On Sliding Mode Control of Permanent Magnet Synchronous Motor

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Abstract: A Sliding Mode Controller of Permanent Magnet Synchronous Motor (PMSM) is analyzed and designed in this paper. Because of non-linearity, strong couple and disturb sensitivity of PMSM, it is difficult for linear control technology to get perfect control performance of PMSM. Sliding Mode Control (SMC) is an efficient robust control method, which can self-adjust controller structure online, and is non-sensitive to parameter variety and outside disturb. dq Coordinate based rotor reference model of PMSM is built up. Principles and characters of SMC are analyzed in theory. Model and design of SMC of PMSM are deduced in details. Finally, simulation model is built up and various simulations are carried out under MATLAB/Simulink. Simulation results show that SMC is more robust than PI.

Key Words: Permanent Magnet Synchronous Motor, Sliding Mode Control, Robust

1 INTRODUCTION

Permanent Magnet Synchronous Motor (PMSM) has some distinguished merits such as high power density, high efficiency, good reliability, simple structure and light weight. Because of its so many merits, PMSM has been widely used in industry, national defense, aviation and aerospace, etc. Also, PMSM and its control systems attract most of research focus [1, 2]. But, PMSM is multivariable, non-linear, strong couple, and very sensitive to outside disturb and parameter variation, which makes it difficult for conventional linear control technologies to get perfect control performance of PMSM [3, 4]. Rapidly developing robust control theory provides an ideal solution for this problem. Sliding Mode Control (SMC) is a special robust control method, which can self-adjust controller structure online and be non-sensitive to parameter variation and outside disturb. SMC is so fit for solving control problems of non-linear uncertain system [5~8].

In this paper, we build dq coordinate based rotor reference model of PMSM. Also, we analyze principles and characters of SMC. We deduce and design SMC controller of PMSM, including parameters evaluation. At last, we build up simulation model of SMC and PI, and carry out some simulations under MATLAB/Simulink.

2 PMSM

Supposed that back electromotive force of PMSM is sinusoidal and its iron loss is negligible, then, rotor reference model of PMSM can be expressed as follows

$$u_d = \frac{d\psi_d}{dt} - \omega\psi_q + R_s i_d \tag{1}$$

$$u_q = \frac{d\psi_q}{dt} - \omega\psi_d + R_s i_q \tag{2}$$

$$\psi_d = L_d i_d + \psi_f \tag{3}$$

$$\psi_q = L_q i_q \tag{4}$$

$$\begin{aligned} T_e &= \frac{3}{2} p(\psi_d i_q - \psi_q i_d) \\ &= \frac{3}{2} p\left[\psi_f i_q + (L_d - L_q) i_d i_q\right] \end{aligned} \tag{5}$$

$$J\frac{d\omega}{dt} = T_e - T_L - B\omega \tag{6}$$

In equations (1) ~ (6), $u_d, u_q, i_d, i_q, \psi_d, \psi_q, L_d, L_q$ are dq coordinate components of stator voltage, stator current, flux and inductance. R_s is stator resistor. ψ_f is permanent magnet flux of rotor. T_L is load torque. T_e is electromagnetic torque. B is frictional coefficient. J is rotational inertia. ω is mechanical angular velocity. p is poles pair of rotor.

3 SMC

SMC is a kind of special non-linear control method while its control is discontinuous. The structure of SMC control system is not fixed, but varying purposefully during dynamic process according to current system state. In consequence, control system will run with state trajectory of preset sliding mode. Because this sliding mode can be designed and it is irrelative with controlled object parameters and outside disturb, SMC has some prominent features such as rapid response, robust and non-sensitive to parameter variation and outside disturb, convenient to be realized physically.

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3.1 Definition

Supposed that there is a control system which is shown in equation (7),

$$\dot{x} = f(x,u,t), \quad x \in R^n, u \in R^m, t \in R \tag{7}$$

Here, we should confirm a switching function $S(x), S \in R^m$, and solve variable structure control function as shown in equation (8).

$$u = \begin{cases} u^+(x), S(x) > 0 \\ u^-(x), S(x) < 0 \end{cases} \tag{8}$$

In equation (8), $u^+(x) \neq u^-(x)$, which will lead to that:

(1) Phase path outside of switching surface $S(x) = 0$ will enter into switching surface in finite time;

(2) Switching surface is region of sliding mode;

(3) Sliding mode running is gradually stable and has nice dynamic performance.

