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Tuned mass dampers for response control of torsional buildings

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SUMMARY

This paper presents an approach for optimum design of tuned mass dampers for response control of torsional building systems subjected to bi-directional seismic inputs. Four dampers with fourteen distinct design parameters, installed in pairs along two orthogonal directions, are optimally designed. A genetic algorithm is used to search for the optimum parameter values for the four dampers. This approach is quite versatile as it can be used with different design criteria and definitions of seismic inputs. It usually provides a globally optimum solution. Several optimal design criteria, expressed in terms of performance functions that depend on the structural response, are used. Several sets of numerical results for a torsional system excited by random and response spectrum models of seismic inputs are presented to show the effectiveness of the optimum designs in reducing the system response. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: seismic response; response control; torsional buildings; tuned mass dampers; genetic algorithms; optimal design

INTRODUCTION

For seismic response control of building structures, several passive and active control schemes are being considered. Among the active and passive schemes, the latter have gained a wider acceptance and are already being used in practice. The passive systems can be divided into two basic categories: (1) base isolation systems and (2) energy dissipation systems. In energy dissipation systems, several different damping systems have been considered (1) friction dampers, (2) visco-elastic dampers, (3) viscous dampers, (4) yielding metallic dampers, and (5) tuned mass dampers. (Although, the tuned mass damper is put in the category of the

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energy dissipation system here, the initial concept of this system was not based on the dissipation of energy but rather on the transfer of energy from the system to be protected to the tuned mass absorber.) All these systems have their special features and attributes that make them suitable for particular response control applications. The focus of this paper is on the use of the tuned mass dampers.

Since its invention in 1909 by Frahm, the concept of a tuned mass damper has attracted special and continued attention of several researchers and practitioners for its application to control vibrations caused by different types of forces. Since it will not be possible to give a complete account of the vast literature available on these devices, here only a few relevant studies are cited. Den Hartog has lucidly described the working principle of the device in his monograph [1], providing simple formulas to obtain the optimum tuning and damping parameters of a tuned mass damper to control the displacement of an undamped single degree-of-freedom system subjected to a harmonic force. Since its early initial application to control the displacement response caused by a harmonic force, now several other excitation and response control conditions have been analyzed. Warburton [2] among others has extended the solution repertoire by covering several other cases of excitation and responses to be controlled. The use and limitation of these formulas with multi-degree of freedom systems is also discussed. Among other researchers, Tsai and Lin [3] have extended the classical solution to a damped primary system. Using curve fitting, they have provided formulas to obtain the optimal parameters.

Although tuned mass dampers have been quite successfully used in several structures to control the wind induced vibrations, their use for seismic response control has not been that convincing. Some investigators [4–6] have shown that tuned mass dampers can be used to control the seismic response, yet there are others [7–9] that show otherwise. Without scrutinizing the details of these studies, it is not straightforward to identify the precise reason (or reasons) for these conflicting conclusions. However, Villaverde and his associate [10–12] offer, perhaps, the most convincing argument for these different findings. They observe that the primary reason for ineffectiveness of the dampers is the use of the classical solutions that are not necessarily optimal for the particular situation under study. They suggest that the damper parameters must be tuned such that the damping ratios of the dominant modes are increased. For this, the damper must be in resonance with its supporting structure and its damping ratio must be equal to the structural damping ratio plus a term that depends on the generalized mass ratio and the modal displacement at the point where the damper is attached [11]. Several numerical results were presented to show the effectiveness of this tuned mass damper design procedure.

Sadek *et al.* [13], however, further examined the approach suggested by Villaverde and Koyama [11] with an example of a tuned mass damper attached to a single degree of freedom system. They observe that that, except for very small mass ratios, Villaverde and Koyama's approach usually leads to unequal damping ratios in the two modes of the combined system, which is not as efficient as having two equal damping ratios in these modes. Based on an exhaustive numerical search of the eigenvalues of the state matrix of the combined structure and damper system for different values of the system parameters, they were able to identify the optimum values of the tuning and damping ratio parameters that would produce two modes with nearly equal damping ratios. By curve fitting they propose simple formulas to calculate these optimum parameters in terms of the mass ratio and the damping ratio of the primary mass. They present several sets of numerical results to demonstrate the effectiveness of their design procedure.

The effective use of the tuned mass damper formulas developed for the single degree of freedom system with the multi degree of freedom systems has also been well demonstrated by several investigators, e.g. in References [10–13]. In such applications, the multidegree of freedom system is represented by an equivalent single degree of freedom system. Such equivalent representations can be successfully used when the response of the multidegree of freedom system is dominated by a single mode, usually the fundamental mode. However, to further improve the robustness and reduce the sensitivity of a design caused by miss-tuning or variability in the system parameter values, and also to control structures with closely spaced frequencies, several researchers [14–18] have proposed the use of a cluster of tuned mass dampers (usually called multiple tuned mass dampers). The frequencies of the cluster are distributed within a frequency bandwidth, usually centered around the frequency of the dominant mode. The objective of using a cluster of dampers is usually not to control several modes of the structure but to improve the control characteristics of the system. However, Rana and Soong [19] have also examined the use of different tuned mass dampers for controlling different modes of a system.

The building systems with accidental or intended eccentricities between their mass and stiffness centers respond with coupled lateral and torsional motions under seismic excitation. It is of interest to study the response control of such structures using several tuned mass dampers, and this paper is precisely concerned with this topic. The writers are familiar with the study by Jangid and Dutta [20] and Lin *et al.* [21] on the control of torsional systems with tuned mass dampers. Jangid and Dutta [20] study the response control of a two degrees of freedom torsional system by a cluster of multiple tuned mass dampers. The input to the main system was white noise excitation. The optimum frequency bandwidth value—the value corresponding to the maximum reduction in the root mean square value of the main system response—was obtained by a parametric variation study. Lin *et al.* [21] study a multi-story torsional building system with one and two tuned mass dampers. They propose a method to identify the dominant modes and critical orientation of the damper track. The optimal parameters are obtained by minimization of the root mean square response of displacement of the dominant mode for a random input.

In this study, four tuned mass dampers, placed along two orthogonal directions in pairs, are considered to control the coupled lateral and torsional response of a multistory building structures subjected to bi-directional earthquake induced ground motions. The objective here is not to target a particular mode of the system for control, but to maximize a performance function that quantifies reduction in a particular response or an overall system response. The performance function may be defined in terms of the floor accelerations, story drifts and shears, and other response quantities of interest. The seismic motion at the base is represented in a more general form either by a stationary random model or by a set of design response spectra. A genetic algorithm is used to search for the optimal design parameters of the four dampers. Several sets of numerical results are presented to demonstrate the effectiveness of such optimally designed tuned mass dampers.

