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A simple power factor calculation for electrical power systems

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ABSTRACT

The accurately and fast estimation of Phase Difference (*PD*) is required between the voltage and current of an AC electrical power system to calculate the Power Factor (*PF*) for defining how effectively the electrical energy is converted into the useful form. Many complex methods based on difficult mathematical equations are presented by the researchers to estimate the *PD*. In this study, a new and simple algorithm derived by using the trigonometric functions is proposed for *PD* estimation to calculate *PF* of a power system. With this method, the fast-time and unaffected by distorted sinusoids of *PD* estimation are carried out by decreasing the number of mathematical equations. The performance of the proposed method is evaluated under the various system conditions by performing the simulation case studies. The results of these studies are given to verify its effectiveness under the distorted system conditions.

Introduction

The rapid developments of the power electronics technology rise the using of controlled AC drives, power converters and non-linear loads, causing the Power Quality Disturbances (PQD). These disturbances can be defined in terms of waveform distortion at the power signals and increasing of the reactive power consumption. They can cause the power loses, terminal voltage drops and negatively affects the safety, quality and economic efficiency of the electricity service. The new electrical politicizes and strategies are required for the operation and management of the electricity service in order to prevent these negative effects and to maintain the reliability and quality.

The term *Power Factor* (*PF*) has emerged with the increasing of the reactive power consumption in the electrical distribution systems recently. This term is used to express how effectively the electrical energy is converted into useful form and simultaneously indicating the quality of the service to the electricity authorized and end-users. *PF* is the ratio between the active (*P*) and the apparent (*S*) powers. It is also defined as the cosine value of the *Phase Difference* (*PD*) between the voltage (*V*) and current (*I*) of an AC electrical power system. It is given in (1).

$$PF = \cos(\varphi) = P/S \tag{1}$$

where φ is the angle value of *PD* between the two sinusoids signals.

The industrial companies try to maintain *PF* of their system close to unity. In other word, an ideal value of *PF* is 1.0, known as unity power factor. If the value of *PF* is under 0.95, it is considered to be poor. Then, its correction can be typically achieved by increasing this value to 0.95–1.0. A poor power factor is generally caused by the distorted current and inductive loads, drawing the reactive power from the utility distribution system. This situation results with the increasing of the phase difference between the voltage and current at the load terminals. Hence, the fast and accurately estimation of *PD* is required to calculate the *PF* for taking the precautions how effectively the value of *PF* can be set to the unity. It has also important significance in the engineering area, such as system model identification, intelligent control, industrial automation, and system characteristic analysis and failure diagnosis [1].

In the literature, the need for the estimating of *PD* between the power signals is justified by numerous papers for the engineering area. In these studies, several types of Zero Crossing [2,3] with their electronic circuit implementations, Discrete Fourier Transform [4–6], Sine-Wave Fit [7] and Ellipse-Fit Methods [8] are proposed. These estimation methods use some form of interpolation between points by using the correlation function to obtain a better resolution. The other types are based on Quadrature Delay Estimator (QDE) if the source is a real-valued sinusoid. These techniques utilize the in-phase and quadrature-phase components of one of the receiver outputs and provide a high-resolution phase-shift estimate [9–11]. The applications of QDE based methods are simple according to the other types of *PD* estimation methods.

The quadrature-phase component is obtained from delayed versions of the input signal such as phase-shifting as the integer number times of $\pi/2$ [9] and time-shifting as the integer number samples [10,11]. If the frequency of the signal is known, phase-shifting could be achieved as exactly $\pi/2$ by an integer number

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of sampling periods. However, calculating QDE with a non-integer number of signal periods causes a bias in estimation [11]. The time-shifting methods are not enough for the estimation of *PD* due to that the additive methods are used for improving their performance [10,11]. Also, these methods are lack of why the quadrature-phase components are used and where the equations of *PD* estimation are derived.

This paper presents a new and simple method derived by using the trigonometric functions is proposed for *PD* estimation to calculate the *PF* of a power system. The much faster and accurate of *PD* estimation are provided by minimizing the number of mathematical equations. The theoretical explanation is given in detail by - supporting its implementation by using simulation program named as Power System Computer Aided Design (PSCAD). The proposed method is evaluated by performing simulation cases under the various system conditions. The results of case studies are given to demonstrate its effectiveness under the distorted system conditions.

