

# Seismic Analysis of Asymmetric Buildings with Flexible Floor Diaphragms

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**Abstract:** Even though a rigid floor diaphragm is a good assumption for seismic analysis of most buildings, several building configurations may exhibit significant flexibility in floor diaphragm. However, the issue of static seismic analysis of such buildings for torsional provisions of codes has not been addressed in the literature. Besides, the concept of center of rigidity needs to be formulated for buildings with flexible floor diaphragms. In this paper, the definition of center of rigidity for rigid floor diaphragm buildings has been extended to unsymmetrical buildings with flexible floors. A superposition-based analysis procedure is proposed to implement code-specified torsional provisions for buildings with flexible floor diaphragms. The procedure suggested considers amplification of static eccentricity as well as accidental eccentricity. The proposed approach is applicable to orthogonal as well as nonorthogonal unsymmetrical buildings and accounts for all possible definitions of center of rigidity.

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## Introduction

Buildings are seldom, if ever, perfectly symmetric and frequently building vibrations involve coupling of lateral and torsional motion (*torsional coupling*). Besides the *static eccentricity* ( $e_{sj}$ ) [defined as the distance between the center of mass (CM) and the center of rigidity (CR) at respective floors], codes often require the designer to incorporate *accidental eccentricity*. Accidental eccentricity accounts for factors such as the rotational component of ground motion about the vertical axis, the difference between computed and actual values of the mass, stiffness or yield strength, and an unfavorable distribution of live load mass. Codal provisions on torsion differ significantly. However, the provisions of most codes for design eccentricity  $e_{dj}$  at the  $j$ -th floor for static analysis of buildings can be expressed in the following general form:

$$e_{dj} = \alpha e_{sj} + \beta b_j \quad (1a)$$

$$e_{dj} = \delta e_{sj} - \beta b_j \quad (1b)$$

where  $e_{sj}$ =static eccentricity at  $j$ -th floor;  $b_j$ =plan dimension of the  $j$ -th floor normal to the direction of ground motion; and  $\alpha$ ,  $\beta$  and  $\delta$  are specified constants. For instance, as per UBC (1997) and NEHRP (1997a,b)  $\alpha=1.0$ ,  $\beta=0.05$ , and  $\delta=1.0$ ; as per NZS 4203 (1992)  $\alpha=1.0$ ,  $\beta=0.1$ , and  $\delta=1.0$ ; as per NBCC (1995)

$\alpha=1.5$ ,  $\beta=0.1$ , and  $\delta=0.5$ ; and as per MCBC (1995)  $\alpha=1.5$ ,  $\beta=0.1$ , and  $\delta=1.0$ . In Eqs. (1a) and (1b), the first term on the right hand side accounts for the coupled lateral torsional effect that arises from lack of symmetry as well as amplification due to dynamic effects whereas the second term incorporates the accidental torsional effect. The response obtained from Eqs. (1a) and (1b), whichever is higher, should be used in the design.

Wherever building codes specify  $\alpha=\delta=1.0$  (e.g., in UBC 1997; NZS 4203 1992), the location of the CR need not be explicitly calculated to implement torsional provisions. In such cases, a rigid floor diaphragm building can be analyzed by applying design lateral force at a point away from the CM by  $+\beta b_j$  or  $-\beta b_j$ , as the case may be. However, where the codes specify values of  $\alpha$  and  $\delta$  different from 1.0 (e.g., in NBCC 1995; MCBC 1995) the general impression used to be that one needs to locate the CR. There are some difficulties in explicitly calculating the location of the CR in a simple manner. Hejal and Chopra (1987) have proposed a stiffness matrix based formulation to locate the CR and the approach is applicable to an arbitrarily shaped diaphragm with arbitrary orientation of the principal planes of lateral load resisting elements. However, the approach is restricted to buildings with rigid floor diaphragms and cannot be implemented with available standard building analysis software. At about the same time, a similar method was documented by Alcocer (1986) and Damy-Rios and Alcocer (1987). A superposition based approach was proposed (Goel and Chopra 1993) to implement Eq. (1), which does not require locating the CR explicitly. However, this approach is applicable only for buildings with rigid floor diaphragms that have an orthogonal system.

