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A macro stress test model of credit risk for the Brazilian banking sector☆

Francisco Vazquez[a], Benjamin M. Tabak[b,*], Marcos Souto[a]

[a] International Monetary Fund, United States
[b] Central Bank of Brazil and Universidade Católica de Brasília, Brazil

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ABSTRACT

This paper proposes a model to conduct macro stress test of credit risk for the banking sector based on scenario analysis. We employ an original bank-level data set that splits bank credit portfolios in 21 granular categories, covering household and corporate loans. The results corroborate the presence of a strong procyclical behavior of credit quality, and show a robust negative relationship between the logistic transformation of non-performing loans (NPLs) and GDP growth, with a lag response of up to three quarters. The results also indicate that the procyclical behavior of loan quality varies across credit types. This is novel in the literature and suggests that banks with larger exposures to highly procyclical credit types and economic sectors would tend to undergo sharper deterioration in the quality of their credit portfolios during an economic downturn. Lack of sufficient portfolio granularity in macro stress testing fails to capture these effects and thus introduces a source of bias that tends to underestimate the tail losses stemming from the riskier banks in a system.

1. Introduction

There has been a growing literature on stress testing in the recent years. The importance of these exercises has been highlighted by the recent crisis and the cascade of bank failures in many countries. A deep understanding of the resilience of a banking sector to adverse macroeconomic scenarios is of crucial importance for the proper evaluation of systemic risk and has a direct connection with the development of new regulatory and prudential tools.

This paper describes a model to conduct macro stress test of credit risk for the Brazilian banking sector based on scenario analysis. The proposed framework comprises three independent, yet complementary modules that are combined in sequence. The first module uses time series econometrics to estimate the relationship between selected macroeconomic variables, and uses the results to simulate distressed, internally consistent, macroeconomic scenarios spanning two years. The second module uses panel data econometrics to estimate the sensitivity of non-performing loans (NPLs) to GDP growth, and uses the results to simulate the evolution of credit quality for individual banks and credit types under distressed scenarios.[1] This module exploits a rich database that tracks the evolution of NPLs for 78 individual banks and 21 categories of credit for the 2001–2009 period.[2] The third module uses the predicted NPLs as a proxy for distressed probabilities of default (PDs) and combines this information with data on the exposures and concentration of bank credit (gross loans) portfolios to estimate tail credit losses, using a credit value-at-risk (VaR) framework.

This paper makes three main contributions to the literature on stress testing. First, it exploits a rich partition of bank credit portfolios by borrower types (i.e., consumer versus corporates) and economic sectors, and assesses the extent of differences in the sensitivity of credit quality to macroeconomic conditions across credit types. Second, it illustrates that macro stress test models based

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* Corresponding author. Tel.: +55 613414 2045; fax: +55 613414 1169.
E-mail addresses: benjamin.tabak@bcb.gov.br, benjaminm.tabak@gmail.com (B.M. Tabak).

[1] Non-performing loans (NPLs) for each credit type are computed as the ratio of loans past due in excess of 90 days relative to the total loans in the corresponding category.

[2] The data comes from information reported by the supervised institutions to the credit registry of the Central Bank of Brazil. In general, the credit portfolios analyzed in this paper cover virtually all the bank credit to the private sector under market conditions. This represents about (2/3) of total bank credit, due to the exclusion of credit operations granted under statutory conditions (the so-called directed lending).

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on insufficiently granular data on banks' credit portfolios may be biased in a material way. In particular, macroeconomic stress test models based on undifferentiated credit data may tend to underestimate the credit losses stemming from the highly procyclical credit types (and overestimate the losses associated with the relatively safer credit types). To the extent that the composition of bank credit portfolios varies across institutions, the use of insufficiently granular credit data would tend to underestimate the tail losses of riskier banks, which runs against prudent principles. Third, we present and discuss the results for the Brazilian banking system, which is one of the largest banking systems in Latin America.

The results corroborate the presence of a strong procyclical behavior of credit quality, as indicated by a robust negative relationship between (the logit transformation of) NPLs and GDP growth, with a lag response of up to three quarters. Comparative static exercises indicate that a 2 percentage point drop in yearly GDP growth, which is akin to the maximum drop observed in Brazil during 1996–2008, would cause a twofold increase in NPLs from their March 2009 levels, to about 7 percent. In addition, credit quality displays a strong inertial behavior across all credit types, with autoregressive coefficients implying that a one percentage point increase in NPLs in a given quarter produces a 0.4 percentage increase in NPLs in the next quarter. Credit to individuals, vehicles, and retail commerce were found to be relatively more sluggish.

The models also indicate substantial variations in the cyclical behavior of NPLs across credit types, with no statistically significant differences across state-owned (public) and private banks, suggesting that the results are not due to likely differences in credit origination practices across these two types of banks.[3] At the same time, some credit types appeared to be more sensitive to changes in economic activity, particularly agriculture, sugar and alcohol, livestock, small consumer credit, and textile. Consequently, the quality of these credit types would likely undergo more severe erosion under a protracted drop in economic activity. Banks with higher exposures to these credit types may need to be followed up more closely.

Overall, the stress tests suggest that the Brazilian banking sector is well prepared to absorb the credit losses associated with a set of distressed macroeconomic scenarios without threatening financial stability. Four alternative macroeconomic scenarios, each one projected over two years, were analyzed. These comprised a Baseline reflecting the expected path of GDP growth, and three distressed scenarios that were deemed to be extreme, but nevertheless likely, under current circumstances. Overall, the results of the baseline scenario indicate that NPLs peak to 6.7 percent in the fourth quarter of 2010, before recovering. The simulated NPLs for the distressed scenarios are higher than for the baseline. The more severe deterioration in credit quality is associated with a slowdown in GDP growth akin to two standard deviations below its 2001–09 mean, with overall NPLs reaching a maximum of 8.5 percent, which is about a twofold increase from their starting levels.

The remainder of the paper is structured as follows: Section 2 presents a brief literature review, whereas Section 3 discusses the methodology. Section 4 presents the empirical results. Finally, Section 5 concludes the paper.

2. Literature review

Since the seminal works of Wilson (1997a,b), which present a framework to examine credit risk under distressed macroeconomic conditions, several papers have applied macro stress test tools to assess the resilience of various banking systems to adverse macroeconomic shocks (Gerlach et al., 2003; Pesola, 2001, 2005; Frøyland and Larsen, 2002; Barnhill et al., 2006; Misina and Tessier, 2007; Berkowitz, 1999; van den End et al., 2006; Hoggarth and Whitley, 2003; Boss et al., 2007; Virolainen, 2004; Sorge, 2004, among others).[4] In this literature, the main objective is to gauge the vulnerability of a portfolio (market, credit, or both) to adverse macroeconomic scenarios, or to extreme but plausible events or shocks. The objective of such tests is to make risks more transparent, assessing the potential losses of a given portfolio under abnormal markets. These tools are commonly used by financial institutions as part of their internal models and management systems and to inform decisions regarding risk taking and capital allocation. In addition, these tools have become increasingly more used by financial regulators to evaluate the soundness of the financial systems under their control.

Typically, macro stress tests of credit risk involve three major tasks. First, the development of a model to capture the interrelationships between selected macroeconomic and financial variables. Second, the calibration of parameter vectors linking macroeconomic and financial variables to specific measures of loan performance. Third, the design of adverse macroeconomic scenarios, and the computation of their impact on credit quality and banks' solvency. Usually, the macroeconomic variables used in stress test models include measures of economic activity (i.e., GDP growth, the output gap, and unemployment), and measures of monetary conditions and key prices (i.e., interest rate, exchange rate, inflation, money growth and property prices).

The investigation of how adverse scenarios may impact asset quality and solvency in the banking sector can be done using two approaches: top-down or bottom-up. The first one builds on aggregated data on bank credit portfolios, sometimes split by credit types or economic sectors, and simulates evolution of aggregated credit quality under distressed macro scenarios with the help of time series analysis (see for example Virolainen, 2004; Wong et al., 2006). A key shortcoming of this approach is its limited capacity to assess the financial conditions of individual institutions, which are frequently the focus of the analysis. The bottom-up approach addresses this shortcoming by resorting to the use of bank-level data.[5] Typically, models based on this approach use panel data econometrics to gauge the evolution of asset quality under distressed macroeconomic scenarios, and the results are then mapped into banks' solvency and aggregated to get a systemic picture. Possibly due to data constraints, however, bottom-up models fail to exploit granular data on the characteristics of individual banks' credit portfolios (i.e., portfolio concentration and loan performance by credit types).[6]

Our paper contributes to this literature by presenting a macro stress test model of credit risk that combines the use of bank-level information, with a granular partition of banks' credit portfolios

[3] Recent literature for the Brazilian banking system suggests that there may be important differences across banks due to ownership (Staub et al., 2010; Tabak and Staub, 2007; Tabak et al., forthcoming; Tecles and Tabak, 2010).

[4] See Sorge and Virolainen (2006) for an overview of stress test methodologies. See also Illing and Liu (2006), Blank et al. (2009), Rodriguez and Trucharte (2007), Castrén et al. (2010) and Cardarelli et al. (2011). Foglia (2009) provides a very interesting review of current approaches to stress testing employed by supervisory authorities.

[5] An example is Duellmann and Erdelmeier (2009) which stress-test the credit portfolio of German banks using a different approach (Merton-type multi-factor credit risk model) from ours. The authors show that it is crucial to capture credit risk dependencies between sectors. The focus is on the automobile sector (key sector) and its interdependencies with other sectors.

[6] There is to date little research using the bottom-up approach. Interesting examples are the works of Coffinet and Lin (2010) and Coffinet et al. (2009), which perform a bottom-up stress test for French banks profitability and income subcomponents, respectively.

between consumer and corporate loans, classifying the former by size and the latter by economic sectors. In particular, we assess the sensitivity of credit quality to macroeconomic conditions using a bank-level dataset that keeps track of 21 credit categories during 2001–09. The estimated parameter vectors are used to simulate the evolution of credit quality for individual banks and specific credit types, under adverse macroeconomic scenarios. This information is then combined using a credit portfolio approach to estimate the bank-specific capital needs conditional on the realization of the adverse macroeconomic scenarios.

