Local and Post-Buckling Behavior of Tubular Beam-Columns

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Abstract: A simple analytical model is proposed to describe the effects of local buckling of circular cross section on the maximum strength and behavior of tubular beam-columns. The behavior is presented in the form of load-deflection and load-shortening relationships. These relationships are developed on the basis of an assumed deflection method coupled with the moment-thrust-curvature relationship including the softening branch of the relationship due to the local cross-sectional distortion. The analytically obtained maximum strength interaction curves of beam-columns show a reasonably good agreement with the available experimental results. The trend of the analytical load-deflection and load-shortening curves is very similar to that of the available experimental results. It is found that the effects of the local buckling on the behavior and strength of tubular beam-columns become more severe with an increase in diameter-to-thickness ratio, and with a decrease in slenderness ratio.

Introduction

The post-buckling behavior of tubular beam-columns has been the subject of intensive research in recent years (Chen and Han 1985; Han and Chen 1983a, 1983b; Toma and Chen 1983). In these analytical studies, it has been assumed that the cross section of the circular tubular members remains circular even in the post-buckling range. However, at large deformations, significant local buckling or distortion of the cross section may occur. The local buckling of the cross section will significantly reduce the maximum load-carrying capacity and the energy absorption capacity of these thin-walled tubular members. In this paper, a simple analytical method is proposed to describe the effects of this local buckling on the maximum strength and energy absorption capacity of the tubular beam-columns.

The maximum load-carrying capacity of an ideal column can be taken as its bifurcation load. The bifurcation load of an imperfect column provides an upper limit on its maximum load-carrying capacity. The load-deflection method of analysis is used to include the effect of imperfections and lateral loads. To determine the energy absorption capacity of members, both the pre- and post-maximum load-deflection curve should be traced. In the load-deflection approach, the section moment-thrust-curvature relationship must be known before any beam-column analysis can be performed. The local buckling of thin-walled tubular sections causes a significant

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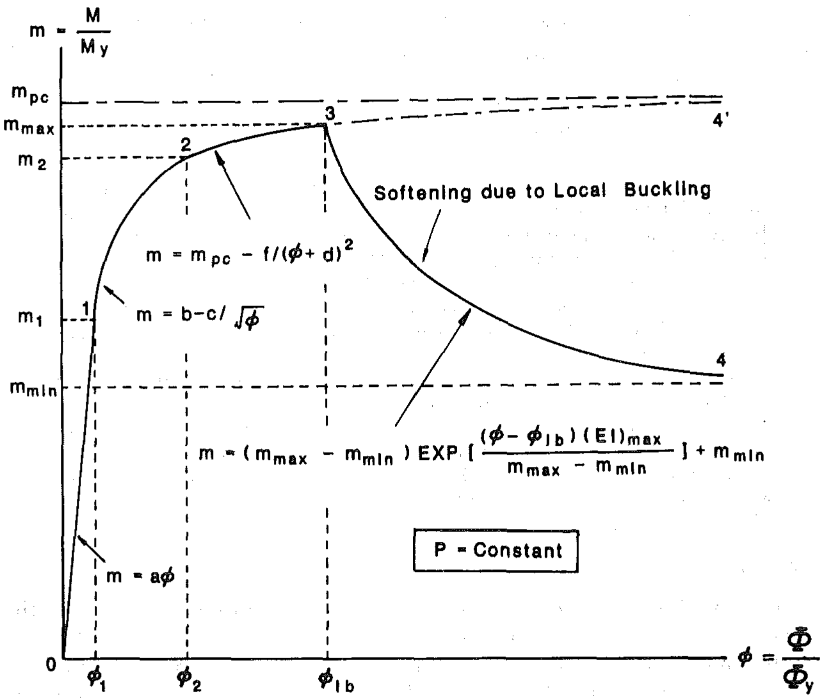

FIG. 1. Closed Form Expressions for Complete M-P-Φ Curves

softening in sectional moment-curvature behavior (Fig. 1). A simple kinematic model (Sohal and Chen 1987) was used to develop the softening branch (Fig. 1, branch 3-4) of a complete moment-thrust-curvature relationship.

The behavior of beam-columns is presented in the form of load-deflection and load-shortening relationships. The analytical expressions for these relationships are developed by supplementing the modified assumed deflection method with a complete moment-thrust-curvature relationship including the softening branch due to the local buckling of the circular cross section (Sohal and Chen 1987). These analytical expressions significantly reduce the computational time required to trace the load-deflection and load-shortening relationships and yet give reasonably accurate results.

The maximum strength interaction curves for tubular beam-columns are compared both with the available experimental results (Sherman 1981) and with the analytical results in which local buckling of the cross section was not considered (Toma and Chen 1979). The load-deflection and load-shortening behavior curves are also compared with the available experimental results (Sherman 1980) and with the analytical results obtained without considering the local buckling of the cross section (Toma and Chen 1983). The effects of various parameters on the behavior of beam-columns are also investigated.

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The loading conditions considered here are the combinations of lateral loads, equal moment at both ends, and axial load. The support condition of the beam-column is either pin-ended or fix-ended.

Generalized Stress-Strain Relationships

In any structural analysis, knowledge of the stress-strain relationship is essential. For the beam-column analysis, moment-thrust-curvature (M-P-Φ) and thrust-curvature-axial strain (P-Φ-ε_0) relations are used as the generalized stress-strain relationships (Chen and Han 1985).