A rigid floor diaphragm is a good assumption in most buildings. However, floor diaphragms in some buildings may have considerable flexibility in their own plane (e.g., buildings that are long and narrow or buildings with stiff end walls). In such buildings, design force for a particular floor cannot be applied at one single point (say, the CM or at some eccentricity) of that floor. If the floor slabs are completely flexible, the lateral load distribution is governed by the tributary mass concept and the issue of torsion does not enter the picture. However, when the floor slabs have

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intermediate flexibility, i.e., floor diaphragms that are neither rigid nor completely flexible, floor diaphragm flexibility must be explicitly accounted for in the analysis. A considerable amount of literature is available on the dynamics of buildings with flexible floor diaphragms (e.g., Shepard and Donald 1967; Jain and Jennings 1984; Jain 1984a,b; Lu et al. 1984; Jain and Jennings 1985; Celbi et al. 1989; Jain and Mandal 1992; Tena-Colunga 1992; Jain and Mandal 1995; Tena-Colunga and Abrams 1995, 1996; Tremblay and Stiemer 1996; Medhekar 1997; de la Colina 1999; Ju and Lin 1999; Tremblay et al. 2000). However, only a limited number of published work on dynamic analysis of unsymmetrical flexible floor diaphragm buildings could be located (e.g., Tena-Colunga 1992; Tena-Colunga and Abrams 1995, 1996; de la Colina 1999). Further, static seismic analysis of unsymmetrical buildings with flexible floor diaphragms has not yet been addressed in the literature. Moreover, the concept of CR for flexible floor system needs to be formulated.

This paper proposes a procedure for static seismic analysis of buildings with significant floor diaphragm flexibility (but not completely flexible) as well as torsional coupling. The approach proposed by Goel and Chopra (1993) for rigid floor system has been modified and suitably extended to buildings with flexible floor diaphragms. Two definitions of center of rigidity, namely, *all floor* and *single floor*, are proposed for buildings with flexible floor diaphragms. The proposed method is such that, as the floor diaphragm rigidity increases, the results are close to those applicable for buildings with rigid floor diaphragms.

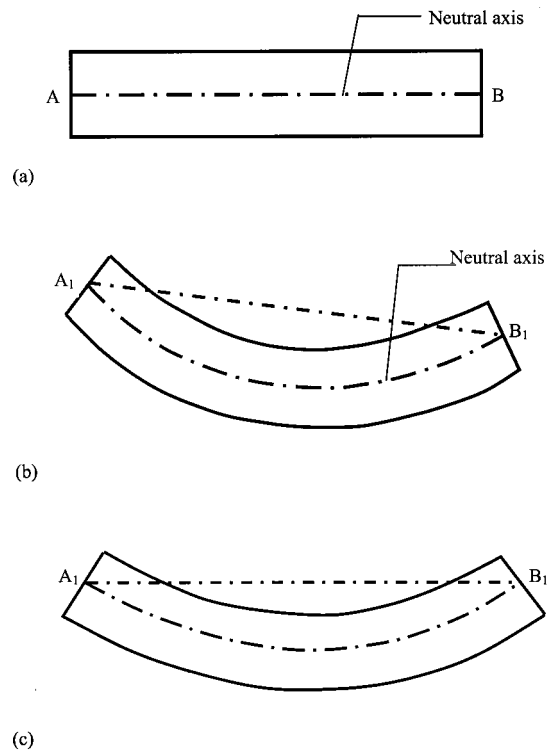
The building is assumed to have a single wing only (e.g., rectangular or trapezoid in plan) and buildings having multiple wings (e.g., L, V, Y, etc. shaped) are not considered. However, lateral load resisting elements need not be oriented orthogonally. The structure is considered linear and elastic. Further, it is assumed to be rigidly held to the ground, so soil–structure interaction effects and flexibility of the foundation are ignored.

## Center of Rigidity

Unlike a single-story building, a multistory building with a rigid floor diaphragm does not have a unique or generally accepted single definition of the CR. The following two definitions are commonly used.

1. Centers of rigidity are the set of points located on each floor, through which application of lateral load profile would cause no rotation in any floor (Poole 1977; Cheung and Tso 1986). According to this definition, the location of the CR is dependent on building stiffness properties as well as on the profile of the applied lateral load. This definition is referred to as “*all floor CR*” in this paper.
2. The center of rigidity of a floor is defined as the point on the floor where application of lateral load passing through that point does not cause any rotation of that particular floor although the other floors may rotate (Humar 1984). This definition is independent of the magnitude of the applied lateral load, and is referred to as “*single floor CR*” in this paper.

