Group bargaining and conflict

N. Quérou*

Abstract

We consider a situation where groups negotiate over the allocation of a surplus (which is used to fund group specific goods). Each group is composed of agents who have differing valuations for public goods. Members choose a representative to take decisions on their behalf. Specifically, representatives can either negotiate cooperatively or use conflict to appropriate the surplus. In the cooperative negotiations disagreement corresponds to a pro rata allocation (as a function of the size of the groups). We analyse the conditions (on the internal composition of the groups) under which conflict will be preferred to negotiated agreements (and vice versa), and we derive welfare implications. Finally, we provide results of comparative statics that highlight the influence of changes in the internal composition of groups and in their relative size on the profitability of negotiated agreements.

1 Introduction

The issue of the prevalence of negotiated agreements over conflict has received a lot of attention in the recent decades. Even though the use of conflict is typically inefficient as it implies some waste of resources (a social cost), conflictual situations are widely observed in practice (union strikes ending negotiations with management, international conflicts over water allocation,...). More specifically, there are situations where two options exist: an agreement can result from a collective bargaining procedure or it can be the outcome of a conflict between the different parties. Let us focus on one illustrative example of such a situation. In national sport leagues (US basketball, hockey as examples) collective bargaining agreements are basic contracts between the National League team owners and the Players

*Address: Queen's University Management School, Queen's University Belfast, 25 University Square, Belfast BT7 1NN (UK). E-mail: n.querou@qub.ac.uk.; Phone number: +44 (0)28 9097 3281.

Association. They are designed to be arrived at through the typical labour-management negotiations of collective bargaining. Regarding the National Hockey league, one of the most recent agreements was reached on July 2005 after a labor dispute which caused the cancellation of the 2004-2005 season. The contract was eventually ratified by the NHLPA membership and by the league's Board of Governors. In this example there are two groups, the team owners and the Players association. Even though negotiations resumed on July 2005, one could understand the final contract mainly as an outcome of a conflictual process between the two groups. Moreover this was obviously a conflict where both sides exerted costly efforts in order to influence the outcome. There are numerous examples of such situations from a national (collective bargaining/strikes and healthcare contracts) or international (negotiation/conflict and natural resource management) point of view. In such situations, a negotiated agreement is socially preferable but conflict can still occur. That is why it is important to understand the conditions under which one type of arrangement (cooperative or conflictual) will dominate the other.

The existing literature has mostly focused on situations where individuals are involved in the process, or where groups are uniform with respect to their members' individual preferences. In the present paper we consider a situation where two groups are involved in the process. A surplus has to be allocated either via a negotiated agreement, which is modeled by using the Nash bargaining solution (Nash (1950)), or by using conflict (which is modeled by a costly contest as defined by Tullock (1980), and analysed in Hirshleifer (1989) or Skaperdas (1992)). Conflict will prevail if one group is willing to use it, while negotiation requires mutual consent. Members of a given group have differing valuations (either high or low) for the surplus. Each group chooses a representative who is given the authority to either negotiate cooperatively or to decide to engage the group in conflict. If conflict occurs then each group's member follows their leader's decision and individually decides how much effort to exert. We would like to stress that conflict should be understood in a quite wide perspective. Conflict should not be simply understood as military, but rather as a metaphore for unproductive and costly activities focused on the appropriation of the surplus (costly litigation, lobbying activities, strikes).

We provide conditions (linking individuals' characteristics with the internal structure and the relative size of each group) under which a cooperative agreement will prevail over conflict (and vice versa). Specifically, the main conclusions are:

- Conflict is more likely to occur when groups have the same size;
- The emergence of negotiated agreements or conflict depends on the ratio of the valuations of members who value the most the surplus in both groups (that is, the high type members in both groups);
- When this ratio is sufficiently low (for one of the groups), a negotiated agreement is not sustainable;
- The situation is more complex when this ratio has moderate or large values. Whether cooperative bargaining is optimal or not depends qualitatively on structural conditions linking the internal composition of each group (whether members with high valuations of the surplus constitute the majority) with their relative size.

We conclude the analysis by providing results of comparative statics that highlight the influence of variations of the internal composition of each group on the range of situations for which negotiated agreements will prevail, and by deriving some implications regarding the structure of groups.

Our contribution can be related to two types of contribution. First, the present paper focuses on a problem of bargaining between groups. As in most of this literature we make the assumption that, once the representative is chosen for each group, his preferences become those of the group.[1] However, the topic of this paper is quite different from most of the contributions on group bargaining that focus on non cooperative approaches (see Horn and Wolinsky (1988), Jun (1989), or Cai (2000)). Unlike these papers, we focus here on the conditions (regarding the groups' characteristics) under which negotiated agreements or conflict will emerge as equilibrium outcomes. Haller and Holden (1997) use the Nash bargaining solution and analyse the interplay between ratification requirement and a group's bargaining power. Again their perspective is quite different from the main points of the present analysis.

Secondly, this paper is related to the literature on negotiations and conflict. Various issues have been analysed so far. Anbarci et al. (2002) provide

[1]Notable exceptions are Ponsati and Sakovics (1996), who analyse how different preference aggregation protocols affect the bargaining outcomes, and Manzini and Mariotti (2005), who highlight the influence of differential information between members on the alliance behavior. By contrast the present paper focuses on the optimality of cooperative or conflictual behavior. It is assumed that each group uses majority voting, as this is the most widely used protocol.

a comparison of several bargaining solutions (in terms of their relative efficiency) under the shadow of conflict. Baliga and Sjostrom (2004) consider a situation where two agents simultaneously decide whether or not to go to war, and they highlight the potential benefit of cheap talk to maintain cooperation. Jackson and Morelli (2007) analyse a model where decision makers or two countries can either go to war or avoid it (by negotiating transfers), and they show how the incentives to rely on conflict depend on the decision makers' risk/reward ratio from a war. Ellingsen and Miettinen (2008) (pursuing the line of research initiated by Schelling (1956, 1963) and later on by Crawford (1982)) analyse a Nash demand game where negotiators might adopt different types of commitments. They show that, when commitment technologies are highly credible, and there is a small (vanishing) cost of commitment, there is a unique equilibrium where demands are incompatible. Sanchez-Pages (2009) analyse a one-sided incomplete information bargaining model where the game ends with an agreement or an absolute conflict. However agents can opt for another option (limited conflict) in order to get information about the outcome of the absolute conflict. It is shown that war may have positive or negative effects on the whole process. Specifically, it can either create room for an agreement or lead to inefficiencies while bargaining is a feasible option.

We focus on understanding the reasons for the emergence of conflict or negotiated agreements as equilibrium outcomes. As such the present paper departs substantively from papers which focus on the analysis of how the shadow of conflict influences the nature of negotiated agreements (as in Anbarci et al. (2002)). In these papers agents anticipate the costs resulting from conflict. As a result they always agree on a negotiated agreement. By contrast we analyse (as in Jackson and Morelli (2007, 2009)) the incentives of decision makers to go to war or to rely on a negotiated agreement. We will show (as they do) that there are cases where each type of allocation becomes optimal. The main difference is that we allow explicitly for heterogeneous populations within each group (members with differing preferences), and we will show that the emergence of negotiation or conflict depends on conditions that relate individuals' characteristics (members' valuations of the surplus) to group characteristics (internal composition, relative size). As such the present contribution complements those of the above papers by analysing the influence of collective characteristics on the emergence of one pattern of behavior at the equilibrium. Moreover, it extends the growing literature analysing the influence of collective structures on economic activities (see Brandts et al. (2009) for such a study in the context of rent seeking activities).

Finally, we would like to mention another paper which is somehow related to the issue of conflict between groups. Brandauer and Englmaier (2009) analyse a contest between two groups where members have different valuations for the contested good. They highlight the influence of this intra group heterogeneity on the delegation problem. The present paper highlights that the emergence of negotiated agreements or conflict depends on an interplay between intra and inter group characteristics.

The paper is organized as follows. The model is introduced in Section 2. The payoffs resulting from cooperative bargaining and conflict are analysed in section 3. The full characterization of the equilibrium set and results of comparative statics are provided in section 4. The case where both groups have the same size and the general case are provided. Section 5 concludes. Some proofs are relegated in an appendix in section 6.

2 The model

We consider a situation where a surplus Φ has to be allocated between two groups G_1 and G_2 (of respective sizes n and m). The allocation process can be specified as follows. First, each group chooses a representative who has full authority during the allocation process. Specifically, it is assumed that the members of a group are fully committed to the decision taken by their representative. Representatives are chosen by simple majority within each group.[2] Members choose the representative such that the payoff resulting from this choice is maximal.