For the moment-thrust-curvature relationship, the closed-form expressions have been developed previously (Sohal and Chen 1984, 1987). The general form of the M-P-Φ curve was divided into four parts (Fig. 1): (1) Elastic (curve 0–1); (2) primary yielded (curve 1–2); (3) secondary yielded (curve 2–3); and softened (curve 3–4). The constants a, b, c, d and f in Fig. 1 were determined such that the curves fit closely to the more rigorous solutions (Sohal and Chen 1984). In the figure, $m_{pc} = M_{pc}/M_y$ is the plastic moment capacity of the tubular section, normalized by the yield moment (M_y) of the section, and reduced for a given axial thrust P. The curvature at which the local buckling of the cross section starts is $\phi_{lb} = \Phi_{lb}/\Phi_y$, which is normalized by the initial yield curvature (Φ_y) of the section. The slope of the softening branch at the start of the local buckling of the cross section is $(EI)_{max}$.

The thrust-curvature-strain (P-Φ-ε_0) relationship for a circular tube can be obtained theoretically by assuming that the wall thickness of the section is thin in comparison with its diameter (Chen and Han 1985). Here, for simplicity, it is assumed that for a given curvature Φ, the P-ε_0 relationship is not affected significantly by the local buckling of the cross section. Thus, the P-Φ-ε_0 relationship for a circular tube, the one used in the pre-local-buckling regime, is used throughout the analysis.

Modified Assumed Deflection Method

The behavior of steel structures has been investigated in the past only under a monotonically increasing moment-curvature condition. The softening part of a moment-curvature relationship is encountered only when the effect of local buckling is considered. For concrete beam-columns, however, the softening part has been encountered due to the cracking and spalling of the concrete. Therefore, in recent years, analytical methods have been proposed to include the softening branch of a complete moment-curvature relationship (Darvall 1984; Darvall and Mendis 1985). Most of these analytical methods require the use of a hinge length. No analytical method is yet available to determine the hinge length; it must be determined from empirical relations.

Here, in the beam-column analysis, the softening branch of a complete moment-curvature relationship is included by assuming the deflected shape of a beam-column, without the use of a hinge length. In this method, we need to consider equilibrium of external loads and internal resistance at only one critical section, which leads to analytical expressions of the load-deflection relation. These analytical expressions drastically reduce the computational time required for tracing the load-deflection and load-shortening relationships.

To obtain accurate results, the assumed deflected shape should be as close to the actual deflected shape as possible. In the linear portion of M-P-Φ relationship, Fig. 1, the exact deflected shapes of the beam-columns are well-known. For example, the deflected shape of an axially loaded beam-column is sinusoidal. Deflected shapes for beams subjected to lateral loads or end moments are polynomial functions. These shapes will be called elastic deflected shapes from here onwards. In the nonlinear portions of M-P-Φ relationship, the deflected shape is a combination of the elastic deflected shape and the failure mechanism shape.

Herein, the deflected shape is assumed to be the elastic deflected shape in the pre-buckling region. In the post-buckling region, the deflected shape is assumed to vary smoothly from the elastic deflected shape to the failure mechanism shape. The deflection W, at a given distance x, from the end of a beam-column can be written as

$$W = W_m[f_w F_e + (1 - f_w)F_m] \quad (1)$$

in which W_m is the maximum deflection at mid-span of the beam-column, F_e is a function representing the elastic deflected shape, F_m is a function representing the failure mechanism shape, and f_w is a weighting factor.

The elastic deflection shape function F_e, for a pin-ended beam-column under axial load can be written as

$$F_e = \sin \frac{\pi x}{L} \quad (2a)$$

For a fix-ended beam-column

$$F_e = \frac{1}{2}\left(1 - \cos \frac{2\pi x}{L}\right) \quad (2b)$$

in which L is the length of the beam-column.

The failure mechanism shape for a beam-column is close to a bilinear shape (Fig. 2(b)), and can be simulated by assuming the curvature distribution as follows:

$$\phi = \frac{\phi_m}{1 + \left(\dfrac{\dfrac{x}{L} - \dfrac{1}{2}}{\beta_1}\right)^2} \quad (3)$$

in which ϕ_m is the maximum curvature at mid-span and β_1 is a parameter representing the spread of curvature at failure (Fig. 2(a)). The failure mechanism shape function corresponding to the curvature distribution given by Eq. 3 can be integrated and written as

$$F_m = 1 - \frac{\left(\dfrac{x - L/2}{\beta_1 L}\right)\tan^{-1}\left(\dfrac{x - L/2}{\beta_1 L}\right) - \dfrac{1}{2}\ln\left[1 + \left(\dfrac{x - L/2}{\beta_1 L}\right)^2\right]}{\left(\dfrac{1}{2\beta_1}\right)\tan^{-1}\left(\dfrac{1}{2\beta_1}\right) - \dfrac{1}{2}\ln\left(1 + \dfrac{1}{4\beta_1^2}\right)} \quad (4)$$

The curvature distribution and the failure mechanism function for $\beta_1 = 0.1$, 0.01, 0.001, are shown in parts (a) and (b) of Fig. 2.

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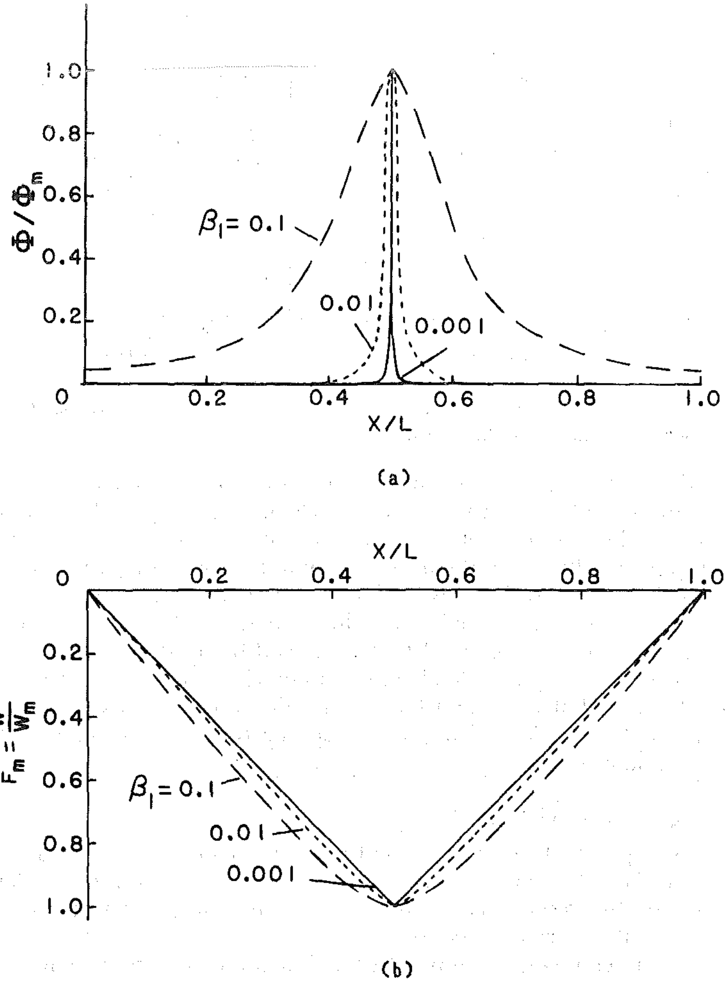

