuzzification process

In this process, the corresponding membership functions for qualitative data in (31) are taken from the linguistic value and membership function module in Fig. 1. These membership functions are mathematically denoted in (21)–(27). Using (15), for example, the final membership functions for basic events b_5 and b_{18} are obtained as follows.

$$\begin{bmatrix} u^{b_5}(x)\\ u^{b_{18}}(x)\end{bmatrix}=\begin{bmatrix} u_{VH}(x) & u_{VH}(x) & u_{VH}(x) & u_{VH}(x) & u_{VH}(x) & u_{VH}(x) & u_{VH}(x)\\ u_M(x) & u_{RL}(x) & u_{RL}(x) & u_M(x) & u_{RL}(x) & u_M(x) & u_M(x)\end{bmatrix}\begin{bmatrix}1/7\\1/7\\1/7\\1/7\\1/7\\1/7\\1/7\end{bmatrix}$$

$$Q_n=\begin{bmatrix} u^{b_5}(x)\\ u^{b_{18}}(x)\end{bmatrix}=\begin{bmatrix}(0.92, 0.96, 1.00)\\ (0.27, 0.40, 0.53)\end{bmatrix} \tag{32}$$

The complete basic event final membership functions generated in this step are given in Table 8 in Appendix A.

6.2.4. Defuzzification process

By substituting (16) into (17), the failure possibility scores for basic events, for example, b_5 and b_{18} are generated as follows.

$$\begin{bmatrix} R_s^{b_5}\\ R_s^{b_{18}}\end{bmatrix}=\begin{bmatrix} ADT(0.92, 0.96, 1.00)\\ ADT(0.27, 0.40, 0.53)\end{bmatrix}=\begin{bmatrix}0.313333\\ 0.112381\end{bmatrix} \tag{33}$$

Table 5
Expert justification results.

Basic events	Basic event qualitative data assessed by						
	e_1	e_2	e_3	e_4	e_5	e_6	e_7
b_1	*M*	*RL*	*M*	*RL*	*M*	*RL*	*RL*
b_2	*RH*	*M*	*M*	*M*	*RH*	*M*	*RH*
b_3	*VH*	*VH*	*VH*	*VH*	*VH*	*VH*	*VH*
b_4	*RH*	*H*	*RH*	*H*	*RH*	*H*	*RH*
$\boldsymbol{b_5}$	***VH***	***VH***	***VH***	***VH***	***VH***	***VH***	***VH***
b_6	*VH*	*VH*	*VH*	*VH*	*VH*	*VH*	*VH*
b_7	*RH*	*M*	*RH*	*M*	*M*	*M*	*M*
b_8	*H*	*VH*	*H*	*VH*	*VH*	*H*	*H*
b_9	*M*	*RH*	*M*	*M*	*RH*	*M*	*M*
b_{10}	*M*	*RL*	*M*	*M*	*RL*	*RL*	*RL*
b_{11}	*RH*	*RH*	*RH*	*RH*	*M*	*RH*	*M*
b_{12}	*L*	*L*	*RL*	*L*	*RL*	*RL*	*L*
b_{13}	*L*	*L*	*RL*	*RL*	*RL*	*L*	*L*
b_{14}	*L*	*RL*	*L*	*RL*	*L*	*RL*	*RL*
b_{15}	*RL*	*RL*	*RL*	*RL*	*RL*	*RL*	*M*
b_{16}	*M*	*RL*	*M*	*M*	*M*	*M*	*M*
b_{17}	*M*	*RL*	*M*	*M*	*RL*	*M*	*M*
$\boldsymbol{b_{18}}$	***M***	***RL***	***RL***	***M***	***RL***	***M***	***M***
b_{19}	*RL*	*RL*	*M*	*RL*	*RL*	*RL*	*RL*
b_{20}	*RL*	*L*	*RL*	*RL*	*RL*	*L*	*RL*
b_{21}	*L*	*RL*	*L*	*RL*	*L*	*L*	*RL*
b_{22}	*H*	*RH*	*RH*	*RH*	*RH*	*RH*	*RH*
b_{23}	*M*	*RH*	*RH*	*M*	*RH*	*M*	*M*
b_{24}	*M*	*RH*	*M*	*RH*	*M*	*RH*	*M*
b_{25}	*M*	*M*	*M*	*M*	*RL*	*M*	*M*
b_{26}	*RL*	*RL*	*RL*	*M*	*RL*	*RL*	*L*
b_{27}	*L*	*L*	*RL*	*L*	*L*	*RL*	*M*
b_{28}	*M*	*M*	*M*	*M*	*RL*	*RL*	*M*
b_{29}	*RL*	*RL*	*RL*	*M*	*RL*	*M*	*RL*
b_{30}	*RL*	*M*	*RL*	*RL*	*L*	*RL*	*RL*
b_{31}	*RL*	*RL*	*RL*	*RL*	*RL*	*RL*	*L*
b_{32}	*L*	*VL*	*L*	*L*	*L*	*L*	*L*
b_{33}	*L*	*L*	*L*	*L*	*L*	*RL*	*L*
b_{34}	*L*	*L*	*L*	*L*	*L*	*L*	*VL*
b_{35}	*L*	*RL*	*L*	*RL*	*L*	*L*	*L*
b_{36}	*L*	*L*	*RL*	*L*	*L*	*L*	*L*
b_{37}	*RL*	*RL*	*RL*	*RL*	*RL*	*RL*	*RL*

