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EFFECT OF EMBEDMENT ON VIBRATION AND DYNAMIC SOIL PARAMETERS FOR BLOCK FOUNDATION OF RECIPROCATING TYPE OF MACHINES

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ABSTRACT

Block Foundation is normally used for reciprocating type of machines (e.g- Pumps, IC engines, Compressors etc). A reciprocating machine is usually associated with six degrees of freedom, viz. Vertical vibration, Sliding vibration considered separately in lateral and longitudinal directions, rocking vibration considered separately in lateral and longitudinal directions, Yawing motion. The two most important parameters for designing the block foundation are (i) Frequency of Vibration and (ii) Amplitude of Vibration, of the machine-foundation system individually for each of the above mentioned degrees of freedom. The popular methods of analysis include Linear elastic weightless spring method and Elastic half space method. The effect of embedment on the above mentioned parameters have been studied extensively by researchers.

This theoretical study, based on Linear Elastic Weightless Spring Method has been carried out on varying sizes of square blocks (3 m X 3 m X 3 m, 4 m X 4 m X 3 m, 5 m X 5 m X 3 m) to determine the effect of embedment ratio (ɖ) (defined as the depth of embedment to height of foundation block) on the Frequency ratio (Ꞙ) (defined as the ratio of machine frequency and natural frequency of machine- soil combination) and amplitude ratio (Ϫ) (defined as the amplitude of vibration at a particular embedment ratio to the amplitude of vibration at zero embedment). The effect of variation of dynamic soil parameter (C_u) has also been studied on the above parameters. The range of C_u has been carefully selected to cover the entire spectrum of soil types usually encountered at sites. This analysis has been done for Vertical vibration, Pure Sliding and Pure Rocking vibration. It has been observed that both, the Frequency Ratio and the Amplitude Ratio decrease with increase in Embedment Ratio, for a particular C_u and the percentage decrease is significantly higher for smaller sized blocks in comparison to larger blocks. Moreover, the effect of dynamic soil parameter was significant on variation of Frequency Ratio but negligible on variation of Amplitude Ratio for a particular block size.

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Based on the noted observations, a statistical regression analysis was done to develop simplified relations between (Ƒ) and (ȡ), (Ϫ) and (ȡ), with variations in C_u. The relations were used to compute the parameters for additional block sizes (2 m X 2 m X 3 m, 4.5 m X 4.5 m X 3 m, 6 m X 6 m X 3 m).The percentage error introduced, between the values obtained from proposed simplified relations and actual values, have been tabulated to give a indicative idea about the variations to be expected.

The simplified equations are expected to give a indicative guideline to practicing construction engineers at site level, where the availability of in-hand simple relations between the controlling parameters shall help engineers to modify the size and depth of embedment of machine block foundation as per site conditions, if necessary.

Keywords:

Block Foundation, Reciprocating Machine, Frequency Ratio, Amplitude Ratio, Embedment Ratio, Linear Elastic Weightless Spring Method, Coefficient of linear elastic uniform compression (Cu), Vertical Vibration, Pure Sliding vibration, Pure Rocking vibration

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ABSTRACT: This theoretical analysis, based on the linear weightless elastic spring approach, has been made to study the effect of embedment ratio (d_r) on the two controlling parameters usually encountered during the design of block foundation for reciprocating types of machines, frequency ratio (Ғ) and amplitude ratio (Ϫ). The effect of variation of dynamic soil parameter, C_U has also been studied. A parametric study has been done with three square block sizes 3m, 4m, 5m each with 3m height. Statistical regression analysis has been done on the results to develop simplified relations between the above mentioned parameters, with the error percentages, so as to facilitate easier applicability at site level. The study has been carried out for Uncoupled Vertical Vibration, Pure Sliding and Pure Rocking Vibration.

INTRODUCTION

In the current backdrop of rapid industrial development in India and across the globe, the design and installation of heavy machinery across sites has gained momentum and is expected to increase. In the above context, the application of reciprocating machines (e.g. pump, internal combustion machine, compressors) commonly supported on block foundation is also expected to rise.

The dynamic soil parameters usually encountered for machine block foundations are Co-efficient of Elastic Uniform Compression (C_u), Co-efficient of Elastic Uniform Shear (C_τ), Co-efficient of Elastic Non-Uniform Compression (C_Φ) and Co-efficient of Elastic Non-Uniform Shear (C_ψ). The relation between the above parameters are given in IS-5249:1992 [1]. The controlling parameters usually encountered for the geotechnical design of these block foundations are (i) Frequency of Vibration (ii) Amplitude of Vibration of the machine – foundation system. The effect of embedment on above mentioned parameters are of interest to researchers .Notable findings on this topic were given by Beredugo and Novak (1972) [2], Stokoe (1972) [3], Stokoe and Richart (1974) [4] .The main finding was that with increase in embedment depth, the Natural frequency of vibration increased, whereas the Amplitude of vibration decreased .Numerous other research have been done on this topic (Prakash and Puri (1971) [5], Vijayvergiya (1981) [6], Swamisaran [7], etc) and continue to do so .The popular methods usually used for the analysis of machine foundations are (i) Linear Elastic Weightless Spring Method (Barkan 1962) [8] (ii) Elastic Half Space Method (Richart 1962) [9] .