ANALYTICAL MODEL

A multistory shear building consisting of rigid floors supported on deformable columns, with coupled lateral and torsional motions, shown in Figure 1, is considered here for its control

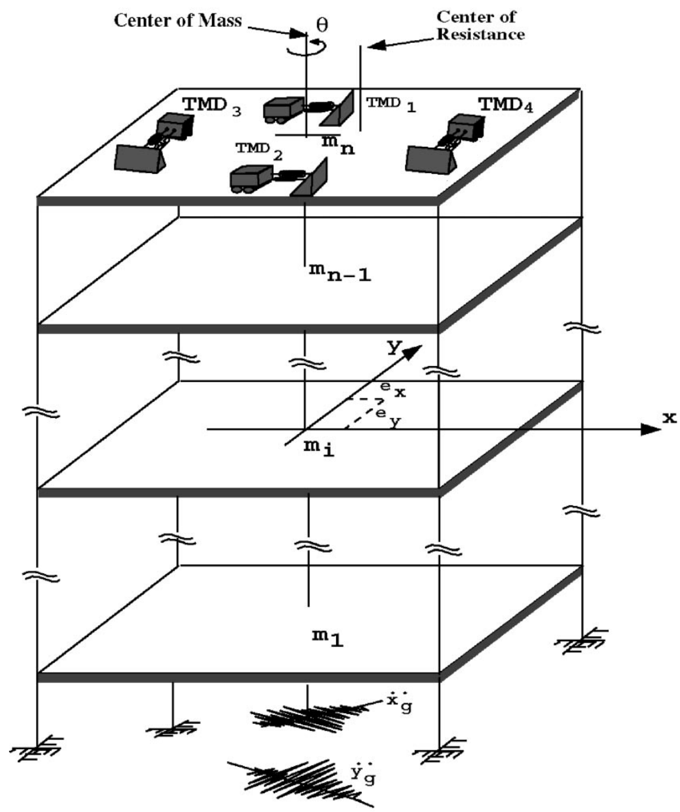


Figure 1. Building frame model with tuned mass dampers at top.

by tuned mass dampers. It is assumed that the mass and stiffness centers of a story do not coincide. However, without any loss of generality it is assumed that the mass centers of all floor diaphragms lie on the same vertical axis and likewise the stiffness centers for each story also lie on another vertical axis. Each floor has three degrees-of-freedom: two displacements in the x- and y-directions and a rotation θ about the vertical axis. The equations of motion of such a building excited by bi-directional seismic inputs in the horizontal plane can be written as

$$\mathbf{M}\ddot{\mathbf{D}} + \mathbf{C}\dot{\mathbf{D}} + \mathbf{K}\mathbf{D} = -\mathbf{r}_{\mathrm{g}}\ddot{\mathbf{g}} \tag{1}$$

where **M**, **C**; and **K**, respectively are the mass, damping and stiffness matrices of the system; **D** is the relative displacement vector consisting of the relative displacements and rotations of each floor $\mathbf{D}_i = \{x_i, y_i, \theta_i\}^{\mathrm{T}}$; $\mathbf{r}_{\mathrm{g}}$ is the influence of the ground excitation; and $\ddot{\mathbf{g}}^{\mathrm{T}} = \{\ddot{x}_{\mathrm{g}}\ \ddot{y}_{\mathrm{g}}\}$ is the vector of the ground acceleration components in x- and y-directions. Although the damping matrix could take any form, here it is defined in terms of assumed values of the modal damping ratios.

On the roof of the above system are installed two pairs of tuned mass dampers. The mass, damping, and stiffness parameters of the two dampers in x-direction are denoted by: (m_{x1}, m_{x2}), (c_{x1}, c_{x2}), and (k_{x1}, k_{x2}). Similar parameters for the dampers along the y-direction are denoted by, (m_{y1}, m_{y2}), (c_{y1}, c_{y2}), and (k_{y1}, k_{y2}). The combined equations of motion of the structure-TMD system can be written as

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_\mathrm{d} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{D}} \\ \ddot{\mathbf{d}} \end{Bmatrix} + \begin{bmatrix} \mathbf{C} + \mathbf{C}' & \mathbf{C}_{dp} \\ \mathbf{C}'' & \mathbf{C}_\mathrm{d} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{D}} \\ \dot{\mathbf{d}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} + \mathbf{K}' & \mathbf{K}_{dp} \\ \mathbf{K}'' & \mathbf{K}_\mathrm{d} \end{bmatrix} \begin{Bmatrix} \mathbf{D} \\ \mathbf{d} \end{Bmatrix} = \begin{bmatrix} \mathbf{r}_\mathrm{g} \\ \mathbf{r}_\mathrm{d} \end{bmatrix}_{(3n+4)\times 2} \begin{Bmatrix} \ddot{x}_\mathrm{g} \\ \ddot{y}_\mathrm{g} \end{Bmatrix} \tag{2}$$

where matrix $\mathbf{M}_\mathrm{d} = \mathrm{diag}(m_{x1}, m_{x2}, m_{y1}, m_{y2})$, $K_\mathrm{d} = \mathrm{diag}(k_{x1}, k_{x2}, k_{y1}, k_{y2})$, and $C_\mathrm{d} = \mathrm{diag}(c_{x1}, c_{x2}, c_{y1}, c_{y2})$, respectively, are the mass, stiffness and damping matrices of the damper system; $\mathbf{d}^\mathrm{T} = \{x_{d1}, x_{d2}, y_{d1}, y_{d2}\}$ is the vector consisting of the relative displacements of the dampers measured with respect to the floor on which they are placed. The influence coefficient matrix associated with TMDs is represented by $\mathbf{r}_\mathrm{d}$. In Equation (2), the coupling matrices $\mathbf{K}_{dp}$ and $\mathbf{C}_{dp}$ and stiffness and damping contribution to the system matrices represented by $\mathbf{K}'$ and $\mathbf{C}'$ are given in the Appendix A. Besides the damper stiffness and damping coefficients, these matrices also depend upon the relative placement position of the dampers on the floor. It is assumed that the perpendicular distances from the center of mass of the damper tracks along the x- and y-directions are l_1 and l_2, respectively.

Performance criteria

The objective is to calculate the values for the mass, damping, stiffness, and distances l_1 and l_2 of the tuned mass dampers that will maximize a performance function which measures the reduction in some response quantity (or quantities) of design interest. The response quantities of the design interest could be the base shear, floor accelerations, etc. In this study, the following four forms of performance functions are used, primarily because they are easy to evaluate for the models of the ground motions (the spectral density function and design response spectra) that are used in this study:

(i) Drift-based criterion (DBC)

$$f_1 = 1 - \frac{\mathrm{Max}[(\sqrt{d_x^2 + d_y^2})_{i,\,\mathrm{controlled}}]}{\mathrm{Max}[(\sqrt{d_x^2 + d_y^2})_{j,\,\mathrm{uncontrolled}}]} \quad i, j = 1, 2, 3, \ldots, n \tag{3}$$

where d_x and d_y are the drifts of a story in the x- and y-directions, respectively.

(ii) Acceleration-based criterion (ABC)

$$f_2 = 1 - \frac{\mathrm{Max}[(\sqrt{a_x^2 + a_y^2})_{i,\,\mathrm{controlled}}]}{\mathrm{Max}[(\sqrt{a_x^2 + a_y^2})_{j,\,\mathrm{uncontrolled}}]} \quad i, j = 1, 2, 3, \ldots, n \tag{4}$$

where a_x and a_y are the accelerations of a floor in the x- and y-direction, respectively.