The complete basic event failure possibility scores generated in this step are given in Table 8 in Appendix A. Using (19), for example, the failure probabilities for basic events b_5 and b_{18} are generated as follows.

$$\begin{bmatrix} R^{b_5} \\ R^{b_{18}} \end{bmatrix} = \begin{bmatrix} 1.03\text{E}{-03} \\ 2.62\text{E}{-05} \end{bmatrix} \tag{34}$$

The complete basic event failure probabilities generated in this step are given in Table 8 in Appendix A. From (34), we can see that the proposed algorithm generates basic event failure probabilities that are similar to the one probabilistically calculated using historical failure data.

6.3. Analysis of results

Relative errors in Table 6 and Table 7 in Appendix A are calculated using the generated and the best estimate failure probabilities. From Table 6, it can be seen that the failure probabilities generated by the proposed algorithm for the basic event b_{18} is very close to the best estimate reliability value collected from the operating experiences. However, the failure probabilities generated for the basic event b_5 is very close to the lower bound reliability value.

Table 6
b_5 and b_{18} failure probabilities comparison.

Basic events	Generated failure probabilities	Known failure probabilities			Relative error
		Lower bound	Best estimate	Upper bound	
b_5	1.03E−03	1.6E−3	7.6E−3	2.0E−2	0.864983
b_{18}	2.62E−05	1.1E−6	2.6E−5	9.5E−5	0.006028

From Table 7 and Figs. 5–6, we can see that the failure probabilities generated by the proposed algorithm for 35 basic events are very close to the best estimate reliability value collected from the operating experiences. However, the failure probabilities generated for the other two basic events, i.e. b_5 and b_6, are very close to the lower bound reliability values. These two exceptions might be caused by the incapability of the proposed algorithm to generate failure probabilities greater than 1.03E−03. It will be interesting to see, in the future research, how the proposed fuzzy reliability algorithm will perform for different membership functions and/or different applications.

Generally, these results have demonstrated that the proposed fuzzy reliability algorithm can be feasibly used as an alternative approach for the conventional probabilistic reliability approach to assess basic event failure probabilities. However, if the expertise disparities of the experts on the system under evaluation are very substantial, the weights amongst experts will be different and, consequently, the basic event failure possibilities justified by them will also be very different. This condition will cause the proposed algorithm generating higher relative errors. Hence, it is important to note that the selection of the experts to subjectively evaluate basic event failure possibilities will affect the generation of the basic event failure probabilities to some extents.

We also have to acknowledge that if basic events to be evaluated have quantitative probability distribution of their lifetime to failures, conventional probabilistic reliability approach should be used. The calculation results of this conventional approach will represent the actual reliability values of those basic events. To deal with vagueness and imprecise information involved in statistical data, Hryniewicz [19] has proposed a fuzzy Bayesian method. On the other hand, if the subjective justification is the only method to evaluate basic event failures, the proposed fuzzy reliability algorithm offers a feasible and effective solution to generate basic event failure probabilities through the qualitative data processing. Experts can intuitively and easily use their expertise and working experience to evaluate basic event failure possibilities using qualitative linguistic values. From the illustrative character of the expert justification that we have done in this case study, the distribution of membership functions used in this experiment produce failure probabilities, which are closely match the actual failure probabilities collected from operating experiences.

