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Convection-radiation thermal analysis of triangular porous fins with temperature-dependent thermal conductivity by DTM

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ABSTRACT

Increasing performance of porous fins is one of the common priorities nowadays. Hence attention here is given to the convection and radiation effects in the analysis of performance of a porous triangular fin with temperature-dependent thermal conductivity. Mathematical problem is solved by differential transformation method (DTM). The DTM results are compared with the numerical results obtained by fourth-order Runge–Kutta method in order to confirm the accuracy of the analytical solution. The dimensionless temperature distributions, fin efficiency and fin effectiveness are studied with respect to emerging parameters involved in the analysis. It is noted that there is an increase in fin performance in view of porous constraint.

1. Introduction

Heat transfer has a pivotal role in various engineering problems. More, the fins or extended surfaces are important in enhancement of the rate of heat transfer from the primary surface. The profile of a fin is generally selected on the basis of the cost of material and manufacturing as well as on the ease of fabrication. There is a wide use of rectangular fins in such situations. This perhaps is related to ease in its design and manufacturing. The attempts highlighting the analysis of fin performance may be mentioned in the studies [1–7]. In these researches, the optimum profile has also been determined to obtain maximum rate of heat transfer. A triangular fin due to reducing volume (fin material) than a rectangular fin for an equal heat transfer is attractive. Thermal analysis of a viscoelastic fluid past a triangular fin was numerically studied by Hsiao and Hsu [8]. They found that the elastic effect in the flow can increase local heat transfer and enhance the heat transfer of a triangular fin. Kundu et al. [9] presented an analytical method for predicting fin performance of triangular fins subject to simultaneous heat and mass transfer. They observed that the performance of wet fins is almost independent on the relative humidity. Performance analysis and optimization of straight taper fins with variable heat transfer coefficient was analytically studied by Kundu and Das [10]. They applied the Frobenius series method to obtain temperature profiles in longitudinal fin.

There is an increasing interest of the recent researchers through heat transfer in porous media for increasing heat transfer rate of fins. Increasing the effectiveness area through which heat is convected to surrounding fluid is an original mechanism just to improve heat transfer using porous fins. Thermal analysis of porous fin for increasing heat transfer rate from a given surface was numerically investigated by Kiwan and Al-Nimr [11]. Kiwan [12] analyzed the performance of porous fins in natural convection environment. Gorla and Bakier [13] numerically investigated the thermal analysis of natural convection and radiation in a rectangular porous fin. They concluded that porous fins provide much higher heat transfer rate than the conventional solid fins. An analytical model for determination of the performance and optimization of porous fins with respect to the different models of prediction was presented by Kundu and Bhanja [14]. MHD effect on a rectangular porous fin was investigated by Taklifi et al. [15]. They concluded that by imposing MHD in system except near the fin tip, heat transfer rate from the porous fin decreases. Bhanja and Kundu [16] analytically investigated thermal analysis of a constructal T-shape porous fin with radiation effects. An increase in heat transfer is found by choosing porous medium condition in the fin.

Thermal radiation effects are quite significant in various engineering and industrial processes especially in the design of reliable equipments, nuclear plants, gas turbines and various propulsion devices for aircrafts, missiles, satellites and space vehicles. The

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Nomenclature

b	width of fin
C_p	specific heat of air
h	convection heat transfer coefficient
K	permeability of the porous fin
k	thermal conductivity
k_r	thermal conductivity ratio, k^*/k_f
k_1, k_2	constants in Eq. (16)
k^*	constant in Eq. (13)
L	length of the fin
$\dot{m}$	mass flow rate
q	actual heat transfer
Ra^*	modified Rayleigh number, $Ra^* = \frac{g\beta KLT_b}{\alpha \upsilon k_r \psi}$
Rd	radiation parameter, $Rd = \frac{\sigma \varepsilon T_b^3 L^2}{k^* y_b}$
T	temperature
υ_w	average velocity of the fluid passing through the fin at any point
x,y	coordinate starting from the fin tip
X	dimensionless coordinate
Z	fin parameter, $\sqrt{hL^2/k^* y_b}$

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
β^*	constant in Eq. (14)
β_R	Rosseland extinction coefficient
ε	emissivity
ε^*	fin effectiveness
ϕ	fin tip temperature
φ	porosity
γ	constant in Eq. (13)
η	fin efficiency
λ	ratio of semi-fin thickness at the base to width, y_b/b
θ	dimensionless temperature, $(T - T_a)/(T_b - T_a)$
ρ	density
σ	Stefan–Boltzmann constant
υ	kinematic viscosity
ω	dimensionless parameter, $1/2\lambda$
ψ	dimensionless semi-thickness at the base, y_b/L

Subscripts

a	ambient
b	fin base
eff	effective properties
f	fluid

concept of thermal radiation is especially important in processes involving high temperature. The optimum dimensions of convecting-radiating fins was analytically determined by Razelos and Kakatsios [17]. Determination and measurement of the temperature along a fin cooled by natural convection and radiation was numerically and experimentally performed by Mueller and Abu-Mulaweh [18]. They observed that the heat loss due to radiation is typically 15–20% of total. Kiwan [19] analytically studied the effect of radiation on the heat transfer from porous fins. He observed that the radiation heat transfer becomes important as the surface temperature parameters increase, which the radiation effect becomes less important when the Rayleigh number increases. Heat transfer analysis from a horizontal fin array by natural convection and radiation was studied by Rao et al. [20] with the alternating direction implicit (ADI) method. Aziz and Green [21] analytically investigated performance and optimum design of convective-radiative rectangular fin with convective base heating, wall conduction resistant and contact resistant between the wall and the fin base. An approximate analytical solution for convection-radiation from a continuously moving fin with temperature-dependent thermal conductivity was developed by Aziz and Khani [22]. They employed the homotopy analysis method [HAM] for the development of solution. They concluded the radiative cooling becomes more effective for stronger radiation and consequently this lowers the temperature in the fin.