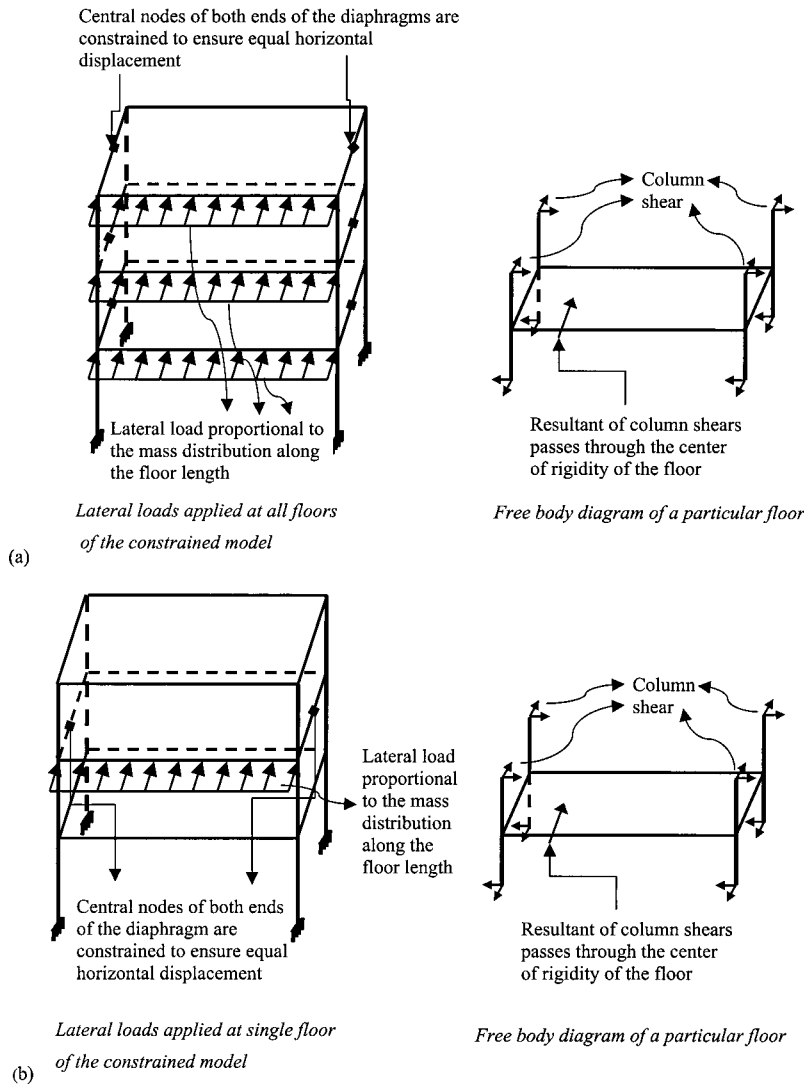
It should be mentioned here that application of torsion provisions based on different definitions of CR could result in different member forces. Since the all floor definition is more convenient to implement in a design office, it is most frequently used. Some codes also define eccentricity with respect to the shear center (SC); see e.g., MCBC (1995). Discussion in this paper is limited to torsional provisions based on center of rigidity but the treatment presented here can be conveniently extended to provisions based on the shear center also.



**Fig. 1.** No-torsion condition in buildings with flexible floor diaphragm: (a) Undeformed floor diaphragm; (b) deflected shape of floor slab under in-plane loading with torsion; (c) deflected shape of floor slab under in-plane loading without torsion

In order to apply Eq. (1) to flexible floor buildings, the CR needs to be defined for such buildings. In the case of a rigid floor system, “no-torsional rotation of floor” in the definition of CR can be viewed as no twisting of the line connecting the center node of both ends of the diaphragm. Hence, in the case of a flexible floor diaphragm system, identical horizontal displacement of center nodes at both ends of the diaphragm can be considered the same as a no-torsion response. For the floor slab shown in Fig. 1(a),  $AB$  is the line that connects the center nodes at both ends of the undeformed diaphragm. Under lateral loads, the floor slab is deformed, translated and rotated [Fig. 1(b)] and line  $A_1B_1$  forms an angle with line  $AB$ . Thus, to obtain the no-torsion situation, it is necessary that points  $A$  and  $B$  must undergo equal horizontal displacement so line  $A_1B_1$  must remain parallel to  $AB$  [Fig. 1(c)]. Therefore, the two definitions of CR can be extended to buildings with flexible floor diaphragms as follows.

1. Apply the design lateral load at all floors such that the lateral load at each floor is distributed along the length of the floor in proportion to the mass distribution. Next, at each floor, constrain the center nodes of both ends of the diaphragm such that they undergo equal horizontal displacement at each respective floor level. The resultant shear forces of all lateral load resisting elements in the stories immediately above and below a particular floor pass through the CR of that floor [all floor CR definition; Fig. 2(a)].
2. Apply lateral load distributed along length of one of the floors such that it is proportional to the floor mass distribution. Next, constrain the center nodes of both ends of that floor diaphragm so that they undergo equal horizontal displacement. The resultant shear forces of all lateral load re-



**Fig. 2.** Procedure for locating (a) all floor center of rigidity and (b) single floor center of rigidity

sisting elements in the stories immediately above and below that floor pass through the CR of that floor [single floor CR definition; Fig. 2(b)].

### Review of Goel and Chopra's Approach to Rigid Floor Diaphragm

Here we review a procedure for analysis for implementation of Eq. (1a) for rigid floor diaphragm buildings following the procedure suggested by Goel and Chopra (1993). Goel and Chopra's procedure is applicable for the all floor definition of CR only. Further, this approach is restricted to orthogonal systems. Goel and Chopra (1993) have pointed out that lateral force profile  $\{F_j\}$  applied at design eccentricity [Fig. 3(a)] according to Eq. (1a) can be considered superposition of the following three cases:

1. A force profile  $\{F_j\}$  acting through the CM [Fig. 3(b)];
2. A moment profile  $\{-(\alpha - 1)e_{sj}F_j\}$  (anticlockwise moment is considered positive) [Fig. 3(c)]; and
3. A moment profile  $\{-\beta b_j F_j\}$  [Fig. 3(d)].