(iii) Drift-based second norm

$$f_3 = 1 - \frac{\sqrt{\sum_{i=1}^{n} (d_x^2 + d_y^2)_{i,\,\mathrm{controlled}}}}{\sqrt{\sum_{i=1}^{n} (d_x^2 + d_y^2)_{i,\,\mathrm{uncontrolled}}}} \tag{5}$$

(iv) Acceleration-based second norm

$$f_4 = 1 - \frac{\sqrt{\sum_{i=1}^{n} (a_x^2 + a_y^2)_{i,\text{controlled}}}}{\sqrt{\sum_{i=1}^{n} (a_x^2 + a_y^2)_{i,\text{uncontrolled}}}} \tag{6}$$

The performance functions in Equations (3) and (4) tend to minimize the maximum story drift and the maximum acceleration values. The drift will mostly be largest in the bottom story. The acceleration will usually be the largest on the top floor, but it may not be the case always. In general, thus maximum controlled and uncontrolled values in Equation (4) could possibly be in different floors of the structure. The performance functions in Equations (5) and (6), which include the responses from all stories or the floors, focus on reducing all involved response quantities. It is mentioned that the optimal search procedure to be used, however, is not restricted to these special forms of the performance functions; it can be used with any input and function as long as the performance function can be calculated.

Optimal search approach

The optimal search for the damper parameters can be conveniently made by the (1) gradient-based methods or (2) genetic algorithm (GA) methods. Here we will use the GA approach primarily for the following reasons. This approach is highly flexible as it can be used with any form of the performance function and any seismic input as long as one can analyze the structure and calculate the performance function. The approach usually leads to a globally optimum solution as it starts with many possible choices that evolve according to the rules of genetic propagation. Earlier, Hadi and Arfiadi [22] have used the genetic algorithm to obtain the optimum parameters of a tuned mass damper installed on a planer structure. A brief description of the basic elements of this very powerful and effective approach is provided below.

Since their development by Holland [23], the genetic algorithms have enjoyed a growing interest in the combinatorial optimization community and have been used in many different applications. Recently, Singh and Moreschi [24] and Moreschi [25] have successfully used GA for optimal sizing and placement of energy dissipation devices in multistory buildings subjected to earthquake induced ground motions. Genetic algorithms are robust search and optimization techniques that are based on the principles of natural biological evolution. In the context of our problem of optimal design parameters of the dampers, all feasible designs of the dampers represent the individuals in the search space. A design is considered the best (fittest individual) if the performance function associated with this design has the highest value. The objective is to identify the best design in this search space. A genetic algorithm operates on a population of the potential solutions (designs) which successively evolves through random genetic changes, involving mating with crossover and mutations that emulate natural biological evolution, to produce a successively better approximation to a design solution. The selection of pairs for reproduction exploits the current knowledge of the solution space by propagating better designs (individuals) and discouraging the poorer ones. The crossover and mutation operators are the two basic mechanisms of a genetic algorithm; they create new designs for further exploration in the search space. The rules for mating, crossover and mutation are defined in a probabilistic manner. As a new population is created, the performance function is evaluated for each new design to determine its fitness with respect to other designs in the population. This process is repeated for a number of cycles (generations) until no further improvement is observed in the best individual in the subsequent generations. The genetic algorithms, thus, differ from conventional optimization techniques in the following ways: (1) They consider simultaneously many design points in the search space and therefore have a reduced chance of converging to local optima. (2) They do not require any computations of gradients of complex functions to guide their search; the only information needed is the response of the system to calculate the objective or fitness function. (3) They use probabilistic transition rules (genetic operators) instead of deterministic transition rules. There are now several monographs [26–29] on the subject and papers dealing with applications in structural engineering [30–32] along with those mentioned earlier.

In the context of our problem of selecting the damper parameters, we proceed as follows. For the four tuned mass dampers, there are total of 14 parameters to be calculated: four parameters each for the mass, damping and stiffness coefficients plus two parameters for the distances l_1 and l_2. First a workable range for each parameter value is selected. These ranges are discretized to identify various discrete values the parameters can take. The discrete values, randomly selected, define a set of four tuned mass dampers, representing a possible control system design. In GA terminology, each design represents a chromosome identifying an individual of the population. The individual is made up of genes consisting of the selected parameter values. The optimization process starts with a population of these individuals. For the problem at hand, 30 individuals were selected to form the population. There is no hard and fast rule to determine the size of a population. A larger population perhaps will go through fewer evolutions of generations than a smaller population to converge to the final optimum solution. However, in each generation a larger population will also require more calculation of the performance function than the smaller population. The structural system is next analyzed for each individual design of the population, and the individual designs are rank-ordered according to the value of their performance functions. The design with highest performance index is the best, and so on. The individual designs are then paired for reproduction of new designs to create a whole new generation of designs through mating. Not all but a high percentage of the individuals in the population are paired for reproduction. Herein we chose to pair 95 per cent of the individuals. The pair selection for mating is done according to the roulette-wheel scheme the details of which can be found in Reference [25]. Each pair of individuals procreates a new pair of the off-springs through crossover. The crossover involves intermixing of the parental genes. A simple crossover scheme may consist of switching of parental genes above a randomly selected gene location. More complex crossover schemes with multiple point switchovers can also be used. A fraction of the new population is also subjected to mutation to introduce new designs in the population. It involves a simple exchange of an existing gene by a randomly selected new gene. In this study, in each generation a 5 per cent of the population was subjected to mutation. Besides mutation, elitist selection schemes are also introduced in the formation of a new generation. In this scheme, the last ranked individuals in the current generation is dropped and replaced by the first ranked individual from the previous generation. This tends to retain the best characteristics of the previous generation in the current generation. The newly formed generation then goes through the whole cycle of genetic evolution to create a new generation. After a few cycles of generation, the best design in the population usually converges to the optimum design. To implement this scheme in this study, the genetic algorithm module written in FORTRAN 90 by McMahon *et al.* [33] was modified and used.

Performance function evaluation

In this genetic search process, the majority of computational effort is spent on the analysis of structure to calculate the performance function for each individual design in each generation. For the linear multidegree of freedom system and the seismic input considered in this study, the response quantities required for calculating the performance function were obtained by a generalized modal analysis approach. Although the damping of the structure is assumed to be classical, the combined structure and damper system represented by Equation (2) will be non-classically damped. To analyze a non-classically damped system, it is convenient to work with the system of first order state equations

$$\mathbf{A}\dot{\mathbf{y}}(t) + \mathbf{B}\mathbf{y}(t) = \mathbf{D}' \begin{Bmatrix} \mathbf{0} \\ \mathbf{r}_s \end{Bmatrix} \ddot{\mathbf{g}} \tag{7}$$

where, for a N degree-of-freedom dynamic system (2) $\mathbf{y}(t)$ is the $2N \times 1$ state vector consisting. The system matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{D}'$ of dimension $2N \times 2N$ are defined as

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{M}_s \\ \mathbf{M}_s & \mathbf{C}_s \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} -\mathbf{M}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_s \end{bmatrix}; \quad \mathbf{D}' = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_s \end{bmatrix} \tag{8}$$

where, $\mathbf{M}_s$, $\mathbf{C}_s$, and $\mathbf{K}_s$ are the mass, damping and stiffness matrices of the combined structure and damper system defined by Equation (2). Following the procedure described by Singh and Maldonado [34, 35] one can calculate the means square value of a response quantity R for bi-direction seismic input defined by spectral density functions as follows:

$$E[R^2(t)] = \sum_{l=1}^{2} \left\{ \begin{array}{l} \sum_{j=1}^{N} \gamma_{jl}^2 I_j^l + 4\sum_{j=1}^{N} a_{jl}^2 I_{2j} + \\ 2\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \left[W_{jk}\left(I_{ij}^l - \dfrac{I_{1k}^l}{\Omega_{jk}^4} \right) + Q_{jk}(I_{2j}^l - I_{2k}^l) + \dfrac{\gamma_{jl}\gamma_{kl}}{\Omega_{jk}^2} I_{1k}^l + 4a_{jl}a_{kl} I_{2k}^l \right] \end{array} \right\} \tag{9}$$

where various terms in this expression are defined in terms of the complex modal properties for the system and seismic inputs, as described in Appendix B. I_{1j}^l, and I_{2j}^l are the frequency integrals that express the mean square values of the relative displacement and relative velocity responses of an oscillator of frequency ω_j and damping ratio β_j subjected to the base motion defined by the spectral density function $\Phi_l(\omega)$ as follows:

$$I_{1j}^l = \int_{-\infty}^{\infty} \frac{\Phi_l(\omega)}{(\omega_j^2 - \omega)^2 + 4\omega_j^2\beta_j^2\omega^2} \, d\omega, \qquad I_{2j}^l = \int_{-\infty}^{\infty} \frac{\Phi_l(\omega)\omega^2}{(\omega_j^2 - \omega)^2 + 4\omega_j^2\beta_j^2\omega^2} \, d\omega \tag{10}$$