Overall, the results suggest that the procyclical behavior of credit quality varies across credit types. By failing to account for these differences, current macro stress test models may be biased in a material way, underestimating the riskiness of banks that are more heavily exposed to highly procyclical credit types and economic sectors. We illustrate this bias by running parallel simulations of bank-level NPLs under adverse macroeconomic scenarios, using two approaches. The first, akin to typical macro stress test models of credit risk, uses bank-level data on credit quality, without allowing for differences in the behavior of credit quality across credit types. The second, following the approach presented in this paper, exploits granular information on the characteristics of bank credit portfolios. The findings provide strong evidence of a data aggregation bias that tends to underestimate the impact of macro shocks on the quality of bank credit portfolios. Papers reporting that banking systems were resilient to adverse macroeconomic scenarios may have been partly influenced by this underestimation of credit risk.

3. Methodology

3.1. Overview of the methodology

The stress test framework presented in this paper comprises three components that are integrated in sequence:

- A macroeconomic model to estimate the relationship between selected macroeconomic variables with the help of times-series analysis. This model is used to simulate distressed, internally consistent, macroeconomic scenarios, projected over a two-year horizon.
- A microeconomic model to assess the sensitivity of loan quality to macroeconomic conditions with the help of dynamic panel econometrics. The model is based on bank-level data, using separate equations for 21 credit types. The results are used to simulate the path of NPLs for each bank and for each of the 21 categories of credit, under the distressed macroeconomic scenarios produced in the previous stage.
- A credit VaR model to estimate the banks' capital needs to cover tail credit losses under the distressed scenarios. The model uses the simulated distributions of NPLs for each bank and credit type as a proxy for the distribution of distressed PDs, and combines this information with data on the credit exposures of individual banks using the Credit Risk+ approach with the programs developed by Avesani et al. (2006).

3.2. The macro model

Macroeconomic data on key target series are available at a quarterly frequency, from the first quarter of 2001 to the first quarter of 2009.[7] While the length of the time series is somewhat short, the period covers some important macro events, including a substantial shock in 2002–03, when the referential interest rate shoot up by almost 10 percentage points to 26.5 percent and the exchange rate depreciated from 2.3 to almost 4 Brazilian Real (BRL) per US Dollar (USD). The memory of this shock is important to help model the dynamics of the global financial crisis, which also impacted Brazil, particularly since the third quarter of 2008. The substantial contraction in GDP is an important consideration for the VAR specification as it will, mechanically, force the factor to rebound in a way that may not be completely consistent with macroeconomic dynamics going forward. We present credit growth, GDP growth and changes in the yield curve for the Brazilian economy in Fig. 1.

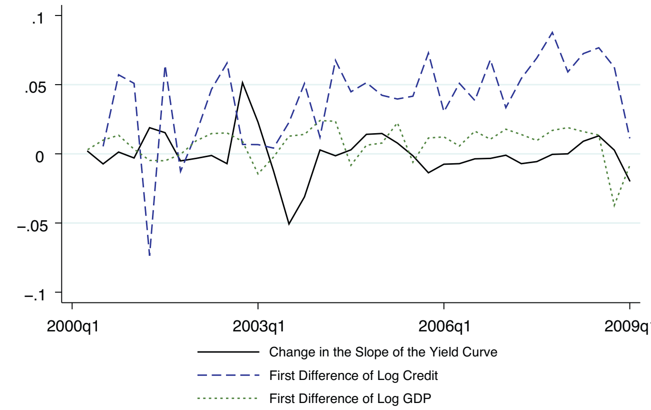

Fig. 1. Selected macroeconomic variables, first differences, 2000–09.

Table 1
Summary statistics of selected variables.

Variable	Obs	Mean	Std. Dev.	Min	Max
D.yc	35	−0.0005	0.0164	−0.0507	0.0514
D.Lncr	35	0.0399	0.0318	−0.0736	0.0878
D.Lngdp	35	0.0075	0.0124	−0.0372	0.0241

The selected specification captures linkages between GDP growth, credit growth, and changes in the slope of the domestic yield curve. We choose a parsimonious specification given the relatively short length of the time series. The variables were selected after exploring the relationships between a larger set of macroeconomic variables restricting the factors to those that were statistically more relevant to the VAR specification, also yielding tighter error bands.[8] The selected variables are defined as follows: (i) GDP growth, *GDP*, computed by taking the first difference to the natural log of the seasonally adjusted GDP series; (ii) credit growth, *Credit*, computed by taking the first difference to the natural log of total (gross) loans in bank credit portfolios; and (iii) the slope of the domestic yield curve, *YC*, measured by the difference between the monetary policy rate (i.e., the Selic), and the long-term interest rate. Summary statistics of the selected variables are presented in Table 1. In order to control for the impact of the global financial crisis in the system, we add a dummy variable that equals one for the last two quarters of the sample (i.e., Q4 2008 and Q1 2009) and

[7] Before 2001 we had the peg regime in exchange rate and a transition to the floating rate regime. After 2001, floating rate regime was in permanent regime.

[8] The set of variables used in the selection of the specification include: the short-term policy rate (i.e., Selic), the spread between bank lending and deposit rates, the US yield curve (measured by the difference between the 7-year and 3-month treasury bill rates, the Chicago VIX index, the EMBI spreads, a commodity price index (proxied by the Commodity Research Bureau index), the unemployment rate, and the exchange rate. We have estimated the correlation for slope between estimations with the 7 years −3 months and 10 years −3 months, and it is above 99 percent.

Table 2
Macro model specification.

Variables	Unrestricted model			Restricted model		
	D.yc	D.lncr	D.lngngdp	D.yc	D.lncr	D.lngdp
LD.yc	0.594***	−0.575**	−0.263***	0.618***	−0.595***	−0.259***
	[0.000]	[0.022]	[0.004]	[0.000]	[0.007]	[0.004]
L2D.yc	−0.027	−0.16	−0.135			−0.054
	[0.885]	[0.580]	[0.205]			[0.536]
L3D.yc	−0.089	0.178	−0.207**			−0.269***
	[0.605]	[0.511]	[0.038]			[0.002]
L4D.yc	−0.03	0.316	−0.059			
	[0.868]	[0.261]	[0.566]			
LD.lncr	0.209**	−0.391***	0.148***	0.239***	−0.306**	0.159***
	[0.013]	[0.003]	[0.002]	[0.001]	[0.014]	[0.000]
L2D.lncr	0.167*	0.051	0.177***	0.180***	0.197*	0.197***
	[0.054]	[0.705]	[0.000]	[0.006]	[0.074]	[0.000]
L3D.lncr	−0.119	0.212	0.079	−0.137*	0.315***	0.065
	[0.162]	[0.112]	[0.106]	[0.074]	[0.008]	[0.135]
L4D.lncr	−0.264***	0.261**	0.04	−0.304***	0.230**	
	[0.001]	[0.032]	[0.371]	[0.000]	[0.028]	
LD.lngdp	0.039	1.100***	−0.514***		0.918***	−0.504***
	[0.856]	[0.001]	[0.000]		[0.000]	[0.000]
L2D.lngdp	0.182	1.129**	−0.524***		0.656*	−0.482***
	[0.557]	[0.020]	[0.003]		[0.089]	[0.002]
L3D.lngdp	−0.001	0.779*	−0.425**			−0.436***
	[0.997]	[0.097]	[0.014]			[0.001]
L4D.lngdp	−0.107	0.606	−0.279*			−0.304**
	[0.696]	[0.159]	[0.078]			[0.019]
Dummy_crisis	−0.01	−0.004	−0.044***			−0.044***
	[0.280]	[0.788]	[0.000]			[0.000]
Constant	−0.002	0.009	0.008***	−0.001	0.014*	0.009***
	[0.674]	[0.264]	[0.005]	[0.802]	[0.061]	[0.001]
Observations	31	31	31	31	31	31
R-Squared	0.63	0.63	0.63	0.56	0.56	0.56
AIC	−16.9	−16.9	−16.9	−16.5	−16.5	−16.5
HQIC	−16.2	−16.2	−16.2	−15.8	−15.8	−15.8
SBIC	−14.9	−14.9	−14.9	−14.5	−14.5	−14.5

p-Values in brackets.
* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

zero otherwise.[9] This variable is treated as exogenous. Unit root tests indicate that GDP growth and credit growth are stationary, but fail to reject the null for the slope of the yield curve, probably due to the short size of the sample. We therefore use the first difference of the series to achieve stationarity. All variables are end of period and in real terms. The model is of the form:

$$y_t = c + \sum_{s=1}^{p} A_s y_{t-s} + Bx_t + \varepsilon_t \qquad (1)$$

where $y = \begin{bmatrix} D.yc \\ D.Ln(Credit) \\ D.Ln(GDP) \end{bmatrix}$, D represents the first difference, $Ln(.)$ represents the natural logarithm of the variables, and x stands for the exogenous regressors.

The ordering of the variables reflects the conjecture that credit markets play a role in the transmission of interest rate shocks to economic activity. The number of lags is set to four, taking into account the frequency of the data and the results of alternative lag order selection criteria (which indicate 2–5 lags).

The estimated coefficients are consistent with *a priori* expectations on the relationship between the selected variables. The results of an unrestricted VAR are presented in columns [1] to [3] of Table 2. According to these, a tightening in monetary policy is associated with a drop in credit growth and GDP growth, and there is a strong positive relationship between the last two variables. There is also evidence that the decline of GDP growth during the last quarter of 2008 and the first quarter of 2009 was larger than otherwise explained by the interaction between the endogenous variables included in the model, as indicated by the coefficient of the dummy variable, which is negative and statistically significant. The results also indicate that the domestic credit markets were somehow isolated from the effects of the global financial crisis, which is likely attributable to the strong expansion of credit by state-owned banks to compensate for the collapse of credit growth by private banks during this period. Similar conclusions can be extracted from the results of a restricted VAR, presented in columns [4] to [6]. Post-estimation tests (not reported to save space), indicate that the models are stable, and that the errors are not autocorrelated and pass standard normality tests. The impulse response functions, together with 95 percent confidence error bands are presented in Fig. 2.