Here, the moment in case (2) can be written in terms of a load profile  $\{(\alpha - 1)F_j\}$  that acts at the CM and load profile  $\{-(\alpha - 1)F_j\}$  that acts through the CR [Fig. 3(e)]. The load cases of

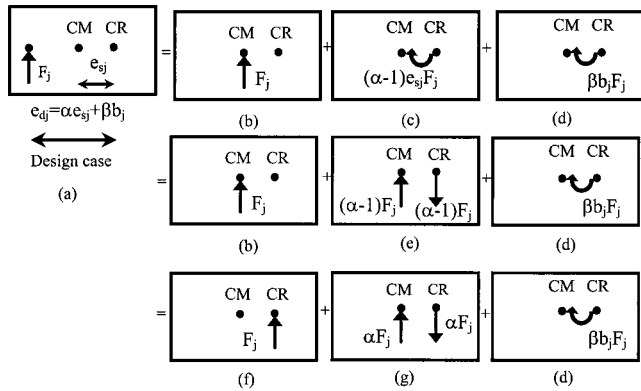
Figs. 3(b and e) can be replaced by the load cases of Figs. 3(f and g). Let  $F_{cr}$  = response when the load profile acts through the CR;  $F_{cm}$  = response when the load profile acts through the CM; and  $R_{ac}$  = response due to the accidental torsional moment vector  $\{\beta b_j F_j\}$ . Now following Eq. (1a), design response [Fig. 3(a)] can be expressed as summations of response to Figs. 3(f, g, and d);

$$F_d = F_{cr} + \alpha(F_{cm} - F_{cr}) - R_{ac} \quad (2a)$$

In a similar way, following Eq. (1b) one can express the design response as

$$F_d = F_{cr} + \delta(F_{cm} - F_{cr}) + R_{ac} \quad (2b)$$

Here,  $F_{cr}$  reflects the pure translational response while  $F_{cm}$  considers translational response as well as the effect of torsion due to the eccentricity  $e_{sj}$ . Hence,  $(F_{cm} - F_{cr})$  is the torsional effect due to the calculated static eccentricity, and  $\alpha(F_{cm} - F_{cr})$  implies the torsional effect is due to amplified static eccentricity. It should be noted that, in Eq. (2), the sign of the response obtained from  $R_{ac}$  should be considered in that it increases the magnitude of the response obtained from the first two terms. This is to account for the location of the CR on either side of the CM. Hence, Eq. (2) may be rewritten as



**Fig. 3.** Alternate form of code specified design eccentricity: (a) Design case; (b) force profile applied through CM; (c) additional torsional moment due to dynamic amplification; (d) accidental torsional moment; (e) alternate representation of additional torsional moment due to dynamic amplification; (f) pure translation case; and (g) total amplified torsion with dynamic amplification.

$$F_d = F_{cr} + \alpha(F_{cm} - F_{cr}) \pm R_{ac} \quad (3a)$$

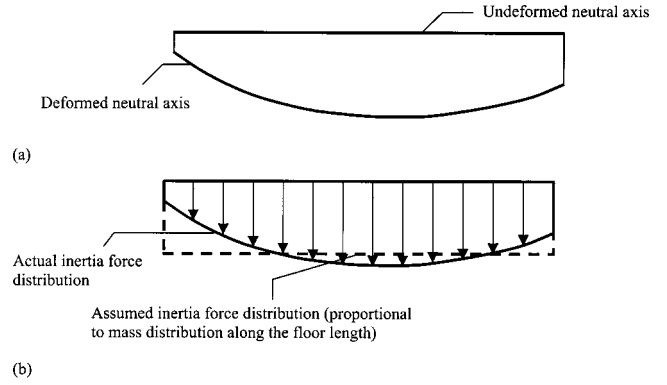
$$F_d = F_{cr} + \delta(F_{cm} - F_{cr}) \pm R_{ac} \quad (3b)$$

Eq. (3) can easily be implemented with the help of any standard building analysis software.  $F_{cm}$  can be calculated by applying the lateral load profile at the CM of the floor slabs of the building. To obtain  $F_{cr}$ , the lateral load profile is applied at the CMs but the floor slabs are constrained to move along the direction of applied loading. This can be achieved by placing roller supports at the floors in the direction normal to the direction of applied load. However, when the load profile is applied through single floor centers of rigidity the building may translate along both orthogonal directions. A similar situation also arises if the building is nonorthogonal. Therefore, the “roller model” restricts the approach from being applicable to these two cases.