3.1 Basic Characters

(1) Existence and accessibility of sliding mode

If there is a non-zero dimension region on switching surface $S(x) = 0$, whose vectors $f^+ = f(x,u^+,t)$ and $f^- = f(x,u^-,t)$ have normal projections with different sign and opposite direction, system with equation (7) will maybe have sliding mode running. That's to say, mathematical expression of sliding mode existence is

$$\begin{cases} \lim\limits_{s \to +0} \dot{S} < 0 \\ \lim\limits_{s \to -0} \dot{S} > 0 \end{cases} \tag{9}$$

Equation (9) indicates that in scope of switching surface, system running path will enter into switching surface in finite time. Equivalent form of accessing condition is shown in equation (10).

$$S\dot{S} < 0 \tag{10}$$

Switching function $S(x)$ must be derivative and zero-crossing, i.e. $S(0) = 0$.

(2) Dynamic Quality

Sliding mode variable structure control system running consists of two components, normal running and sliding mode running.

When normal running, system paths are always out of $S = 0$ or crossing switching surface in finite times. In order to improve dynamic qualities of normal running, reaching law is specified for running path in some ways as

$$\frac{ds}{dt} = -\xi sign(s) - f(s) \tag{11}$$

In equation (11), $f(0) = 0$. If $s \neq 0$, $sf(s) > 0$, which meets existence condition of generalized sliding mode.

When sliding mode running, system paths are always on sliding mode region of switching surface. System will run in sliding mode beside switching surface and along switching surface $S(x) = 0$. Dynamic qualities of sliding mode running are determined by switching function and its parameters.

$$\lim_{s \to 0} s\frac{ds}{dt} = 0, s(0,0,\ldots 0) = 0 \tag{12}$$

4 PMSM-SMC

When motor running, its inner temperature will arise, and its parameters such as winding resistor will vary. Also, load variation will affect system's stability and robustness. Therefore, characters of controller are so critical for performance of system.

For SPMSM, $L_d = L_q = L$, motor's mathematical model can be expressed as state equation (13).

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & p\omega & 0 \\ p\omega & -\frac{R}{L} & -\frac{p\psi_f}{J} \\ 0 & \frac{p\psi_f}{J} & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{u_d}{L} \\ \frac{u_q}{L} \\ -\frac{T_L}{J} \end{bmatrix} \tag{13}$$

And we adopt $i_d = 0$ vector control to decouple, so state equation (13) can be rewritten as equation (14).

$$\begin{bmatrix} \dot{i}_q \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{p\psi_f}{J} \\ \frac{p\psi_f}{J} & 0 \end{bmatrix} \begin{bmatrix} i_q \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{u_q}{L} \\ -\frac{T_L}{J} \end{bmatrix} \tag{14}$$

In order to introduce SMC into system of PMSM, we set some state variables:

(a) $x_1 = \omega_{ref} - \omega_r$, which is angular velocity error;

(b) $x_2 = \dot{x}_1$, which is input of SMC controller.

Reference current i_{qref} is output of SMC controller. If viscous coefficient is $B = 0$, then system mathematical model is rewritten as equation (15).

$$\begin{cases} \dot{x}_1 = x_2 = -\frac{p}{J}(1.5p\psi_f i_q - T_L) \\ x_2 = -\frac{1.5p^2\psi_f}{J}\dot{i}_q \end{cases} \tag{15}$$

If we set these variables $a = \frac{1.5p^2\psi_f}{J}$, $b = \frac{pT_L}{J}$, $U = \dot{i}_q$, then equation (15) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -a \end{bmatrix} U \tag{16}$$

In order to let system running in steady state directly without overshoot, and system non-sensitive to parameter variation, first order sliding mode switching function is selected, as expressed as equation (17).

$$s = x_1 + cx_2 \tag{17}$$

Straight line $s = 0$ is switching line, which is discontinuous and controls u. For sliding, two sides of switching line must meet the condition of $s\dot{s} < 0$, which can ensure that state variable x always faces switching line whenever it lies on both sides of switching line $s = 0$. When system is sliding, state of phase surface path is considered to meet switching line equation, that is, s holds up with zero.

$$s = x_1 + cx_2 = x_1 + c\dot{x}_1 = 0 \tag{18}$$

And its solution is

$$x_1 = x_1(0)e^{-\frac{t}{c}} \tag{19}$$

Sliding mode variable structure controller fed by complete state variables is

$$U = \varphi_1 x_1 + \varphi_2 x_2 \tag{20}$$

In equation (20),

$$\varphi 1 = \begin{cases} \alpha 1, x_1 s < 0 \\ \beta 1, x_1 s < 0 \end{cases} \tag{21}$$

$$\varphi 2 = \begin{cases} \alpha_2, x_2 s > 0 \\ \beta_2, x_2 s > 0 \end{cases} \tag{22}$$