Second, the process enters the allocation stage. The representatives can either negotiate cooperatively (using the Nash bargaining solution) or they can decide that the groups will enter a conflict over the appropriation of the surplus. While cooperative negotiations require the consent of both parties, conflict will occur if only one representative is willing to use it. The surplus is allocated either by cooperative bargaining or the outcome of the costly contest (following recent contributions on the analysis of conflict, see Esteban and Ray (2008) among other examples). We will be more specific about the modeling of benefits and costs in the next section.

The share of the surplus secured is then used to fund group specific goods. More specifically, the problem analysed here focuses on situations where the

[2]It is implicitly assumed that there is one candidate from each sub-population of agents in each group.

outcome of the allocation process (shares of surplus for both parties) is characterized by rivalry in consumption and excludability at the inter-group level, but is non rival in consumption and non excludable at the intra-group level. This assumption is met in social settings where all members of a given group benefit from the share of the surplus secured. For instance, interest groups might negotiate or lobby to influence the allocation process; all members of one group benefit from the outcome. Unions and senior managers might bargain over wage increases or use conflict to force the decision. Wage increases benefit all workers: in case of conflict, they benefit both strikers and non strikers. From now on, thinking about groups as communities, we will assume that the surplus secured by each group is spent on (strictly) local public goods.

Members of each group have two possible types. They have either low or high valuation for local public goods. Specifically, let θ^i_H (respectively, θ^i_L) denote the valuation of members of group i that are high type. The valuation of low type members in group i is denoted by θ^i_L. It is assumed that $\theta^i_H > \theta^i_L$ for each group i. Their internal composition is defined by a parameter n_i ($i = 1, 2$), which denotes the number of high type members of group i.

At this stage of the description we would like to stress one important remark regarding the above process. During the second stage representatives can either choose to use a negotiation process or a conflict. In other words it is assumed that representatives have two possible choices: to go to or to avoid war. If both representatives decide to avoid war then this corresponds to a situation wheere they can sign some enforceable treaty so that they do not go to war provided that a mutuallly suitable allocation is chosen via negotiations. Even though we model negotiations as a cooperative bargaining process by using the Nash bargaining solution, we have to define what are usually called the threat points or disagreement payoffs. These would correspond to perpetual disagreement during negotiations. In the present case we assume a proportional situation, that is, we assume that the disagreement payoff of respectively groups 1 and 2 corresponding to the negotiation process are equal to

$$\frac{n}{n+m}\Phi_{dis}$$

and

$$\frac{m}{n+m}\Phi_{dis},$$

where Φ_{dis} is the disagreement surplus, which is smaller than the surplus Φ both groups can secure by negotiating successfully. In other words, it is assumed that, if both groups decide to avoid war but are yet not able to agree on a division of the surplus then they receive a share of the surplus according to a pro rata rule (or any division that is increasing in the relative size of a group). This implicitly assumes some sort of proportional representation, which makes sense when the surplus to be allocated is used for public spendings on (local) public goods. In such situations the pro rata allocation rule can be thought about as some sort of status quo allocation rule in case both parties agree to negotiate. In local funding related issues (healthcare for instance), it makes sense to use the relative population size to determine the status quo allocation rule.

There are several features of the above model that deserve further comment. First, the above specification might correspond to a situation where there is a third party, which is acknowledged informally by both groups as having some ability to intervene in case a negotiation process is chosen but fails to provide an agreement. At the same time, this third party does not have any formal authority, which means that both groups can decide to go to war if they are willing to do so. With such an interpretation in mind, the question would become: If groups can either delegate their authority to a regulator (whose principles of distributive justice correspond to the Nash bargaining solution) or remain uncommitted and appropriate the surplus by conflict, what is the influence of collective characteristics on the emergence of a social contract or conflict as an equilibrium outcome? It is important to notice that this third party would never intervene actually, since a mutually suitable (efficient) agreement would always result when both countries are willing to rely on a negotiation process.

Secondly, there is an important assumption about the payoffs of the groups corresponding to the situation where they agree to commit to the negotiation protocol but would fail to reach an agreement. Specifically, the disagreement payoffs corresponding to the negotiation process do not (necessarily) correspond to the payoffs resulting from conflict. Why is it so? The main starting point is that disagreement is not equivalent to conflict in the present setting. From a general point of view, there are many situations where agents could disagree on an issue but would not enter an open conflict since the latter could be very destructive. The present model would thus refer to situations where group representatives understand that there are circumstances where it is preferable to avoid war, even though negotiations do not necessarily lead to an (efficient) agreement. Another potential interpretation exists if one relies on the idea that the Nash bargaining solution would correspond to the principles of distributive justice of a regulator. In such a case the specifics of negotiations would rely on this regulator's principles, and the definition of the disagreement points would depend on his specification of the protocol. It could well be the case that the regulator would define what happens in case of disagreement during negotiations by relying on his knowledge about the characteristics of the groups. The relative size of the groups would then be a reasonably natural criterion. With this perspective in mind, the group representatives could decide to use conflict if they reject the course of action specified by the social contract offered by the regulator.

There is a final comment to be made. As explained above the specification of the disagreement payoffs adopted in the present setting seems to be reasonable and to make sense in certain situations, but one might argue that it could be the outcome of some endogenous process too. A first possibility would be to assume that it would follow from the agents' anticipation of (future) costs of conflict. In this case the situation would correspond to a problem under the shadow of conflict. This is not the kind of situation we want to analyse, as it takes the fact that both parties would seat at the negotiation table as granted. Moreover the usual conclusion is that conflict never occurs in such problems. We would like to stress that the main purpose of the present analysis is rather to understand the structural conditions (on collective characteristics) that would explain the emergence of either negotiated agreements or conflict. We want to understand why it is the case that agents might decide to seat at the negotiation table or refuse to do so. As such we do consider a setting where the options to negotiate and to use conflict are simultaneous rather than sequential.

A second possibility would be to consider a dynamic problem where both parties would have the opportunity to use either pattern of behavior at each period. This would allow for an endogenous definition of the disagreement payoffs corresponding to the option to negotiate. In such a situation the analysis of conditions under which one pattern of behavior or the other will dominate would be clearly interesting. However a first problem would be to analyse if there are conditions that could explain the existence of conflict. We want to focus on the influence of collective characteristics in the present setting. That is why we consider the present framework as appropriate since it enables us to focus specifically on this problem. Nonetheless, extending the present work to a dynamic setting is obviously an important point for future research.

To summarize, the timing of the game is as follows. First, a representative is chosen by the members of a group.[3] Members choose a representative according to the (expected) payoff that will result from this choice. Secondly, the representatives of both groups decide whether they will negotiate cooperatively or they will fight over the appropriation of the surplus. Thirdly, the allocation is the Nash bargaining solution (where the disagreement point is defined by using a pro rata allocation rule), or the outcome of a costly contest. Finally, the resulting allocation is used to provide local public goods which are group specific.

Before concluding this section, let us be slightly more specific about the choice of the representatives. In the analysis there will be several instances where the equilibria of the overall game will not depend on the choice of the representative. In some other cases they will only depend on which type of agents constitutes the majority in the group considered. This depends on the following fact. The problem is similar to a voting process where there are only two alternatives, namely negotiating or using conflict. This implies that there is no difference (concerning the equilibrium outcomes) between the cases where agents use sincere or strategic voting to choose their representative in the present setting. Indeed there will be two options: either one pattern of behavior is clearly optimal for both types of members (in a given group), and the choice of the representative does not matter; or the preferred option differs within the group, which means that the agents' type which constitutes the majority will rule the election process. In the analysis we will not make a distinction between sincere and strategic voting, and we will mainly characterize the equilibrium outcomes (emergence of negotiated agreements or conflict).

In the next section we will solve the above game. Specifically, we will solve for the subgame perfect equilibrium (SPE) of the four stage game, and we will analyse the conditions under which negotiated agreements (or conflict) will prevail at the equilibrium.

[3]The specific voting process that is used does not really matter, as it will become obvious in the analysis. As said previously, we will consider that the representative is chosen by simple majority.

3 Preliminaries: The contest and the cooperative bargaining games

As usual when dealing with subgame perfect equilibria, we will use backward induction arguments to analyse the model. In the present section we analyse the last stage of the process. We first derive the payoffs of each type of members in a given group when groups are involved in conflict or in cooperative bargaining.