### Analysis for Buildings with Flexible Floor Diaphragms

Here the procedure proposed by Goel and Chopra (1993) is extended to buildings with flexible floor diaphragms. The proposed approach considers all floor as well as single floor definitions of CR and is applicable to orthogonal and nonorthogonal systems.

Analysis of flexible floor diaphragm buildings is different from that of rigid floor diaphragms in several respects. Lateral load cannot be applied at a single point (at the CM or at an offset) on the diaphragm of the flexible floor building; it has to be distributed along the length of the floor. Further, earthquake inertia force associated with any vibration mode is proportional to the mass distribution along the floor length and depends on the displacement profile of that particular mode (Fig. 4). Estimation of the actual inertia force distribution that takes into consideration the displacement profile may be tedious. However, the static force procedure is an approximation of the building response obtained from dynamic analysis; thus in most cases, it is adequate to assume that the inertia force is in proportion to the mass distribution along the length of the floor. In extreme cases of large aspect ratio where the effect of the displacement profile is important, an iterative approach may be used to obtain the inertia force profile. First, a distribution of the load profile is assumed and the displacement

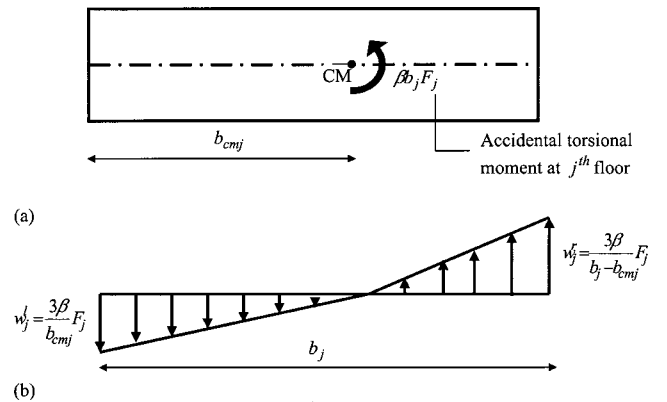


**Fig. 4.** Inertia force distribution for flexible floor diaphragm: (a) In-plane floor displacement and (b) actual and assumed inertia force distribution (for uniform mass distribution)

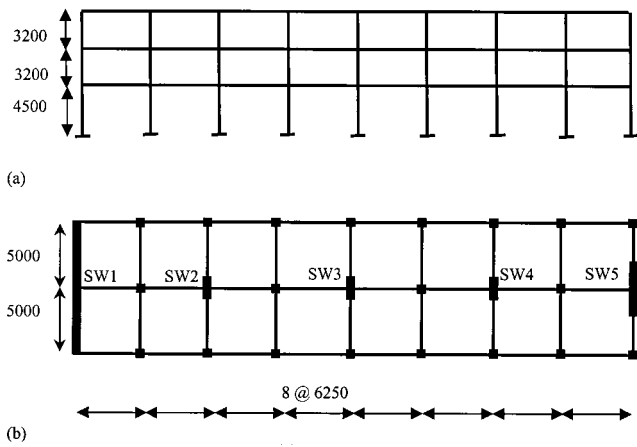
profile is calculated. Second, using this calculated displacement profile the load distribution is modified, and so on.

It is shown in Eq. (2a) that the design torsional response can be expressed as superposition of the following three cases: (i) no-torsion case ( $F_{cr}$ ), (ii)  $\alpha$  times the difference in response of natural torsion and no-torsion cases [ $\alpha(F_{cm} - F_{cr})$ ], and (iii) accidental torsion case ( $R_{ac}$ ). This concept of superposition is also applicable to buildings with flexible floor diaphragms except that details for each case are different. The natural torsion case ( $F_{cm}$ ) is the response with the load profile applied in proportion to the mass distribution along the floor length. For the no-torsion case ( $F_{cr}$ ) the same force profile is applied while the floor diaphragms of the building are constrained accordingly. Finally, to obtain a response for the accidental torsional moment, a linearly varying force distribution can be applied such that it meets the code-specified accidental torsional moment (Fig. 5).

The proposed approach differs from that of Goel and Chopra (1993) in defining the no-torsion case. It defines the no-torsion case in terms of constraining the center nodes of both ends of the diaphragm so that they undergo equal horizontal displacement; this allows the building to translate along both orthogonal directions. This also enables the proposed approach to become applicable for all definitions of center of rigidity and to orthogonal as well as nonorthogonal buildings.



**Fig. 5.** Estimate of response due to accidental torsional moment: (a) floor slab at  $j$ -th floor and (b) load profile to account for positive accidental torsional moment



**Fig. 6.** Example of building (a) elevation and (b) its plan (all dimensions are in millimeters)

The step-by-step algorithm for the proposed static analysis method is as follows.