The first difference of the slope of the yield curve would represent a change in the yield curve slope from one period to the next. Changes in yield curve slope are associated with investors' perception about future monetary policy, vis-à-vis current interest rates. For example, if GDP decreases from one period to the next,

9 We tried a number of variables to capture external effects (GDP growth in the US and the EU, commodity prices, EMBI, VIX) with poor (insignificant) t-statistics. Therefore, this may suggest that a decline in foreign demand was not the main reason affecting Brazil. One likely cause is the sharp exchange rate depreciation following the collapse of Lehman Brothers, partly triggered by the combination of heavy capital outflows with the sudden unwinding of corporate positions in foreign exchange rate derivatives, which prompted a number of monetary policy responses by the Central Bank.

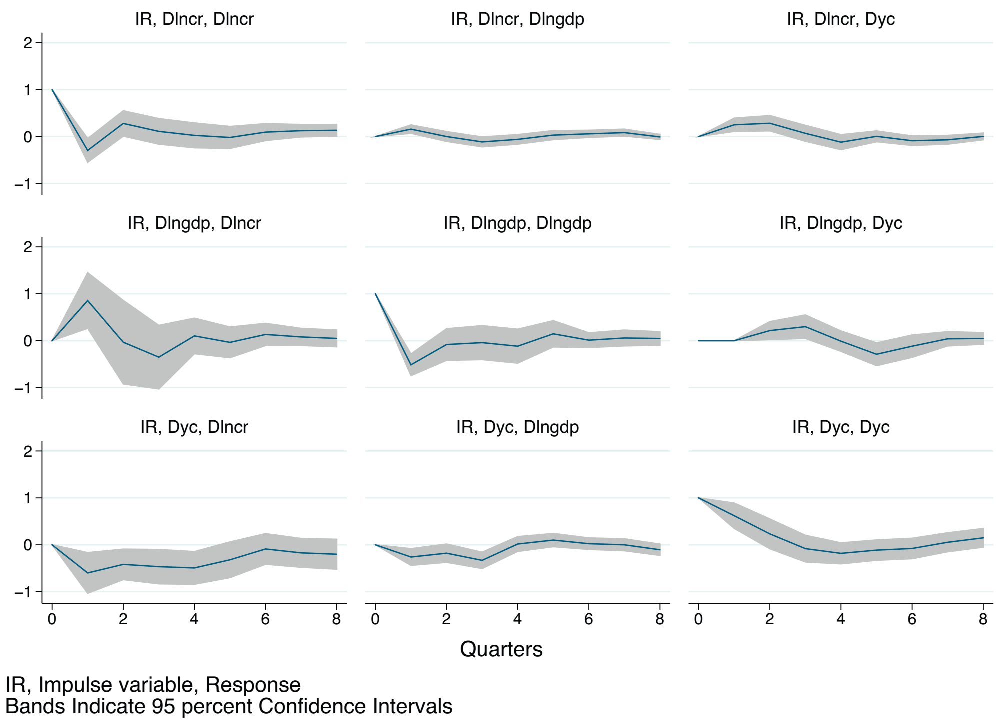

Fig. 2. Macro model impulse response functions. *Note*: This figure presents the impulse-responses of the VAR model described in equation [1]. Dlncr is the first difference in the natural logarithm of bank's credit growth, where credit is estimated as the total loans in the aggregate banking sector portfolio, at the end of the period, and growth is estimated quarter-on-quarter. Dlngdp is the first difference in the natural logarithm of GDP growth, where GDP growth is computed as the natural logarithm of the seasonally adjusted GDP series, quarter-on-quarter, using end of the period numbers for GDP. Dyc is the first difference in the yield curve slope, measured by the difference between the monetary policy yield curve (i.e. Selic), and the long-term interest rate.

investors may expect the interest rates to go down in the future, causing a drop in the slope of the yield curve.

3.3. Microeconomic model

Data on credit portfolios were gathered from the credit registry of the Central Bank of Brazil, which contains rich information on individual credit operations granted by the supervised banks. The registry covers the bulk of credit in the system, leaving aside operations lower than a minimum reporting threshold, and credits granted by unsupervised entities (such as non-financial corporations).[10] The data used in this exercise, however, focuses on lending granted with non-earmarked resources, which accounts for about 70 percent of total credit, as information on directed lending was not available.[11] For the purposes of the analysis, the data were aggregated at the level of individual banks and classified in 21 categories (Table 3). For each one, we have: (i) total (gross) loans, (ii) non-performing loans (NPLs), (iii) number of loan operations, (iv) number of loan operations in default, and (v) (specific) loan-loss provisions.

Overall, the database covers the credit operations of 78 banks, at the quarterly frequency, between 2003q1 and 2009q1. The size of the credit portfolios included in the analysis is rather continuous throughout the sampled period (Table 4). The sample, however, is unbalanced due to the exit or merge of some banks and the incorporation of new ones. As of March 2009, the sample included 49 banks jointly accounting for about 85 percent of total bank credit.[12] The time coverage was dictated by data availability. In particular, the construction of time series going further back in time was not possible due to a change in accounts and data reporting definitions introduced in 2002. The quality of the data was deemed to be good. Several filters were applied to identify potential inconsistencies, and a few data reporting issues were found to be (generally) associated with a specific subgroup of banks.

A look at the bank-level data indicates that credit quality has been relatively poor and extremely heterogeneous across credit types. Overall, NPLs averaged 3.6 percent during the sampled period, which is relatively high considering the favorable macroeconomic environment and the rapid expansion of credit portfolios. Furthermore, credit quality has been dispersed across banks and throughout time, as indicated by the size of the standard deviations of NPLs, which are generally 2–3 times larger than their corresponding mean values (Table 5). The extent of the dispersion of credit quality and the severity of loan nonperformance in some institutions is also illustrated by the NPL ratios of banks in the 90th percentile of the distribution, which exceeded 10 percent in many sectors. Across credit types, the higher average rates of NPLs have been associated with credit to individuals (particularly small and medium-sized loans), firms operating in the services sector, producers of livestock, and electric and electronic equipment.

The evolution of NPLs was also diverse across bank types. Overall, state-owned banks displayed better loan quality during the sampled period, only interrupted by a sharp increase in NPLs on exposures to the petrochemical and food industries in 2005–06

[10] It is important to highlight that the credit registry covers operations which represent more than 80 percent of the total volume of credit. Also, in Brazil most credit operations are performed within the regulated banking system. Therefore, the database is highly representative of the credit operations in Brazil.

[11] Non-earmarked resources are credit granted by financial institutions without implicit or explicit subsidies from the government.

[12] Credit is highly concentrated in Brazil with the largest 5 banks accounting for approximately 70 percent of total credit.

Table 3
Structure of loan portfolios across bank ownership, March 2009, in percent.

	Non-performing loans Private			Share in loan portfolio Private		
	Domestic	Public	Foreign	Domestic	Public	Foreign
Consumer (large)	2.9	2.0	3.3	1.4	5.8	2.5
Consumer (medium)	6.5	2.0	7.1	7.5	13.4	10.7
Consumer (small)	8.9	2.9	7.2	28.3	20.3	25.8
Agriculture	2.7	1.0	3.8	2.0	2.2	2.7
Food	3.2	1.4	2.8	2.2	2.7	2.5
Livestock	2.4	1.2	3.9	3.0	3.9	3.2
Vehicles	4.4	2.3	5.1	3.0	2.6	2.5
Electrical and electronic	6.8	2.9	5.0	1.4	1.5	1.5
Electricity and gas	0.0	0.0	1.1	3.2	3.0	3.7
Wood and furniture	2.9	2.5	2.8	8.8	6.0	8.8
Recreation services	4.7	3.3	4.8	1.8	1.6	1.8
Petrochemicals	2.3	0.7	2.4	3.1	5.6	2.6
Chemicals	3.8	1.6	2.3	1.5	1.6	2.2
Health services	2.7	1.9	2.6	1.9	1.6	2.5
Other services	3.9	3.0	4.0	3.8	1.9	3.2
Metal products	1.3	0.4	1.5	3.2	4.4	2.6
Sugar and alcohol	1.2	1.4	1.4	3.8	1.5	3.1
Textile	6.5	3.1	5.5	2.5	3.3	3.0
Transportation	1.8	1.0	2.2	6.5	3.2	4.2
Retail trade	3.8	1.8	2.9	2.7	3.4	2.6
Other	1.4	0.8	1.3	8.5	10.3	8.2

Source: Central bank of Brazil and authors estimates.

(Fig. 3). Remarkably, the segments of private and foreign banks experienced a moderate, but sustained increase in NPL ratios after 2005, despite rapid credit growth and the supportive economic environment. More recently, since the third quarter of 2008, credit quality deteriorated rapidly and across-the-board, reflecting the impact of the global financial crisis on the macroeconomic and financial environment. As mentioned before, however, these aggregate figures mask large differences in loan quality across individual banks and credit types. In general, the smaller banks have tended to underperform, also displaying higher concentration in their loan exposures to specific credit types.

Table 4
Sample coverage.

	Number of sampled bank				
	Public	Private	Foreign	Total	Total loans (in million BRL)
2003q1	6	37	25	68	214,838
2003q2	7	39	23	69	214,368
2003q3	7	39	22	68	219,499
2003q4	7	38	20	65	239,102
2004q1	6	38	20	64	242,760
2004q2	6	38	19	63	258,230
2004q3	6	37	19	62	268,066
2004q4	6	37	20	63	277,670
2005q1	6	37	21	64	291,032
2005q2	6	36	21	63	303,805
2005q3	6	36	21	63	316,163
2005q4	6	35	21	62	343,966
2006q1	5	36	21	62	357,901
2006q2	5	35	20	60	380,806
2006q3	5	35	19	59	401,241
2006q4	5	35	19	59	438,637
2007q1	5	35	19	59	456,863
2007q2	5	34	18	57	490,680
2007q3	5	33	18	56	533,389
2007q4	3	27	15	45	533,458
2008q1	5	33	18	56	619,536
2008q2	5	32	18	55	676,095
2008q3	4	32	17	53	733,894
2008q4	4	32	16	52	767,665
2009q1	4	29	16	49	779,501

Source: Central bank of Brazil and authors estimates.