3.1 The contest game

If at least one representative has decided to use conflict as a way to appropriate the surplus, then a contest game occurs. As explained in the previous section, a representative has full authority over the decision to rely on bargaining or conflict. This means that all members of the group are committed to follow his decision. This has the following implication in the contest game. All members of a group are assumed to exert (costly) efforts to influence the overall probability of appropriating the surplus. We use a form of group contest success function that has been recently axiomatized (Munster (2009)). Specifically, if e_i^j denotes the effort exerted by member i of group j, then the probability that group j appropriates the surplus through conflict is $\frac{\sum_{l \in G_j} e_l^j}{\sum_{l \in G_1} e_l^1 + \sum_{k \in G_2} e_k^2}$, where $j = 1, 2$ respectively. If no member exerts effort, then each group has probability $\frac{1}{2}$ of appropriating the surplus. If group j does not win, it gets a zero payoff.

Concerning the equilibrium definition used in the contest game, we follow Esteban and Ray (2007). Specifically, an equilibrium in the contest game is defined as a vector of (non negative) levels of effort $(e_1^{j*}, ..., e_{|G_j|}^{j*})$ for any group $j = 1, 2$ (where $|G_1| = n$ and $|G_2| = m$) such that, for any $j = 1, 2$ and any $i \in \{1, ..., |G_j|\}$, we have:

$$e_i^{j*} = \max_{e_i^j \geq 0} \frac{e_i^j + \sum_{l \neq i, l \in \{1,...,|G_j|\}} e_l^{j*}}{e_i^j + \sum_{l \neq i, l \in \{1,...,|G_j|\}} e_l^{j*} + \sum_{l \in \{1,...,|G_{-j}|\}} e_l^{-j*}} \theta_k^j \Phi - e_i^j, \qquad (1)$$

where $-j \neq j$, and $k = L, H$ denotes the type of member i of group j. In other words, member i of group j exerts the level of effort that maximizes his expected payoff in the contest, assuming that all other agents (other members of the same group, and members of the opposing group) exert their equilibrium level of effort.

We need several steps to derive the solution of this stage of the game. First, each group has incentives to exert a positive level of (aggregated) effort at the equilibrium. Indeed, if it is not the case then each group has the same chance to appropriate the surplus. It is straightforward to check that any member has incentives to increase his effort (assuming that all other agents exert no effort) by a small amount, as this increases the probability that his group wins the contest from $\frac{1}{2}$ to 1.

A second point is that we will focus on symmetric equilibrium. Specifically, we will focus on equilibria where, for a given group, members of the same type exert the same level of effort. This implies that such an equilibrium is described by vectors $(e_H^1, ..., e_H^1, e_L^1, ..., e_L^1)$ (for group 1) and $(e_H^2, ..., e_H^2, e_L^2, ..., e_L^2)$ (for group 2) where the n_1 (respectively, n_2) first terms of the first (respectively, second) vector refer to the level of contribution of high type members of group 1 (respectively, of group 2), and the remaining terms to the level of contribution of low type members of group 1 (respectively, of group 2).

The next step is to consider the possible types of symmetric equilibrium. They can be described as follows (focusing on group 1):

1. $e_H^1 > 0$ and $e_L^1 > 0$
2. $e_H^1 > 0$ and $e_L^1 = 0$
3. $e_H^1 = 0$ and $e_L^1 > 0$
4. $e_H^1 = 0 = e_L^1$.

Case 4 has already been ruled out by the argument presented in the first point discussed above.

Let us focus on the first case. If such a case is valid, then the equilibrium levels of effort satisfy the following first order conditions (that are necessary and sufficient from (strict) concavity of expected payoffs (1)):

$$\frac{n_2 e_H^2 + (m - n_2) e_L^2}{[(n_1 - 1) e_H^1 + (n - n_1) e_L^1 + n_2 e_H^2 + (m - n_2) e_L^2 + e]^2} \theta_H^1 \Phi = 1,$$

for a high type member of group 1, and

$$\frac{n_2 e_H^2 + (m - n_2) e_L^2}{[n_1 e_H^1 + (n - n_1 - 1) e_L^1 + n_2 e_H^2 + (m - n_2) e_L^2 + e]^2} \theta_L^1 \Phi = 1,$$

for a low type member of group 1. Because $\theta_H^1 \neq \theta_L^1$ a simple inspection of the above two first order conditions leads to the conclusion that they cannot be satisfied simultaneously. This rules out case 1.

This conclusion deserves further comment. It implies that intra group equilibrium levels of effort are characterized as follows. One type of members exert positive levels of effort while members from the other type do not contribute (they free ride on the efforts of others). This implication relies on the specific form of the rent seeking functions used in (1). With functions where the rent seeking technology is more general (for instance, with convex costs of effort) all members will exert positive levels of effort, but the members who previously did not contribute will still free ride on the efforts of others, that is, they will still exert a lower level of effort. This is intuitively perfectly consistent with the kind of situations we consider. Let us consider the example of union - management relationships as an illustrative example. If wage increases are the outcome of a conflict, they benefit both stricking and non stricking workers. As such workers have incentives to free ride and to not participate in strike. This is the exact meaning of the above conclusion. We will keep the above form of rent seeking technology as this will enable us to present results in the simplest possible form.

Let us come back to the analysis. We now analyse the conditions under which the equilibrium levels of effort could correspond to either case 2 or case 3. Let us first consider that case 2 is satisfied, that is, we have:

$$e_H^1 > 0, e_L^1 = 0.$$

Then this is equivalent to assuming that the expected payoff of low type member of group 1 is a decreasing function of his own level of effort. Thus, differentiating expression (1) with respect to e_L^1, we have:

$$\frac{n_2 e_H^2 + (m - n_2)e_L^2}{[n_1 e_H^1 + (n - n_1)e_L^1 + n_2 e_H^2 + (m - n_2)e_L^2]^2}\theta_L^1 \Phi \leq 1.$$

Moreover, assuming positive levels of effort for high type members of group 1, e_H^1 is then characterized by the following condition:

$$\frac{n_2 e_H^2 + (m - n_2)e_L^2}{[n_1 e_H^1 + (n - n_1)e_L^1 + n_2 e_H^2 + (m - n_2)e_L^2]^2}\theta_H^1 \Phi = 1.$$

Combining the above two conditions, we obtain that

$$[n_2 e_H^2 + (m - n_2)e_L^2]\theta_L^1 \Phi \leq [n_2 e_H^2 + (m - n_2)e_L^2]\theta_H^1 \Phi.$$

Simplifying, we deduce the equivalent expression:

$$\theta_L^1 \leq \theta_H^1,$$

which is always satisfied. Thus case 3 is never valid at the equilibrium.

Now we are in a position that enables us to characterize the optimal levels of effort. We know that necessarily $e_L^1 = e_L^2 = 0$ at the equilibrium. Moreover, (e_H^1, e_H^2) is the solution to the system defined by the following conditions (for $j = 1, 2$):

$$\frac{n_{-j} e_H^{-j}}{[n_1 e_H^1 + n_2 e_H^2]^2} \theta_H^j \Phi = 1.$$

We derive the following equality:

$$n_2 e_H^2 \theta_H^1 = n_1 e_H^1 \theta_H^2$$

which leads to the following relation between e_H^1 and e_H^2:

$$e_H^2 = \frac{n_1 \theta_H^2}{n_2 \theta_H^1} e_H^1.$$

Plugging it into the first order condition and solving for e_H^1, we obtain:

$$e_H^1 = \frac{\frac{\theta_H^2}{n_1}}{[1 + \frac{\theta_H^2}{\theta_H^1}]^2} \Phi.$$

We finally obtain the following expression of the expected payoff of a high type member of group 1:

$$EU_H^1 = \frac{\theta_H^1 + \frac{n_1 - 1}{n_1} \theta_H^2}{[1 + \frac{\theta_H^2}{\theta_H^1}]^2} \Phi,$$

and the expected payoff of a low type member of group 1 is as follows:

$$EU_L^1 = \frac{\theta_L^1}{1 + \frac{\theta_H^2}{\theta_H^1}} \Phi.$$

To understand the difference between these two expressions, one needs to keep in mind that low type members do not exert effort at the equilibrium. Thus, they do not bear the burden of conflict.

We conclude the section by summing up their main findings.

Proposition 3.1. *If conflict occurs at the final stage of the game, then a high type member of group j obtains the following expected payoff (for $j = 1, 2$):*

$$EU_H^j = \frac{\theta_H^j + \frac{n_j - 1}{n_j}\theta_H^{-j}}{[1 + \frac{\theta_H^{-j}}{\theta_H^j}]^2}\Phi.$$

The (expected) payoff secured by low type members is:

$$EU_L^j = \frac{\theta_L^j}{1 + \frac{\theta_H^{-j}}{\theta_H^j}}\Phi.$$

In the next section we will analyse the outcome of the game at the final stage if representatives decide to allocate the surplus by using a negotiated arrangement.