By relating these frequency integrals with the ground response spectrum values of the input motions, one can obtain the design value for the response quantity R_d of interest as follows:

$$R_d^2 = \sum_{l=1}^{2} \left[\begin{array}{l} \sum_{j=1}^{N} \left(\dfrac{\gamma_{jl}^2}{\omega_j^4} P_{jl}^2 + 4a_{jl}^2 V_{jl}^2 \right) + \\ 2\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \left[\dfrac{W_{jk}}{\omega_j^4}(P_{jl}^2 - P_{kl}^2) + Q_{jk}(V_{jl}^2 - V_{kl}^2) + \dfrac{\gamma_{jl}\gamma_{kl}}{\omega_j^2\omega_k^2} P_{kl}^2 + 4a_{jl}a_{kl} V_{kl}^2 \right] \end{array} \right] \tag{11}$$

where, P_{jl} and V_{jl}, respectively, are the pseudo-acceleration and relative-velocity response spectrum values for an oscillator with parameters (ω_j, β_j) excited by the lth component of the base motion.

NUMERICAL STUDY

System model

The 6-story shear building model shown in Figure 1 is considered for the numerical study. Properties of the building are: mass of each floor $= 40\,000$ kg, flexural story stiffness in x- and y-direction $(k_x, k_y) = 4.5 \times 10^7$ N/m, torsional stiffness of a story $(k_\theta) = 1.18 \times 10^9$ N m/rad and radius of gyration $(r) = 11.7$ m. The distribution of stiffness is symmetrical about the y-axis, thus $(e_x/L_x = 0)$. However, in the y-direction an eccentricity of magnitude $e_y/L_y = 0.075$ exists between the mass and stiffness centers. The first nine natural frequencies of the building are 7.98, 8.09, 11.36, 23.46, 23.79, 33.59, 37.59, 38.11, and 49.53 rad/s. It is noted that in these nine frequencies there are three pairs of closely spaced frequencies. Modal damping ration of the building is assumed to be 0.03 in all modes.

Ground excitation models

Both, the stochastic model and the design response spectra have been used to define the base input motion. For the stochastic model, the ground motions in x- and y-directions are described by two identical but uncorrelated zero-mean stationary processes with power spectral density functions $\Phi_l(\omega)$ of the Kanai–Tajimi form:

$$\Phi_l(\omega) = \frac{\omega_g^4 + 4\omega_g^2\zeta_g^2\omega^2}{(\omega^2 - \omega_g^2)^2 + 4\omega_g^2\zeta_g^2\omega^2} S^2 \tag{12}$$

The parameters of this function are: $\omega = 18.85$ rad/s, $\beta = 0.65$ and $S^2 = 0.0618$ m^2/s^3/rad. The response spectrum model is represented in terms of the pseudo acceleration and relative velocity response spectra shown in Figures 2a and 2b, respectively. They are the average values of the response spectra of an ensemble of 50 simulated ground acceleration time histories.

Tuned mass damper parameters

As mentioned before, two pairs of tuned mass dampers are installed at the top of the building (Figure 1). The objective of the study is to design the optimum parameters of these TMDs that would maximize the performance functions stated earlier. The possible ranges for the four design parameters are fixed as follows:

(a) Mass ratio, m_r: The mass ratio is defined as the ratio of the damper mass to the total building mass. It is assumed that each damper mass ratio can vary in the range of 0.1 per cent to 1 per cent of the building mass. Thus the maximum mass of the damper system consisting of four dampers could be as high as 4 per cent of the building mass.

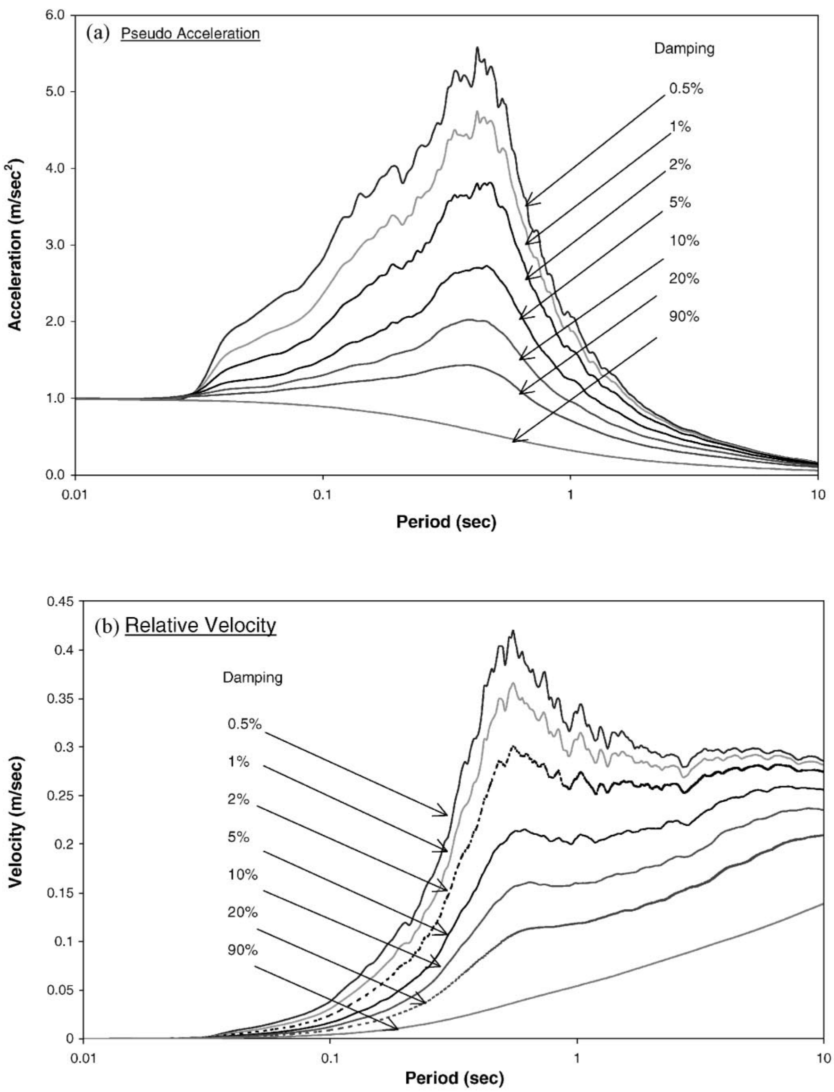


Figure 2. Input response spectra for: (a) pseudo-acceleration; (b) relative velocity.