By accessible condition $S\dot{S} < 0$ of sliding mode running, equation (23) can be got.

$$\begin{aligned} s\dot{s} &= (\dot{x}_1 + c\dot{x}_2)s \\ &= [x_2 + c(\phi_1 x_1 + \phi_2 x_2)] \\ &= [(1 - ac\phi_2)x_2 - ac\phi_1 x_1] < 0 \end{aligned} \tag{23}$$

Solution of equation (23) is

$$\begin{cases} \phi_1 = \begin{cases} \alpha_1, x_1 s > 0 \\ \beta_1, x_1 s < 0 \end{cases} \\ \phi_2 = \begin{cases} \alpha_2 > \dfrac{1}{ac}, x_2 s > 0 \\ \beta_2 < -\dfrac{1}{ac}, x_2 s < 0 \end{cases} \end{cases} \tag{24}$$

Because of non-linearity of sliding mode control, i.e. switching performance, system definitely has vibration. While output of SMC is filtered by an integrator, system vibration may be weakened evidently, and system steady error can be eliminated.

Output of SMC is

$$i_{qref} = \frac{1}{s}(\phi_1 x_1 + \phi_2 \dot{x}_1) = \phi_1 \frac{1}{s} x_1 + \phi_2 x_1 \tag{25}$$

From above, framework of SMC is shown in Figure 1.

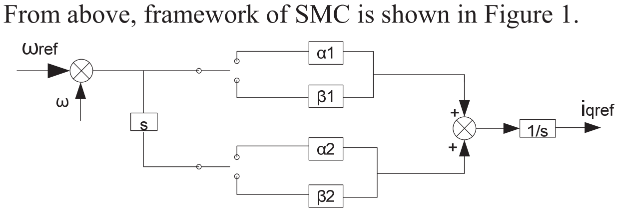


Fig 1. Framework of SMC

And simulation model of SMC is built up in Matlab/Simulink, as shown in Figure 2.

5 Simulation

Parameters of PMSM for simulation are shown in Table1.

$i_d = 0$ Based vector control of PMSM simulating model is built up under MATLAB/Simulink. Speed controllers are PI and SMC individually. Parameters of PI are set with $K_p = 10$, $K_i = 0.5$. And parameters of SMC are set with $\alpha_1 = 4000$, $\beta_1 = 50$, $\alpha_2 = 65$, $\beta_2 = -1$. Both output values of PI and SMC are limited with saturation.

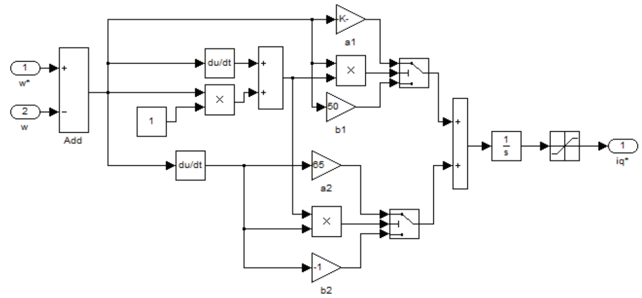


Fig 2. MATLAB/Simulink Model of SMC

Table1 Parameters of PMSM

$R_s(\Omega)$	2.875
$L_d(mH)$	0.85
$L_q(mH)$	0.85
$\psi_f(Wb)$	0.175
p	4

Angular velocity reference of PMSM is set at 100rad/s. Startup with no load, and load $T_L = 5N.m$ is added at $t = 0.2s$. Whole simulating time is 0.4s, which means that system runs in steady state after 0.4s. Simulating results are shown in Figure 3 and Figure 4.

By Figure 3 compared with Figure 4, some conclusions may be got:

(1) Dynamic performance of PI is better than that of SMC. Startup time with no load of SMC is about 0.03s, while that of PI is much shorter;

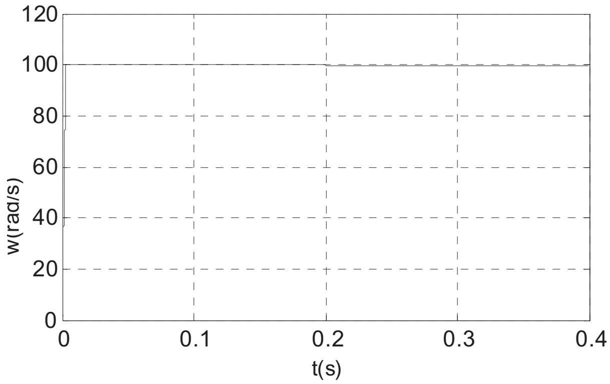
(2) When load added, SMC runs steadily at angular velocity reference but PI has a bigger steady error;

(3) Startup current and torque of SMC is much less than that of PI.