3.2 The cooperative bargaining game

If both representatives decide to choose a negotiated arrangement, then the allocation corresponds to the Nash bargaining solution. To define the solution we need to specify the disagreement point, that is, the allocation that results from (perpetual) disagreement.

First, it is assumed that the disagreement surplus is $\Phi_{dis} < \Phi$, that is, the surplus shrinks down in case of disagreement. This might correspond to a situation where groups are allocated funding by local authorities. To provide incentives to reach an agreement the surplus available is larger if the two communities agree on an allocation than if they disagree perpetually.

Second, each representative obtains a share of the disagreement surplus that is a function of the relative size of his own group. Again, this is an intuitive assumption as there are several real life examples where communities are allocated funding on the basis of their (relative) size.

Now we are in a position to derive the optimal allocation of the surplus if representatives rely on a negotiated arrangement. Let us denote this allocation by $(\alpha^*, 1 - \alpha^*)$ where $\alpha^* \in [0, 1]$ denotes the share of the surplus allocated to the representative of the first group. Then α^* is the solution to the following problem:

$$\max_{\alpha \in [0,1]} \{\theta_i^1 \alpha \Phi - \theta_i^1 \frac{n}{n+m}\Phi_{dis}\}\{\theta_j^2(1-\alpha)\Phi - \theta_j^2 \frac{m}{n+m}\Phi_{dis}\},$$

where $i, j = L, H$ denotes the type of the representative of group 1 (respectively, group 2). It can be immediately checked that $0 < \alpha^* < 1$ because of the specification of the disagreement point. Thus the optimal share is characterized by the following first order condition:

$$\theta_i^1 \Phi\{\theta_j^2 (1-\alpha^*)\Phi - \theta_j^2 \frac{m}{n+m}\Phi_{dis}\} - \theta_j^2 \Phi\{\theta_i^1 \alpha^* \Phi - \theta_i^1 \frac{n}{n+m}\Phi_{dis}\} = 0.$$

Simplifying, we obtain:

$$\{(1-\alpha^*)\Phi - \frac{m}{n+m}\Phi_{dis}\} = \{\alpha^* \Phi - \frac{n}{n+m}\Phi_{dis}\}.$$

Now solving for α^* we obtain:

$$\alpha^* = \frac{1}{2} + \frac{1}{2}[\frac{n}{n+m} - \frac{m}{n+m}]\frac{\Phi_{dis}}{\Phi}.$$

We can now recap the main findings of the present section as follows:

Proposition 3.2. *Assume that representatives decide to rely on a negotiated arrangement. Then the resulting payoff for i type members of group 1 (where $i = L, H$) is given by the following expression*

$$\Pi_i^1 = \theta_i^1 \{\frac{1}{2} + \frac{1}{2}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}\}\Phi.$$

Respectively, the payoff for i type members of group 2 (where $i = L, H$) is given by the following expression:

$$\Pi_i^2 = \theta_i^2 \{\frac{1}{2} + \frac{1}{2}\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}\}\Phi.$$

Before concluding the present section let us make an additional comment. The negotiated arrangement described in proposition 2.2 is efficient by definition of the Nash bargaining solution. The allocation resulting from conflict is obviously inefficient due to the social cost that is beared by both groups.

4 Characterization

Going backward, we now proceed with the analysis of the first two stages of the process. Specifically, we will focus in the next section on the incentives of each group to rely on conflict or negotiations by analysing the delegation problem, and we will characterize the equilibria of the full game. In this section we will first provide the analysis for the symmetric case, that is, the situation where both groups have the same size ($n = m$), in order to provide preliminary insights. We will then develop the full analysis of the general case, which will enable us to highlight some qualitative differences between the symmetric and general cases. Finally, we will provide results of comparative statics and some general implications resulting from these results.

4.1 The symmetric case

4.1.1 Conflict or negotiated agreement?

We now go one step backward and we analyse the incentives of each type of member to opt for conflict or negotiation. Again an important feature of the model must be kept in mind. One can highlight it by focusing on the case of the contest game.

If at least one representative decided to use conflict, then we know (by proposition 2.1) that low type members of each group will (to some extent) free ride on the efforts of high type members. This does not depend on the identity of the representative of the group. There is however a main difference when a high type member represents the group and when this is not the case. If such a member is the representative then he can decide to either negotiate or fight, depending on what is optimal for him. If he is not, then he cannot decide and must follow the leader's decision. In other words, one could expect strategic incentives from low type members to rely on conflict as they do not bear most of the burden resulting from conflict. The analysis that follows will highlight this point (among others), and will focus mainly on the conditions (on the intra and inter group compositions, and on individual characteristics as well) that guarantee that negotiated arrangements will prevail over conflict (and vice versa).

Let us now proceed with the analysis. The first step is to understand the conditions under which one allocation method prevails over the other. We will focus on the case of group 1 for the exposition of the conditions (the case of group 2 is symmetric). From proposition 2.2, we know that, provided the representatives decided to negotiate, the payoff of a type i member is then given by:

$$\Pi_i^1 = \theta_i^1 \{ \frac{1}{2} + \frac{1}{2} \frac{n-m}{n+m} \frac{\Phi_{dis}}{\Phi} \} \Phi. \qquad (2)$$

Now, if at least one representative is willing to use conflict, using proposition 2.1 the payoff of a high type member becomes

$$EU_H^1 = \frac{\theta_H^1 + \frac{n_1-1}{n_1}\theta_H^2}{[1+\frac{\theta_H^2}{\theta_H^1}]^2}\Phi, \tag{3}$$

and the expected payoff of a low type member is

$$EU_L^1 = \frac{\theta_L^1}{1+\frac{\theta_H^2}{\theta_H^1}}\Phi. \tag{4}$$

Let us first consider the case of a high type member. Using expressions (2) and (3) and simplifying, we find that whether conflict is preferable than negotiation is equivalent to the following condition:

$$EU_H^1 \geq \Pi_H^1 \iff 1 + \frac{n_1-1}{n_1}\frac{\theta_H^2}{\theta_H^1} \geq [1+\frac{\theta_H^2}{\theta_H^1}]^2\{\frac{1}{2} + \frac{1}{2}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}\}.$$

To highlight the main features of the analysis in a simple benchmark, we focus first on the case where the two groups have the same size, that is, where $n = m$. This has the advantage of simplifying the above condition as follows:

$$EU_H^1 \geq \Pi_H^1 \iff 1 + \frac{n_1-1}{n_1}\frac{\theta_H^2}{\theta_H^1} \geq [1+\frac{\theta_H^2}{\theta_H^1}]^2\frac{1}{2}.$$

The right hand side term is a polynomial function of the ratio $\frac{\theta_H^2}{\theta_H^1}$; developing it leads to the following expression:

$$-(\frac{\theta_H^2}{\theta_H^1})^2 - \frac{2}{n_1}\frac{\theta_H^2}{\theta_H^1} + 1 \geq 0.$$

Solving for $\frac{\theta_H^2}{\theta_H^1}$, we obtain the following properties:

1. When $\frac{\theta_H^2}{\theta_H^1} \in]0, \frac{1}{n_1}[\sqrt{1+(n_1)^2} - 1][$ then high type members prefer conflict over negotiated arrangements;

2. When $\frac{\theta_H^2}{\theta_H^1} \geq \frac{1}{n_1}[\sqrt{1+(n_1)^2} - 1]$ then high type members are willing to rely on negotiations.

The above properties highlight the way both the structure of group 1 (via parameter n_1) and the characteristics of high members of each group (via the ratio $\frac{\theta_H^2}{\theta_H^1}$) influence the profitability of conflict and negotiations for high type members of group 1. This depends on how the valuation of high type members of the other group compares with respect to their own valuation. When θ_H^2 is sufficiently large, then high type members of group 2 are more willing to provide efforts during conflict than similar members in group 1. They realize that they will lose more by relying on conflict than on negotiations. When the comparison of the valuations is reversed then high type members of group 1 have incentives to rely on an agressive strategy.

Let us now consider the case of low type members. Using expressions (2) and (4) it is straightforward to conclude that conflict is preferred to negotiations if and only if the following inequality holds:

$$\frac{1}{2} - \frac{1}{2}\frac{\theta_H^2}{\theta_H^1} \geq 0,$$

or

$$\theta_H^1 \geq \theta_H^2.$$

Now we are in a position to state the main result when groups have the same size.