(b) Frequency tuning ratio, f_r: The frequency ratio for each damper is defined as the ratio its own natural frequency to the fundamental frequency of the building structure. Here it is assumed that this ratio could vary between the range of 0–2.5.
(c) Damping ratio: This is the ratio of the damping coefficient to its critical value. Thus for the ith damper in x-direction it is defined as $\beta_i = c_{xi}/\sqrt{4k_{xi}m_{xi}}$. It is assumed that this ratio can vary in the range of 0–10 per cent.
(d) Damper positions from the mass center, l_1 and l_2: It is assumed that this distance can vary between 1 and 2.5 metres.

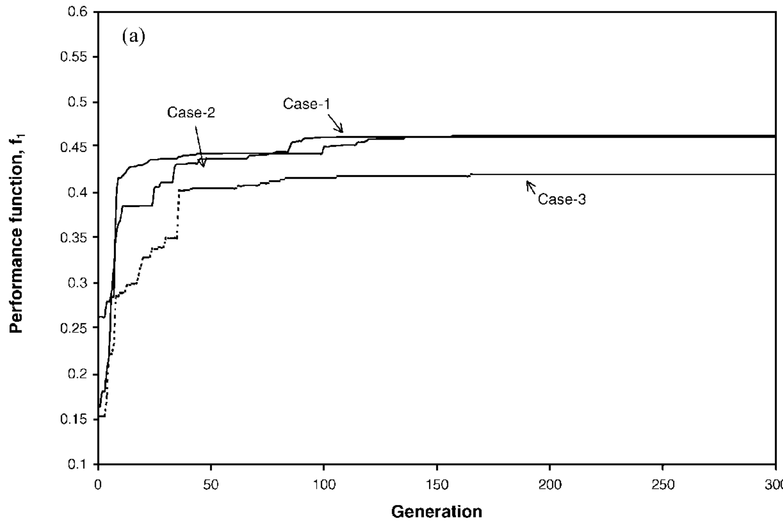
Table I. Damper parameters and their ranges for the three design cases.

Design parameters	Case 1	Case 2	Case 3
Mass ratio (m_r)	$0.001 < m_r \leqslant 0.01$	0.01 (fixed)	0.01 (fixed)
Tuning frequency ratio (f_r)	$0.0 < f_r \leqslant 2.5$	$0.0 < f_r \leqslant 2.5$	$0.0 < f_r \leqslant 2.5$ and $f_{r1} = f_{r2}; f_{r3} = f_{r4}$
Damping ratio (d_r)	$0\% < d_r \leqslant 10\%$	$0\% < d_r \leqslant 10\%$	$0\% < d_r \leqslant 10\%$
Position from center of mass (l)	$1.0 \leqslant l \leqslant 2.5$	$1.0 \leqslant l \leqslant 2.5$	$1.0 \leqslant l \leqslant 2.5$

To use the genetic algorithm, this continuous range is divided into 25 equally spaced discrete values. Since there are 14 design parameters, and each parameter could be chosen independently of others in 25 different ways, there a total of 25^{14} possible design solutions. One could not possibly evaluate each of these valid solutions to identify the optimum solution. The genetic algorithm, however, makes it possible to identify the best solution within a manageable computational effort.

Varying all parameters independently adds flexibility in the optimization process, but it also increases computational effort to achieve the optimum solution. So it might be of interest to fix some parameter, or constrain them to a certain value, to reduce the number of different parameter combinations to be considered. To examine such effects, here three different cases have been considered. In case 1, all design parameters are allowed to vary independently of each other within their specified ranges. In case 2, however, the mass is fixed at its highest value of 1 per cent for each damper, but other parameters are allowed to vary freely in their ranges. Case 3 is similar to case 2, but with an additional constraint that frequencies of the two x-direction dampers are the same, and so are the frequencies of the two y-direction dampers. Except for these constraints on the mass and frequency ratios, the other two parameters can vary freely in their own ranges. The characteristics of these three cases are shown Table I. A comparison of these three cases with regard to their effectiveness in reducing the response will show the benefit of having flexibility in the design process. Case 1 has the highest flexibility but also the largest number of design parameters to be optimized. Case 3 obviously has the least flexibility and correspondingly a smaller number of parameters to be optimized.

For the three cases shown in Table I, Figures 3(a) and 3(b) show the evolution of the optimum solution with the number of generation. These results are for the input defined by the spectral density functions. The performance function values are thus expressed in terms of the mean square values of the drift and acceleration responses. Figure 3(a) is for the drift-based performance function, and Figure 3(b) for the acceleration-based performance function. The vertical axis indicates the value of the performance function for different generations shown on the x-axis. It is seen that the solutions for all cases approach their maximum values after about 150 cycles of generations. The case 1, with most flexibility and most number of parameters, seems to converge the faster than others and also reaches the highest value of the performance function. The case 3 converges more slowly and also does not reach as high in its performance as the other two cases. Thus the case having the more flexibility in the choice of parameters is better than other cases. The final optimum parameters for the three cases for



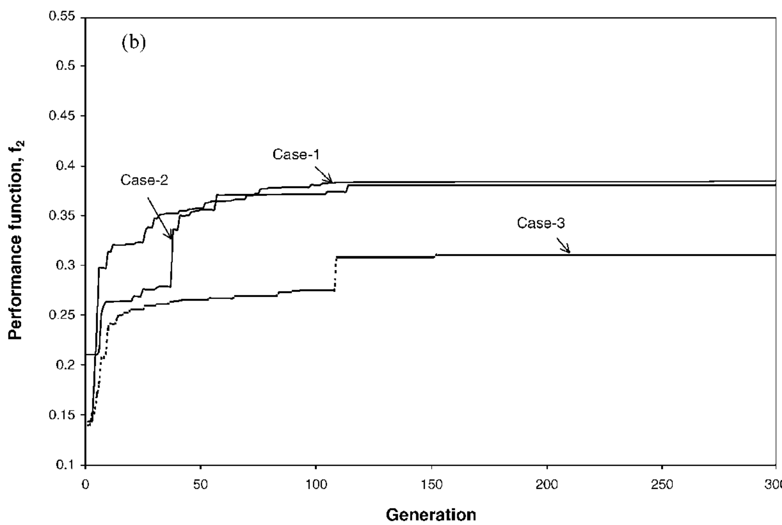


Figure 3. Performance improvement with generations for: (a) drift-based performance function, f_1; (b) acceleration based performance function, f_2; Kanai–Tajimi PSDF input.

the two optimum design criteria are give in Table II. The table also shows the final values of the performance functions obtained for these three cases. For the following numerical results, only case 1 is considered.

Figures 4 and 5 show the mean square values of the story drift and floor accelerations, respectively, for the uncontrolled and controlled structures. The drifts and accelerations in the x- and y-directions are shown separately. In both figures, the controlled values for the damper parameters obtained according to the drift-based and acceleration-based performance

Table II. Optimum design parameters of tuned mass dampers for different cases: Kanai–Tajimi PSDF input.