FIG. 2. Distribution of Curvature and Displacement Along Beam-Columns at Failure: (*a*) Curvature; and (*b*) Displacement

The weighting factor f_w is equal to one in the pre-buckling regime. In the post-buckling regime, f_w decreases exponentially with the decrease in the load-carrying capacity of the beam-column and can be written as follows.

$$f_w = \exp\left[\left(1 - \frac{P_{max}}{P}\right)\beta_2\right] \quad \text{(5)}$$

where β_2 is a parameter representing the rate of change of the deflected shape. The P_{max} in Eq. 5 is calculated by the average flow moment method as proposed previously by Chen and Atsuta (1972).

In order to determine the parameters β_1 and β_2, the load-deflection and load-shortening curves of several beam-columns, obtained from the pro-

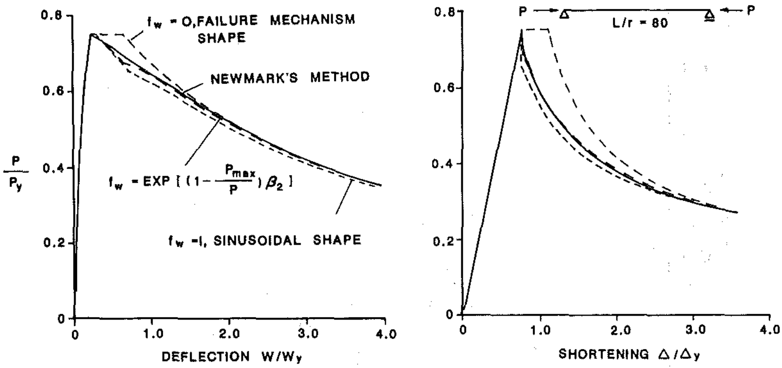

FIG. 3. Comparison of Load-Deflection and Load-Shortening Curves by Newmark's and Modified Deflection Methods

posed deflected shape were compared with those obtained from Newmark's numerical integration method (Chen and Atsuta 1976), a more rigorous method. For beam-columns with an effective slenderness ratio (KL/r) between 40 and 120, the most suitable values of β_1 and β_2 were found to be 0.04 and 0.3, respectively; where K = the effective length factor; L = the length of the beam-column; and r = the radius of gyration of the section. These values of β_1 and β_2 will be used in this paper from here onwards. For a pin-ended beam-column with a slenderness ratio equal to 80, the load-deflection and load-shortening relationships, obtained by the proposed deflection shape, are compared to those obtained by Newmark's method in Fig. 3.

Load-Deflection Relation

The mid-span deflection W_0, due to end moments and/or lateral load is calculated from the conventional beam theory. The additional deflection W_m due to the axial load is determined by considering the moment equilibrium at the critical section.

For pin-ended beam-columns, the bending moment at mid-span induced by external loads is

$$M_{ext} = M_{MQ} + P(W_i + W_0 + W_m) \quad (6)$$

where $M_{MQ} = M_0 + (QL/4)$ = bending moment due to the end moment M_0 and/or the lateral load Q; W_i = initial imperfection at mid-span; and P = axial load. In nondimensionalized form, we have

$$m_{ext} = m_{MQ} + m_i + m_{0P} + \frac{P}{M_y} W_m \quad (7a)$$

where $m_{ext} = M_{ext}/M_y$, $m_{MQ} = M_{MQ}/M_y$, $m_i = P\,W_i/M_y$, and $m_{0P} = P\,W_0/M_y$. For fix-ended beam-columns, the non-dimensionalized external moment can be expressed as

$$m_{ext} = m_{MQ} + m_i + m_{0P} + \frac{P}{M_y}\frac{W_m}{2} \quad (7b)$$

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For a given curvature, the internal resisting moment can be obtained from the closed form moment-curvature expressions (Sohal and Chen 1984) in terms of the parameters: a, b, c, d, f, m_1, m_2, m_{pc}, ϕ_1, ϕ_2, ϕ_{lb}, m_{max}, m_{min} and $(EI)_{max}$, Fig. 1. However, in order to obtain the internal resisting moment corresponding to a given mid-span deflection, a relationship between the mid-span curvature and the mid-span displacement is needed. This relationship can be obtained by a double differentiation of the assumed deflected shape given by Eq. 1. For pin-ended beam-columns, the resulting relationship can be expressed as

$$\phi_P = \frac{W_m}{L^2}\frac{EI}{M_y}\left\{\pi^2 f_w + \frac{1-f_w}{\beta_1^2\left[\frac{1}{2\beta_1}\tan^{-1}\frac{1}{2\beta_1} - \frac{1}{2}\ln\left(1+\frac{1}{4\beta_1^2}\right)\right]}\right\} \quad \text{(8)}$$

For $\beta_1 = 0.04$, the above expression reduces to

$$\phi_P = W_m \frac{P_{cr}}{M_y}[f_w + 3.9(1-f_w)] \quad \text{(9a)}$$

in which P_{cr} is Euler's buckling load. For fix-ended beam-columns, the resulting relationship is

$$\phi_P = \frac{W_m}{2}\frac{P_{cr}}{M_y}[f_w + 1.95(1-f_w)] \quad \text{(9b)}$$

where the parameter f_w is taken the value one in the pre-buckling regime, and is given by Eq. 5 in the post-buckling regime.