In this paper, the differential transform method (DTM) is employed for solving the nonlinear energy equation of a triangular porous fin with temperature-dependent thermal conductivity. It is evident from the existing literature that the convection-radiation heat transfer in triangular porous fins has not been pursued analytically. As mentioned above, Kundu et al. [9] just developed an analytical model via the Adomain decomposition method (ADM) to solve energy equation for a triangular wet fin without radiation and porosity effects. In present study, after obtaining explicit expression of temperature profiles, the efficiency of fin and fin effectiveness for different values of emerging parameters are also obtained and discussed. The temperature distributions obtained from the present model are compared with the numerical results obtained by fourth-order Runge–Kutta method. The concept of differential transformation method (DTM) was originally proposed by Zhou [23] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. The main benefit of this method is that it can be applied for both linear and nonlinear initial and boundary value problems without the need of linearization, discretization, or perturbation. DTM has been already applied for solution of both linear and nonlinear differential equations by many researchers [24,25]. Joneidi et al. [26] applied DTM to solve heat transfer problem from straight fin with temperature-dependent thermal conductivity. An analytical approach for predicting fin performance of three different fin profiles was conducted by Moradi and Ahmadikia [27] using DTM. Franco [28] analytically investigated optimum thermal design of convection longitudinal fin arrays via the differential transformation method. In present investigation, the obtained results are plotted and analyzed for these parameters.

2. Basic idea of DTM

The DTM is based on the Taylor series expansion and is a useful tool to develop analytical solutions of the differential equations. In this method, the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions. The solution of these algebraic equations gives the desired solution of the problem.

Consider a function $f(x)$ which is analytic in a domain D and let $x = x_0$ represents any point in D. The function $f(x)$ is then represented by a power series whose center is located at x_0.The differential transform of the function $f(x)$ is

$$F(k) = \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{1}$$

in which $f(x)$ is an original function and $F(k)$ is the transformed function. The inverse transformation is given by:

$$f(x)=\sum_{k=0}^{\infty}(x-x_0)^kF(k) \tag{2}$$

Combination of Eqs. (1) and (2) yields

$$f(x)=\sum_{k=0}^{\infty}\frac{(x-x_0)^k}{k!}\left(\frac{d^kf(x)}{dx^k}\right)_{x=x_0} \tag{3}$$

Having Eq. (3) in mind, it is noticed that the concept of differential transform is developed from Taylor series expansion. However, the method does not evaluate the derivatives symbolically. In actual applications, the function $f(x)$ is expressed by a finite series and Eq. (3) can be written as follows:

$$f(x)=\sum_{k=0}^{m}\frac{(x-x_0)^k}{k!}\left(\frac{d^kf(x)}{dx^k}\right)_{x=x_0} \tag{4}$$

which means that $f(x)=\sum_{k=m+1}^{\infty}\frac{(x-x_0)^k}{k!}(\frac{d^kf(x)}{dx^k})_{x=x_0}$ is negligibly small. Here the value of m depends on the convergence rate of the series coefficients. Some of the original functions and transformed functions are shown in Table 1.

3. Mathematical formulations

Fig. 1 shows the physical model of the triangular porous fin attached to an isothermal vertical plate. In the present study, we consider the following assumptions:

(1) The thermal conductivity and diffusivity of the fin and convective heat transfer coefficient are constant.
(2) The porous medium is homogenous, isotropic and saturated with a single phase fluid.
(3) The temperature variation through the fin is one-dimensional and it varies along the length of fin.
(4) The Darcy model is employed to simulate the porous medium and fluid interaction.
(5) All the physical properties of the wall and fluid are taken constant except there is density variation of fluid which may affect the buoyancy term.

In view of above mentioned assumptions, the energy equation gives

$$\frac{dq}{dx}+\dot{m}C_p(T_a-T)+\sqrt{1+\left(\frac{dy}{dx}\right)^2}(2y+b)\{h(1-\varphi)(T_a-T)+\sigma\varepsilon(T_a^4-T^4)\}=0 \tag{5}$$

in which y and b are the local semi-thickness and width of the triangular fin respectively. The mass flow rate of fluid $\dot{m}$ passing through the porous material is [19]:

Table 1
The fundamental operations of differential transform method.

Original function	Transformed function
$f(x)=\alpha g(x)\pm\beta h(x)$	$F(k)=\alpha G(k)\pm\beta H(k)$
$f(x)=g(x)h(x)$	$F(k)=\sum_{l=0}^{k}G(l)H(k-l)$
$f(x)=g(x)^{(n)}$	$F(k)=(k+1)(k+2)\ldots(k+n)G(k+n)$
$f(x)=x^n$	$F(k)=\delta(k-n)=\begin{Bmatrix}1 & k=n\\ 0 & k\neq n\end{Bmatrix}$
$f(x)=\exp(\alpha x)$	$F(k)=\frac{\alpha^k}{k!}$
$f(x)=(1+x)^n$	$F(k)=\frac{k(k-1)\ldots(k-m-1)}{k!}$
$f(x)=\sin(\gamma x+\chi)$	$F(k)=\frac{\gamma^k}{k!}\sin(\pi\frac{k}{2!}+\chi)$
$f(x)=\cos(\gamma x+\chi)$	$F(k)=\frac{\gamma^k}{k!}\cos(\pi\frac{k}{2!}+\chi)$