	Performance function in Equation (3)				Performance function in Equation (4)			
TMD	Tuning frequency ratio (f_r)	Mass ratio (m_r)	Damping (d_r) (in %)	Position (l) (in m)	Tuning frequency ratio (f_r)	Mass ratio (m_r)	Damping (d_r) (in %)	Position (l) (in m)
		Case 1; $f_1 = 0.4616$				Case 1; $f_2 = 0.3847$		
1	0.80	0.0020	9.2	2.25	1.00	0.0084	10	1.06
2	1.00	0.0048	10	2.25	2.50	0.0100	10	1.06
3	0.70	0.0092	10	1.56	2.50	0.0100	10	1.94
4	1.00	0.0080	10	1.56	1.00	0.0096	10	1.94
		Case 2; $f_1 = 0.46312$				Case 2; $f_2 = 0.3807$		
1	1.00	0.0100	10	1.00	2.50	0.0100	10	2.50
2	0.90	0.0100	10	1.00	1.00	0.0100	10	2.50
3	1.20	0.0100	10	2.50	2.50	0.0100	10	1.94
4	1.00	0.0100	10	2.50	1.00	0.0100	10	1.94
		Case 3; $f_1 = 0.4203$				Case 3; $f_2 = 0.3105$		
1	1.00	0.0100	10	2.50	1.00	0.0100	10	2.50
2	1.00	0.0100	10	2.50	1.00	0.0100	10	2.50
3	1.00	0.0100	10	2.50	1.00	0.0100	10	2.50
4	1.00	0.0100	10	2.50	1.00	0.0100	10	2.50

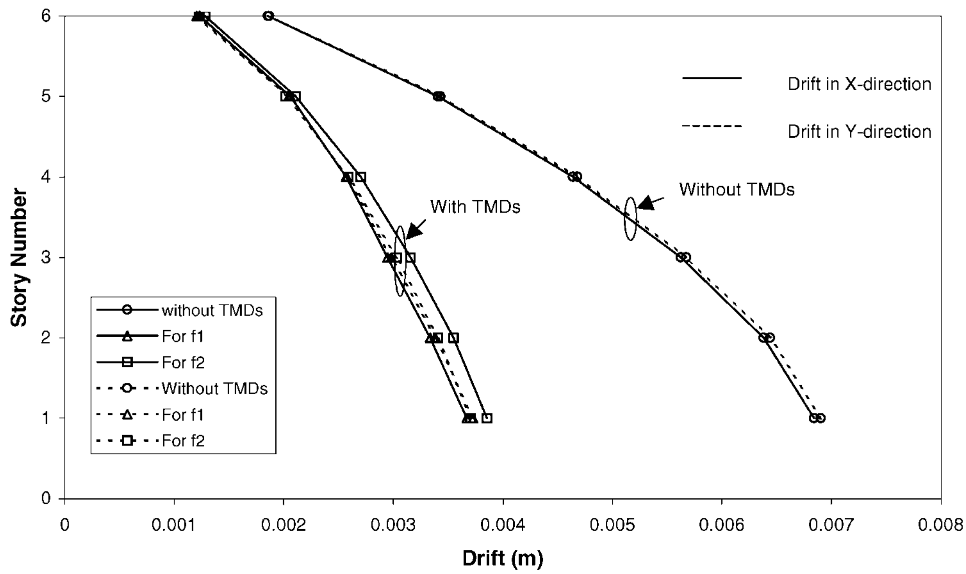


Figure 4. Inter-story drift responses in X- and Y-directions with and without tuned mass dampers designed for f_1 (Equation (3)) and f_2 (Equation (4)) performance functions; Kanai–Tajimi PSDF input.

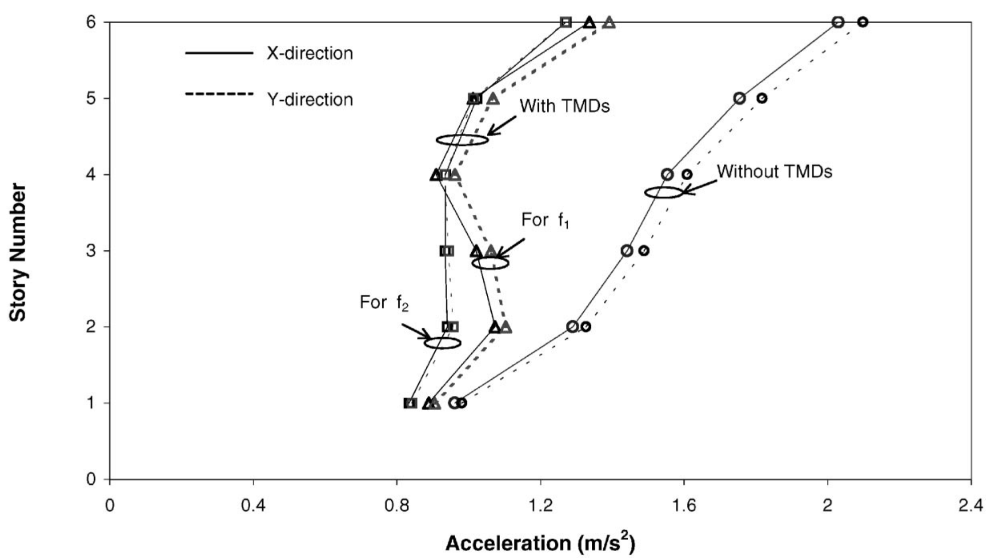


Figure 5. Floor acceleration responses in X- and Y-directions with and without tuned mass dampers designed for f_1 (Equation (3)) and f_2 (Equation (4)) performance functions; Kanai–Tajimi PSDF input.

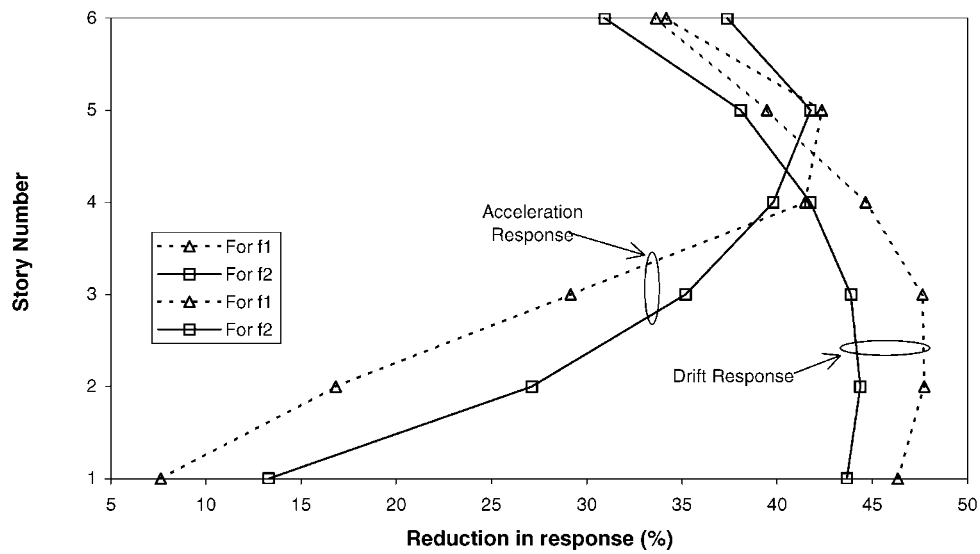


Figure 6. Percent reduction in story drifts and floor accelerations in X-direction for tuned mass dampers designed for f_1 (Equation (3)) and f_2 (Equation (4)) performance functions; Kanai–Tajimi PSDF input.