#### Procedure with All Floor Definition

1. Apply the design lateral load at all floor levels such that lateral load at each floor is distributed along the length of the floor in proportion to the mass distribution. Analyze the structure to obtain response  $F_{cm}$ .
2. At all floor levels constrain the center nodes of both ends of the diaphragm so that they undergo equal horizontal displacement at each respective floor level. Analyze the structure for the same force profile as that in step 1 above to obtain response  $F_{cr}$ .
3. Apply the force couple that represents accidental torsional moments (Fig. 5) at all floors. Analyze the original model of the structure subjected to this force distribution to obtain response  $R_{ac}$ .
4. Combine the above responses following Eqs. (3a) and (3b).

#### Procedure with Single Floor Definition

1. Apply the lateral load profile at all floor levels to the original model of the building in a way similar to that in Procedure with all Floor Definition and calculate the response  $F_{cm}$ .
2. Constrain the center nodes of both ends of the diaphragm only at  $j$ -th floor so that they undergo equal horizontal displacement. Next, apply the distributed lateral load at the  $j$ -th floor in proportion to the mass distribution and calculate response  $F_{cr}^j$ .
3. Repeat step 2 for each floor,  $j=1, \dots, n$  and obtain  $F_{cr} = \sum_{j=1}^n F_{cr}^j$ .
4. Calculate the response due to accidental torsion in a way similar to in the all floor case and combine the responses following Eqs. (3a) and (3b).

#### Illustration of Methodology

A three-story building (Fig. 6) with floor diaphragms that are neither rigid nor completely flexible is analyzed to study its torsional response. The building consists of two end shear walls and seven interior frames, three of which include a small shear wall. The left shear wall (SW1) is 10 m in length while right shear wall

**Table 1.** Shear Force in Shear Walls Using the “All Floor” Definition of Center of Rigidity

Member	Wall	Story	$F_{cm}^a$ (kN)	$F_{cr}^b$ (kN)	$R_{ac}^c$ (kN)	Shear	Shear	Design
						from Eq. (3a) (kN)	from Eq. (3b) (kN)	
SW1		3	142.62	270.10	-38.27	117.20	180.89	180.89
		2	228.96	430.96	-58.00	185.96	286.96	286.96
		1	263.18	463.56	-64.44	227.43	327.62	327.62
SW2		3	4.57	4.64	-0.30	4.84	4.87	4.87
		2	8.85	7.40	0.42	10.00	9.27	10.00
		1	13.86	11.57	0.59	15.60	14.45	15.60
SW3		3	6.70	5.24	0.42	7.85	7.12	7.85
		2	14.19	9.43	1.44	18.01	15.63	18.01
		1	20.27	12.63	2.33	26.42	22.60	26.42
SW4		3	8.58	4.61	1.44	12.01	10.02	12.01
		2	16.14	7.63	2.52	22.92	18.66	22.92
		1	23.30	11.37	3.80	33.07	27.10	33.07
SW5		3	104.32	24.74	23.88	167.99	128.20	167.99
		2	196.22	45.79	42.64	314.08	238.86	314.08
		1	245.71	84.97	52.23	378.31	297.94	378.31

<sup>a</sup> $F_{cm}$ =response due to earthquake force profile applied in proportion to the mass distribution along the floor length.

<sup>b</sup> $F_{cr}$ =response in no-torsion case.

<sup>c</sup> $R_{ac}$ =response due to accidental torsional moment.

(SW5) is 4 m in length; the length of each intermediate shear wall (SW2–SW4) is 1.5 m. The thickness of all the shear walls is 150 mm. All columns are 400 mm×400 mm, the longitudinal beams are 250 mm×550 mm, and the transverse beams are 250 mm×400 mm. The floor slabs are taken as 200 mm thick. The aspect ratio of the floor plan (50 m×10 m) is chosen as 5, slightly more than that of typical buildings to show the effect of floor flexibility. The first story height is taken as 4.5 m whereas the height of the other two stories is 3.2 m. The modulus of elasticity and shear modulus of concrete are taken as  $2.55 \times 10^7$  and  $1.06 \times 10^7$  kN/m<sup>2</sup>, respectively. The gross moment of inertia is used in the analysis and the extra rigidity in the rigid end zones of the members is not modeled. Design eccentricity is taken according to Eqs. (1a) and (1b) with constants  $\alpha$ ,  $\beta$  and  $\delta$  equal to 1.5, 0.1 and 1.0, respectively.

The building is analyzed using the SAP2000 program. Beams and columns are modeled as *line elements*. Shear walls are modeled as *shell elements* by considering the membrane as well as the plate behavior while floor slabs are considered pure *membrane elements*. The design earthquake force on the building is calculated using Indian code provisions for seismic zone V; this gives the design seismic load profile as {62.0 181.6 363.9}<sup>T</sup> kN starting from the bottommost floor. Resultant force (shear force) in the shear walls is presented in Tables 1 and 2 in which all floor and single floor definitions of CR, respectively, are considered. It is observed that for this example the building response obtained from both the definitions is practically the same. Therefore, the all floor definition of CR, being easier to implement, is used in this paper as the basis for further comparisons. It should be noted that the sum of the shear forces in the walls will not be equal to the story shear applied since the columns also share some lateral load.