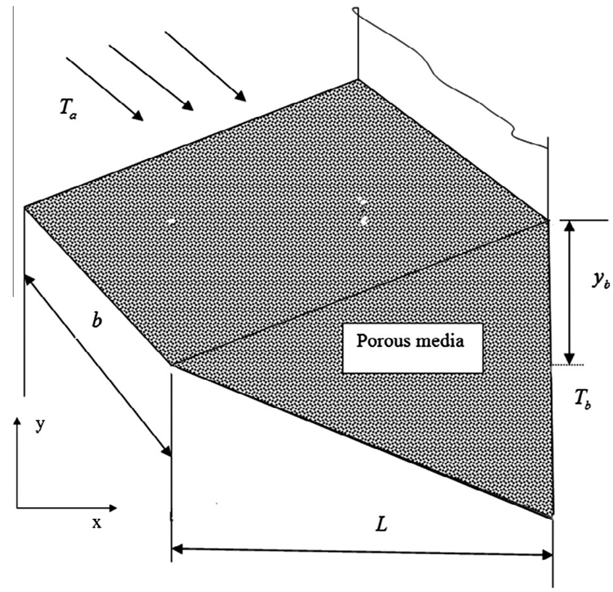

Fig. 1. Schematic diagram of a triangular porous fin.

$$\dot{m}=\rho v_w\sqrt{1+\left(\frac{dy}{dx}\right)^2}(2y+b) \tag{6}$$

where v_w in accordance with the Darcy's model is given by

$$v_w(x)=\frac{gK\beta}{\upsilon}(T-T_a) \tag{7}$$

The energy transfer rate of combined conduction and radiation at any section of the fin can be written as follows

$$q=q_{conduction}+q_{radiation} \tag{8}$$

where by Fourier's law one has

$$q_{conduction}=-k_{eff}\,y\,b\frac{dT}{dx} \tag{9}$$

and the radiation heat flux term in view of Rosseland's [29] approximation is presented as

$$q_{radiation}=-\frac{4\sigma}{3\beta_R}y\,b\frac{dT^4}{dx} \tag{10}$$

From Eqs. (5)–(10) we have

$$\frac{d}{dx}\left(yb\left(k_{eff}\frac{dT}{dx}+\frac{4\sigma}{3\beta_R}\frac{dT^4}{dx}\right)\right)+\sqrt{1+(\frac{dy}{dx})^2}(2y+b)\left\{-\frac{\rho C_pgK\beta}{\upsilon}(T-T_a)^2+h(1-\varphi)(T_a-T)+\sigma\varepsilon(T_a^4-T^4)\right\}=0 \tag{11}$$

When the temperature differences through the flow are assumed to be sufficiently small then

$$T^4\cong 4T_a^3T-3T_a^4 \tag{12}$$

and for most materials, the thermal conductivity of the material k_{eff} increases linearly with temperature. Thus we consider

$$k_{eff}=k^*[1+\gamma(T-T_a)] \tag{13}$$

Defining

$$\theta=\frac{T-T_a}{T_b-T_a},\ X=\frac{x}{L},\ \psi=\frac{y_b}{L},\ \lambda=\frac{y_b}{b},\ Z^2=\frac{hL^2}{k_{eff}y_b},\ \beta^*=\gamma(T_b-T_a)$$
$$Rd=\frac{4\sigma T_a^3}{3\beta_Rk^*},\ Ra^*=\frac{g\,\beta\,K\,L\,(T_b-T_a)}{\alpha\upsilon k_r\psi},\ R^*=\frac{4\,\sigma\,\varepsilon\,T_a^3\,L}{k^*\,\psi} \tag{14}$$

the energy equation can be rewritten as

$$(1 + 4Rd + \beta^*\theta)\left(X\frac{d^2\theta}{dX^2} + \frac{d\theta}{dX}\right) + \beta^* X\left(\frac{d\theta}{dX}\right)^2 = (X + \omega)(k_1\theta^2 + k_2\theta) \tag{15}$$

with

$$\begin{aligned} \omega &= 1/2\lambda, \\ k_1 &= 2\lambda\sqrt{1+\psi^2}Ra^*, \\ k_2 &= 2\lambda\sqrt{1+\psi^2}[(1-\varphi)Z^2 + R^*] \end{aligned} \tag{16}$$

The above second-order energy equation is solved by using boundary conditions, i.e. the temperature at the fin base, T_b, is maintained at a constant value and the temperature has a finite value at fin tip.

The actual heat transfer through the triangular fin can be determined by applying Fourier's law of heat conduction as follows:

$$q_{actual} = k_{eff}A_b\frac{dT}{dx}|_{x=L} + \frac{4\sigma A_b}{3\beta_R}\frac{dT^4}{dx}|_{x=L} = \frac{k^* y_b b(T_b - T_a)}{L}[1 + \beta^* + 4Rd]\frac{d\theta}{dX}|_{X=1} \tag{17}$$

The ideal heat transfer of the fin is obtained when the entire of the fin surface maintaining at its base temperature i.e.

$$q_i = (bL\sqrt{1+(y_b/L)^2} + y_bL)\left[h(1-\varphi)(T_b - T_a) + \sigma\varepsilon(T_b^4 - T_a^4) + \frac{\rho C_p g K\beta}{\upsilon}(T_b - T_a)^2\right] \tag{18}$$

the fin efficiency can be determined by the following expression

$$\eta = \frac{q_{actual}}{q_i} = \frac{(2\lambda\sqrt{1+\psi^2})(1+\beta^*+4Rd)}{(\sqrt{1+\psi^2}+\lambda)[k_1+k_2]}\frac{d\theta}{dX}|_{X=1} \tag{19}$$