The rest of the paper is organized as follows. Section 'Mathematical derivation of the proposed method' presents the mathematical derivation of the proposed method. Section 'Improvement of the proposed method for Power Quality Disturbances' describes the improvement of the proposed method for power quality disturbances. Section 'Case studies' reports the evaluation of the case studies by comparing the simulation results with their real values. Section 'Conclusions' concludes this paper.

Mathematical derivation of the proposed method

The main variables of a single-phase power system are *V* and *I*, drawn by the electrical circuit elements. If there are no harmonic source and non-linear loads in the system, these variables can be expressed by using the trigonometric functions given in the following.

$$V = V_m \cdot \sin(w_0 t + \varphi_1) \tag{2}$$

$$I = I_m \cdot \sin(w_0 t + \varphi_2) \tag{3}$$

where w_0 is the angular velocity calculated by taking the derivative of the angular displacement in a per-unit time ($w_0 = d\varphi/dt$), $V_m - I_m$ and $\varphi_1 - \varphi_2$ are the maximum values and unknown initial phases of *V* and *I*, respectively.

The calculation of main variables known as reference voltage (V_r) and current (I_r) gets complicated for three-phase -phase power system. They are also used to calculate the other variables such as *P* and reactive power (*Q*), *PF*, and phase angle. If there are no non-linear loads and harmonic source in the system, the instantaneous value of V_r and I_r can be expressed by using complex functions given in the following.

$$V_r \cdot e^{j(w_0 t+\varphi_1)} = \frac{2}{3}(V_a \cdot e^{jw_0 t} + V_b \cdot e^{j(w_0 t+2\pi/3)} + V_c \cdot e^{-j(w_0 t+2\pi/3)}) \tag{4}$$

$$I_r \cdot e^{j(w_0 t+\varphi_2)} = \frac{2}{3}(I_a \cdot e^{jw_0 t} + I_b \cdot e^{j(w_0 t+2\pi/3)} + I_c \cdot e^{-j(w_0 t+2\pi/3)}) \tag{5}$$

where $V_{a,b,c} - I_{a,b,c}$ are the phase-voltages and the line-currents of a three-phase system, respectively.

In the complex $(x - jy)$ space, V_r and I_r rotate with the speed of w_0. According to the load characteristic of the power system, either

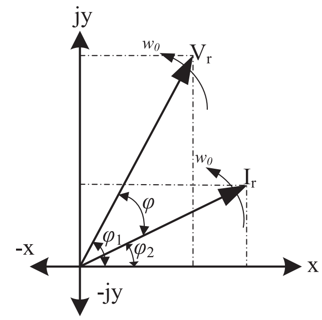

Fig. 1. Vector positions of V_r and I_r in the $x - jy$ space having the inductive characteristic.

I_r tracts V_r with the positive *PD* (φ, where $\varphi_1 > \varphi_2$) known as inductive characteristic or V_r tracts I_r with the negative *PD* ($-\varphi$, where $\varphi_1 < \varphi_2$) known as capacitive characteristic. For instance, the vsituation of the inductive characteristic is illustrated in Fig. 1.

The velocities of V_r and I_r equal to each other due to the having the same frequencies. This situation supplies the constant *PD* between these variables. Then, *PD* can be calculated by subtracting the initial phases (φ_1, φ_2) with each other for $t = 0$. It is expressed in the following form.

$$\varphi = \varphi_1 - \varphi_2, \quad \text{where } t = 0 \tag{6}$$

For $t > 0$, (6) can be written in the form of time-bounded by adding and subtracting of the term "*wt*" to the right side of the equation. Then, if a simple arrangement is made for the trigonometric functions, it can be rewritten in the following form.

$$\begin{aligned} \varphi &= wt - wt + \varphi_1 - \varphi_2, \qquad \text{where } t > 0 \\ &= (wt + \varphi_1) - (wt + \varphi_2) \end{aligned} \tag{7}$$