The building is then analyzed for torsion without accounting

**Table 2.** Shear Force in Shear Walls Using the Single Floor Definition of Center of Rigidity

Member		$F_{cm}^a$	$F_{cr}^1{}^b$	$F_{cr}^2{}^b$	$F_{cr}^3{}^b$	$F_{cr}^c$	$R_{ac}^d$	Shear from Eq. (3a)	Shear from Eq. (3b)	Design shear
Wall	Story	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)
SW1	3	142.62	0.75	16.58	250.42	267.75	-38.27	118.33	180.89	180.89
	2	228.96	5.85	122.16	287.44	415.45	-58.00	193.72	286.96	286.96
	1	263.18	34.66	137.00	292.95	464.61	-64.44	226.91	327.62	327.62
SW2	3	4.57	0.00	-7.52	12.31	4.79	-0.30	4.76	4.87	4.87
	2	8.85	-2.85	9.69	0.41	7.25	0.42	10.07	9.27	10.07
	1	13.86	4.97	4.45	2.39	11.81	0.59	15.48	14.45	15.48
SW3	3	6.70	-0.21	-9.35	14.88	5.32	0.42	7.81	7.12	7.81
	2	14.19	-3.43	11.79	1.24	9.60	1.44	17.93	15.63	17.93
	1	20.27	6.25	5.51	1.10	12.86	2.33	26.31	22.60	26.31
SW4	3	8.58	0.00	-6.56	11.28	4.72	1.44	11.95	10.02	11.95
	2	16.14	-2.50	9.09	1.67	8.26	2.52	22.60	18.66	22.60
	1	23.30	4.76	4.28	2.38	11.42	3.80	33.04	27.10	33.04
SW5	3	104.32	0.17	13.17	11.98	25.32	23.88	167.70	128.20	167.70
	2	196.22	5.04	7.23	46.81	59.08	42.64	307.43	238.86	307.43
	1	245.71	5.42	22.19	55.39	83.00	52.23	379.30	297.94	379.30

<sup>a</sup> $F_{cm}$ =response due to the earthquake force profile applied in proportion to the mass distribution along the floor length.

<sup>b</sup> $F_{cr}^1, F_{cr}^2, F_{cr}^3$ =response from no-torsion case of first, second and third floors, respectively.

<sup>c</sup> $F_{cr}=F_{cr}^1+F_{cr}^2+f_{cr}^3$ .

<sup>d</sup> $R_{ac}$ =response due to the accidental torsional moment.

for accidental torsion, i.e.,  $\beta=0$  in Eqs. (1a) and (1b); the resultant force is shown in column 3 of Table 3. A comparison with results when accidental torsion is also considered (column 4, Table 3) indicates that accidental torsion contributes up to 26.8% of the wall shear for this example. The accidental torsional moment is directly proportional to the dimension of the building

normal to the direction of applied force. Therefore, accidental torsion may contribute significantly to the resultant design force for long buildings. It should be mentioned, however, that the value of  $\beta$  in the codes that specify accidental torsion is based on studies of rigid floor diaphragm buildings; its applicability for flexible floor diaphragm buildings needs to be established.

**Table 3.** Comparison of the Contribution of Torsion and Diaphragm Flexibility to Design Response

Member		Flexible floor diaphragm without accidental torsion	Flexible floor diaphragm with accidental torsion		Rigid floor diaphragm without accidental torsion		Flexible floor diaphragm with lateral loads applied proportionally to floor mass	
Wall	Story	Shear force (kN)	Shear force (kN)	Difference (%)	Shear force (kN)	Error (%)	Shear force (kN)	Error (%)
SW1	3	142.62	180.89	26.8	150.84	5.8	142.62	0
	2	228.96	286.96	25.3	243.24	6.2	228.96	0
	1	263.18	327.62	24.5	273.44	3.9	263.18	0
SW2	3	4.57	4.87	6.6	4.39	-3.9	4.57	0
	2	9.58	10.00	4.4	6.09	-36.4	8.85	-7.6
	1	15.01	15.60	3.9	10.62	-29.3	13.86	-7.7
SW3	3	7.43	7.85	5.7	8.31	11.8	6.70	-9.8
	2	16.57	18.01	8.7	11.57	-30.2	14.19	-14.4
	1	24.09	26.42	9.7	17.63	-26.8	20.27	-15.9
SW4	3	10.57	12.01	13.6	12.23	15.7	8.58	-18.8
	2	20.40	22.92	12.4	17.05	-16.4	16.14	-20.9
	1	29.27	33.07	13.0	24.63	-15.9	23.30	-20.4
SW5	3	144.11	167.99	16.6	154.47	7.2	104.32	-27.6
	2	271.44	314.08	15.7	289.10	6.5	196.22	-27.7
	1	326.08	378.31	16.0	337.72	3.6	245.71	-24.7

The building cited as an example is further analyzed by considering the floor diaphragms as rigid in their own planes and with the all floor definition of CR as the basis of torsional provisions but excluding the contribution by accidental torsion (column 6, Table 3). It is observed that the shear force in the end shear walls (SW1 and SW5) is somewhat overestimated (up to 7.2%) and that in intermediate shear walls (SW2–SW4) is significantly underestimated (up to 36.4%) by assumption of a rigid diaphragm.