Proposition 4.1. *Let us consider the case where groups have the same size, that is, $n = m$. It can be described as follows (where $i, j = 1, 2$ and $i \neq j$):*

- *When $\frac{\theta_H^j}{\theta_H^i} \in]0, \frac{1}{n_i}[\sqrt{1 + (n_i)^2} - 1][$, then all members of group i prefer conflict over negotiations;*
- *When $\frac{\theta_H^j}{\theta_H^i} \in [\frac{1}{n_i}[\sqrt{1 + (n_i)^2} - 1], 1[$, then high type members of group i are willing to rely on negotiations, while low type members prefer conflict;*
- *Finally, when $\frac{\theta_H^j}{\theta_H^i} \geq 1$, then all members of group i prefer a negotiated arrangement.*

Proof. In the Appendix. □

The above proposition describes the situation when groups have the same size. In this case the profitability of conflict over negotiation does not depend on the relative size of the groups, but on a relationship between their internal composition (via parameter n_i) and the members' characteristics (via the ratio $\frac{\theta_H^j}{\theta_H^i}$).

There are several interesting implications. First, for low and high values of ratio $\frac{\theta_H^j}{\theta_H^i}$ the situation is simple as it corresponds to a case of common opinion within each group. For low values of $\frac{\theta_H^j}{\theta_H^i}$ then all members of group i prefer an agressive strategy, while they unanimously prefer a negotiated arrangement when this ratio becomes sufficiently large.

Second, there is internal disagreement over which strategy should be preferred for intermediate values of the ratio. In such cases members with a high valuation would prefer negotiations, while members with a low valuation would prefer an agressive strategy. This means that the choice of the representative will depend on an additional parameter (compared to the cases depicted previously), namely the type of agents who constitute the majority within each group.

Let us proceed with the final step of the analysis of the equilibrium. We will provide conditions under which either conflict or negotiation will prevail by analysing the choice of the representatives for each group, taking the choice of the other representative as given.

4.1.2 Characterization of the equilibrium set

Going one more step backward, we reach the first stage of the game, namely the choice of the representative. This choice will then determine whether conflict or negotiation will prevail at the equilibrium. This will enable us to provide implications regarding the optimal structure of the groups, that is, to assess the conditions (on the internal structure of each group and on their relative size as well) that would ensure that negotiated arrangements prevail at the equilibrium. Since conflict yields a loss of the surplus, it is obvious that negotiated arrangements are (socially) preferable.

The final part of the characterization of the equilibrium is relatively straightforward. Specifically, we characterized in section 2.3 the optimal strategy for each type of members within a single group. Now we need to derive the implications of this characterization when the strategy of the two groups are considered simultaneously.

Let us focus first on the case of groups with the same size. Coming back to the results provided in proposition 2.3 there are three cases to consider. If $\frac{\theta_H^j}{\theta_H^i} \in]0, \frac{1}{n_i}[\sqrt{1+(n_i)^2}-1][$, then we know that all members of group i prefer conflict over negotiations. At the same time we have $\frac{\theta_H^i}{\theta_H^j} \in]\frac{n_i}{\sqrt{1+(n_i)^2}-1}, +\infty[$, which implies that all members of group j would prefer a negotiated arrangement. Indeed, one can check immediately that

$$\frac{n_i}{\sqrt{1+(n_i)^2}-1} > 1,$$

which leads to the above conclusion by using the third case in proposition 2.3 for group j. In such a situation it follows then that conflict will be the outcome of the game, and this is independent from the specific choice of representative for each group.

When $\frac{\theta_H^j}{\theta_H^i} \in [\frac{1}{n_i}[\sqrt{1+(n_i)^2}-1], 1[$, then we know that high type members of group i are willing to rely on negotiations, while low type members prefer conflict. Again, since this implies that $\frac{\theta_H^i}{\theta_H^j} \in]1, \frac{n_i}{\sqrt{1+(n_i)^2-1}}]$ then it follows from the third case in proposition 2.3 that all members of group j would prefer a negotiated arrangement. This has an important implication, namely that the outcome of the game depends on the internal composition of group i. Specifically, if high type members constitute the majority, then we know that a high type representative will be chosen in group i, who would be willing to use a cooperative agreement. In such a case negotiation is favored by both groups, and a negotiated arrangement will prevail. Now if high type members are a minority in group i, then conflict will be chosen by group i and this will be the outcome of the game.

Finally, when $\frac{\theta_H^j}{\theta_H^i} \geq 1$, then all members of group i prefer a negotiated arrangement. This implies that $\frac{\theta_H^i}{\theta_H^j} \in]0, 1]$ and this has two possible implications for the optimal strategy within group j. Specifically, we have:

1. Either $\frac{\theta_H^i}{\theta_H^j} \in]0, \frac{1}{n_j}[\sqrt{1+(n_j)^2}-1][$ and we conclude that all members of group j prefer conflict over negotiations;
2. Or $\frac{\theta_H^i}{\theta_H^j} \in]\frac{1}{n_j}[\sqrt{1+(n_j)^2}-1], 1]$ and then high type members of group j are willing to rely on negotiations, while low type members prefer conflict.

In the first sub case we can conclude that conflict will be the outcome of the game, as this would be the optimal strategy for group j. In the second sub case a negotiated arrangement will prevail if high type members constitute the majority in group j, while conflict will emerge if they are a minority.

To summarize, we obtain:

Theorem 1. *Let us consider the case where groups have the same size. The situation can be characterized as follows:*

1. *If* $\frac{\theta_H^2}{\theta_H^1} \in]0, \frac{1}{n_1}[\sqrt{1+(n_1)^2} - 1][$ *then conflict is the outcome of the game. The conclusion does not depend on the choice of the representatives.*

2. *If* $\frac{\theta_H^2}{\theta_H^1} \in [\frac{1}{n_1}[\sqrt{1+(n_1)^2} - 1], 1[$ *then two sub cases can occur:*

   (a) *if high type members constitute the majority in group* 1*, then a cooperative agreement will prevail.*

   (b) *Otherwise conflict is the outcome of the game.*

3. *If* $\frac{\theta_H^2}{\theta_H^1} \geq 1$ *then two sub cases can occur:*

   (a) *Either* $\frac{\theta_H^1}{\theta_H^2} \in]0, \frac{1}{n_2}[\sqrt{1+(n_2)^2} - 1][$ *and then conflict is the outcome of the game;*

   (b) *Or* $\frac{\theta_H^1}{\theta_H^2} \in]\frac{1}{n_2}[\sqrt{1+(n_2)^2} - 1], 1]$ *and then a negotiated arrangement will prevail if high type members constitute the majority in group* 2*, while conflict will be the outcome of the game otherwise.*

There are several interesting implications to the above result.

First, one might expect conflict to be a frequent outcome of the game. More precisely, when the ratio of high type members' valuations $\frac{\theta_H^j}{\theta_H^i}$ is either sufficiently low or high then a negotiated agreement is not sustainable. This implication was quite expected as conflict will prevail as soon as at least one party will find it profitable, while a negotiated agreement requires mutual consent.

The intuition is as follows. Let us focus on the case where $\frac{\theta_H^j}{\theta_H^i}$ is low, the second case is symmetric once the role of each group is reversed. In such a situation high type members of group i value the funding for local public goods much more than high type members in group j, which increases the incentives of high type members in group i to exert efforts to appropriate the surplus. This increases the chance that group i will indeed appropriate the surplus, which in turn increases the incentives for Low type members of group i to choose conflict as well. Thus all members of group i favor conflict over negotiation, and conflict will prevail.

Second, the optimality of negotiated arrangements crucially depend on the internal composition of the groups. Specifically, if members with a high valuation for (local) public goods constitute the majority in both groups, then a negotiated agreement will be optimal provided that $\frac{\theta_H^j}{\theta_H^i}$ satisfies the following condition:

$$\frac{\theta_H^j}{\theta_H^i} \in [s_i, s_j] := [\frac{1}{n_i}[\sqrt{1+(n_i)^2}-1], \frac{n_j}{\sqrt{1+(n_j)^2}-1}]. \tag{5}$$

The above condition depends obviously on the specific values of parameters n_i and n_j, namely, the number of high type members in each group. In other words, whether cooperative bargaining is optimal or not depends qualitatively on whether high type members constitute the majority in at least one group or not. Then, provided it is the case, negotiation will emerge at the equilibrium for intermediate values of the ratio $\frac{\theta_H^j}{\theta_H^i}$.

The intuition is as follows. For intermediate values of this ratio the situation is balanced in that the valuations of the high type members in each group are approximately the same. As such conflict becomes less attractive compared to negotiation. More precisely, for values of the ratio $\frac{\theta_H^j}{\theta_H^i}$ close to, but below 1, the loss of surplus resulting from conflict outweighs the potential gains for all members of group j, and for high type members of group i. This decreases the incentives of high type members to exert effort in group i. However, conflict would still be preferable for low type members of group i (the potential gain has decreased compared to the first case, but they mostly free ride during conflict). As such for cooperative bargaining to occur it is necessary that high type constitute the majority in group i.