In a three-phase power system, *P* and *Q* are calculated by using the line currents and the phase voltages in (8) and (9) without using φ in the following form [12].

$$P = V_a \cdot I_a + V_b \cdot I_b + V_c \cdot I_c \tag{8}$$

$$Q = (I_a \cdot (V_c - V_b) + I_b \cdot (V_a - V_c) + I_c \cdot (V_b - V_a))/\sqrt{3} \tag{9}$$

The derivation of φ equals to w_0 in the complex space and gives the ratio between *Q* and *P* in *PF* triangle known as "$S^2 = P^2 + Q^2$". Also, this ratio can be calculated by taking the tangent value of φ. In the proposed method, the tangent value of (7) is taken to calculate φ and written in the following form.

$$\tan(\varphi) = \tan((wt + \varphi_1) - (wt + \varphi_2)) \tag{10}$$

The right side of (10) can be separated to its sinus and cosines components and rearranged by using the sum and difference formulas of the trigonometric functions. Hence, $\tan(\varphi)$ becomes an equation expressed by using sinus and cosines functions. After the simplifications, the last form of (10) is given in the following.

$$\tan(\varphi) = \frac{\sin(wt+\varphi_1)\cdot\cos(wt+\varphi_2) - \cos(wt+\varphi_1)\cdot\sin(wt+\varphi_2)}{\cos(wt+\varphi_1)\cdot\cos(wt+\varphi_2) + \sin(wt+\varphi_1)\cdot\sin(wt+\varphi_2)} \tag{11}$$

The right side of (11) are multiplied and divided with the terms V_m and I_m and then arranged to make it similar with (2) and (3). The last statement of $\tan(\varphi)$ is written in the following form.

$$\tan(\varphi) = \frac{(V_m \cdot \sin(wt + \varphi_1))(I_m \cdot \cos(wt + \varphi_2)) - (V_m \cdot \cos(wt + \varphi_1))(I_m \cdot \sin(wt + \varphi_2))}{(V_m \cdot \cos(wt + \varphi_1))(I_m \cdot \cos(wt + \varphi_2)) + (V_m \cdot \sin(wt + \varphi_1))(I_m \cdot \sin(wt + \varphi_2))} \tag{12}$$

The multiplication of V_m with cosine value of φ_1 and the multiplication of I_m with cosine value of φ_2 are the new terms required to be expressed by utilizing (2) and (3). It can be accomplish by using the trigonometric relationships and taking the first derivations of these equations. For instance, the multiplication of V_m with cosine value of φ_1 can be obtained by taking the first derivations of (2). This formulation is the novelty of the proposed method according to QDE based methods. The derivation of new terms by using the proposed method is demonstrated in the following.

$$V' = V_m \cdot \frac{\partial}{\partial t}(\sin(wt + \varphi_1) \tag{13}$$

$$V' = \omega \cdot V_m \cdot \cos(wt + \varphi_1) \tag{14}$$

$$V_m \cdot \cos(wt + \varphi_1) = V'/w \tag{15}$$

The multiplication of I_m with cosine value of φ_2 can be also obtained by taking the first derivations of (3). It is given in the following.

$$I_m \cdot \cos(wt + \varphi_2) = I'/w \tag{16}$$

The right side of (12) is arranged by considering (15), (16) and then φ is calculated by taking the arc-tangent of last statement of $\tan(\varphi)$. It is given in the following.

$$\varphi = \tan^{-1}\left(\frac{V \cdot (I'/w_0) - (V'/w_0) \cdot I}{(V'/w_0)(I'/w_0) + V \cdot I}\right) \tag{17}$$

Improvement of the proposed method for Power Quality Disturbances

The harmonics of V and I are the most common PQDs in the electrical distribution systems. They generally emerge during the short circuit, grounding problems and working of the switching mode power supplies. For this reason, (2) and (3) should be modified by adding the harmonics [13]. Then, they can be rewritten in the following form.

$$V_{m+h} = V_m \sin(w_0 t + \varphi_1) + \sum_h = 2^{\infty} V_h \sin(hw_0 t + \varphi_1) \tag{18}$$

$$I_{m+h} = I_m \sin(w_0 t + \varphi_2) + \sum_h = 2^{\infty} I_h \sin(hw_0 t + \varphi_2) \tag{19}$$

where h is the harmonic indices, V_h and I_h are the amplitude of each harmonic for V and I, respectively.