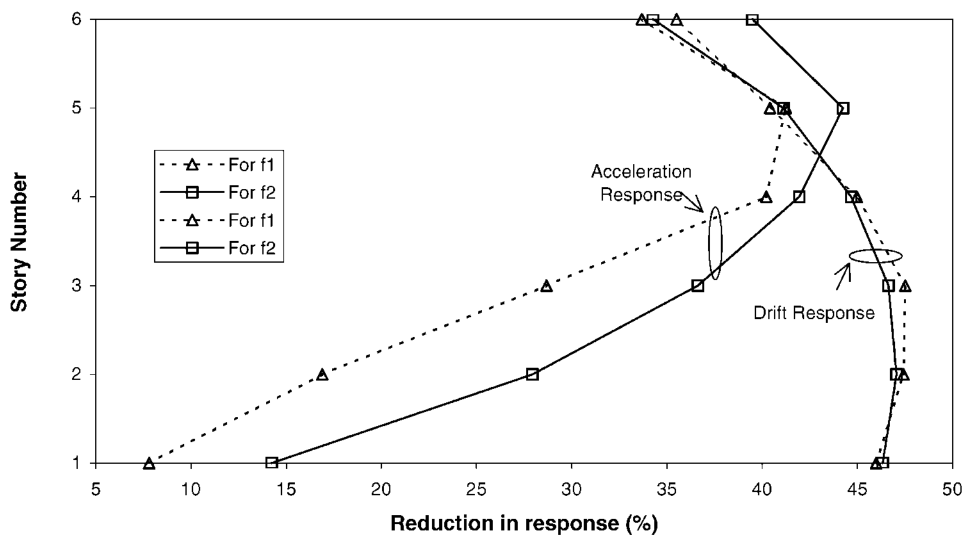


Figure 7. Percent reduction in story drifts and floor accelerations in Y-direction for tuned mass dampers designed for f_1 (Equation (3)) and f_2 (Equation (4)) performance functions; Kanai–Tajimi PSDF input.

functions are shown. A significant reduction in the two response quantities is noted for both design criteria. The percent reductions in the responses are, however, more clearly seen from Figures 6 and 7. Figure 6 is for the response values in the x-direction and Figure 7 for the values in the y-direction. From these figures it is noted that design based on a particular

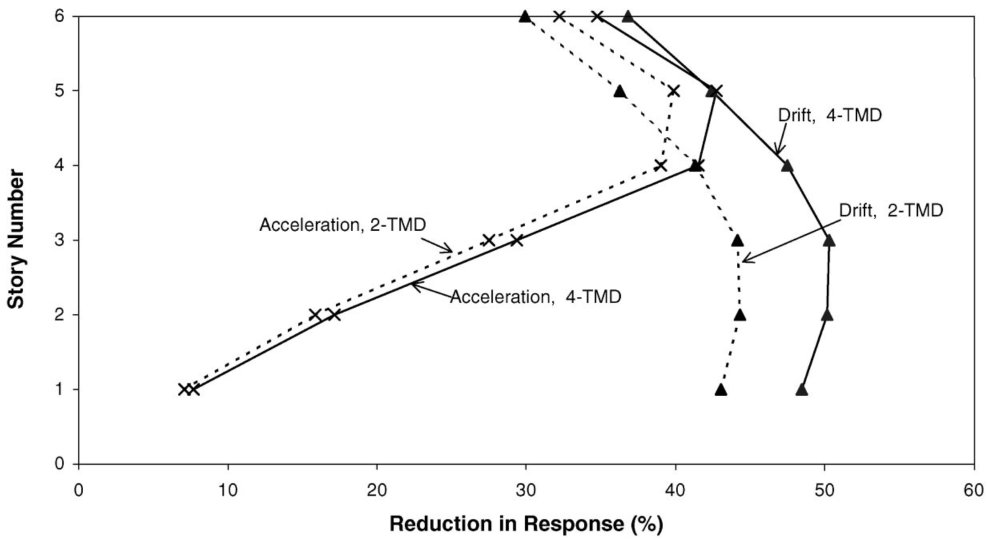


Figure 8. Percent reduction in total drifts and accelerations for 2 and 4 tuned mass dampers designed with performance function, f_1 (Equation (3)); Kanai–Tajimi PSDF input.

response quantity-based criterion may also be effective in reducing other response quantities. However, in general, the use of the acceleration-based optimum design will be effective in reducing the acceleration response more than the drift response, and vice versa. As one would expect, the acceleration reduction at the base due to the tuned mass dampers installed on the top is not as high as at the other locations.

In Figure 8 we compare the effectiveness of two versus four tuned mass dampers. The response reductions in the total root mean square values of the story shears and floor accelerations are shown. Here, the total response means the square root of the sum of the squares of the x- and y-direction responses. The optimum designs were based on the drift-based performance function. It is clear from this figure that four dampers are more effective than two dampers. Obviously, having more dampers provides additional flexibility to adjust parameters for a better response control, especially in a multi-degree of freedom system.

Figure 9 is similar to Figure 6 or 7, except that the seismic input for these results is defined by the pseudo-acceleration and relative velocity response spectra of Figures 2. These results are presented to simply show that one can effectively use the commonly used form of seismic input—design response spectra—for optimum design of tuned mass dampers. The response quantities in Figure 9 are calculated by the response spectrum approach mentioned earlier. The percent reduction in the total story shear and floor accelerations are shown for the two designs based on acceleration and drift norms of Equations (5) and (6). The optimum parameters of the dampers in the two designs are shown in Table III. Also shown are the final performance function values achieved in the optimization process. The observations based on the results of Figure 9 are similar to those given for Figures 6 and 7.

The calculated optimal parameter values can be further refined by dividing the parameter ranges into finer discrete values. This will of course increase the computational effort. However, knowing an optimal parameter value with a coarser division of the range, one can

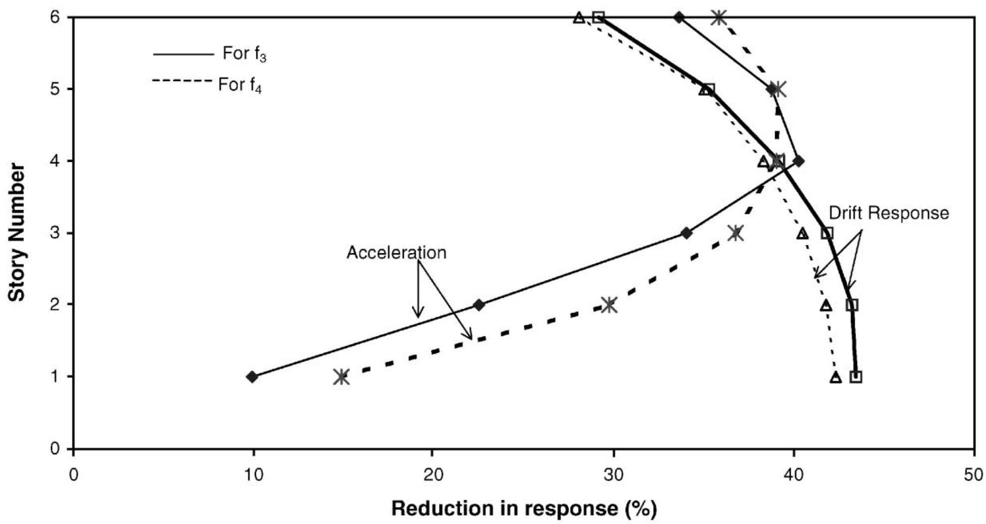


Figure 9. Percent reductions in total drifts and accelerations for tuned mass damper designs based on f_3 (Equation (5)) and f_4 (Equation (6)) performance functions; response spectrum input.

redefine a narrower range around the calculated optimal value. This narrower range can be further divided into finer discrete values to calculate a refined optimal value. This refinement was carried out here. The results in lower part of Table III show the new optimal values of the parameters, and their corresponding performance function values. The refined parameter values, indeed, provide a better performance as indicated by the higher values of the performance function realized.