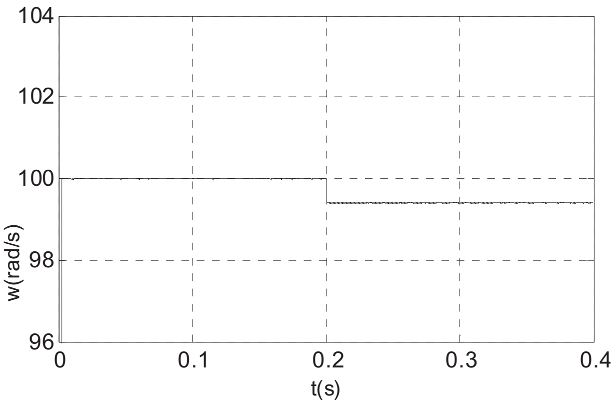
Furthermore, performances of PI and SMC are compared when overload of PMSM. Angular velocity reference of PMSM is set at 100rad/s. Startup with no load, and load $T_L = 5N.m$ is added at $t = 0.2s$. Whole simulating time is 0.4s, which means that system runs in steady state after 0.4s. Simulating results are shown in Figure 5. The simulating results show that when overload, PI has a bigger steady error, but SMC has little steady error.

6 Conclusions

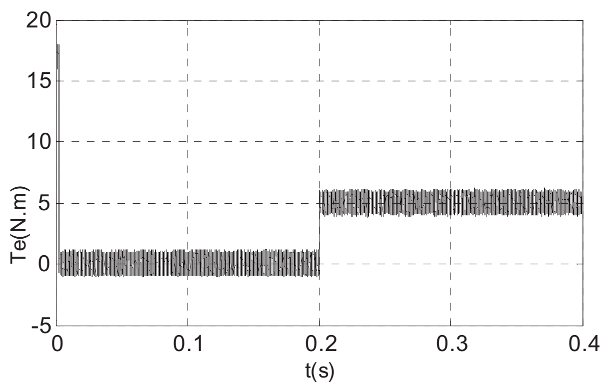
In this paper, model of PMSM in dq coordinate rotor reference and fundamentals of SMC are analyzed. SMC control system of PMSM is deduced in details and built up