In the real power system, the derivation of (17) is inadequate for the precise calculation of PD due to the harmonics. It should be more robust by considering (18) and (19). According to this situation, the improvement can be carried out by eliminating the negative effects of the disturbances. Therefore, the digital filters are used in the application of the proposed method. The realization of the proposed method is illustrated in Fig. 2.

The robustness of the prosed method is made by considering (18) and (19) equations. Its application can be divided into five parts. In the first-part, the Butterworth Filters having a third-order with the cutoff frequency 35 Hz is used to eliminate the harmonics and DC offsets of the input signals. Then, the filter outputs are multiplied with the coefficient (K_f) to equalize the input and output of the each filter. For the Butterworth Filters, K_f is taken as 2.1795 in the prosed method.

According to (15) and (16), the innovation of the proposed method is actualized by taking the derivation of the filter outputs and then dividing of the results to the term "w_0" in the second-part. The instant value of PD is obtained by using the required outputs of the first and second parts in (17) at the third-part. However, the fluctuation can be occurred on the curve of PD if the input signal contains the high harmonic level. Due to this reason, the time average (*mean*) of the result is taken to remove the ripples in the fourth-part [9]. In this application, 100 samples are taken in one period of the input signal to obtain the constant value of φ by minimizing the ripples. Then, it is given to the fifth-part for calculating PF by using φ in the cosine function.

Case studies

To demonstrate the validation of the proposed method, a simple test system consisted of uncontrolled rectifier and non-linear load is constructed in the simulation program and then the proposed method is examined with several case studies by considering the balanced and unbalanced system conditions.

In the test system, the voltage source is selected as an ideal type with the rating of 0.38 kV. The uncontrolled-rectifier is added to the power circuit for obtaining the harmonic source and the three-phase fault blocks are used for creating the unbalanced system conditions. These blocks and the rectifier can be activated or deactivated by using the two breakers to make the different case studies. The reference PD of case studies is calculated by taking the arc-tangent of (8) divided by (9) and the reference PF is calculated by using (8) and (9) in (1). The power circuit of the simulation test system is illustrated in Fig. 3.

Case 1: Ideal test system

The Breaker-A (BRKA) of the rectifier and Breaker-B (BRKB) of the fault block are taken into the open positions to leave the loads, having a value of 127, 171Ω∠44.99° for each phase, alone in the test system. The phase-voltage (v_a) and current (i_a) of line-A are measured from the source side and their values are used in the algorithm given in Fig. 2 to calculate the values of PD and PF. Then, the results of these variables are compared with their real values in Fig. 4.

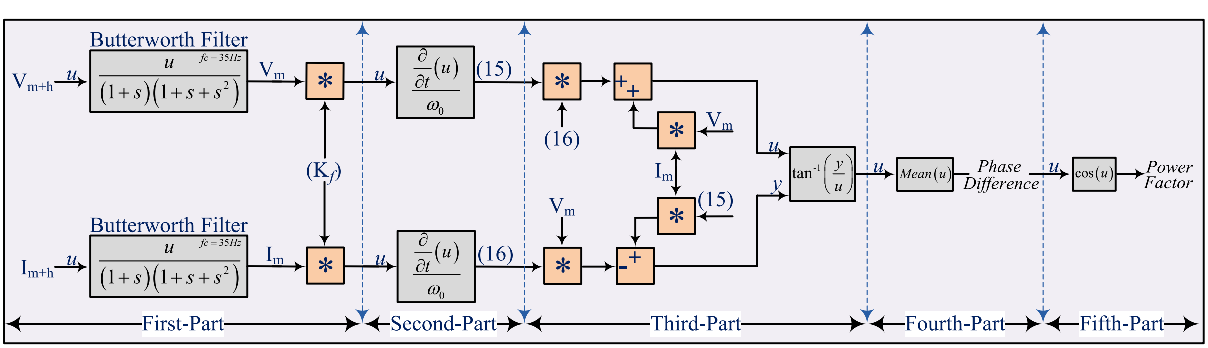

Fig. 2. Application of the proposed method.