In the following sections, load-deflection expressions are obtained by equating the external moments due to the applied loads on the beam-column to the internal resisting moments of the cross section in the elastic, primary yielded, secondary yielded and post-local buckling regimes of the M-P-Φ relationship.

Elastic Regime

In the elastic range, the internal resisting moment at mid-span can be expressed as

$$m_{int} = a\phi_m = a(\phi_{MQ} + \phi_P) \quad \text{(10)}$$

where $m_{int} = M_{int}/M_y$; a = stiffness constant; $\phi_m = \Phi_m/\Phi_y$, and $\phi_{MQ} = m_{MQ}$. For pin-ended beam-columns, by equating Eqs. 7a and 10 and by substituting ϕ_p from Eq. 9a, for $W_m \le W_{1P}$, the mid-span deflection W_m can be expressed as

$$W_m = \frac{m_{MQ}(1-a) + m_i + m_{0P}}{\left\{a[f_w + 3.9(1-f_w)] - \frac{P}{P_{cr}}\right\}\frac{P_{cr}}{M_y}} \quad \text{(11a)}$$

For fix-ended beam-columns, for $W_m/2 \le W_{1F}$, we have

$$W_m = 2 \frac{m_{MQ}(1-a) + m_i + m_{0P}}{\left\{a[f_w + 1.95(1-f_w)] - \frac{P}{P_{cr}}\right\} \frac{P_{cr}}{M_y}} \quad \text{(11b)}$$

in which W_{1P} and W_{1F} are the elastic limit deflections for pin-ended and fix-ended beam-columns, respectively. The following expressions for these elastic limit deflections are obtained by substituting ϕ_P from Eq. 9 and $\phi_m = \phi_1$ in Eq. 10.

$$W_{1P} = (\phi_1 - m_{MQ})\left(\frac{M_y}{P_{cr}}\right) \frac{1}{[f_w + 3.9(1-f_w)]} \quad \text{(12a)}$$

$$W_{1F} = (\phi_1 - m_{MQ})\left(\frac{M_y}{P_{cr}}\right) \frac{1}{[f_w + 1.95(1-f_w)]} \quad \text{(12b)}$$

Primary Yield Regime

Similarly, by equating the external bending moment to the internal resisting moment in the primary yield range, the expressions for the mid-span displacement are obtained. For a pin-ended beam-column, for $W_{1P} \le W_m \le W_{2P}$ and $W_m \le W_{lbP}$, the resulting expression is

$$\left[m_{MQ} + m_i + m_{0P} - b + \frac{P}{M_y} W_m\right]^2$$

$$\left\{m_{MQ} + \frac{P_{cr}}{M_y} W_m[f_w + 3.9(1-f_w)]\right\} = c^2 \quad \text{(13a)}$$

For fix-ended beam-columns, for $W_{1F} \le W_m/2 \le W_{2F}$ and $W_m/2 \le W_{lbF}$, the resulting expression is

$$\left[m_{MQ} + m_i + m_{0P} - b + \frac{P}{M_y}\frac{W_m}{2}\right]^2$$

$$\left\{m_{MQ} + \frac{P_{cr}}{M_y}\frac{W_m}{2}[f_w + 1.95(1-f_w)]\right\} = c^2 \quad \text{(13b)}$$

in which W_{2P} and W_{2P} are the primary yield limit deflections for pin-ended and fix-ended beam-columns, respectively, and W_{lbP} and W_{lbF} are the deflections at the start of the local buckling for pin-ended and fix-ended beam-columns, respectively. W_{2P}, W_{2F}, W_{lbP} and W_{lbF} can be expressed as follows.

$$W_{2P} = (\phi_2 - m_{MQ})\left(\frac{M_y}{P_{cr}}\right) \frac{1}{[f_w + 3.9(1-f_w)]} \quad \text{(14a)}$$

$$W_{2F} = (\phi_2 - m_{MQ})\left(\frac{M_y}{P_{cr}}\right) \frac{1}{[f_w + 1.95(1-f_w)]} \quad \text{(14b)}$$

$$W_{lbP} = (\phi_{lb} - m_{MQ})\left(\frac{M_y}{P_{cr}}\right) \frac{1}{[f_w + 3.9(1-f_w)]} \quad \text{(15a)}$$

$$W_{lbF} = (\phi_{lb} - m_{MQ})\left(\frac{M_y}{P_{cr}}\right)\frac{1}{[f_w + 1.95(1 - f_w)]} \quad \text{(15b)}$$

Secondary Yield Regime

For pin-ended beam-columns, for $W_{2P} \leq W_m \leq W_{lbP}$, we have

$$\left[m_{pc} - m_{MQ} - m_i - m_{0P} - \frac{P}{M_y} W_m\right]$$

$$\left\{m_{MQ} + d + \frac{P_{cr}}{M_y} W_m[f_w + 3.9(1 - f_w)]\right\}^2 = f \quad \text{(16a)}$$

For fix-ended beam-columns, for $W_{2F} \leq W_m/2 \leq W_{lbF}$, we have

$$\left[m_{pc} - m_{MQ} - m_i - m_{0P} - \frac{P}{M_y}\frac{W_m}{2}\right]$$

$$\left\{m_{MQ} + d + \frac{P_{cr}}{M_y}\frac{W_m}{2}[f_w + 1.95(1 - f_w)]\right\}^2 = f \quad \text{(16b)}$$