7. Conclusions and further studies

In real-world applications, when quantitative historical failure data are scarce or are not available at all, linguistic values are often used by decision makers to assess system reliability. This study has proposed a fuzzy reliability algorithm to handle qualitative data in order to assess basic events of fault trees through qualitative data processing. Those data are described in terms of failure possibilities and represented by fuzzy numbers, to characterize basic event failure likelihood. The key advantage of using linguistic values in system reliability assessment is that the developed framework can intuitively and easily express expert opinions, which otherwise cannot be represented by quantitative data. Using the case study, we demonstrated the performance of the algorithm by comparing the generated failure probabilities with the actual failure probabilities collected from the operating experiences of the Combustion Engineering reactor protection system. The results show that the proposed fuzzy reliability algorithm offers a very good alternative approach to assess event reliability data when historical quantitative failure data is insufficient or unavailable to invoke probabilistic methods.

While the study has offered an alternative modeling approach, there are still a number of interesting avenues to pursue. First, the underlying model can be further refined and enriched by admitting various classes of fuzzy sets (membership functions). Second, more experimentation using various data sets coming from other nuclear power plants fault tree analysis would be advantageous to gain a better assessment of the performance of the model and the linguistic analysis.

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Appendix A

Table 7
Basic event failure probabilities.

Basic events	Generated failure probability	Known failure probability			Relative error
		Lower bound	Best estimate	Upper bound	
b_1	1.82E−05	4.3E−6	1.8E−5	4.5E−5	0.009559
b_2	1.48E−04	6.3E−6	1.5E−4	5.5E−4	0.016080
b_3	1.03E−03	1.4E−4	1.1E−3	3.5E−3	0.067153
b_4	4.77E−04	3.4E−5	5.0E−4	1.8E−3	0.045884
b_5	1.03E−03	1.6E−3	7.6E−3	2.0E−2	0.864983
b_6	1.03E−03	6.5E−4	2.7E−3	6.8E−3	0.619951
b_7	1.14E−04	1.1E−5	1.1E−4	3.5E−4	0.040376
b_8	8.32E−04	4.2E−4	8.4E−4	1.5E−3	0.009576
b_9	1.14E−04	4.1E−5	1.3E−4	2.8E−4	0.119682
b_{10}	1.82E−05	3.4E−7	1.7E−5	6.4E−5	0.068945
b_{11}	2.32E−04	2.2E−5	2.6E−4	8.8E−4	0.107907
b_{12}	7.59E−07	1.9E−7	1.0E−6	2.7E−6	0.240724
b_{13}	7.59E−07	8.0E−8	7.1E−7	2.2E−6	0.069403
b_{14}	1.28E−06	3.9E−7	1.1E−6	4.0E−5	0.163532
b_{15}	7.54E−06	2.5E−7	8.7E−6	3.3E−5	0.133485
b_{16}	4.87E−05	5.1E−6	5.4E−5	1.8E−4	0.097310
b_{17}	3.63E−05	2.3E−6	3.7E−5	1.3E−4	0.019914
b_{18}	2.62E−05	1.1E−6	2.6E−5	9.5E−5	0.006028
b_{19}	7.54E−06	1.4E−7	7.2E−6	2.8E−5	0.047039
b_{20}	2.02E−06	3.7E−8	1.7E−6	6.6E−6	0.189768
b_{21}	7.59E−07	7.1E−9	7.7E−7	2.9E−6	0.013927
b_{22}	3.85E−04	4.9E−5	3.8E−4	1.2E−3	0.012703
b_{23}	1.48E−04	1.3E−5	1.7E−4	5.6E−4	0.131835
b_{24}	1.48E−04	2.3E−5	1.4E−4	3.8E−4	0.054200
b_{25}	4.87E−05	6.3E−6	5.7E−5	1.8E−4	0.144820
b_{26}	5.54E−06	3.0E−7	5.0E−6	1.8E−5	0.107479
b_{27}	1.78E−06	4.0E−8	1.5E−6	5.8E−6	0.186810
b_{28}	3.63E−05	8.0E−6	3.7E−5	9.8E−5	0.019914
b_{29}	1.21E−05	7.5E−7	1.0E−5	3.5E−5	0.205248
b_{30}	5.54E−06	7.4E−7	5.0E−6	1.5E−5	0.107479
b_{31}	3.04E−06	2.3E−7	2.5E−6	8.3E−6	0.214396
b_{32}	4.26E−08	7.5E−10	3.6E−8	1.4E−7	0.182478
b_{33}	2.03E−07	4.8E−9	1.6E−7	6.0E−7	0.267381
b_{34}	4.26E−08	5.3E−10	4.3E−8	1.7E−7	0.010018
b_{35}	4.15E−07	4.8E−9	4.7E−7	1.8E−6	0.117941
b_{36}	2.03E−07	8.2E−10	2.0E−7	7.2E−7	0.013905
b_{37}	4.37E−06	5.7E−7	4.8E−6	1.5E−5	0.089537