CONCLUDING REMARKS

The paper describes an approach for calculating the optimum design parameters of several tuned mass dampers to control the seismic response of torsionally coupled building structures. As an example, two damper pairs are used on the top floor of a torsionally coupled building. The optimal parameters are obtained by maximizing several different forms of the performance functions that measure in different ways the reduction in seismic response. A genetic algorithm is used to search for the optimal solution. This optimal approach can be conveniently used with any form of the performance function and seismic inputs as long as the performance function can be calculated. For linear systems, the commonly used design spectra can be used in optimal designs. Having more flexibility in the choice of parameters can be beneficial for producing a better design. The effectiveness of the optimally designed tuned dampers is demonstrated by several numerical examples.

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Table III. Optimum design parameters of tuned mass dampers for case 1: response spectrum input.

TMD	Performance function in Equation (5)				Performance function in Equation (6)			
	Tuning frequency ratio (f_r)	Mass ratio (m_r)	Damping (d_r) (in %)	Position (l) (in m)	Tuning frequency ratio (f_r)	Mass ratio (m_r)	Damping (d_r) (in %)	Position (l) (in m)
	$f_3 = 0.4151$				$f_4 = 0.3292$			
1	1.10	0.0052	8.80	2.50	2.50	0.0100	10.00	1.0625
2	1.00	0.0056	9.20	2.50	1.00	0.0076	10.00	1.0625
3	1.20	0.0048	10.00	2.38	2.50	0.0100	10.00	1.4375
4	1.00	0.0092	10.00	2.38	1.00	0.0100	10.00	1.4375
	Refined parameters $f_3 = 0.4537$				*Refined parameters* $f_4 = 0.3439$			
1	0.95	0.0048	10.00	2.40	2.49	0.0100	10.00	1.0000
2	1.04	0.0060	9.85	2.40	0.98	0.0065	9.96	1.0000
3	1.06	0.0060	10.00	2.30	2.50	0.0100	9.67	1.1167
4	0.94	0.0100	10.00	2.30	0.99	0.0098	9.96	1.1167

APPENDIX A

The coupling matrices involved with matrix $\mathbf{C}_s$ and $\mathbf{K}_s$ in Equation (2) are as follows:

$$\mathbf{C}_{dp} = \begin{bmatrix} -c_{x1} & -c_{x2} & 0 & 0 \\ 0 & 0 & -c_{y1} & -c_{y2} \\ l_1 c_{x1} & -l_1 c_{x2} & l_2 c_{y1} & -l_2 c_{y2} \end{bmatrix}, \quad \mathbf{K}_{dp} = \begin{bmatrix} -k_{x1} & -k_{x2} & 0 & 0 \\ 0 & 0 & -k_{y1} & -k_{y2} \\ l_1 k_{x1} & -l_1 k_{x2} & l_2 k_{y1} & -l_2 k_{y2} \end{bmatrix} \tag{A1}$$

$$\mathbf{C}_{pd} = \mathbf{C}_{dp}^{\mathrm{T}} \quad \text{and} \quad \mathbf{K}_{pd} = \mathbf{K}_{dp}^{\mathrm{T}} \tag{A2}$$

$$\mathbf{C}' = \begin{bmatrix} [\mathbf{0}]_{(3n-3)\times(3n-3)} & [\mathbf{0}]_{(3n-3)\times 3} \\ [\mathbf{0}]_{3\times(3n-3)} & \begin{bmatrix} c_{x1}+c_{x2} & 0 & (c_{x2}-c_{x1})l_1 \\ 0 & c_{y1}+c_{y2} & (c_{y2}-c_{y1})l_2 \\ (c_{x2}-c_{x1})l_1 & (c_{y2}-c_{y1})l_2 & (c_{x1}+c_{x2})l_1^2+(c_{y1}+c_{y2})l_2^2 \end{bmatrix}_{3\times 3} \end{bmatrix} \tag{A3}$$

$$\mathbf{K}' = \begin{bmatrix} [\mathbf{0}]_{(3n-3)\times(3n-3)} & [\mathbf{0}]_{(3n-3)\times 3} \\ [\mathbf{0}]_{3\times(3n-3)} & \begin{bmatrix} k_{x1}+k_{x2} & 0 & (k_{x2}-k_{x1})l_1 \\ 0 & k_{y1}+k_{y2} & (k_{y2}-k_{y1})l_2 \\ (k_{x2}-k_{x1})l_1 & (k_{y2}-k_{y1})l_2 & (k_{x1}+k_{x2})l_1^2+(k_{y1}+k_{y2})l_2^2 \end{bmatrix}_{3\times 3} \end{bmatrix} \tag{A4}$$

$$\mathbf{C}'' = [[0]_{4\times(3n-3)} \quad [\mathbf{C}_{pd}]_{4\times 3}], \quad \mathbf{K}'' = [[\mathbf{0}]_{4\times(3n-3)} \quad [\mathbf{K}_{pd}]_{4\times 3}] \tag{A5}$$

APPENDIX B

Various terms and coefficients required in Equations (9), (10) and (11) are defined in terms of the eigenproperties of the system obtained from the solution of the following eigenvalue problem:

$$-\boldsymbol{\lambda}_j \mathbf{A}\boldsymbol{\varphi}_j = \mathbf{B}\boldsymbol{\varphi}_j, \quad j = 1, \ldots, 2N \tag{B1}$$

where φ_j is the eigenvector. The eigenvalue λ_j is defined as

$$\lambda_j = -\beta_j \omega_j + \mathrm{i}\omega_j \sqrt{1-\beta_j^2} \tag{B2}$$

In terms of these, we define

$$\omega_j = |\lambda_j|, \quad \beta_j = -\mathrm{Re}(\lambda_j)/\omega_j, \quad \Omega_{jk} = \frac{\omega_j}{\omega_k} \tag{B3}$$

$$a_{jl} = \mathrm{Re}(\rho_j F_{jl}), \quad b_{jl} = \mathrm{Im}(\rho_j F_{jl}) \tag{B4}$$

$$\gamma_{jl} = 2\omega_j (b_{jl}\sqrt{1-\beta_j^2} - a_{jl}\beta_j) \tag{B5}$$

In Equation (B4), F_{jl} is the participation factor defined as

$$F_{jl} = \{\phi_j^{\mathrm{L}}\}^{\mathrm{T}} \mathbf{M}_s \mathbf{r}_s \tag{B6}$$

where $\{\varphi_j^{\mathrm{L}}\}$ is the lower half of the eigenvector φ_j. The quantity ρ_j is the modal response quantity of interest obtained from the eigenvector φ_j as

$$\rho_j = \mathbf{T}^{\mathrm{T}}\varphi_j \tag{B7}$$

The transformation vector T transforms the eigenvector into the desired response quantity. This vector depends upon the system properties. Influence matrix of ground excitation is defined as

$$[r_{\mathrm{s}}]_{(3n+4)\times 2} = [[r_{\mathrm{s1}}]_{(3n+4)\times 1} \quad [r_{\mathrm{s2}}]_{(3n+4)\times 1}] \tag{B8}$$

The participation factor coefficients W_{jk} and Q_{jk} used in Equations (9) and (11) are defined in the appendix of paper by Singh *et al.* [36].

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