Post-Local-Buckling Regime

For pin-ended beam-columns, for $W_m \geq W_{lbP}$, we have

$$(m_{\max} - m_{\min}) \exp\left[\frac{\left\{\frac{P_{cr}}{M_y} W_m[f_w + 3.9(1 - f_w)] - \phi_{lb}\right\}(EI)_{\max}}{(m_{\max} - m_{\min})}\right] + m_{\min}$$

$$= m_{MQ} + m_i + m_{0P} + \frac{P}{M_y} W_m \quad \text{(17a)}$$

For fix-ended beam-columns, for $W_m/2 \geq W_{lbF}$, we have

$$(m_{\max} - m_{\min}) \exp\left[\frac{\left\{\frac{P_{cr}}{M_y}\frac{W_m}{2}[f_w + 1.95(1 - f_w)] - \phi_{lb}\right\}(EI)_{\max}}{(m_{\max} - m_{\min})}\right]$$

$$+ m_{\min} = m_{MQ} + m_i + m_{0P} + \frac{P}{M_y}\frac{W_m}{2} \quad \text{(17b)}$$

Eqs. 11, 13, 16, and 17 are used to determine both the pre- and post-maximum deflections of a beam-column with a given value of axial thrust. Eqs. 11, 13, and 16 can be solved directly without any iteration. However, a few iterations are required to solve Eq. 17.

Load-Shortening Relationship

The total axial shortening of a beam-column consists of two parts: (1) The axial shortening due to the axial strain; and (2) the axial shortening due to the lateral deflection. Herein, the axial shortening of the beam-column is calculated by dividing the beam-column into a number of segments (N).

The axial shortening due to the axial strain is calculated by (Chen and Han 1985; Toma and Chen 1983)

$$\Delta_s = \sum_{i=1}^{N} \left(\frac{L}{N}\right) \varepsilon_{0i} \quad (18)$$

in which ε_{0i} is the axial strain in segment i. The axial strain ε_{0i} is determined further in two additional steps. First, curvature is determined from the known mid-span deflection and weighting factor f_w. Then, this curvature and the given axial thrust are used to determine the axial strain from the thrust-curvature-strain relationship. The thrust-strain (p-ε) relationship, for a given curvature, is assumed to be unaffected by the change in the shape of the cross section.

The axial shortening due to the lateral deflection is calculated by (Chen and Han 1985)

$$\Delta_g = \sum_{i=1}^{N} \left[\frac{L}{N} - \sqrt{\left(\frac{L}{N}\right)^2 - (W_{1i} - W_{2i})^2}\right] \quad (19)$$

in which W_{1i} and W_{2i} are the total deflections at the two ends of the segment i. The deflections W_{1i} and W_{2i} in Eq. 19 are determined by using Eq. 1.

Numerical Studies

In this section, Newmark's method (Chen and Atsuta 1976) is used first to investigate the maximum strengths of concentrically loaded columns and of beam-columns with end moments. The modified assumed deflection method is used next to investigate the pre-buckling, post-buckling, and post-local-buckling behavior of various beam-columns. The load-deflection and load-shortening relationships computed in the present studies are compared with those in which the effect of the local buckling of the cross section was not considered (Toma and Chen 1983). Comparisons are also made with the experimental results. The effect of important parameters on the behavior and strength of beam-columns is also studied.

Maximum Strength of Beam-Columns with Equal End Moments

The maximum strength interaction curves computed both with and without considering the local buckling of cross section are compared with the available experimental data (Sherman 1981) in Fig. 4. The beam-column with the effective slenderness ratio (KL/r) equal to 30 and the diameter-to-thickness ratio (D/t) equal to 77 is chosen for the comparison. In Fig. 4, M_p is the full plastic moment capacity of the tubular section, and P_y is the yield axial load of the tubular section. There is a reasonably good agreement between the experimental results and the present interaction curve in which the effect of the local buckling of the cross section is considered.

For beam-columns with KL/r = 40, 80, and 120, the maximum strength interaction curves obtained by considering the local buckling of the cross

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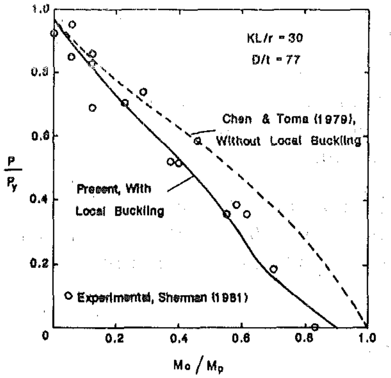

FIG. 4. Comparison of Experimental Data with Numerical Maximum Strength Interaction Curve for Beam-Columns with Equal End Moments (*KL*/*r* = 30)

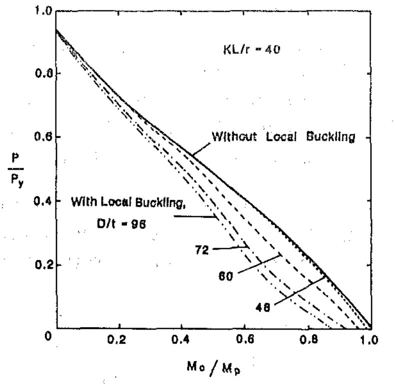

FIG. 5. Effect of Local Buckling on Maximum Strength Interaction Curve of Beam-Columns with Equal End Moments (*KL*/*r* = 40)

section are compared with those obtained without considering the local buckling of the cross section, in Figs. 5–7 respectively. In general, the effect of the local buckling on the maximum strength increases as the diameter-to-thickness ratio increases and as the slenderness ratio decreases. The effect of the local buckling on the beam-column strength (see Figs. 5–7) is found to be more severe for the axial load nondimensionalized with yield axial load (P/P_y) between 0.1 and 0.4. This happens because sectional strength is more affected for P/P_y = 0.1 to 0.4 (Sohal and Chen 1987).