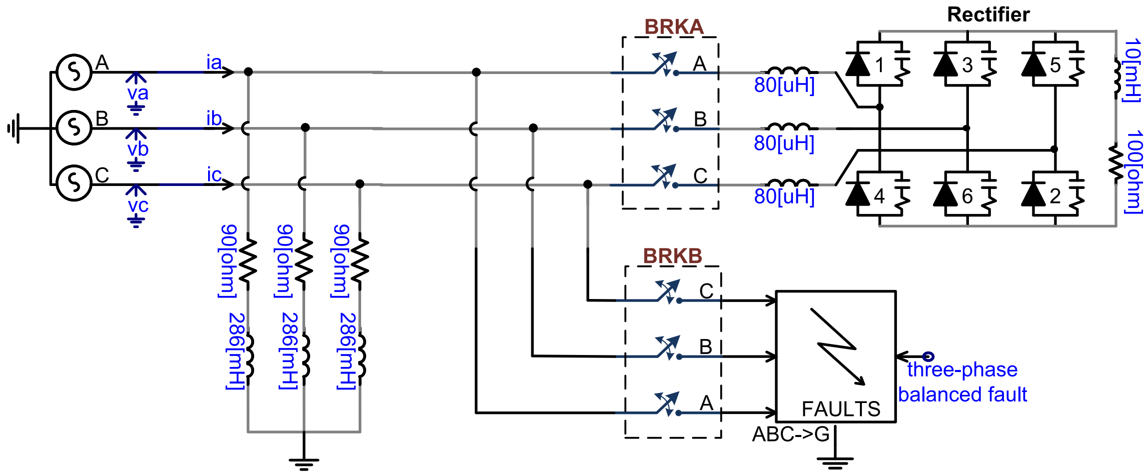

Fig. 3. Power circuit of the simulation test system.

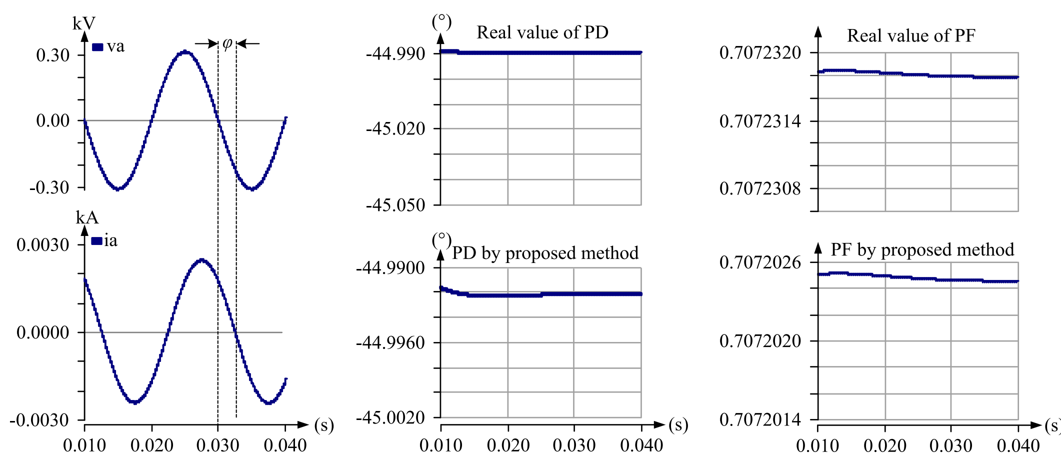

Fig. 4. Simulation results of the Case 1.