Now if there is no fin, the heat transfer through the same base area can be expressed as follows

$$q_e = A_b[h(T_b - T_a) + \sigma\varepsilon(T_b^4 - T_a^4)] \tag{20}$$

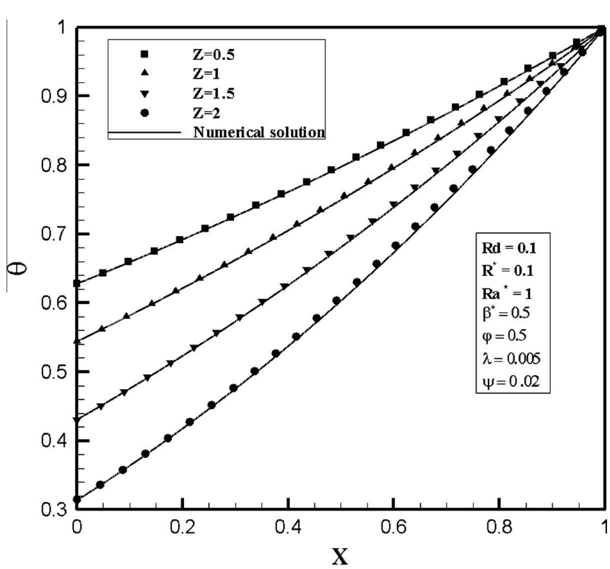

Fig. 2. Temperature distribution obtained by DTM in comparison with the numerical solution for different values of the fin parameter *Z*.

The fin effectiveness is given in view of the following relation

$$\varepsilon^* = \frac{q_{actual}}{q_e} = \frac{(1+\beta^*+4Rd)}{\psi[Z^2+R^*]}\frac{d\theta}{dX}|_{X=1} \tag{21}$$

4. Solution by differential transformation method

Obviously the nonlinear second-order ordinary differential Eq. (15) is not solved by a conventional analytical technique. Hence for an explicit form of temperature profile, the differential transformation method is employed. Taking the differential transform of Eq. (15) considered by using the related definitions in Table 1, we have:

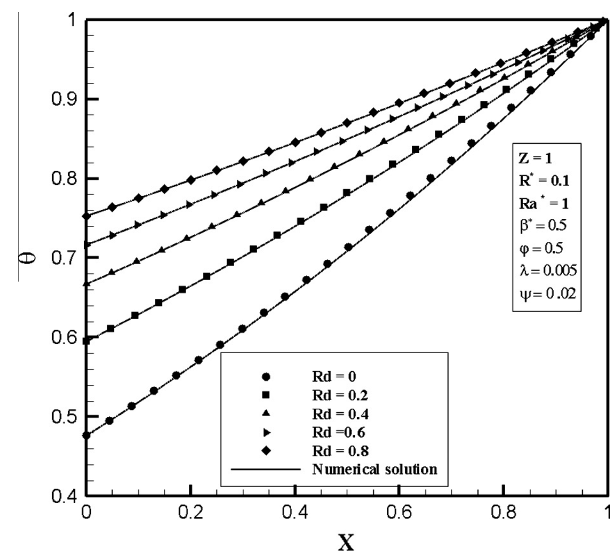

Fig. 3. Temperature distribution obtained by DTM in comparison with the numerical solution for different values of the radiation-conduction parameter *Rd*.

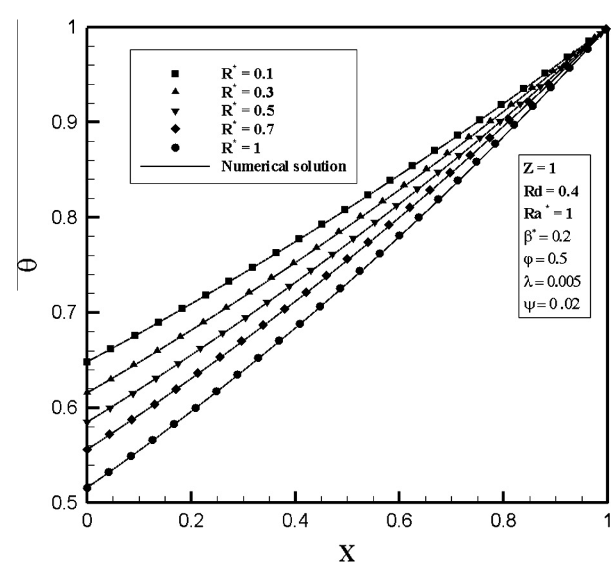

Fig. 4. Temperature profile as a function of fin length predicted by DTM in comparison with the numerical solution for different values of the surface radiation parameter.

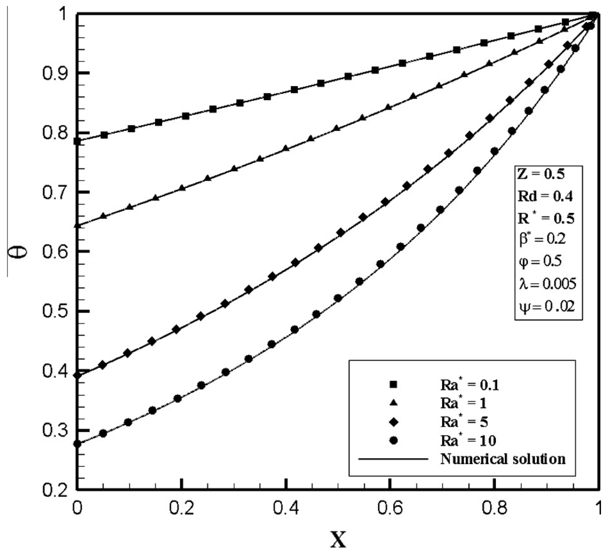

Fig. 5. Temperature profile as a function of fin length obtained by DTM in comparison with the numerical solution for different values of the Rayleigh numbers.