Table 8
Algorithm generated data summary.

Basic events	Final membership functions	Failure possibility scores	Generated failure probabilities
b_1	(0.25, 0.37, 0.49)	0.102619	1.82E−05
b_2	(0.47, 0.60, 0.73)	0.178095	1.48E−04
b_3	(0.92, 0.96, 1.00)	0.313333	1.03E−03
b_4	(0.71, 0.79, 0.87)	0.249524	4.77E−04
b_5	(0.92, 0.96, 1.00)	0.313333	1.03E−03

(*continued on next page*)

Table 8 (*continued*)

Basic events	Final membership functions	Failure possibility scores	Generated failure probabilities
b_6	(0.92, 0.96, 1.00)	0.313333	1.03E−03
b_7	(0.43, 0.57, 0.70)	0.165952	1.14E−04
b_8	(0.86, 0.91, 0.96)	0.294286	8.32E−04
b_9	(0.43, 0.57, 0.70)	0.165952	1.14E−04
b_{10}	(0.25, 0.37, 0.49)	0.102619	1.82E−05
b_{11}	(0.55, 0.66, 0.78)	0.202381	2.32E−04
b_{12}	(0.13, 0.21, 0.29)	0.056190	7.59E−07
b_{13}	(0.11, 0.19, 0.27)	0.050476	7.59E−07
b_{14}	(0.13, 0.21, 0.29)	0.056190	1.28E−06
b_{15}	(0.20, 0.30, 0.41)	0.083095	7.54E−06
b_{16}	(0.32, 0.47, 0.61)	0.131905	4.87E−05
b_{17}	(0.30, 0.43, 0.57)	0.122143	3.63E−05
b_{18}	(0.27, 0.40, 0.53)	0.112381	2.62E−05
b_{19}	(0.20, 0.30, 0.41)	0.083095	7.54E−06
b_{20}	(0.14, 0.23, 0.32)	0.061905	2.02E−06
b_{21}	(0.11, 0.19, 0.27)	0.050476	7.59E−07
b_{22}	(0.66, 0.75, 0.84)	0.234286	3.85E−04
b_{23}	(0.47, 0.60, 0.73)	0.178095	1.48E−04
b_{24}	(0.47, 0.60, 0.73)	0.178095	1.48E−04
b_{25}	(0.32, 0.47, 0.61)	0.131905	4.87E−05
b_{26}	(0.18, 0.28, 0.38)	0.077381	5.54E−06
b_{27}	(0.14, 0.22, 0.31)	0.060238	1.78E−06
b_{28}	(0.30, 0.43, 0.57)	0.122143	3.63E−05
b_{29}	(0.22, 0.34, 0.45)	0.092857	1.21E−05
b_{30}	(0.18, 0.28, 0.38)	0.077381	5.54E−06
b_{31}	(0.16, 0.25, 0.34)	0.067619	3.04E−06
b_{32}	(0.06, 0.12, 0.17)	0.029524	4.26E−08
b_{33}	(0.08, 0.15, 0.22)	0.039048	2.03E−07
b_{34}	(0.06, 0.12, 0.17)	0.029524	4.26E−08
b_{35}	(0.10, 0.17, 0.24)	0.044762	4.15E−07
b_{36}	(0.08, 0.15, 0.22)	0.039048	2.03E−07
b_{37}	(0.17, 0.27, 0.37)	0.073333	4.37E−06