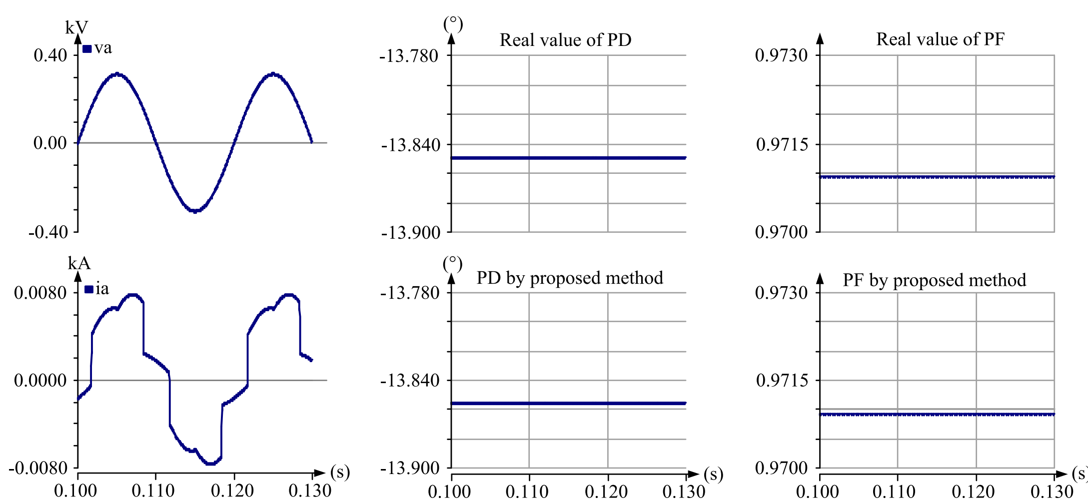

Fig. 5. Simulation results of the Case 2.

Case 2: Test system with harmonic source

In this case study, a power system consisted of a harmonic source is created by taking the close and open positions of BRKA and BRKB, respectively. Hence, the rectifier is only connected to the test system. During the simulation, the Total Harmonic Distortion (THD) of the line current is calculated as 21.87% by using Fast Fourier Transform (FFT). The simulation results of *PD* and *PF* calculated by proposed method are compared with their real values in Fig. 5.

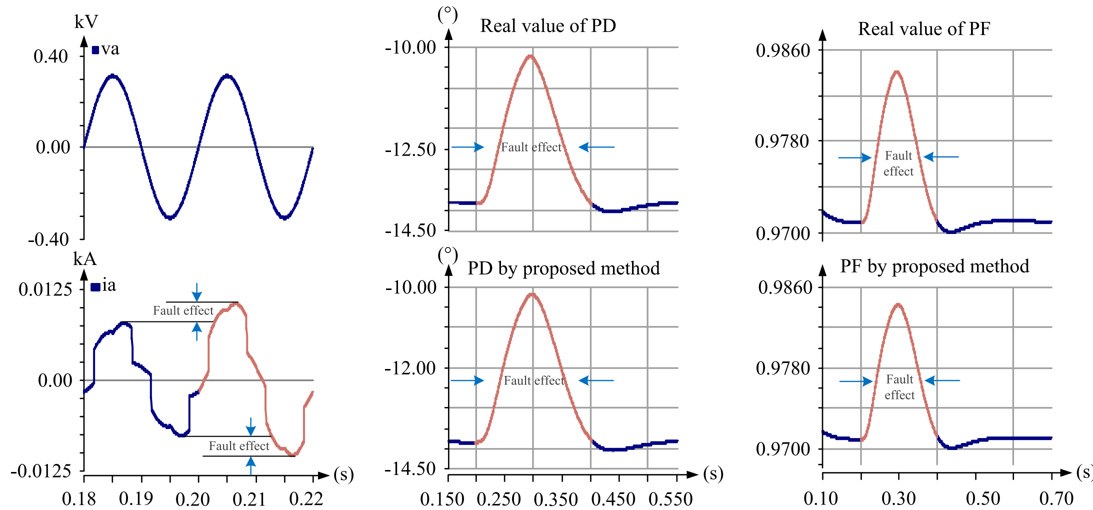

Fig. 6. Simulation results of the Case 3.

Table 1
Simulation results of the Case 4.