4.2 The general case

Now we come back to the general case where the two groups may have different sizes. The next result will highlight the main difference in such a situation. Specifically it becomes more complex as there is an added dimension, that is, whether the first or second group is the largest one. This will make the analysis more complex, as the description of the ranges over which conflict/negotiation is preferred will be less straightforward.

4.2.1 Conflict or negotiated agreement?

From expressions (2) and (3) conflict is profitable for high type members of group 1 if and only if the following inequality holds:

$$\frac{1+\frac{n_1-1}{n_1}\frac{\theta_H^2}{\theta_H^1}}{[1+\frac{\theta_H^2}{\theta_H^1}]^2} \geq \{\frac{1}{2}+\frac{1}{2}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}\},$$

or

$$1+\frac{n_1-1}{n_1}\frac{\theta_H^2}{\theta_H^1} \geq [1+\frac{\theta_H^2}{\theta_H^1}]^2\{\frac{1}{2}+\frac{1}{2}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}\}.$$

Developping the above expression and simplifying, high type members of group 1 prefer an agressive strategy if and only if the ratio $\frac{\theta_H^2}{\theta_H^1}$ satisfies the following inequality:

$$-\frac{1}{2}[1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}](\frac{\theta_H^2}{\theta_H^1})^2-[\frac{1}{n_1}+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}]\frac{\theta_H^2}{\theta_H^1}+\frac{1}{2}[1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}] \geq 0. \quad (6)$$

This is a polynomial expression of the form $a(\frac{\theta_H^2}{\theta_H^1})^2+b\frac{\theta_H^2}{\theta_H^1}+c$; thus, we know that the above inequality will never hold if

$$\Delta = b^2-4ac = \frac{1}{(n_1)^2}+\frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1<0.$$

Second, provided that $\Delta \geq 0$ then it can be checked that the following conclusion holds:

- Condition (6) is satisfied on $[0, \frac{\sqrt{\frac{1}{(n_1)^2}+\frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_1}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}]$;
- Condition (6) does not hold on $]\frac{\sqrt{\frac{1}{(n_1)^2}+\frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_1}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}, +\infty[$ (negotiation is preferred over conflict).

Now it remains to assess the sign of Δ. The first thing to note is that Δ is positive if the first group is larger than the second one. Indeed, if $n \geq m$ then $\Delta = \frac{1}{(n_1)^2}+\frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1 > 0$ since it is the sum of three positive terms.

Let us then consider that $m > n$; this is not sufficient to ensure that Δ be negative. More precisely, we must have

$$\frac{1}{n_1}+n_1 < 2\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}. \quad (7)$$

Since $2\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} < 2$, then the only case where the above inequality could hold is $n_1 = 1$. As this would depend on the specific value of the ratio $\frac{\Phi_{dis}}{\Phi}$, we will assume instead that either the internal composition of group 1 or the value of $\frac{\Phi_{dis}}{\Phi}$ is such that condition (7) does not hold. Thus, we consider from now on that Δ is positive.

Let us now analyse the strategy of low type members in the first group. From expressions (2) and (4) conflict is profitable if and only if the following inequality holds:

$$\frac{1}{1+\frac{\theta_H^2}{\theta_H^1}} \geq \{\frac{1}{2} + \frac{1}{2}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}\},$$

or

$$1 \geq [1+\frac{\theta_H^2}{\theta_H^1}]\{\frac{1}{2} + \frac{1}{2}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}\}.$$

Developping the above expression and simplifying, low type members of group 1 prefer an agressive strategy if and only if the ratio $\frac{\theta_H^2}{\theta_H^1}$ satisfies the following inequality:

$$\frac{\theta_H^2}{\theta_H^1} \leq \frac{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}. \tag{8}$$

Combining the conclusions obtained for the two types of agents, we obtain the following result:

Proposition 4.2. *In the general case ($n \neq m$) the situation can be described as follows (where $i, j = 1, 2$ and $i \neq j$):*

- *When $\frac{\theta_H^j}{\theta_H^i} \in]0, \frac{\sqrt{\frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_i}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}[$, then all members of group i prefer conflict over negotiations;*

- *When $\frac{\theta_H^j}{\theta_H^i} \in [\frac{\sqrt{\frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_i}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}, \frac{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}[$, then high type members of group i are willing to rely on negotiations, while low type members prefer conflict;*

- *Finally, when $\frac{\theta_H^j}{\theta_H^i} \geq \frac{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}$, then all members of group i prefer a negotiated arrangement.*

Proof. In the Appendix. □

The qualitative implications of the above proposition are similar than in the case of groups of identical size. However, there is an additional property when groups have different sizes. It follows from inspection that the set of values of the ratio $\frac{\theta_H^j}{\theta_H^i}$ for which a negotiated agreement is unanimously preferred within group i becomes larger compared to the case where groups have the same size. More precisely, the lower bound of this set becomes smaller as soon as $n \neq m$ (this is obvious looking at the third case in proposition 2.4).

4.2.2 Characterization of the equilibrium set

Now we would like to check if the conclusions obtained in section hold in the general case. For instance, it would be interesting to understand what would happen if one group dominates the other in terms of its size. Would conflict be more likely to occur than in the benchmark case? From a general point of view it is important to understand the specific influence of the relative size of the groups. This is the aim of the present section.

Let us first describe the outcome of the game. We can rely on the same arguments than in the proof of Theorem 1 to obtain the following result:

Theorem 2. *Let us consider the general case ($n \neq m$). The situation can be characterized as follows:*

1. *If* $\frac{\theta_H^2}{\theta_H^1} \in]0, \frac{\sqrt{\frac{1}{(n_1)^2} + \frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_1} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}[$ *then conflict is the outcome of the game. The conclusion does not depend on the choice of the representatives.*

2. *If* $\frac{\theta_H^2}{\theta_H^1} \in [\frac{\sqrt{\frac{1}{(n_1)^2} + \frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_1} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}, \frac{1 - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}[$ *then two sub cases can occur:*

   (a) *if high type members constitute the majority in group* 1*, then a cooperative agreement will prevail.*

   (b) *Otherwise conflict is the outcome of the game.*

3. *If* $\frac{\theta_H^2}{\theta_H^1} \geq \frac{1 - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}$ *then three sub cases can occur:*

   (a) *If* $\frac{\theta_H^1}{\theta_H^2} \in]0, \frac{\sqrt{\frac{1}{(n_2)^2} + \frac{2}{n_2}\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_2} - \frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}[$ *then conflict is the outcome of the game;*

(b) *In case* $\frac{\theta_H^1}{\theta_H^2} \in]\frac{\sqrt{\frac{1}{(n_2)^2}+\frac{2}{n_2}\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_2}-\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}, \frac{1-\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}[$ *then a negotiated arrangement will prevail if high type members constitute the majority in group* 2*, while conflict will be the outcome of the game otherwise;*

(c) *Finally, provided that* $\frac{\theta_H^1}{\theta_H^2} \in [\frac{1-\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}, \frac{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}]$ *a negotiated agreement will prevail.*

Proof. In the Appendix. □

The qualitative implications of the general case are similar to those of the case where groups have the same size for most cases. There is a notable difference, namely sub case 3, point c. When groups have different sizes there now exists a set of situations where the optimality of negotiated agreements does not depend on their internal composition. This set does not exist when groups have the same size. In other words, conflict is less likely to occur compared to the benchmark case.

4.3 Comparative statics

We conclude the analysis by providing results of comparative statics regarding the influence of the fundamentals of the problem on the range of situations for which a negotiated agreement will prevail. We will provide separate results for the symmetric and general cases (because of the differences in the characterization of equilibria), and we will discuss potential implications.

4.3.1 The symmetric case

In the present sub section we will try to understand how the number of high type members in each group influences the range of parameters over which negotiated arrangements will prevail. To answer this question we now provide a few results of comparative statics.

The main result can be stated as follows.

Proposition 4.3. *Let us consider* s_i *and* s_j *as defined in (5). Then we have the following properties:*

- s_i *increases with an increase in the proportion of high type members in group* i *(as measured by* n_i*);*

- *s_j decreases with an increase in the proportion of high type members in group j (as measured by n_j).*

Proof. In the Appendix. □

The above proposition has several interesting implications. Let us first consider that high type members constitute the majority in both groups. In such a case negotiated arrangements prevail for values of the ratio $\frac{\theta_H^j}{\theta_H^i}$ lying in the interval $[s_i, s_j]$. From proposition 2.5 we deduce that this interval shrinks down as the number of high type members increases in either groups.