The model discussed in this section analyzes the sensitivity of non-performing loans to macroeconomic conditions with the help of dynamic panel econometric techniques. The specification was selected after exploring the sensitivity of NPLs to a combination of candidate macroeconomic and bank-level variables encompassing, inter alia, GDP growth, the unemployment rate, credit growth (both aggregated and bank-specific), long-term and short-term interest rates, bank lending spreads, and the change of the exchange rate (both in nominal and real terms). The inclusion of credit growth in the exploratory specifications was motivated by the observation that credit quality tends to improve in the early stages of an episode of accelerating credit expansion. However, after exploring with various lag structures and using both aggregate and bank-specific credit growth, this variable turned out to be not significant in the regressions. Similarly, and more in line with expectations, the real exchange rate was also not significant, likely reflecting the lack of material dollarization in the credit portfolios of Brazilian banks.

The main criteria guiding model selection was the precision of the parameter estimates and the robustness of the results, reflect-

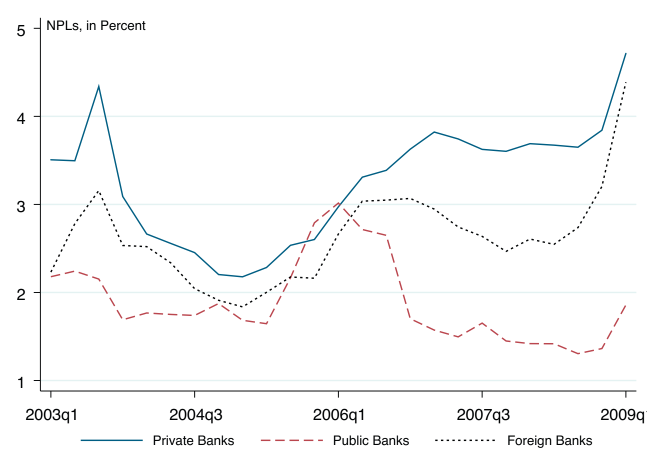

Fig. 3. Evolution of NPLs across bank types.

Table 5
Selected statistics of NPLs across credit types and bank ownership, 2003q1–2009q1 in percent.

	Private domestic			Public banks			Foreign banks			Total sample		
	Mean	St. dev.	Pct. 90	Mean	St. dev.	Pct. 90	Mean	St. dev	Pct. 90	Mean	St. dev.	Pct. 90
Consumer (large)	4.6	14.7	7.3	4.0	6.2	14.0	1.5	4.7	2.7	3.6	11.8	7.1
Consumer (medium)	7.4	12.2	17.6	3.3	3.9	7.9	4.3	7.6	9.9	6.1	10.6	14.7
Consumer (small)	6.9	9.0	14.0	3.0	1.7	4.8	4.7	8.2	10.2	5.9	8.4	12.9
Wood and furniture	5.0	11.1	12.7	3.6	4.8	7.4	1.3	4.1	2.8	3.8	9.1	8.5
Transportation	4.7	13.6	8.9	5.5	11.5	12.2	1.5	6.9	2.1	3.8	11.8	7.7
Petrochemicals	3.9	10.1	9.6	9.7	23.6	26.8	0.7	2.4	1.8	3.6	11.4	7.4
Metal products	2.9	12.4	4.2	2.8	6.8	8.8	0.3	1.6	0.8	2.1	10.0	2.9
Electricity and gas	1.8	7.9	3.1	1.3	5.9	1.5	0.6	3.9	0.7	1.3	6.6	1.5
Livestock	5.4	16.7	8.0	5.8	11.4	17.2	1.4	4.5	2.6	4.2	13.8	6.9
Other services	6.3	14.6	19.3	5.7	8.4	13.6	1.8	5.2	3.1	5.0	12.3	12.7
Sugar and alcohol	0.5	2.9	0.5	0.3	1.2	0.7	0.8	6.7	0.2	0.6	4.3	0.5
Retail trade	4.5	13.0	9.0	5.2	8.9	15.5	1.4	7.5	2.3	3.7	11.3	7.1
Textile	4.2	10.1	10.1	5.3	9.2	11.5	2.8	10.7	4.4	3.9	10.2	9.2
Vehicles	3.8	11.5	7.2	3.2	9.1	6.0	0.9	2.1	2.5	3.0	9.6	5.5
Food	4.0	11.7	8.2	14.0	27.2	60.3	1.2	3.9	2.7	4.3	13.5	7.7
Agriculture	2.2	8.9	4.0	2.3	7.3	4.0	0.6	2.5	1.0	1.7	7.3	2.9
Health services	3.9	12.2	6.7	2.2	3.8	5.2	1.8	7.5	2.1	3.2	10.5	5.0
Chemicals	2.5	9.6	4.3	3.3	4.1	8.8	0.9	3.2	2.3	2.2	7.8	4.1
Recreation services	5.4	14.6	15.3	4.7	5.4	10.0	2.4	7.1	5.3	4.5	12.4	9.9
Electrical and electronic equipment	5.9	16.1	13.3	5.4	6.1	16.6	2.2	7.1	3.4	4.9	13.4	11.1
Other	3.6	10.7	7.2	4.7	10.5	11.5	1.3	6.2	1.4	2.9	9.5	6.6

Source: Central bank of Brazil and authors estimates.

ing the purpose of the exercise (i.e., simulating loan quality under alternative macroeconomic scenarios). In particular, we postulate that the logit-transformed NPLs of each credit type of bank *i* follow an AR(1) process and are influenced by past GDP growth, with up to S lags:

$$\ln\left(\frac{NPL_{i,t}}{1-NPL_{i,t}}\right) = \mu_i + \alpha\ \ln\left(\frac{NPL_{i,t-1}}{1-NPL_{i,t-1}}\right) + \sum_{s=0}^{S}\beta_{t-s}\ \Delta\ln(GDP)_{t-s} + \varepsilon_{i,t} \quad (2)$$

where $NPL_{i,t}$ stands for the ratio of non-performing loans to total gross loans of each credit type of bank *i* in period *t*, and GDP_t stands for GDP in quarter *t*.[13] The inclusion of the lagged dependent variable is motivated by the persistence of NPLs. The term μ_i refers to the bank-level fixed effects, which are treated as stochastic, and the idiosyncratic disturbances $\varepsilon_{i,t}$ are assumed to be independent across banks and serially uncorrelated (i.e., after the inclusion of the lagged dependent variable).[14] The coefficient α is expected to be positive but less than one, and the β coefficients are expected to be negative, reflecting deteriorating loan quality during the economic downturn.

Under this specification, the short-term effect of a change in quarter-on-quarter GDP growth on the logit of NPLs is given by the sum of the estimated β coefficients. By the chain rule, the effect of a shock to GDP growth on the untransformed NPL ratios, evaluated at the sample mean of NPLs is given by:

$$\text{Short-term effect}: \quad \frac{\Delta NPL}{\Delta\ln(GDP)} = \overline{NPL}\times(1-\overline{NPL})\times\sum_s\beta_{t-s} \quad (3)$$

$$\text{Long-term effect}: \quad \frac{\Delta NPL}{\Delta\ln(GDP)} = \frac{1}{1-\alpha}\times\overline{NPL}\times(1-\overline{NPL})\times\sum_s\beta_{t-s} \quad (4)$$

As a first approximation, we estimate equation [2] for the overall NPL ratios of individual banks, without distinguishing between credit types, which is the typical approach used in macro stress test models. The estimation was carried out using several alternative methods to assess the robustness of the results. We then selected a preferred estimation method and re-estimated equation [2] for each of the 21 credit types. All the models were computed over the entire sample of banks and separately for state-owned, private domestic, and foreign banks with the help of interacting dummies. The latter were used to explore for differences in the sensitivity of loan quality to macroeconomic conditions across bank types, possibly induced by systematic differences in loan origination practices and bank clientele across state-owned, private, and foreign banks. However, since the results showed no evidence of systematic differences across bank types, the final specification was computed over the entire sample to increase efficiency.

The results of the exploratory regressions were consistent with expectations, and extremely robust under alternative estimation methods, including pooled OLS, Within Group estimation, and two alternative applications of Generalized Method of Moments (GMM) estimators, treating GDP growth as either predetermined or strictly exogenous for the panel variables (Table 6). After exploring with various lag structures, we selected four lags of GDP growth, also reflecting the frequency of the data. Overall, the coefficients of the lagged dependent variable are around 0.6, reflecting the strong persistence of NPLs. In turn, the coefficients of the lagged GDP growth

[13] Since the non-performing loan ratio is bounded in the interval [0,1], the dependent variable was subject to the logit transform $\log(NPL/(1-NPL))$, to avoid problems associated with non-Gaussian errors.

[14] Therefore, the model assumes that the (positive) correlation of NPLs across individual banks originates exclusively from their common exposure to macroeconomic conditions. It also assumes that the effect of macroeconomic conditions on loan quality is symmetric during the upturn and the downturn of the economic cycle, and neglects possible non-linear dynamics and potential feedback effects running from credit markets to macroeconomic activity. A set of alternative specifications (available upon request) were estimated, exploring for non-linear effects and for potential differences in the sensitivity of loan quality to economic activity throughout the cycle, with non-significant results.

Table 6
Results of exploratory panel regressions.