Events	V (kV)	I (kA)	Real *PD* and *PF*	*PD* and *PF* by proposed method	Relative error (%)
Events 1	0.38 sin(*wt*)	0.08 sin(*wt* + 50°)	50.00° – 0.64270	50.00° – 0.64270	0.0000
Events 2	0.38 sin(*wt*) + 0.14 sin(3*wt*) + 0.05 sin(5*wt*)	0.08 sin(*wt* + 50°) + 0.01 sin(3*wt* + 50°) + 0.005 sin(5*wt* + 50°)	50.00° – 0.64270	49.99° – 0.64280	0.0054
Events 3	0.38 sin(*wt*) + 0.10 sin(3*wt*) + 0.03 sin(5*wt*) + 0.10 sin(7*wt*)	0.08 sin(*wt* + 70°) + 0.01 sin(3*wt* + 70°) + 0.005 sin(5*wt* + 50°) + 0.005 sin(7*wt* + 50°)	70.00° – 0.34200	69.99° – 0.34210	0.0140
Events 4	0.38 sin(*wt*) + 0.10 sin(3*wt* + 50°) + 0.03 sin(5*wt*) + 0.10 sin(7*wt* + 50°)	0.08 sin(*wt* – 35°) + 0.01 sin(3*wt* + 30°) + 0.005 sin(5*wt* + 60°) + 0.005 sin(7*wt* + 80°)	−35.00° – 0.8191	−35.00° – 0.8191	0.0000

Case 3: Non-ideal test system

During the fault conditions, the behavior of the proposed method is observed and examined by creating the non-ideal test system in this case study. For this reason, the rectifier and fault block are connected to the test system by taking the close positions of the breakers. The balanced line-ground fault is applied to the test system between 0.20 and 0.40 s. During the simulation, THD of the line current is measured as 21.87%. The simulation results of *PD* and *PF* calculated by proposed method are compared with their real values in Fig. 6.

Case 4: The sinusoidal signals mixed with noise

The two sinusoidal signals (x_1 and x_2) created by utilizing (2) and (3) are mixed with noises having different frequencies (hw_0) and amplitude (V_h and I_h). The value of hw_0, the amplitudes of x_1 (V_m) and x_2 (I_m), V_h and I_h are selected by considering the most found harmonics in the power system. The value of φ between x_1 and x_2 is manually changed by the users. The simulation results of *PD* and *PF* calculated by proposed method are compared with their real values in Table 1.

Conclusions

In this study, a new and simple algorithm is proposed for *PD* between two sinusoidal signals having a same frequency to calculate the accurately and fast estimation of *PF*. The proposed method is based on the simple trigonometric functions instead of the complex mathematical equations according to the other methods. Its theory is given by supporting mathematical derivation and is improved by considering PQD.

The performance of the proposed method is evaluated under the various system conditions by performing simulation case studies. In the first case study, the ideal system having the inductive load is formed by changing the breakers positions. The value of φ between V and I is easily realized by the readers in Fig. 4 due that there are no any harmonics at the system variables. In the second and third studies, the systems having the current harmonics and the line-ground faults are form to see the negative effects of the harmonics and faults, respectively. In these case studies, φ can be calculated with numerical methods due to the harmonics. Also, the negative effects of the faults are easily realized by the readers in Fig. 6. According to the results, *PD* and *PF* are correctly calculated by proposed method at these studies. In the fourth study, the two sinusoidal signals are mixed with noises having different frequencies and amplitude. The values of *PD* and *PF* are calculated with small errors (less than 0.015%) ignored by the users.

The simulation results show that the proposed method correctly and quickly calculates the values of *PD* and *PF*. In addition, the implementation of its algorithm can be easily integrated to the micro-controllers and the digital signal processors instead of FFT and the other methods due that the less space is required in the memory of these devices. The other advantage of the proposed method is the inputs (u and y) of the fourth-part illustrated in Fig. 2. The values of u and y represent the instant values of the active and reactive powers of the test system, respectively. Hence, the proposed method can be easily integrated to the system's power calculation besides the extraction of *PD* and *PF*. However, the fluctuation can be occurred on the curve of u and y if the input signals contain the high harmonic levels. If the time averages of u

and y are separately taken to remove the ripples, their constant values are obtained at the fourth-part.

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