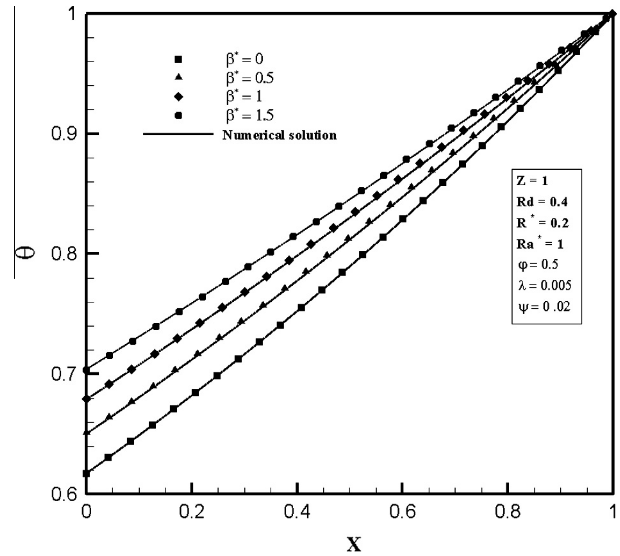

Fig. 7. Temperature distribution as a function of fin length predicted by DTM in comparison with the numerical solution for different values of thermal conductivity parameter.

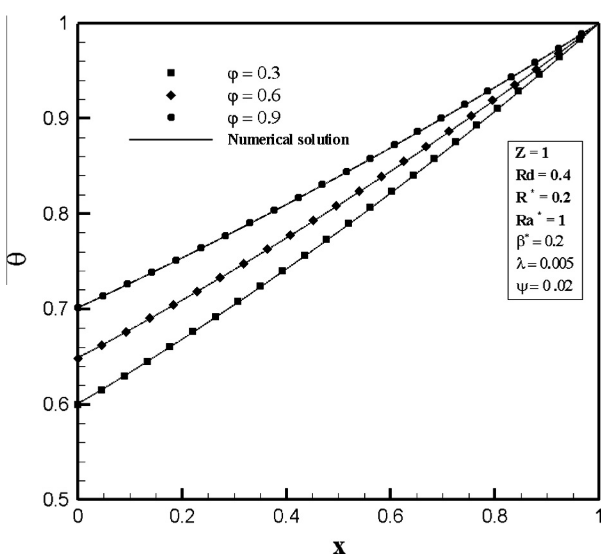

Fig. 6. Temperature distribution as a function of fin length predicted by DTM in comparison with the numerical solution for different values of porosity.

$$\begin{aligned}
&((1+4Rd)(j+1)^2+\beta^*F(0)j(j+1)+\beta^*F(0)(j+1))F(j+1)\\
&\quad=-\beta^*\sum_{i=1}^{j}F(i)(j-i+1)F(j-i+1)\\
&\qquad-\beta^*\sum_{i=1}^{j-1}\sum_{r=0}^{j-i-1}(j-i-r)(j-i-r+1)F(i)F(j-i-r+1)\\
&\qquad-\beta^*\sum_{r=1}^{j-1}(j-r)(j-r+1)F(0)F(j-r+1)\\
&\qquad-\beta^*\sum_{i=0}^{j-1}i(j-i)F(i)F(j-i)+k_1\sum_{i=0}^{j-1}F(i)F(j-i-1)\\
&\qquad+k_2F(j-1)+k_1\omega\sum_{i=0}^{j}F(i)F(j-i)+k_2\omega F(j)
\end{aligned}\tag{22}$$

where $F(j)$ is transformed function of $\theta(X)$.

Putting $j = 0$ in Eq. (22) and considering $F(0) = \varphi$ we obtain

$$F(1)=(k_1\omega\phi^2+k_2\omega\phi)/(1+4Rd+\beta^*\phi)\tag{23}$$

For $j = 1$,

$$\begin{aligned}
F(2)=&(k_1\phi^2+k_2\phi+2k_1\omega\phi F(1)+k_2\omega F(1)\\
&-\beta^*(F(1))^2)/4(1+4Rd+\beta^*\phi)
\end{aligned}\tag{24}$$

Continuation of above process for different integer values of j, a general expression for $F(j)$ can be determined from Eq. (22) as:

$$\begin{aligned}
F(n)=&\frac{1}{n^2(1+4Rd+\beta^*\phi)}\\
&\left[-\beta^*\sum_{i=1}^{n-1}F(i)(n-i)F(n-i)\right.\\
&-\beta^*\sum_{i=1}^{n-2}\sum_{r=0}^{n-i-2}(n-i-r-1)(n-i-r)F(i)F(n-i-r)\\
&-\beta^*\sum_{r=1}^{n-2}(n-r-1)(n-r)F(0)F(n-r)\\
&-\beta^*\sum_{i=0}^{n-2}i(n-i-1)F(i)F(n-i-1)\\
&+k_1\sum_{i=0}^{n-2}F(i)F(n-i-2)+k_2F(n-2)\\
&\left.+k_1\omega\sum_{i=0}^{n-1}F(i)F(n-i-1)+k_2\omega F(n-1)\right],\quad\text{for } n\geqslant 3
\end{aligned}\tag{25}$$

The closed form of fin temperature profile is

$$\theta(X)=\sum_{i=0}^{\infty}X^iF(i)\tag{26}$$

The boundary condition at $X = 1$ is employed for the value ϕ i.e.

$$\theta(1)=\sum_{i=0}^{\infty}F(i)=1\tag{27}$$

The solution of above equation through MATHEMATICA gives ϕ.