The effect of the local buckling for the special cases of zero axial load is found to be the same for all ratios of *KL*/*r*. For this pure bending case, the

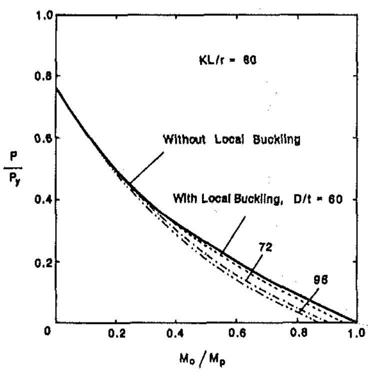

FIG. 6. Effect of Local Buckling on Maximum Strength Interaction Curve of Beam-Columns with Equal End Moments (*KL*/*r* = 80)

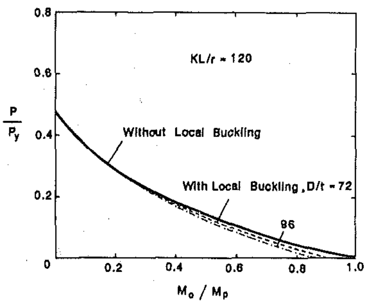

FIG. 7. Effect of Local Buckling on Maximum Strength Interaction Curve of Beam-Columns with Equal End Moments (*KL*/*r* = 120)

maximum load-carrying capacity of members is independent of the length of members, and it depends only on the strength of the critical section.

For the special case for which $M = 0$ (concentrically loaded columns, Figs. 5–7), the maximum load-carrying capacity of columns is not affected by the local buckling of the cross section. For these columns, the local buckling occurs in the post-maximum regions.

Comparison of Behavior with Results without Local Buckling

The beam-column with the slenderness ratio (L/r) equal to 80 and with the diameter-to-thickness ratio (D/t) equal to 48 (most commonly used in

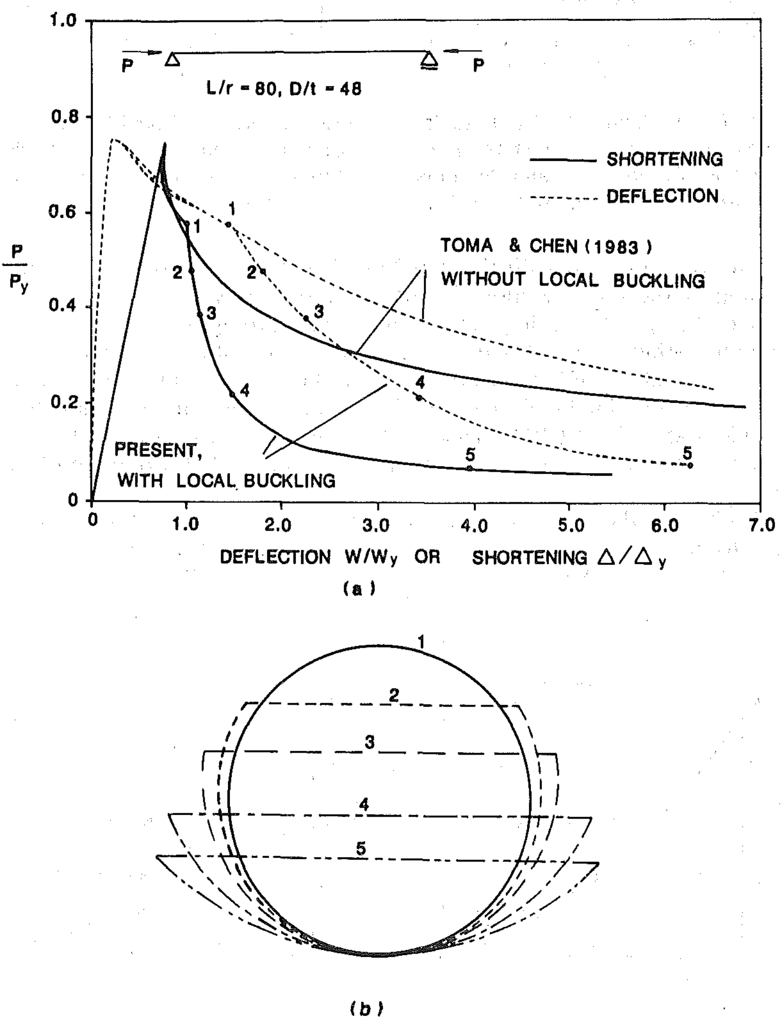

FIG. 8. Changes of Cross Section Along the Load-Deflection and Load-Shortening Curves for Pin-Ended Column: (a) Load-Deflection and Load-Shortening Curves; (b) Changes of Cross Section

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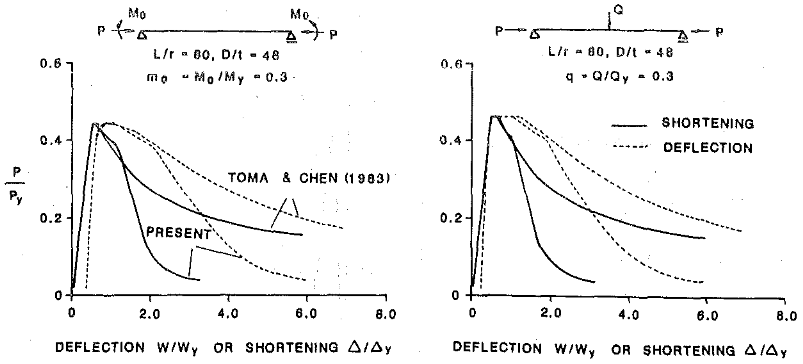

FIG. 9. Comparison of Numerical Results for Beam-Columns with End Moments or Lateral Load

offshore frames) has been chosen for the present comparison. The out-of-straightness is taken as 0.1%.