Consider the case when no special attention is given to issues of torsion and the flexible floor diaphragm building is analyzed for the lateral load profile in proportion to the mass distribution along the floor. This implies consideration only of static eccentricity without dynamic amplification and accidental torsion ( $\alpha = 1.0$ ,  $\beta = 0$  and  $\delta = 1.0$ ). In this case (column 8, Table 3), the wall forces are underestimated by up to 27.7% in the end walls and by up to 20.9% in the intermediate walls. It should be noted here that the shear force induced in SW1 (and also at the top story of SW2) is the same as that in column 3 of Table 3. This is because the resultant design force for the stiff side elements are governed by Eq. (3b) with  $\delta = 1.0$ .

The results for the building given as an example above clearly show that torsional effects may be quite significant in buildings with a flexible floor diaphragm. In such buildings, neither the floor diaphragm flexibility nor the torsional response can be ignored. Moreover, ignoring either accidental torsion or torsional amplification may cause significant differences in design forces. However, when the floor diaphragm is completely or significantly flexible (Tena-Colunga and Abrams 1996), each individual frame responds almost independently without any interference from the others and the torsional contribution may be significantly diminished.

## Summary and Conclusions

In seismic analysis of buildings, the floor slab is usually assumed to be rigid in its own plane. However, for many buildings that are long and narrow or have stiff end walls, floor diaphragm flexibility must be accounted for in the distribution of lateral load. Considerable research has been reported in the literature on the dynamics of flexible floor diaphragm buildings; however, the issue of seismic design of such buildings that takes into consideration torsional provisions of the codes has not yet been addressed. In this paper we developed a framework for analysis of such buildings following usual codal requirements for torsion. The building is assumed to have a single wing only, i.e., buildings with multiple wings (e.g., L, V, Y, etc. shaped) are not considered.

The definition of center of rigidity for rigid floor diaphragm buildings is extended to flexible floor buildings. The no-torsion condition for flexible floor buildings is defined such that center nodes at either end of the diaphragm are constrained so that they undergo equal horizontal displacement. The proposed analysis procedure considers the final response as the superposition of three cases: the no-torsion case, amplification of the static eccentricity, and accidental torsion. The proposed procedure ensures that the resultant member force is close to that of rigid floor buildings as the floor diaphragm rigidity increases.

Analysis results of a sample building clearly show the significance of considering the torsion provisions of design codes for asymmetric flexible diaphragm buildings. It is seen that treating the diaphragms of such buildings as rigid for torsional analysis may cause considerable error. The example also illustrates that the

contribution of accidental torsion as well as the torsional amplification terms can be quite significant. However, the usual codal specification of accidental eccentricity as a fraction of the building dimension may be somewhat conservative for such buildings and this issue needs to be addressed in the future.

Horizontal offset buildings constitute a class of structures that are particularly prone to in-plane floor deformation and torsion occurring simultaneously. The present work may be useful for developing a methodology for the treatment of such buildings.

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## Notation

The following symbols are used in this paper:

- $b_{cmj}$  = distance of CM of  $j$ -th floor from center node of left end of diaphragm;
- $b_j$  = lateral dimension of  $j$ -th floor normal to direction of ground motion;
- $e_{dj}$  = design eccentricity of  $j$ -th floor;
- $e_{sj}$  = static eccentricity of  $j$ -th floor;
- $F_{cm}$  = response with load profile applied in proportion to mass distribution along floor length;
- $F_{cr}$  = response due to design load profile when center nodes of both ends of diaphragm at all floor levels are constrained to undergo equal horizontal displacement;
- $F_{cr}^{(j)}$  = response due to design load at constrained  $j$ -th floor while other floors are unconstrained;
- $F_d$  = design response;
- $F_j$  = lateral load at  $j$ -th floor;
- $\{F_j\}$  = lateral load profile;
- $R_{ac}$  = response due to accidental torsion;
- $w_j^l$  = intensity of load profile at the left end of  $j$ -th floor;
- $w_j^r$  = intensity of load profile at right end of  $j$ -th floor;
- $\alpha$  = torsional amplification factor;
- $\beta$  = normalized accidental eccentricity; and
- $\delta$  = torsional amplification factor.

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