	[1] Pooled OLS	[2] Within groups	[3] Difference GMM GDP exog.	[4] Difference GMM GDP pred.	[5] System GMM GDP exog.	[6] System GMM GDP pred.
L.Logit (NPL)	0.905***	0.569***	0.589***	0.597***	0.602***	0.631***
	[0.024]	[0.064]	[0.124]	[0.123]	[0.088]	[0.082]
D.LnGDP	−7.481***	−7.853***	−9.529***	−8.804***	−7.767***	−6.928***
	[2.032]	[1.903]	[2.198]	[2.132]	[1.927]	[1.939]
LD.LnGDP	−2.569	−4.544**	−6.081***	−5.729***	−3.922*	−3.086
	[2.282]	[1.935]	[2.254]	[1.990]	[2.026]	[2.023]
L2D.LnGDP	−7.482X	−6.877X	−10.675***	−9.152***	−8.123**	−5.971*
	[3.197]	[3.081]	[3.627]	[3.361]	[3.138]	[3.077]
L3D.LnGDP	1.597	1.067	0.423	−0.734	1.225	0.828
	[3.273]	[3.172]	[3.433]	[3.130]	[3.337]	[3.322]
Observations	1201	1201	1121	1121	1201	1201
R-Squared	0.83	0.341				
Hansen test (*p*-value)			0.02	0.13	0.04	0.11
AR(1) (*p*-value)			0.00	0.00	0.00	0.00
AR(2) (*p*-value)			0.184	0.175	0.184	0.191
Number of instruments		70	11	17	13	17
Number of banks			69	69	70	70

Robust standard errors in brackets.
* $p < 0.1$.
** $p < 0.05$.
*** $p < 0.01$.

are negative, as expected, and significant for up to three lags, falling within a relatively narrow interval.

Based on a comparison across estimation methods, we select the specification presented in column [4] as the preferred model. In particular, the estimation in column [1] uses OLS in levels, which produce upward-biased estimates of the coefficients associated with the lagged dependent variable (the $\alpha_i{}'_s$) due to the positive correlation between the latter and the fixed-effects. The Within Groups estimator in column [2] eliminates the fixed-effects by subtracting the mean from the series, but introduces a downward bias stemming from negative correlation between the lagged dependent variable and the transformed errors. Therefore, the consistent estimator of α is expected to fall between the OLS and the Within Groups estimators. This is in fact the case for all the models that follow, which are based on GMM estimators. The results presented in columns [3] and [4] use the Arellano-Bond GMM estimator in first differences, treating GDP growth as strictly exogenous in the first case, and as predetermined in the second (see Arellano and Bond, 1991; Blundell and Bond, 1998). The latter seems to be the preferred treatment, as indicated by the results of the Hansen test presented at the bottom, which fail to reject the null of orthogonality between the instruments and the error term. In turn, the results presented in columns [5] and [6] use the Arellano–Bover System GMM estimator, which exploit additional information from the equations in levels, but require the additional assumption that GDP growth is uncorrelated with the bank-level fixed effects, which may not be realistic. In all the GMM estimations, the number of instruments was limited by setting a maximum of 6 lags, to avoid problems associated with instrument proliferation.

The estimates of a full set of parallel regressions, one for each credit type, are also consistent with expectations and broadly robust. All the coefficients of the lagged dependent variable are positive in the interval [0,1] as expected and statistically significant at conventional levels (Table 7). The average value across all credit types is 0.4, which is slightly below the estimate obtained for the entire loan portfolios, likely reflecting the stronger sluggishness of the latter induced by diversification. The results also indicate that the AR(1) specification is adequate to eliminate the autocorrelation of the errors, as the tests of autocorrelation of order 2 in the first-differenced errors fail to reject the null in all cases. In turn, the sums of the coefficients of lagged GDP growth are negative in all cases, with the exception of credit to transport and “other credits” categories, and statistically significant at the five percent level in about one-half of the cases. The largest autoregressive coefficients are obtained for small credits to consumers, retail, textiles, and vehicles, indicating higher sluggishness in loan quality to these sectors. Furthermore, the largest coefficients for GDP growth are obtained for agriculture, sugar and alcohol, and energy. In order to gauge the sensitivity of NPLs to economic activity, however, these coefficients have to be rescaled by the average NPLs of the corresponding credit types, as shown in equations [3] and [4].

Using these results, we compute “rule-of-thumb” estimates of the impact of a change in GDP growth on NPLs (Table 8). For the overall sample, displayed at the bottom, we go back to the regression presented in column [4] of Table 7, where the coefficients of GDP growth add up to −24.4. Plugging this into equation [2], and using the average NPLs (2.8 percent), we find that a 2 percentage point drop in GDP growth (which is akin to the maximum drop observed between 1996 and 2008) would cause a 1.3 percentage point increase in NPLs in the short-term (i.e., $0.028 \times (1 - 0.028) \times 24.4 \times 2$). Subsequently, using equation [3], the predicted long-term increase in NPLs would be 3.3 percentage points (i.e., $1.3/(1 - 0.6)$), entailing a distressed NPL level of 7.2 percent, which is almost two-times higher than their March 2009 levels. Carrying out similar calculations for each credit type gives a range of results. The higher NPL ratios are obtained for consumer credit, which reaches 7.6 percent for medium-sized loans and 10.4 percent for small loans. Among lending to firms, the sectors reaching the highest NPL levels include textile, electric and electronic equipment, retail trade, and vehicles. In relative terms, the distressed NPL ratios are generally between 1½ and 2 times higher than their March 2009 values, with the most sensitive sectors being electricity and gas, livestock, agriculture, food, sugar and alcohol, and retail trade.

4. Stress tests

This section summarizes the results of stress test exercises of credit risk based on scenario analysis. It describes the criteria used in the construction of the scenarios and provides a brief comparison of their evolution. It also discusses the main characteristics of the

Table 7
Results of the dynamic panel regressions for individual credit types, 2003q1–2009q1.

	[1] Consumer (large)	[2] Consumer (medium)	[3] Consumer (small)	[4] Wood and furniture	[5] Transportation	[6] Petrochemicals	[7] Metal Products	[8] Electricity and gas	[9] Livestock	[10] Other services	[11] Sugar and alcohol
L.Dependent	0.351	0.379	0.665	0.335	0.380	0.398	0.483	0.423	0.498	0.409	0.340
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.004]
D.lngdp	−6.129	−4.186	−5.906	−3.235	3.759	−2.008	−1.526	−16.453	−7.283	−2.102	−6.134
	[0.136]	[0.008]	[0.025]	[0.070]	[0.296]	[0.385]	[0.581]	[0.003]	[0.144]	[0.265]	[0.485]
LD.lngdp	−7.951	−2.931	−2.168	−8.643	−4.182	−8.169	−6.355	−3.671	−14.080	−3.057	−11.806
	[0.071]	[0.032]	[0.148]	[0.005]	[0.045]	[0.010]	[0.044]	[0.609]	[0.271]	[0.143]	[0.117]
L2D.lngdp	−2.797	−6.578	−1.730	−4.565	3.592	−2.753	−6.047	−17.539	−0.978	−0.498	−24.826
	[0.602]	[0.011]	[0.377]	[0.097]	[0.379]	[0.282]	[0.383]	[0.008]	[0.876]	[0.897]	[0.005]
L3D.lngdp	−8.132	−0.023	0.333	−2.059	−3.089	1.284	−3.086	−23.542	8.514	−2.879	−24.430
	[0.258]	[0.992]	[0.887]	[0.498]	[0.538]	[0.705]	[0.486]	[0.006]	[0.245]	[0.442]	[0.122]
Observations	376	889	983	726	561	570	412	287	477	659	184
Number of banks	37	58	61	54	43	41	35	25	38	51	18
Hansen test (*p*-value)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AR(1) (*p*-value)	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.02	0.00	0.01
AR(2) (*p*-value)	0.19	0.25	0.95	0.03	0.14	0.57	0.90	0.39	0.92	0.84	0.18
Number of instruments	146	146	146	146	146	146	146	146	146	146	144
Sum of GDP coeff.	−25.009	−13.72	−9.47	−18.50	0.08	−11.65	−17.01	−61.21	−13.83	−8.54	−67.20
p-Value	0.04	0.02	0.09	0.01	0.99	0.06	0.06	0.00	0.55	0.26	0.01
Long-term effect	−38.535	−22.090	−28.272	−27.823	0.129	−19.346	−32.909	−106.075	−27.544	−14.443	−101.812
p-Value	0.03	0.01	0.14	0.02	0.52	0.07	0.09	0.02	0.25	0.18	0.12

	[12] Retail trade	[13] Textile	[14] Vehicles	[15] Food	[16] Agriculture	[17] Health services	[18] Chemicals	[19] Recreation Services	[20] Electrical and electronic equipment	[21] Other
L.Dependent	0.628	0.543	0.522	0.465	0.451	0.443	0.468	0.172	0.352	0.287
	[0.000]	[0.003]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.042]	[0.000]	[0.000]
D.lngdp	−0.798	−0.684	−2.170	−7.343	−11.447	−1.207	−1.794	−3.641	−1.032	−0.957
	[0.722]	[0.855]	[0.370]	[0.005]	[0.006]	[0.743]	[0.442]	[0.162]	[0.786]	[0.769]
LD.lngdp	−6.651	−10.950	−2.901	−6.508	−11.446	−2.730	−3.751	−5.318	−5.744	0.913
	[0.004]	[0.000]	[0.116]	[0.032]	[0.000]	[0.372]	[0.188]	[0.148]	[0.004]	[0.704]
L2D.lngdp	−4.095	−7.673	−1.249	−0.124	−5.723	−2.194	0.632	−2.213	1.932	1.008
	[0.239]	[0.026]	[0.689]	[0.971]	[0.213]	[0.584]	[0.863]	[0.632]	[0.629]	[0.854]
L3D.lngdp	−3.719	−2.667	−5.113	−1.821	−6.605	−2.649	1.060	−3.243	−9.148	2.825
	[0.189]	[0.557]	[0.040]	[0.628]	[0.029]	[0.512]	[0.670]	[0.381]	[0.003]	[0.450]
Observations	502	577	509	549	377	515	443	521	469	711
Number of banks	41	44	36	42	30	37	39	41	42	51
Hansen test (*p*-value)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
AR(1) (*p*-value)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00
AR(2) (*p*-value)	0.11	0.88	0.42	0.35	0.08	0.54	0.85	0.42	0.92	0.82
Number of instruments	146	146	146	146	146	146	146	146	146	146
Sum of GDP coeff.	−15.26	−21.97	−11.43	−15.80	−35.22	−8.78	−3.85	−14.42	−13.99	3.79
p-Value	0.01	0.01	0.07	0.03	0.00	0.33	0.43	0.07	0.08	0.69
Long-term effect	−41.030	−48.083	−23.918	−29.525	−64.155	−15.763	−7.242	−17.409	−21.593	5.314
p-Value	0.08	0.26	0.20	0.00	0.00	0.44	0.17	0.17	0.42	0.88

Table 8
Effect of a 2 p.p. drop in GDP growth on NPLs, by credit types in percent unless indicated.