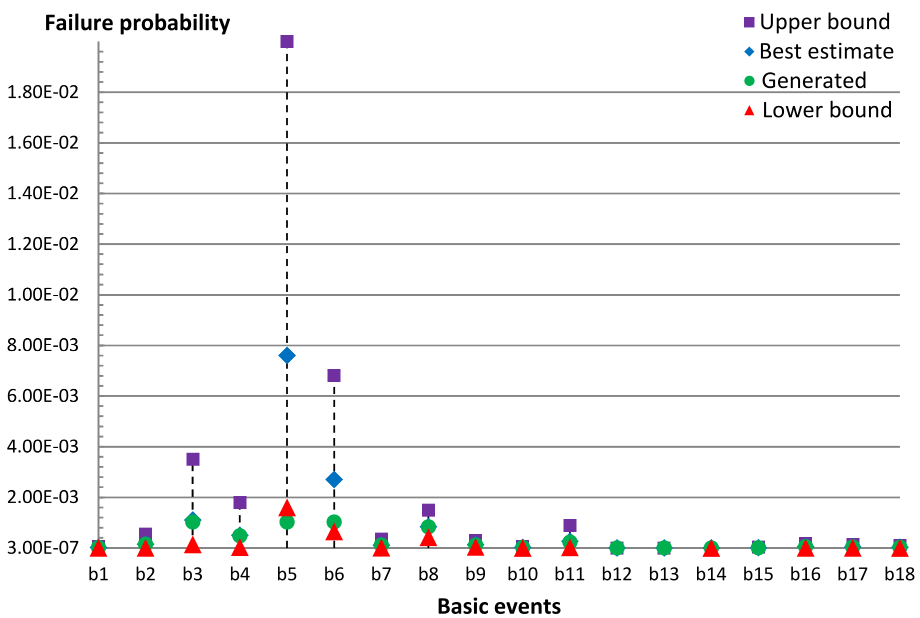

Fig. 5. Failure probability comparison for basic events b_1–b_{18}.

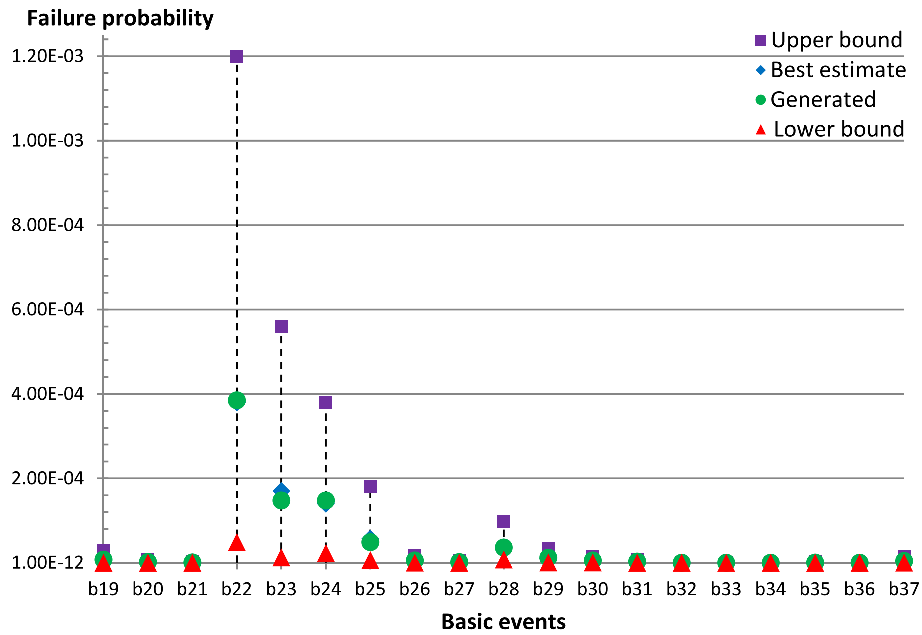

Fig. 6. Failure probability comparison for basic events b_{19}–b_{37}.

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