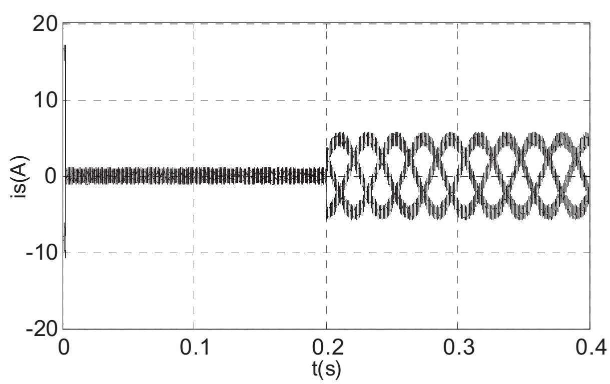
(a) Angular Velocity



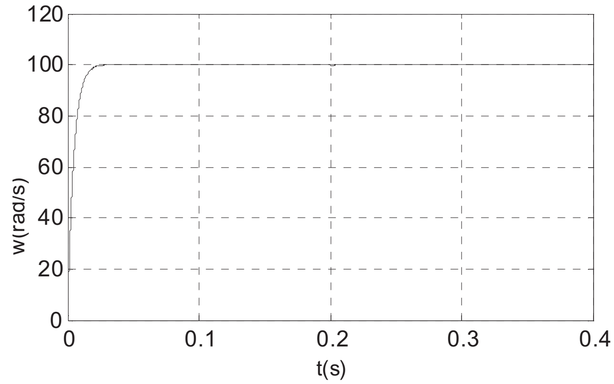
(b) Zoom of Angular Velocity



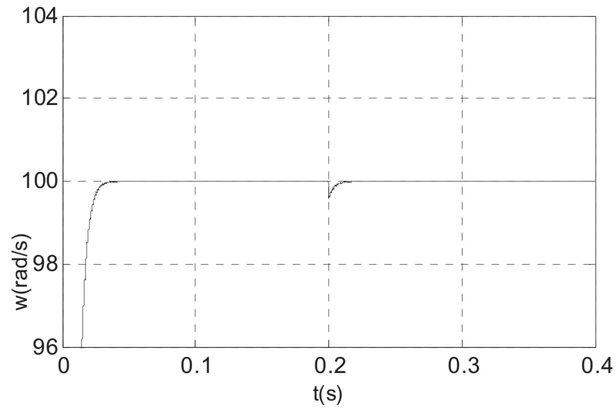
(c) Electromagnetic Torque



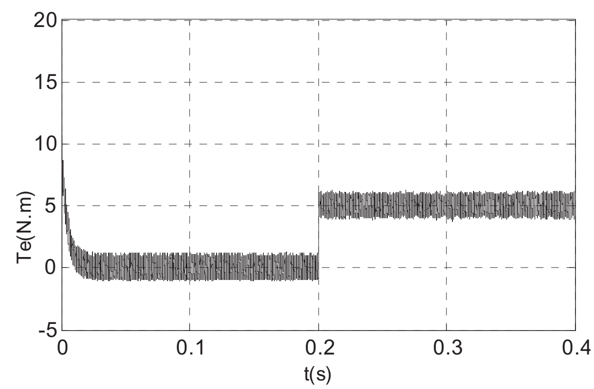
(d) Stator Current
Fig 3. PI



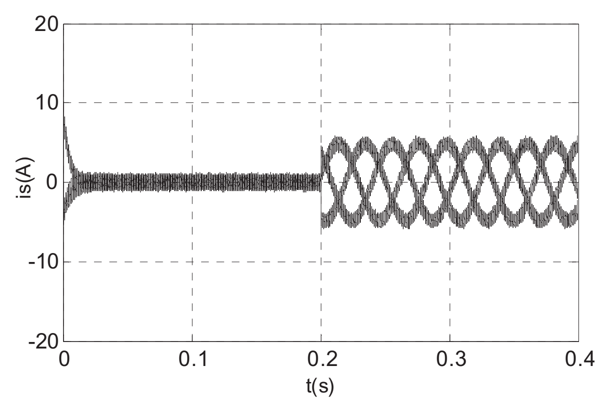
(a) Angular Velocity



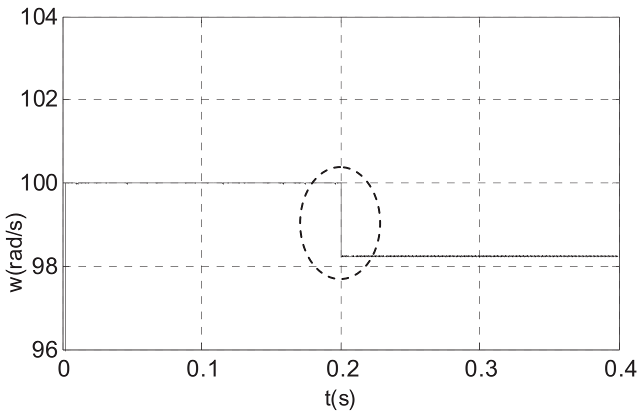
(b) Zoom of Angular Velocity



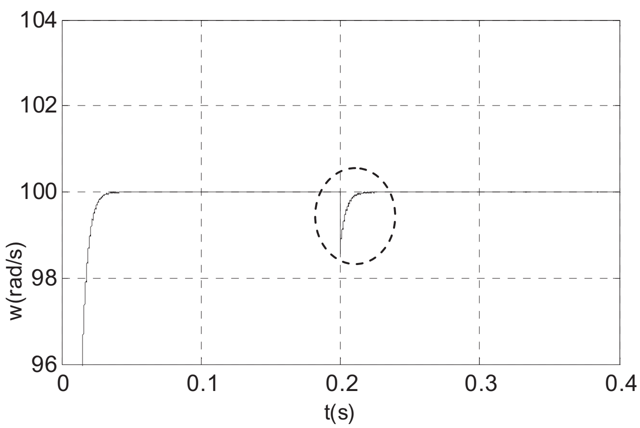
(c) Electromagnetic Torque



(d) Stator Current
Fig 4. SMC



(a) PI



(b) SMC
Fig 5. Overload

under MATLAB/Simulink. Various simulations are carried out. Simulation results are analyzed and compared.

Simulation results indicate that SMC has better steady performance than PI. For load variation when motor running, SMC has very strong robustness, and can enter into steady state in short time, which makes it fit for operating mode of load variation frequently.

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