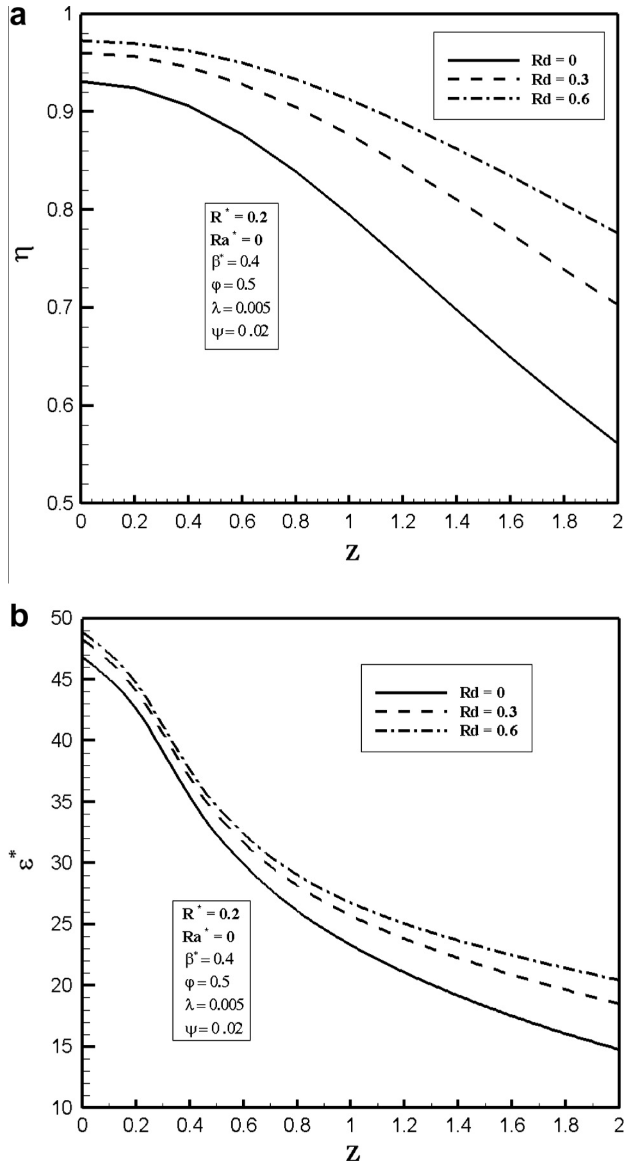

Fig. 8. Fin performances as a function of the fin parameter *Z* for different values of the radiation-conduction parameter *Rd*: (a) fin efficiency; (b) fin effectiveness.

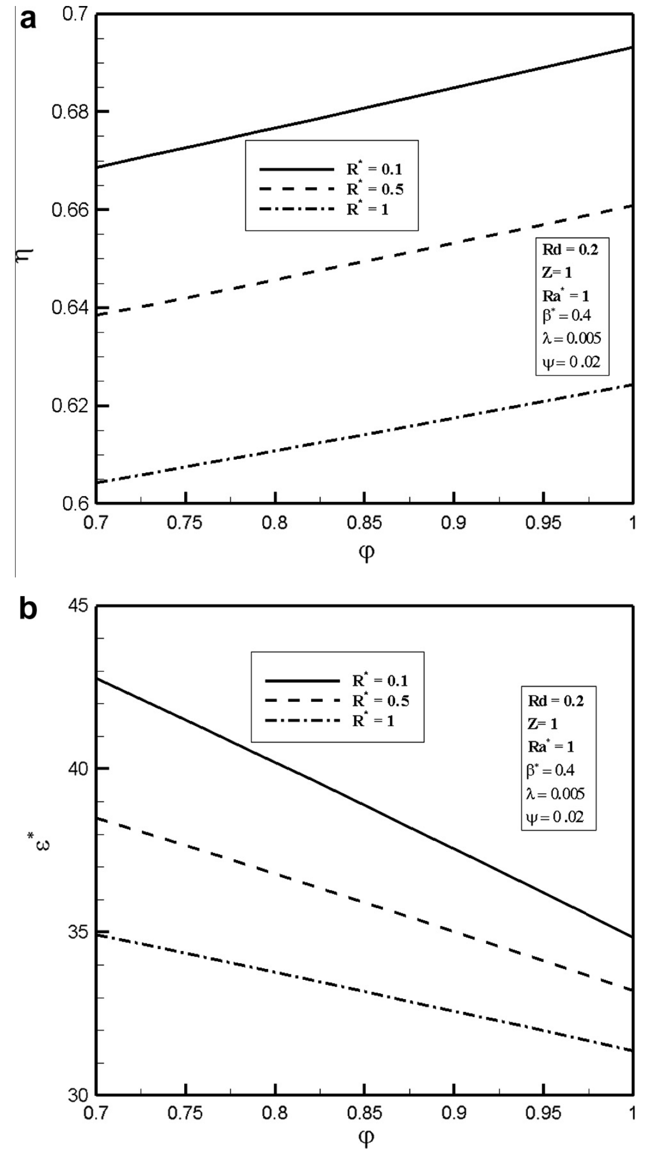

Fig. 9. Fin performances as a function of the porosity for different values of the surface radiation parameter: (a) fin efficiency; (b) fin effectiveness.