	Estimates of panel regressions						Increase in NPLs		Stressed NPLs	
	[1] Average NPLs 2003–09	[2] NPLs March 2009	[3] Coef. lagged NPLs	[4] Sum coef. GDP Growth	[5] Long-term effect[a]	[6] Scale factor[b]	[7] Short-term (percentage points)[c]	[8] Long-term (percentage points)[d]	[9] Level[e]	[10] Times increase
Consumer (large)	3.6	2.5	0.4	−25.0	−38.5	0.035	1.7	2.7	5.2	2.1
Consumer (medium)	6.1	5.0	0.4	−13.7	−22.1	0.057	1.6	2.5	7.6	1.5
Consumer (small)	5.9	7.3	0.7	−9.5	−28.3	0.055	1.0	3.1	10.4	1.4
Wood and furniture	3.8	2.8	0.3	−18.5	−27.8	0.036	1.3	2.0	4.8	1.7
Transportation	3.8	1.7	0.4	0.1	0.1	0.037	0.0	0.0	1.7	1.0
Petrochemicals	3.6	1.7	0.4	−11.6	−19.3	0.035	0.8	1.3	3.0	1.8
Metal products	2.1	1.0	0.5	−17.0	−32.9	0.021	0.7	1.4	2.4	2.4
Electricity and gas	1.3	0.3	0.4	−61.2	−106.1	0.013	1.6	2.8	3.1	10.0
Livestock	4.2	2.4	0.5	−13.8	−27.5	0.041	1.1	2.2	4.6	2.0
Other services	5.0	3.7	0.4	−8.5	−14.4	0.047	0.8	1.4	5.1	1.4
Sugar and alcohol	0.6	1.3	0.3	−67.2	−101.8	0.006	0.8	1.2	2.5	1.9
Retail trade	3.7	3.0	0.6	−15.3	−41.0	0.035	1.1	2.9	5.9	2.0
Textile	3.9	5.2	0.5	−22.0	−48.1	0.038	1.7	3.6	8.8	1.7
Vehicles	3.0	4.0	0.5	−11.4	−23.9	0.029	0.7	1.4	5.4	1.3
Food	4.3	2.6	0.5	−15.8	−29.5	0.041	1.3	2.4	5.0	1.9
Agriculture	1.7	2.6	0.5	−35.2	−64.2	0.017	1.2	2.2	4.7	1.8
Health services	3.2	2.5	0.4	−8.8	−15.8	0.031	0.5	1.0	3.5	1.4
Chemicals	2.2	2.8	0.5	−3.9	−7.2	0.021	0.2	0.3	3.1	1.1
Recreation services	4.5	4.4	0.2	−14.4	−17.4	0.043	1.2	1.5	5.9	1.3
Electrical equipment	4.9	5.3	0.4	−14.0	−21.6	0.046	1.3	2.0	7.3	1.4
Other	2.9	1.2	0.3	3.8	5.3	0.029	−0.2	−0.3	0.9	0.7
Overall sampled credit	2.8	3.9	0.6	−24.4	−60.6	0.027	1.3	3.3	7.2	1.8

Memo: Change in yearly GDP growth: −2.

[a] Computed as [5] = [4]/(1 − [3]).

[b] The scale factor is computed as: [6] = [1]/(1 − [1]) (i.e., NPL × (1 − NPL)).

[c] Assuming a 2 pp drop in GDP growth, the short-term increase in NPLs is computed as: [7] = [4] × [6] × (−2).

[d] The long-term increase in NPLs is computed as: [8] = [7]/(1 − [3]).

[e] The stressed PD are computed as: [9] = [2] + [8].

out-of-sample simulations of NPLs under selected scenarios, and presents an illustration of the bias that can result from inadequate granularity in the credit portfolio data. Finally, the section presents the results of a credit VaR calculation based on these projections.

4.1. *Simulation of NPLs under alternative scenarios*

The exercises to assess credit risk are based on four macroeconomic scenarios, including a Baseline that reflects the expected path of GDP growth, and three distressed scenarios. Designing relevant stress scenarios is not a trivial issue. One can use history as guidance to construct the shocks, but history hardly repeats and the circumstances surrounding the shocks are almost always different, questioning their validity. Alternatively, the shocks can also be constructed more arbitrarily, considering current conditions and incorporating forward-looking considerations. In this paper we abstract from this discussion and use a mix of history and current conditions to shock the framework. The idea is to illustrate the model sensitivity to these various scenarios.

The evolution of GDP growth under the four scenarios considered was determined as follows:

- *Baseline Scenario*: This scenario is taken as reference and aims at capturing the expected evolution of economic activity. Under this, the results of the VAR model are projected onwards, without shocking the system. The resulting GDP growth drops from 5.1 percent in 2008 to −0.6 percent in 2009, followed by a resumption to above 3 percent in the subsequent two years.
- *Scenario 1*: Uses the results of the VAR to simulate the effect of an 11.6 percentage point increase in the slope of the yield curve in Q2 2009. The shock is akin to the mean of the slope during 2001-09 plus 2 standard deviations.
- *Scenario 2*: Uses the results of the VAR to simulate the effects of a negative shock to credit growth equal to 2.4 percentage points in Q2 2009. The shock is akin to the mean quarterly credit growth during 2001-09 minus 2 standard deviations.
- *Scenario 3*: Uses the results of the VAR to simulate the effects of a negative shock to GDP growth equal to 1.9 percentage points in Q2 2009. The shock is akin to the mean quarterly GDP growth during 2001-09 minus 2 standard deviations.

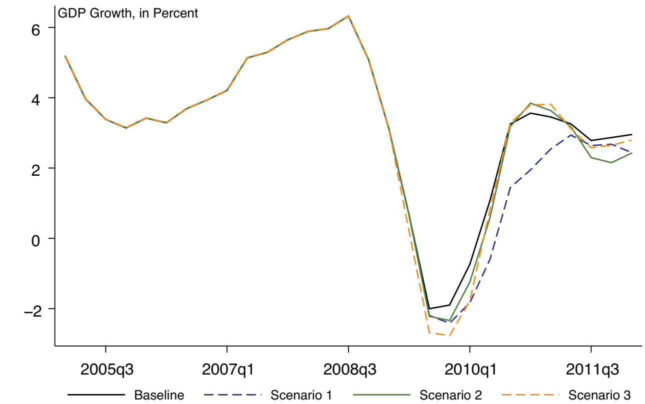

Fig. 4. Evolution of GDP growth y-o-y under alternative scenarios.

A comparison on the evolution of GDP growth under these four scenarios is provided in Fig. 4.

Using the results of the panel estimations we conduct an out-of-sample simulation of NPLs for each bank and credit type under each of the four scenarios. The results indicate deteriorating loan quality during 2009 (Fig. 5), followed by a relatively quick and steady recovery in 2010. For the baseline scenario, NPLs peak at 6.7 percent in the fourth quarter of 2010, before recovering. This out-of-sample simulation tracks reasonably well the ex-post observed data on NPLs for reference credit operations during the second and third quarters of 2009 (NPLs reached 5.8 percent in September 2009). The simulated NPLs for the distressed scenarios are higher than for the baseline, but following a qualitatively similar dynamics. The more severe deterioration in credit quality is associated with Scenario 3, with NPLs reaching a maximum of 8.5 percent, which is high at about two times the maximum observed during the sample period. Across credit types (not shown), the higher levels of NPLs are associated with credit to consumers, sugar and alcohol, textiles,

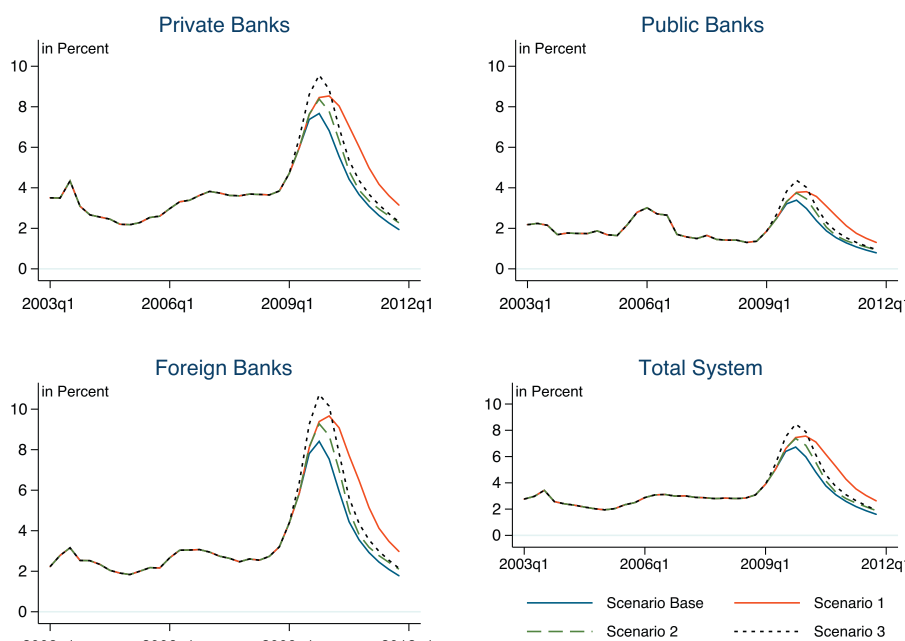

Fig. 5. Evolution of NPLs under alternative scenarios.