Fig. 8(*a*) shows the comparison of the computed load-deflection and load-shortening curves of a pin-ended member under axial load. Before local buckling, there is a slight difference between the present results and those reported by Toma & Chen (1983). This difference is due to the use of the modified assumed deflection shape in the present study. The effect of the local buckling starts after the inelastic buckling of the column. In the post-buckling branch, the local buckling accelerates the decrease in axial load-carrying capacity of the member. The five-stage changes in the shape of the cross section during the post-local-buckling regime as marked in Fig. 8(*a*) are shown in Fig. 8(*b*).

The comparison for the same member in the presence of end moment and lateral load is shown in Fig. 9. For these cases, the post-local-buckling behavior is similar to the case of zero end moment and zero lateral load.

Comparison of Behavior with Experimental Results

The analytical load-deflection and load-shortening curves are compared with the experimental results (Sherman 1980) in Figs. 10 and 11. The fix-ended beam-columns both with and without lateral loads are chosen for the comparison. In both figures, a behavior trend can be observed, in that in the post-buckling regime, the local buckling of the cross section accelerates the reduction in the load-carrying capacity of a beam-column.

The analytical results indicate that the local buckling of the cross section initiates at the higher loads than those indicated by experimental results. This may be due to the fact that the experimental results shown in Figs. 10 and 11 were performed on beam-columns made from ERW (electrical resistance weld) tubes. The moment-curvature relationship used in the present study is based on the critical strains (strains at which the local buckling of the cross section starts) observed for fabricated tubes (Sohal and Chen 1987) which are most commonly used in offshore structures. The ERW tubes usually have lesser imperfections and higher critical strains, which delay the initiation of local buckling and result in higher load-deflection and load-shortening curves in post-local-buckling regime.

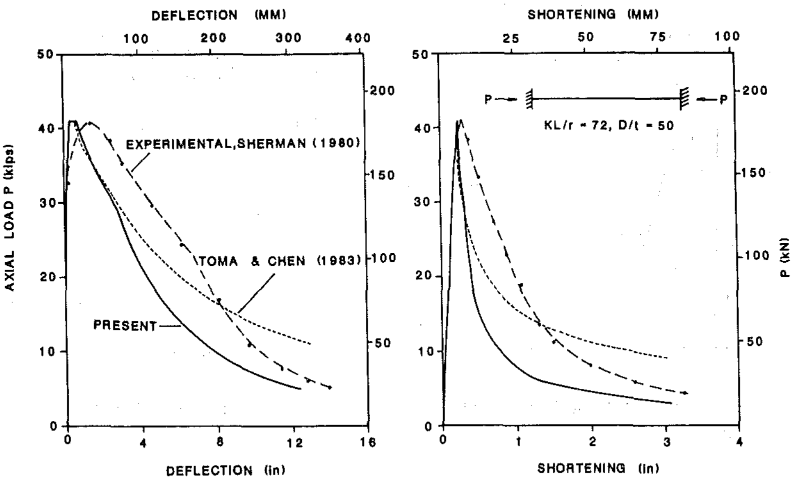

FIG. 10. Comparison of Experimental and Numerical Results for Fix-Ended Column

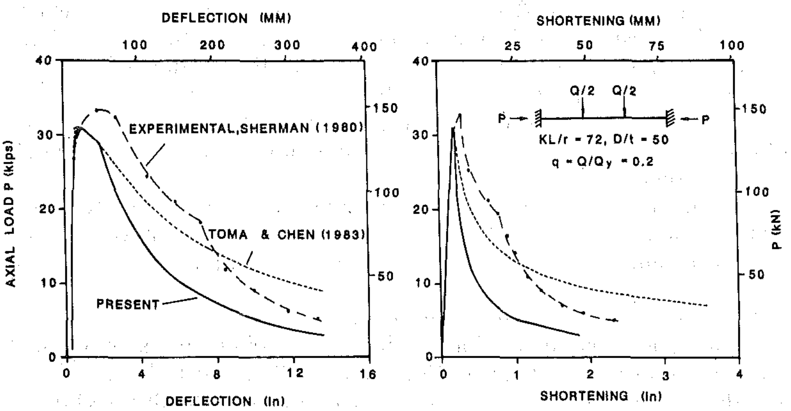

FIG. 11. Comparison of Experimental and Numerical Results for Fix-Ended Beam-Column with Concentrated Lateral Loads

In both figures, in general, the deflections and shortenings from the experiments are higher than those from the analysis. There are many factors contributing to this difference. In the analytical studies, the ends of the members are assumed perfectly restrained. In the experiments, the ends were welded to the tubes split lengthwise. The flexibility of the split tubes has resulted in additional shortenings and lateral deflections. The residual stresses and geometric imperfections produced by the end welds have not been considered in the present analysis. The other contributing factors might be the eccentricity of axial load and strain hardening of the material. Note that if the experimental curves are shifted to match the peak points of the analytical curves, the post-buckling branches of the experimental curves would be much closer to those of the analytical curves obtained by considering the local buckling of the cross section.