5. Results and discussion

The aim here is to see the simultaneous effects of convection and radiation for different values of the governing parameters. The accuracy of differential transformation technique is checked with the help of numerical results obtained by fourth-order Runge–Kutta method. A comparative study between the DTM and numerical results for different emerging parameters is shown in the Figs. 2–7. Fig. 2 shows the temperature profile of the triangular porous fin as a function of fin length predicted by the analytical and numerical solutions for different values of the fin parameter *Z*. It is noticed that the convective cooling becomes stronger which promotes lower temperatures in the fin when *Z* is increased. Fig. 3 provides the temperature profile of the fin along the fin length in both analytical and numerical solutions when the radiation-conduction parameter *Rd* has been assigned the values equal to 0, 0.2, 0.4, 0.6 and 0.8. It is found from these figures that as the radiation-conduction parameter increases, the energy flux of radiation and subsequently the diffusion of heat transfer become more effective which in turn yields an increase in temperatures in the fin. Fig. 4 shows the effect of surface radiation on the temperature distribution of the fin. There is a decrease in fin temperature when thermal radiation parameter R^* increases. It causes stronger cooling which in turn the temperature declines. Fig. 5 depicts the variation of the Rayleigh number Ra^* on the temperature distributions of the fin. The rate of heat removal from the fin surface increases and reduces the effective length of the fin when the Rayleigh number increases. As a consequence the fin temperature decreases. The variation of the temperature distribution along the fin for different values of porosity is illustrated in Fig. 6. Here, the fin temperature enhances when porosity increases. When porosity increases, hear removal rate decreases as well as the effective thermal conductivity of the porous fin decreases due to the removal of the solid material, but in another way, it increases the rate of heat removal by convection and radiation. Fig. 7 shows the temperature profile of a triangular porous fin along the fin length for different values of β^*. As illustrated, the fin temperature is increasing function of β^*. As parameter β^* becomes larger, the heat conduction through fin length amplifies, so that the fin temperature enhances. Finally, it can be concluded that the analytical results coincide exactly with the numerical results. This means that the differential transforma-

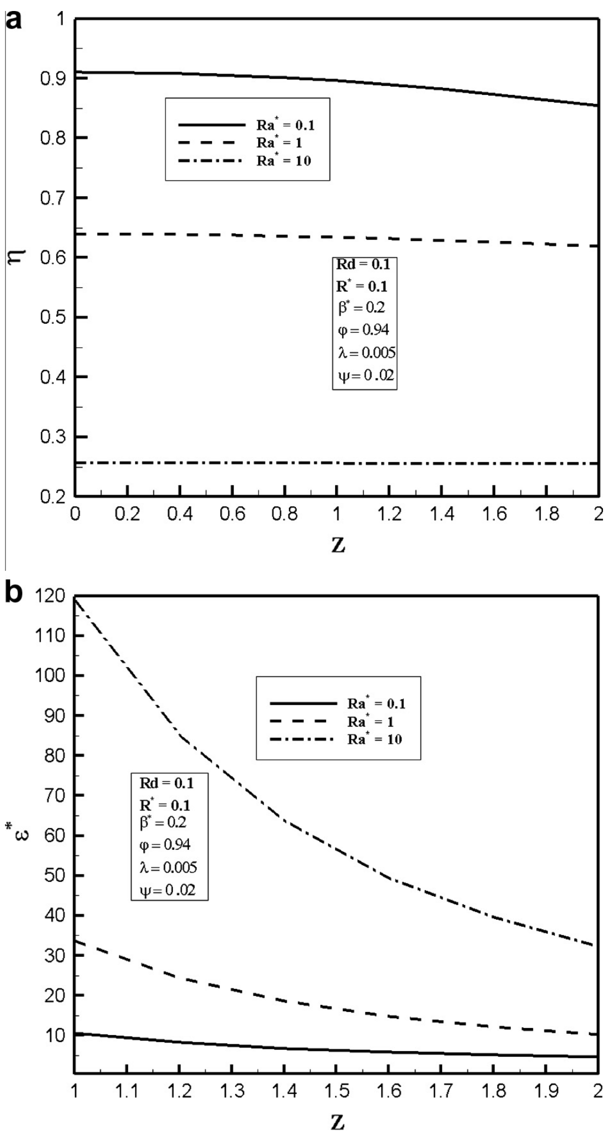

Fig. 10. Variation of the fin performances with the variation in the fin parameter *Z* for different values of the Rayleigh number: (a) fin efficiency; (b) fin effectiveness.