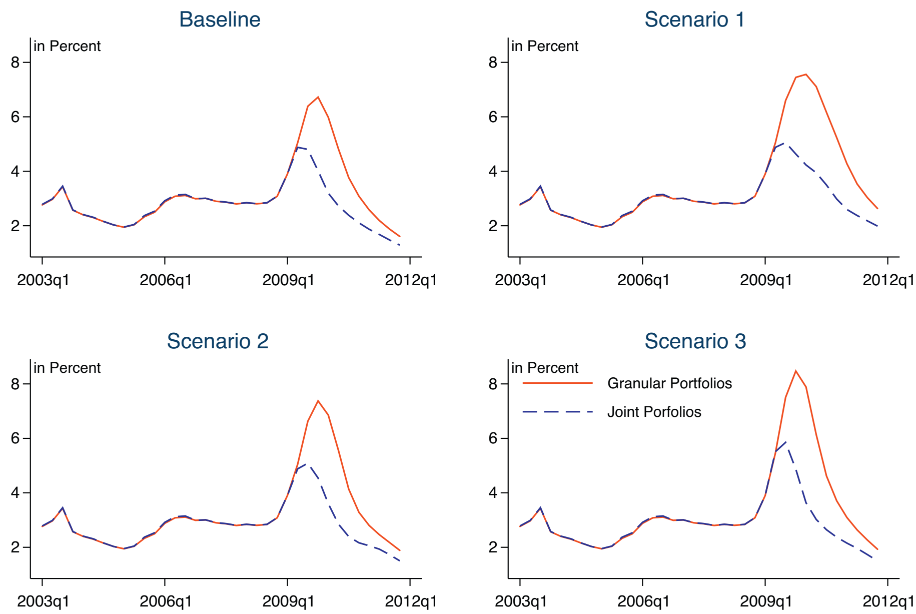

Fig. 6. Evolution of NPLs by scenarios and computation methods.

electricity and gas, and vehicles, which is roughly consistent with the results of the static exercise.

4.2. Portfolio aggregation bias

As mentioned before, the use of granular data on bank credit portfolios is a key contribution of this paper. Existing macro stress test models try to map the impact of the macroeconomic environment on credit quality using two approaches. The first one exploits aggregate data on credit quality, sometimes split by economic sectors, and applies econometric techniques to compute (elasticity) parameters linking macroeconomic conditions to credit quality. This approach is frequently used to assess systemic financial stability, but it has important shortcomings, as the profiles of banks' credit portfolios, and the cushions to absorb credit losses, are likely to differ across banks. The validity of the results under this approach becomes weaker as bank sizes, solvency, and risk profile of their credit portfolios, depart from the population mean.

The second approach exploits bank-by-bank data on credit quality, albeit without differentiating between types of credit, and without taking into account information on large exposures and other measures of portfolio concentration. While this approach allows the assessment of bank solvency at the level of individual institutions, it has its own shortcomings and potential sources of bias. In particular, the estimated elasticities linking macroeconomic conditions to credit quality reflect average values, mixing divergent elasticities between types of credit.

Arguably, the latter approach would tend to lead to biased estimations, as the sensitivity of credit quality to macroeconomic conditions is likely to differ between economic sectors. A bank with larger exposures to highly cyclical sectors would tend to be more vulnerable to credit losses under an adverse economic scenario. Therefore, simulations based on this approach would underestimate the potential losses of riskier banks, which is contrary to a prudent principle. In addition, the lack of information on portfolio concentration is a critical shortcoming, as concentration plays a major role in the risk profile of banks' credit portfolios.

To illustrate the bias stemming from the use of insufficient granularity in banks' credit portfolios, we use the results of the previous section to estimate the bank-specific NPLs under two approaches. The first one, akin to typical macro stress test models of credit risk, simulates the evolution of bank-level NPLs without exploiting information on specific credit types (*Joint Portfolios*), while the second exploits a partition of banks' credit portfolios in specific credit categories (*Granular Portfolios*). The two approaches share the same estimation techniques and the two-year macroeconomic scenarios described above.

The results are consistent with the presence of a bias stemming from inadequate granularity in the credit portfolios. In particular, the weighted average of the simulated NPLs using joint portfolios is always lower than the simulated NPLs under the granular portfolios (Fig. 6). The kernel densities of the simulated NPLs under the two approaches also illustrate this portfolio aggregation bias (Fig. 7). In particular, the mean and the median NPLs under the granular approach exceed those of the joint approach in all cases (Table 9). Similarly, the probability mass at the (right) tail of the distributions is also thicker under the granular approach.[15] On the other hand, the skewness and kurtosis of the granular approach are lower than those of the joint approach, reflecting the bias induced by the latter. Paired t-tests of mean differences confirm that the population mean under the granular approach exceed those resulting from the joint approach in all cases, while differences in their population variances are not statistically significant (Table 10).

4.3. Credit VaR

This section uses the previous results to compute a Credit VaR and come up with an estimate of banks' (unexpected) credit losses under an adverse macroeconomic environment. For each bank, we model the distribution of credit losses using the CreditRisk+ (CSFP, 1997) formulation and the exposures of each bank as of March 2009. Under CreditRisk+ and assuming that default probabilities are random, the probability generating function $G(z)$ of the nor-

[15] The maximum value under the granular approach, however, was smaller than the joint approach due to a small bank that starter the simulation period with very low credit quality.

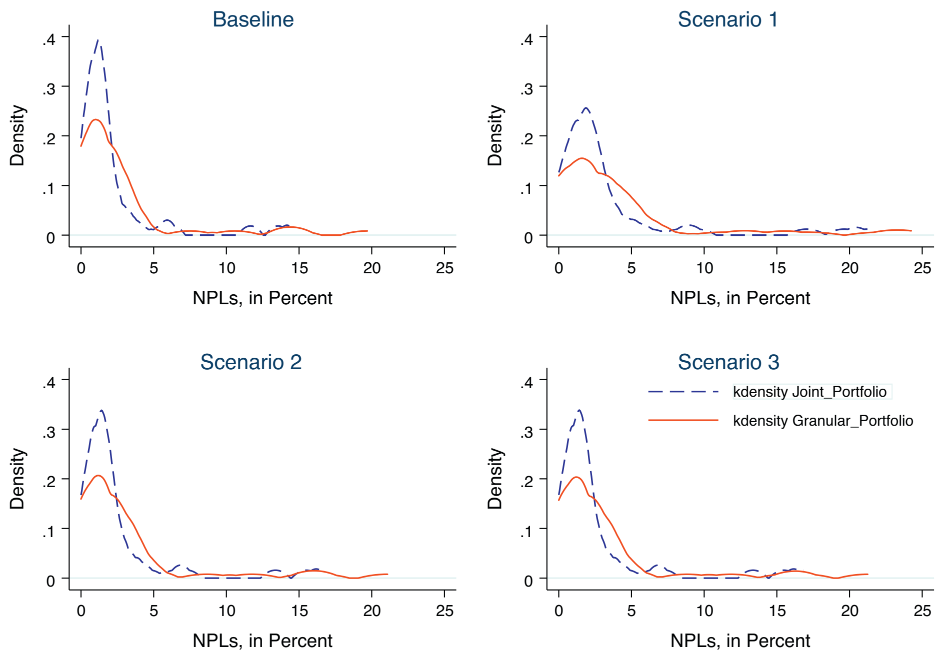

Fig. 7. Kernel densities of simulated NPLs in 2011q4, by estimation methods.

malized total expected losses of a portfolio of n credit types, can be written as (Avesani et al., 2006):

$$G(z) = \exp\left\{-\sum_{k=1}^{n} \frac{1}{\sigma_k^2} \ln[1 - \sigma_k^2 \cdot p_k(z)]\right\} \quad (5)$$

where default rates are assumed to be distributed according to a Gamma distribution, $\Gamma(a_k \cdot \beta_k)$ with $\alpha_k = \mu_k^2/\sigma_k^2$ and $\beta_k = \sigma_k^2/\mu_k$. In turn, μ_k is estimated as $\mu_k = \varepsilon_k/\nu_k$, where ε_k is the expected loss and ν_k the exposure to credit type k. We use the bank-specific estimates of NPLs for each credit type under Scenario 1 as a proxy for distressed PDs. In particular, we take the average of the out-of-sample simulation of NPLs for each bank and credit type over the first simulated year of Scenario 3 (the more severe) as a proxy for distressed PDs of the corresponding credit categories. To account for uncertainty on the true value of the PDs (σ_k) we use the standard deviation of the NPLs over the corresponding first year of the out-of-sample simulation. Admittedly, NPLs are an imperfect proxy for PDs due to their "backward-looking" nature. While PDs are intended to capture the likelihood of borrower's default within a given horizon (i.e., one-year ahead), NPLs typically measure the proportion of loans that are more than 90 days past due in total credit portfolios. Therefore, PDs would vary in response to changes in the repayment capacity of a given borrower, which may not immediately translate into changes in NPLs. Using NPLs as a proxy for PDs is prone to various sources of bias that may operate in opposite directions. During the upturn of the economic cycle, which tends to be associated with strong credit growth, NPLs may underestimate the risk profile of banks' credit portfolios. In contrast, NPLs may overestimate credit risk during the downturn, as banks defer loan write-offs.

For each credit type, we conduct an initial calculation of the average exposures to individual borrowers, by dividing the corresponding total exposures over the number of loan oper-

Table 9
Summary statistics of simulated NPLs by scenarios and estimation methods.