Let us focus on the case of group i. An increase in the number of high type members in the other group has two opposite effects. On one side it has a positive effect as it increases the probability that high type members constitute the majority in group j, which increases the chance of a negotiated agreement (which is preferred by group i). On the other side such an increase has a negative effect on the size of interval $[s_i, s_j]$.

Now we move on to the general case, and we will check whether the above qualitative properties continue to hold. Then we will provide some general implications of the results.

4.3.2 The general case

We can use theorem 2 to characterize the set of situations where negotiated agreements will prevail. We focus on the ideal case where high type members constitute the majority in both groups. Then from points 2a), 3b) and 3c) we can conclude that negotiation is optimal when the ratio $\frac{\theta_H^2}{\theta_H^1}$ satisfies the following condition:

$$\frac{\theta_H^2}{\theta_H^1} \in [\frac{\sqrt{\frac{1}{(n_1)^2} + \frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_1} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}, \frac{1 + \frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{\sqrt{\frac{1}{(n_2)^2} + \frac{2}{n_2}\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_2} - \frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}]. \tag{9}$$

We can now provide results of comparative statics. Let us introduce the following notations:

$$th_1 = \frac{\sqrt{\frac{1}{(n_1)^2} + \frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_1} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}},$$

and

$$th_2 = \frac{\sqrt{\frac{1}{(n_2)^2} + \frac{2}{n_2}\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_2} - \frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}.$$

We can provide interesting insights about the influence of the parameters of the problem on the value of th_i which in turn will enable us to assess the influence of the internal composition and the relative size of each group, and of the specification of the disagreement surplus Φ_{dis}, on the relative optimality of negotiated agreements over conflict.

We obtain the following result:

Proposition 4.4. *The influence of the structural parameters of the game is described below (where $i \neq j$):*

- *Threshold th_i $(i = 1, 2)$ decreases with an increase in the value of the disagreement surplus Φ_{dis}.*
- *It decreases as the size of group i increases.*
- *It increases as the size of group j increases.*
- *It increases as the number of high type members in group i increases.*

Proof. In the Appendix. □

Using proposition 3.4 we can now assess the influence of the structural parameters on the optimality of negotiated agreements.

Lemma 4.1. *We have the following results:*

- *The interval (9) expands as the value of the disagreement surplus Φ_{dis} increases.*
- *It shrinks down as the number of high type members in group i $(i = 1, 2)$ increases.*
- *The effect of an increase in the size of one group is ambiguous.*

Proof.
- From the first point in proposition 4.4 we know that an increase in the value of the disagreement surplus Φ_{dis} will result in a decrease of the value of the first bound defining the interval (9). Moreover, the upper bound is equal to $\frac{1}{th_2}$; as such one can immediately check[4] that an increase in the value of Φ_{dis} will result in an increase in the value of the upper bound. This concludes the proof.

[4]Since $(\frac{1}{th_2})' = -\frac{(th_2)'}{(th_2)^2}$.

- From the last point in proposition 4.4 we know that an increase in the number of high type members in group 1 will result in an increase in the value of the lower bound defining the interval (9). An increase in the number of high type members in group 2 will result in a decrease in the value of the upper bound defining it. This concludes the proof.
- Finally, let us focus on the influence of an increase in n. The first bound defining (9) will then increase, and the second bound will decrease. This concludes the proof of the lemma.

□

There are several implications following from the above result. First, the effects of the disagreement surplus and of the proportion of high type members (in each group) are clear. Increasing Φ_{dis} will contribute to making negotiated arrangements more profitable compared to conflict, while the effect will be exactly the opposite for an increase in the proportion of high type members.

Second, it is not possible to sign the effect of an increase in the size of a particular group. This is quite logical as such an increase has an opposite effect on the payoff secured by agents who have the same type but are members of different groups. Thus, this affects the profitability of negotiation over conflict in two opposite ways.

4.3.3 Implications

From a general point of view we can conclude that the qualitative implications of the benchmark case are reinforced in the general situation. Focusing on the case of group i, theorem 2 and the above lemma suggest that:

- When $n_j \leq \frac{m}{2}$ negotiated arrangements will be more likely (considering the case of group i) as the number of high type members n_j increases;
- When $n_j \geq \frac{m}{2} + 1$ then negotiated arrangements will be more likely as the number of high type members is as small as possible.

This provides welfare implications on the internal composition of the groups as negotiated arrangements are (socially) efficient. We can add the following insights. When the ratio $\frac{\theta_H^j}{\theta_H^i}$ is sufficiently small, that is, when it is less than

$$\frac{\sqrt{\frac{1}{(n_1)^2} + \frac{2}{n_1}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_1} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}$$

then it would be preferable to keep the number of high type members in group i as low as possible (this follows from inspection of the expressions of payoffs used in the proof of proposition 3.4). Respectively, when the ratio is sufficiently large (greater than $\frac{n_j}{\sqrt{1+(n_j)^2}-1}$) then it would be preferable to keep the number of high type members in group j as low as possible. Indeed this would ensure that the interval over which negotiated arrangements would prevail becomes as large as possible.

5 Conclusion

We consider a situation where a resource has to be allocated between different groups whose members have either high or low valuation for the resource. The (two) groups can either negotiate cooperatively or use conflict to solve the allocation problem. We show that the incentives to negotiate or to go to war depend on the ratios of high type members' valuations. Specifically, the situation is described as follows:

- When the valuation of high type members in one group is sufficiently small compared to that of the other group, conflict will be optimal;
- For intermediate values of the ratio $\frac{\theta_H^2}{\theta_H^1}$ the emergence of conflict/negotiation depends on the internal composition of group 1. If high type members constitute the majority then negotiation emerges, otherwise conflict is optimal;
- When $\frac{\theta_H^2}{\theta_H^1}$ is sufficiently large, conflict emerges if $\frac{\theta_H^1}{\theta_H^2}$ is sufficiently low or moderate (if and only if high type members do not constitute the majority in group 2), otherwise negotiation becomes optimal.

Another implication of the analysis is that conflict is more likely to emerge when groups have the same size. We finally provide results of comparative statics. Provided that high type members constitute the majority in both groups, negotiation is less likely to emerge as the proportion of high type members increases. This suggests that it would be important (for economic efficiency) to ensure that groups are balanced regarding their internal composition. Moreover, it seems that situations with groups of the same size are more likely to promote conflictual situations.

Even though the above analysis provides some interesting insights on the influence of internal and external structures of groups on the emergence of conflict or negotiation, several extensions could (and should) be considered.

For instance, we assume that agents have complete information. Even though the assumption that members of a given group have a reasonably good knowledge of the characteristics of their own group makes sense in some situations, it would be useful to analyse a situation where information about the group characteristics is asymmetric. Moreover, we assume that the share of the surplus secured by each group is used entirely to fund (strictly) local public goods. In many real life situations, funds are likely to be divided between private and public goods. It would be very interesting to extend the analysis to such settings. We leave this for future work.

6 Appendix

6.1 Proof of Proposition 4.1

Proof.

- The fact that $\frac{\theta^j_H}{\theta^i_H} \in]0,1[$ implies that $\theta^i_H \geq \theta^j_H$, which leads to the conclusion that low type members of group i prefer conflict over negotiations. Moreover, we know that, provided $\frac{\theta^j_H}{\theta^i_H} \in]0, \frac{1}{n_i}[\sqrt{1+(n_i)^2}-1][$ then high members of group i prefer conflict as well. This concludes the proof of the first part of the proposition.

- It can be immediately checked that $\frac{1}{n_i}[\sqrt{1+(n_i)^2}-1] < 1$ for any $n_i \geq 1$; we deduce that in case $\frac{\theta^j_H}{\theta^i_H} \in [\frac{1}{n_i}[\sqrt{1+(n_i)^2}-1], 1[$ high type members of group i are willing to rely on negotiated arrangements. Since $\theta^i_H \geq \theta^j_H$ low type members still prefer conflict. This concludes the proof of the second part of the proposition.

- Finally, when $\frac{\theta^j_H}{\theta^i_H} \geq 1 > \frac{1}{n_i}[\sqrt{1+(n_i)^2}-1]$ then high type members of group i prefer a negotiated arrangement. Moreover this implies that $\theta^j_H \geq \theta^i_H$, which means that low type members prefer a negotiated arrangement. This concludes the proof of the proposition.

□

6.2 Proof of Proposition 4.2

Proof. It can be immediately checked that

$$\frac{\sqrt{\frac{1}{(n_i)^2} + \frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1} - \frac{1}{n_i} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}} < \frac{1 - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}};$$

indeed we have

$$\frac{\sqrt{\frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_i}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}-\frac{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}$$

$$=\frac{\sqrt{\frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1}-\frac{1}{n_i}-1}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}$$

which is negative as $\frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1<(\frac{1}{n_i}+1)^2$ since $\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}<1$ by assumption. Now the proof follows from the same arguments than that of proposition 2.3. □

6.3 Proof of Theorem 2

Proof. The proof of cases 1, 2, and 3 (sub cases a and b) follows from the same arguments than that of theorem 1 and is thus omitted.