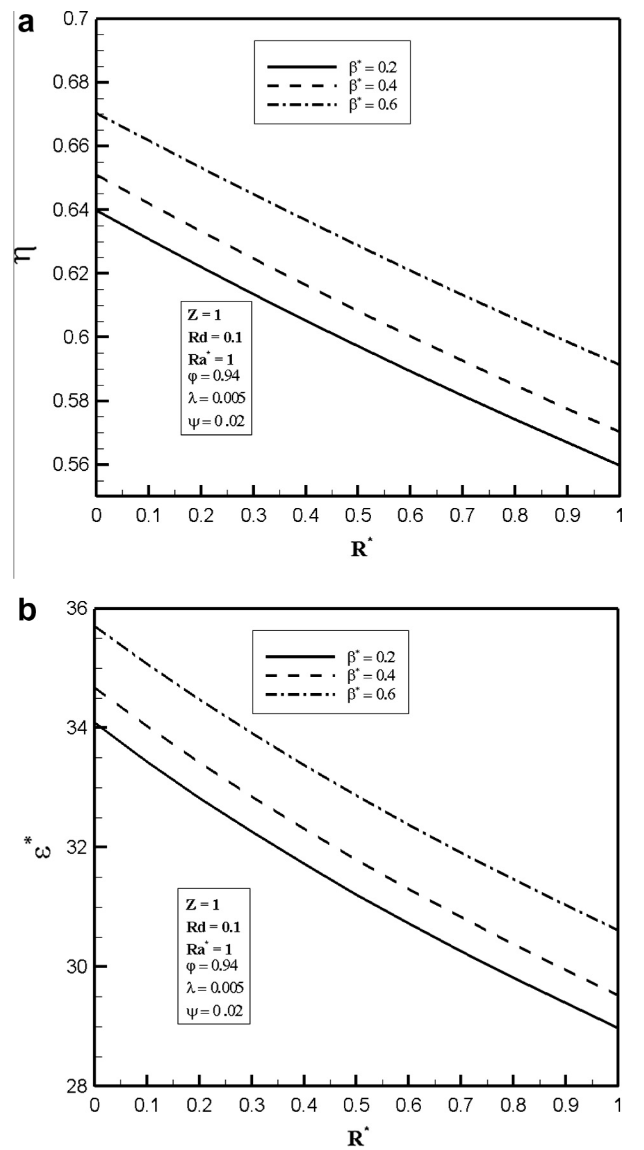

Fig. 11. Fin performances as a function of the surface radiation parameter for different values of thermal conductivity parameters: (a) fin efficiency; (b) fin effectiveness.

tion method has a high ability in solving highly nonlinear initial and boundary value problems without requiring linearization and perturbation.

Having the temperature distribution in the fin, we now concentrate on two significant characteristics of fins, namely fin efficiency and fin effectiveness. The variation of fin efficiency with the variation of fin parameter *Z* for different values of radiation-conduction parameter *Rd* is shown in Fig. 8a and b represents the variation of fin effectiveness as a function of *Z* for different values of *Rd*. It is observed that the fin performance parameters decrease ownіng to an increase in the fin parameter *Z*. In addition, the fin efficiency and fin effectiveness increase when the radiation heat flux becomes stronger. The effects of the surface radiation parameter on the fin efficiency and fin effectiveness as a function of porosity are shown in the Fig. 9a and b respectively. As shown in Fig. 9a, the fin efficiency increases when the value of porosity increases. This nature of result can be completely explained with the help of Fig. 6. A decrease in temperature variation between fin base and fin tip with the porosity gives an increasing fin efficiency, whereas the reverse results are found for the fin effectiveness (see Fig. 9b). For the fin effectiveness, when the porosity increase, as mentioned above the variation of temperature along the fin length decreases (temperature gradient decreases), so that the fin effectiveness declines (see Eq. (21)). Further, both fin performance characteristics decrease as the surface radiation becomes stronger. When the surface radiation enhances, the variation of temperature between fin tip and fin base becomes larger, so that both fin efficiency and fin effectiveness decline. As the porosity increases, the effective surface area of the fin subject to the working fluid increases. As a result the heat transfer rate from the fin surface increases. The similar results for the fin performance have been reported by Kundu and Bhanja [14] for a rectangular porous fin. The influence of the Rayleigh number on the fin performances is investigated by Fig. 10. Fig. 10a illustrates the fin efficiency as a function of fin parameter *Z*. Clearly, the fin efficiency decreases as the natural cooling becomes more effective. For the small values of the Rayleigh number the decline in the fin efficiency is identical, while as the Rayleigh number becomes larger, this decline becomes smaller and the fin efficiency varies gradually with the fin parameter *Z*. Variation of the Rayleigh number on the fin effectiveness is shown in Fig. 10b. It is found that the fin effectiveness increases when the natural convection becomes stronger. Thus the results of fin effectiveness are quite opposite when compared with the fin efficiency. It should be noted that the difference between the fin effectiveness for the different Rayleigh numbers is more severe for the smaller values of the fin parameter Z, whereas such discrepancy declines when the fin parameter Z is increased. For small Rayleigh number, the fin effectiveness changes gradually as a function of fin parameter Z. Fig. 11 plots the fin performance parameters as a function of the surface radiation parameter R^* for different values of β^*. Fig. 11a provides the fin efficiency as function of R^*. It is shown that this performance parameter of porous fins increases as the value of β^* becomes larger. Fig. 11b depicts that the fin effectiveness is a decreasing function of R^*. It is seen that the fin efficiency increases when β^* increases.

6. Conclusions

The thermal performance of straight porous fins of triangular profile with temperature-dependent thermal conductivity in presence of convection and radiation heat transfer was examined. A closed form analytical model was developed by employing the differential transformation method in obtaining temperature distributions through the triangular porous fin. The obtained temperature profiles for different values of emerging parameters with the analytical model were compared with the predicted temperature profiles obtained by forth-order Runge–Kutta method. An excellent agreement is noted between DTM and numerical results. The results indicated that the fin temperature decreases by increasing Z, R^* and Ra^*. On the other hand, the reverse results were found for porosity and radiation-conduction parameter Re. Both the fin efficiency and fin effectiveness decrease when the fin parameter Z and R^* increase. The fin efficiency increases with increased the values of porosity while the contrary outcomes were observed for the fin effectiveness. The results also show that with increasing the Rayleigh number the fin efficiency and fin effectiveness decreases and increases respectively. It was observed that the fin performances highly depend upon the fin parameters Z, R^*, Ra^* and porosity.

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