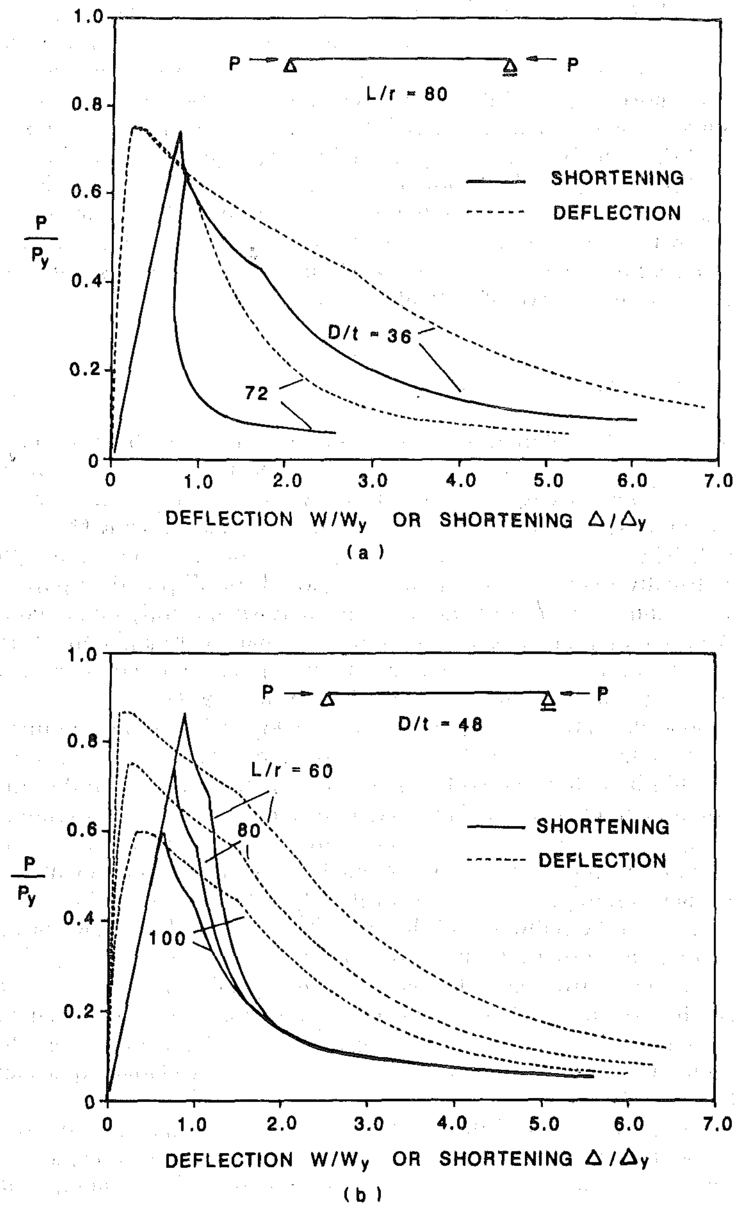

FIG. 12. Effect of: (*a*) Diameter-to-Thickness Ratio; and (*b*) Slenderness Ratio on Member Behavior

Parametric Studies

The parameters considered here are diameter-to-thickness ratio (D/t) and slenderness ratio (L/r). The effect of D/t on the behavior of a pin-ended tubular column is shown in Fig. 12(a). Two diameter-to-thickness ratios are considered: 36 and 72. For the member with D/t equal to 36, the effect of the local buckling starts at shortening approximately equal to 1.75 times the yield shortening Δ_y, and it is very small. For the member with D/t equal to 72, the effect of the local buckling starts at an axial shortening less than the yield shortening, and it is very severe. The steeper negative slopes of softening branches of moment-curvature curves for cross sections with higher D/t's contribute to the severe effect for members with higher D/t's.

Fig. 12(b) shows the behavior of members with slenderness ratios (L/r) equal to 60, 80, and 100. The effect of the local buckling is greater for the column with slenderness ratio equal to 60 and it becomes lesser as slenderness ratio increases. In other words, columns are less sensitive to the decrease in strength of critical section.

Summary and Conclusions

A simple analytical method is proposed to include the effect of the local buckling on the maximum strength and behavior of tubular beam-columns.

The maximum strength of concentrically loaded columns and eccentrically or laterally loaded beam-columns is investigated by using Newmark's method. The effect of the local buckling on the maximum strength of concentrically loaded columns can be ignored for all practical purposes. The maximum strength of beam-columns may significantly be affected by the local buckling of the cross section. The analytical maximum strength interaction curve for beam-columns, shows a reasonably good agreement with the available experimental results (Sherman 1981).

The load-deflection expressions are developed by supplementing the modified deflection method with a complete moment-thrust-curvature relationship including the softening branch of the relationship due to the cross-sectional distortion. The load-deflection and load-shortening relationships of several beam-columns are investigated by using these expressions. In the post-buckling regime, the local buckling of the cross section accelerates the reduction in the strength of beam-columns. This acceleration in the reduction of the strength is mainly due to the softening in the moment-thrust-curvature relationship. The faster change in the deflected shape, from smooth elastic to bilinear mechanism shape, also contributes to the rapid reduction in the post-local-buckling strength of beam-columns. The trend of the analytical load-deflection and load-shortening relationships is the same as that of the available experimental results.

In general, the effect of the local buckling on the behavior and strength of beam-columns increases with an increase in diameter-to-thickness ratio and with a decrease in slenderness ratio. For the beam-columns with a diameter-to-thickness ratio equal to or greater than 48, the local buckling effect becomes significant; therefore, it must be considered in the analysis and design of tubular structures.

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Appendix II. Notation

The following symbols are used in this paper:

a, b, c, d, f = constants defining the shape of moment-curvature curves;
D = diameter of the tubular cross section;
EI = bending rigidity;
F = deflected shape function;
f_w = a weighting factor representing the combination of deflected shapes;
K = effective length factor;
L = length of the beam-column;
M = bending moment;
m = bending moment normalized by the yield moment of the cross section;
N = number of segments of the beam-column;
P = axial thrust;
p = axial thrust normalized by the yield axial thrust of the cross section;
r = radius of gyration;
t = wall thickness of the beam-column;
W = lateral deflection;
x = distance along the beam-column;

β_1 = parameter defining the spread of curvature;

β_2 = parameter defining the rate of change of the deflection shape;

Δ = axial shortening;

ε = axial strain;

Φ = curvature; and

ϕ = curvature normalized by the yield curvature of the cross section.