Finally, to prove sub case 3c it is sufficient to note that

$$\frac{\theta_H^2}{\theta_H^1}\geq\frac{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}\Longleftrightarrow\frac{\theta_H^1}{\theta_H^2}\leq\frac{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}$$

and that

$$\frac{1-\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}<\frac{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}};$$

finally when $\frac{\theta_H^1}{\theta_H^2}\geq\frac{1-\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}$ we know that all members of group 2 prefer a negotiated agreement, which implies that both groups favor such an arrangement when the ratio $\frac{\theta_H^1}{\theta_H^2}$ belongs to the interval $[\frac{1-\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}}{1+\frac{m-n}{n+m}\frac{\Phi_{dis}}{\Phi}},\frac{1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}{1-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}]$. This concludes the proof. □

6.4 Proof of Proposition 4.3

Proof.
- Using that $s_i=\frac{1}{n_i}[\sqrt{1+(n_i)^2}-1]$ and differentiating with respect to n_i, we obtain:

$$\frac{\partial s_i}{\partial n_i}=-\frac{1}{(n_i)^2}[\sqrt{1+(n_i)^2}-1]+\frac{1}{\sqrt{1+(n_i)^2}}$$

$$=\frac{1}{(n_i)^2\sqrt{1+(n_i)^2}}\{-1+\sqrt{1+(n_i)^2}\}$$

which is positive for any non negative value of n_i. This concludes the proof of the first part of the proposition.

- Using that $s_j = \frac{n_j}{\sqrt{1+(n_j)^2}-1}$ and differentiating with respect to n_j, we obtain:

$$\frac{\partial s_j}{\partial n_j} = \frac{\sqrt{1+(n_j)^2} - 1 - \frac{(n_j)^2}{\sqrt{1+(n_j)^2}}}{[\sqrt{1+(n_j)^2} - 1]^2}$$
$$= \frac{1 - \sqrt{1+(n_j)^2}}{\sqrt{1+(n_j)^2}[\sqrt{1+(n_j)^2} - 1]^2}$$

which is negative for any non negative value of n_i. This concludes the proof of the proposition.

□

6.5 Proof of Proposition 4.4

Proof. In each case we come back to the expression of th_i and we differentiate it with respect to the appropriate parameter.

- We obtain after simplification the following expression:

$$\frac{\partial th_i}{\partial \Phi_{dis}} = \frac{n-m}{(n+m)}\frac{1}{\Phi}\frac{[\frac{1}{n_i} - \frac{1}{(n_i)^2} - 1 - \frac{1}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}]\frac{1}{\sqrt{\frac{1}{(n_i)^2}+1+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}} - 1 + \frac{1}{n_i}}{[1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}]^2}.$$

Since $\frac{1}{n_i} - 1 < 0$ for any n_i at least equal to one, it is easily checked that the numerator of the above expression is negative.

- After simplification, we obtain:

$$\frac{\partial th_i}{\partial n} = \frac{2m}{(n+m)^2}\frac{\Phi_{dis}}{\Phi}\frac{[\frac{1}{n_i} - \frac{1}{(n_i)^2} - 1]\frac{1}{\sqrt{\frac{1}{(n_i)^2}+1+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}} - 1 + \frac{1}{n_i}}{[1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}]^2}.$$

Again, as $\frac{1}{n_i} - 1 < 0$ it is easily checked that the above expression is negative.

- Differentiating with respect to m and simplifying, we obtain:

$$\frac{\partial th_i}{\partial m} = -\frac{2n}{(n+m)^2}\frac{\Phi_{dis}}{\Phi}\frac{[\frac{1}{n_i} - \frac{1}{(n_i)^2} - 1]\frac{1}{\sqrt{\frac{1}{(n_i)^2}+1+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}}} - 1 + \frac{1}{n_i}}{[1+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}]^2},$$

which is positive since $\frac{1}{n_i} - 1$ is negative.

- We obtain:

$$\frac{\partial th_i}{\partial n_i} = \frac{-\frac{1}{n_i} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + \sqrt{\frac{1}{(n_i)^2} + \frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1}}{[1 + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}]\sqrt{\frac{1}{(n_i)^2} + \frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1}}\frac{1}{(n_i)^2}.$$

Since we have

$$(\frac{1}{n_i}+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi})^2 = \frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+(\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi})^2 < \frac{1}{(n_i)^2}+\frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+1$$

it follows that

$$-\frac{1}{n_i} - \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + \sqrt{\frac{1}{(n_i)^2} + \frac{2}{n_i}\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} + 1}$$

$$> -\frac{1}{n_i}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+\sqrt{(\frac{1}{n_i} + \frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi})^2} = -\frac{1}{n_i}-\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi}+\frac{1}{n_i}+\frac{n-m}{n+m}\frac{\Phi_{dis}}{\Phi} = 0.$$

Thus we have $\frac{\partial th_i}{\partial n_i} > 0$, which concludes the proof.

$\square$

References

[1] Anbarci, N., Skaperdas, S., and C. Syropoulos (2002), "Comparing Bargaining Solutions in the Shadow of Conflict: How Norms Against Threats Can Have Real Effects, *Journal of Economic Theory* 106, 1-16.

[2] Baliga, S., and T. Sjostrom (2004), "Arms Races and Negotiations," *Review of Economic Studies* 71, 351-369.

[3] Brandauer, S., and F. Englmaier (2009), "A Model of Strategic Delegation in Contests Between Groups", *Review of Economic Design* 13, 205-232.

[4] Brandts, J., Abbink, K., Herrmann, B., and H. Orzen (2009), "Inter-Group Conflict and Intra-Group Punishment in an Experimental Contest Game", *American Economic Review*, forthcoming.

[5] Cai, H. (2000), "Bargaining on Behalf of a Constituency", *Journal of Economic Theory* 92, 234-273.

[6] Crawford, V. (1982), "A Theory of Disagreement in Bargaining", *Econometrica* 50, 607-637.

[7] Ellingsen, T., and T. Miettinen (2008), "Commitment and Conflict in Bilateral Bargaining", *American Economic Review* 98, 1629-1635.

[8] Esteban, J., and D. Ray (2008), "On the Salience of Ethnic Conflict", *American Economic Review* 98, 2185-2202.

[9] Haller, H., and S. Holden (1997), "Ratification Requirement and Bargaining Power", *International Economic Review* 38, 825-851.

[10] Hirshleifer, J. (1989), "Conflict and Rent Seeking Success Functions: Ratio vs Difference Models of Relative Success", *Public Choice* 63, 101-112.

[11] Horn, H., and A. Wolinsky (1988), "Worker Substitutability and Patterns of Unionisation", *Economic Journal* 98, 484-497.

[12] Jackson, M., and M. Morelli (2007), "War, Transfers, and Political Bias", *American Economic Review* 97, 1353-1373.

[13] Jackson, M., and M. Morelli (2009), "Strategic Militarization, Deterrence and Wars", *Quarterly Journal of Political Science*, forthcoming.

[14] Jun, B. (1989), "Non Cooperative Bargaining and Union Formation", *Review of Economic Studies* 56, 59-76.

[15] Manzini, P., and M. Mariotti (2005), "Alliances and Negotiations", *Journal of Economic Theory* 121, 128-141.

[16] Munster, J. (2009), "Group Contest Success Functions", *Economic Theory* 41, 345-357.

[17] Nash, J. (1950), "The Nash Bargaining Problem", *Econometrica* 18, 155162.

[18] Ponsati, C., and J. Sakovics (1996), "Multiperson Bargaining Over Two Alternatives", *Games and Economic Behavior* 12, 226-244.

[19] Sanchez-Pages, S. (2009), "Conflict as a Part of the Bargaining Process", *Economic Journal* 119, 1189-1207.

[20] Schelling, T. (1956), "An Essay on Bargaining", *American Economic Review* 46, 281-306.

[21] Schelling, T. (1963), *The Strategy of Conflict*, London and New York, Oxford University Press.

[22] Skaperdas, S. (1992), "Cooperation, Conflict, and Power in the Absence of Property Rights", *American Economic Review* 82, 720-739.

[23] Tullock, G. (1980), "Efficient Rent Seeking", in **Toward a Theory of the Rent Seeking Society**, J.M. Buchanan, Tollison, R., and G. Tullock (eds), College Station, TX, Texas A and M university Press.