	Baseline		Scenario 1	
	Granular	Joint	Granular	Joint
Mean	5.690	4.630	7.225	5.943
St. dev.	6.643	6.168	7.616	7.361
Minimum	0.007	0.014	0.013	0.021
Median	3.358	2.520	4.963	3.521
90th percentile	14.412	11.114	17.452	13.910
95th percentile	19.694	18.488	22.108	23.804
99th percentile	33.272	32.054	36.733	35.435
Maximum	36.171	41.186	38.682	42.489
Skewness	2.241	2.975	2.049	2.666
Kurtosis	8.529	12.784	7.545	10.138
	Scenario 2		Scenario 3	
	Granular	Joint	Granular	Joint
Mean	6.116	5.010	6.572	5.300
St. dev.	6.954	6.567	7.313	6.939
Minimum	0.008	0.016	0.009	0.016
Median	3.756	2.699	4.011	2.880
90th percentile	15.468	11.476	16.335	12.623
95th percentile	21.083	19.914	21.858	21.009
99th percentile	35.474	34.621	37.529	36.631
Maximum	37.170	42.671	39.367	46.367
Skewness	2.182	2.923	2.122	2.937
Kurtosis	8.281	12.418	8.090	12.670

Table 10
Comparison of simulated kernel distributions of NPLs.

	Baseline	Scenario 1	Scenario 2	Scenario 3
Mean paired t-test				
[1] Granular (mean)	5.690	7.225	6.116	6.572
[2] Joint (mean)	4.630	5.943	5.010	5.300
Difference [1]–[2]	1.060	1.282	1.105	1.273
H1: mean diff < 0 (p-value)	1.000	1.000	1.000	1.000
H1: mean diff ~ 0 (p-value)	0.000	0.000	0.000	0.000
H1: mean diff > 0 (p-value)	0.000	0.000	0.000	0.000
Variance ratio test				
[1] Granular (st. dev.)	6.643	7.616	6.954	7.313
[2] Joint (st. dev.)	6.168	7.361	6.567	6.939
Difference [1]–[2]	0.475	0.254	0.387	0.374
H1: mean diff < 0 (p-value)	0.933	0.754	0.876	0.856
H1: mean diff ~ 0 (p-value)	0.134	0.493	0.247	0.288
H1: mean diff > 0 (p-value)	0.067	0.246	0.124	0.144

Table 11
Selected credit risk parameters used in the calculation of the credit VaR.

	Distressed PDs			Credit exposures		
	Mean	Median	Max.	Median size (million BRL)	Number of operations	Median credit (million BRL)
Agriculture	0.141	0.089	0.859	915	1802	0.508
Food	0.032	0.023	0.110	999	4344	0.230
Livestock	0.088	0.028	0.687	1360	2513	0.541
Vehicles	0.062	0.045	0.215	1185	7728	0.153
Electrical and electronic equipment	0.113	0.062	0.507	624	4944	0.126
Electricity and gas	0.028	0.001	0.442	1376	175	7.867
Wood and furniture	0.049	0.053	0.121	3369	17,700	0.190
Recreation services	0.036	0.034	0.086	742	9708	0.076
Other	0.024	0.009	0.174	3755	7098	0.529
Petrochemicals	0.091	0.039	0.424	1559	4121	0.378
Consumer (large)	0.066	0.043	0.290	1103	1782	0.619
Consumer (medium)	0.109	0.118	0.223	4114	66,188	0.062
Consumer (small)	0.096	0.091	0.228	10,814	814,219	0.013
Chemicals	0.015	0.013	0.066	701	3760	0.187
Health services	0.046	0.036	0.202	809	5741	0.141
Other services	0.068	0.050	0.294	1318	11,036	0.119
Metal products	0.028	0.015	0.167	1421	1770	0.803
Sugar and alcohol	0.222	0.202	0.773	1351	172	7.860
Textile	0.176	0.132	0.667	1184	12,264	0.097
Transportation	0.020	0.019	0.078	2173	7222	0.301
Retail trade	0.100	0.051	0.810	1213	11,631	0.104

ations. This treatment, however, may underestimate portfolio concentration and therefore the results of the Credit VaR. We thus consider an alternative exercise by assuming that 80 percent of the exposures under each credit category are concentrated in 20 percent of the number of credit operations (and the reminder 20 percent of the exposures correspond to 80 percent of the number of credit operations). Since we do not have information on losses given default (LGDs), we choose a generic value of 50 percent for all credit types. We recognize that this is also a critical assumption in our assessment of banks' solvency, as higher LGD would mechanically lead to higher unexpected losses in the credit VaR calculation. Moreover, LGDs are likely to vary across credit types, also depending on the existence of collateral on individual credit operations, which we cannot observe. There is also evidence that LGDs tend to vary throughout the cycle, increasing during the economic downturn. On the other hand, to the extent that potential differences in the sensitivity of LGD's to the economic cycle across credit types go in tandem with the dynamics of their respective PDs, which is likely the case, our relative assessment of individual banks' solvency would not be biased. Furthermore, our assumption of a 50 percent LGD is likely to be conservative.

The parameters used in the Credit VaR are bank-specific. While the sensitivities of credit quality to macroeconomic conditions are the same for each credit type, the quality of individual banks' credit portfolios in each of the distressed scenarios would vary according to their staring conditions. Overall, consumer loans and credit to the textile industry, agriculture, and sugar and alcohol are the most pro-cyclical, and concentration to the latter more pronounced (Table 12). Using these parameters, the distribution of credit losses is computed using the probability generating function defined in equation [5] and the unexpected losses are estimated as the 99th percentile of the expected losses distribution.

The results suggest that the banking sector is well prepared to undergo the credit losses associated with the distressed scenarios considered without threatening financial stability. The (unexpected) credit losses associated with a 99 percent credit VaR

Table 12
Results of the credit VaR, in million BRL, unless otherwise indicated.

Bank number	Credit VaR	Credit VaR/Net exposure (percent)	Credit VaR/gross exposure (percent)	Gross exposure	Share of loans in sample (percent)
1	4689	5.0	2.5	189,052	24.3
2	11,095	12.8	6.4	173,172	22.2
3	6865	10.1	5.1	135,276	17.4
4	8181	14.0	7.0	116,957	15.0
5	1479	9.3	4.7	31,798	4.1
6	906	5.8	2.9	31,378	4.0
7	1152	10.9	5.5	21,118	2.7
8	551	11.4	5.7	9642	1.2
9	286	6.3	3.2	9046	1.2
10	347	10.2	5.1	6839	0.9
11	20	0.6	0.3	6348	0.8
12	506	16.3	8.1	6216	0.8
13	185	10.0	5.0	3675	0.5
14	161	9.1	4.5	3538	0.5
15	200	11.4	5.7	3502	0.4
16	71	4.6	2.3	3113	0.4
17	309	20.7	10.4	2985	0.4
18	116	9.1	4.5	2563	0.3
Total	37,118	9.8	4.9	756,218	97.0

Note: Net exposures are computed by subtracting the estimated recovery values from the gross credit exposures.
Parameters: VaR level: 0.99; Model: Poisson defaults/FFT; LGD: 0.5.

for the 18 banks with the largest credit portfolios in the sample amount to around BRL37 billion, or 4.9 percent of their gross exposures (Table 11). As a reference, these losses are roughly equivalent to about 19 percent of the joint tangible capital of these banks. Our measure of tangible capital equals regulatory capital minus the sum of specific loan loss provisions included in banks' own resources, deferred taxes, and goodwill. Therefore, the capital cushions of the largest banks appear sufficient to absorb the credit losses associated with the scenarios considered without threatening financial stability.

5. Final considerations

The econometric estimations presented in this paper provide strong evidence of a cyclical behavior of loan quality. The estimations substantiate the existence of a robust inverse relationship between GDP growth and NPLs, with the effects operating with up to three quarter lags. The results also indicate differences in the persistence of NPLs across credit types, and in their sensitivity to economic activity. Loan quality in Brazil appears to be more sensitive to GDP growth for small consumer loans, credit to agriculture, sugar and alcohol, livestock, and textile. In addition, credit for vehicle acquisition and electric and electronic equipment displayed high level of NPLs under distressed macroeconomic scenarios. Banks with relatively higher exposures to these sectors are likely to experience larger credit losses under a macroeconomic downturn.

While intuitive, the modeling of differences in the sensitivity of loan quality to macroeconomic conditions at the level of individual banks is novel to the macro stress testing literature. Existing models based on bank-level data do not allow for a differential response of credit quality to macroeconomic conditions across credit types, possibly due to lack of data availability. On the other hand, existing macro stress test models that exploit variations in the sensitivity of loan quality to macroeconomic conditions across credit types are based on aggregated data, and are therefore less suited to assess the adequacy of bank capital at the level of individual institutions.

The results presented in this paper show that the lack of sufficiently granular data on the composition of bank credit portfolios can bias the results in a way that is contrary to a prudent criterion. To the extent that the sensitivity of credit quality to macroeconomic conditions varies between different credit types, the lack of differentiation would tend to underestimate the deterioration of credit quality for the highly procyclical credit types and sectors under a distressed macroeconomic environment (and overestimate the deterioration of credit quality for the relatively safer credit types). These biases would translate in a systematic way into the assessment of bank risk profiles, leading to an underestimation of tail losses in the more vulnerable institutions in the banking system under analysis.

The model presented in this paper represents an improvement over existing literature but is still subject to several important caveats. First, the model assumes a linear relationship between loan quality and macroeconomic conditions which may fail to capture potential non-linear relationships during periods of severe macroeconomic distress. Second, the model assumes that historic correlations between loan quality and macroeconomic conditions are symmetric during the upturn and the downturn of the economic cycle, and remain valid during periods of severe distress. Third, the model fails to capture potential feedback effects between credit quality and economic growth, as it does not fully integrate the macro and microeconomic modules. In particular, the macroeconomic module allows credit volumes to vary over time, while the microeconomic module assumes that individual banks maintain constant credit portfolios. To the extent that credit quality tends to deteriorate during periods of slow credit growth, the model presented in this paper may underestimate banks' loan losses. All these caveats may likely bias the results in the same direction during periods of financial distress, causing a potential underestimation of bank credit losses. Further analysis is needed to address these shortcomings.

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