This theoretical study has been carried out on varying sizes of square blocks to determine the effect of embedment on the above mentioned controlling parameters and to obtain simplified relation between them using statistical regression. The analysis has been done in Linear Elastic Weightless Spring Method for Vertical Vibration, Pure Sliding Vibration and Pure Rocking Vibration. The effect of variation of dynamic soil parameter, simply represented in terms of C_u , has also been studied. The equations have been developed considering the best fit curves obtained. The percentage error for the variation in block sizes and dynamic soil parameter, C_u has been discussed, for better applicability. The simplified equations are expected to give a indicative guideline to practicing construction engineers at site level, where the availability of in-hand simple relations between the controlling parameters shall help engineers to modify the size and depth of embedment of machine block foundation as per site conditions (if necessary), being well aware of the implications.

METHOD OF STUDY

The entire theoretical study has been conducted in the Linear elastic weightless spring method , where soil is considered analogous to weightless springs. The damping of soil is not taken into consideration in this approach. In this particular study, the equations developed by Vijayvergiya (1981) [6] has been used for determination of equivalent spring stiffness of the embedded foundation. The same is utilised to find out the natural frequency and amplitude of vibration, briefly discussed below –

For Uncoupled Vertical Vibration , the equivalent spring stiffness , K_{ze} is given by-

$$K_{ze} = C_{uD}A + 2C_{\tau av}(bD + aD) \qquad (1)$$

The natural frequency (ω_{nze}) and maximum amplitude of motion (A_{ze}) is given as –

$$\omega_{nze} = \sqrt{K_{ze}/m} \qquad (2)$$

$$A_{ze} = F_z/m(\omega_{nze}^2 - \omega^2) \qquad (3)$$

For Pure Sliding Vibration , the equivalent spring stiffness , K_{xe} is given by-

$$K_{xe} = C_{\tau D}A + 2C_{uav}bD + 2C_{\tau av}aD \qquad (4)$$

The natural frequency (ω_{nxe}) and maximum amplitude of motion (A_{xe}) is given as –

$$\omega_{nxe} = \sqrt{K_{xe}/m} \qquad (5)$$

$$A_{xe} = F_x/m(\omega_{nxe}^2 - \omega^2) \qquad (6)$$

For Pure Rocking Vibration, the equivalent spring stiffness , $K_{\Phi e}$ is given by-

$$K_{\Phi e} = C_{\Phi D}I - WL + 0.041667C_{\Phi av}b(16D^3 - 12hD^2) + 2C_{\Phi av}I_0 + 0.5\, C_{\tau av}Dba^2 \qquad (7)$$

The natural frequency ($\omega_{n\Phi e}$) and maximum amplitude of motion (A_{xe}) is given as –

$$\omega_{n\Phi e} = \sqrt{K_{\Phi e}/M_{mo}} \qquad (8)$$

$$A_{xe} = M_y/M_{mo}(\omega_{n\Phi}^2 - \omega^2) \qquad (9)$$

where C_{uD} , $C_{\tau D}, C_{\Phi D}$ = Co-efficient of elastic uniform compression , Co-efficient of elastic uniform shear, Co-efficient of elastic non-uniform compression respectively obtained at base of block foundation embedded at depth D below surface level .(At ground surface , they are simply represented as C_u , C_τ, C_Φ respectively).
C_{uav} , $C_{\tau av}$, $C_{\Phi av}$ = Average co-efficient of above mentioned co-efficients (obtained as mean of the values at ground surface and that of the embedded depth), a , b , h = length, width & height of the block foundation respectively, M_{mo} =Moment of inertia of the mass of machine block combination w.r.t axis of rotation, m = Combined mass of machine and foundation, (F_z, F_x, M_y) = Magnitude of maximum vertical vibration force, sliding vibration force & rocking moment respectively. W = Combined weight of machine and foundation, L = height of the machine-foundation C.G above block base centre.
Moment of Inertia, $I = \frac{ba^3}{12}$, $I_o = \frac{aD^3}{3}$

The inter-relation between C_u , C_τ, C_Φ is given as – IS-5249:1992 [1]

$$C_u = 2\, C_\tau \qquad (10)$$

$$C_\Phi = 3.46\, C_\tau = 1.73\, C_u \qquad (11)$$

PARAMETRIC STUDY

A reciprocating machine is symmetrically placed on a concrete block foundation ($\gamma_{concrete}$ = 25 kN/m^3). The operating speed of the machine is taken as N = 150 r.p.m.

Three square block sizes selected for this theoretical study are of size 3 m by 3 m by 3 m, 4 m by 4 m by 3 m, 5 m by 5 m by 3 m. The depth of embedment is varied such that embedment ratio (ძ) (defined as the depth of embedment to height of foundation block) is 0, 0.167, 0.333, 0.500, 0.667, 0.833, 1.000.

The resultant loads and moments on the block foundation are –
Maximum Un-balanced Vertical Force of vibration (F_z) = 3.5 kN
Maximum Un-balanced Horizontal Force of vibration (F_x) = 1.0 kN at 0.5 m from top of block
Maximum Un-balanced Rocking Moment of vibration (M_y) = (1.5 + 0.5) x 1 = 2.0 kNm

The general arrangement of the problem is shown in Figure-1

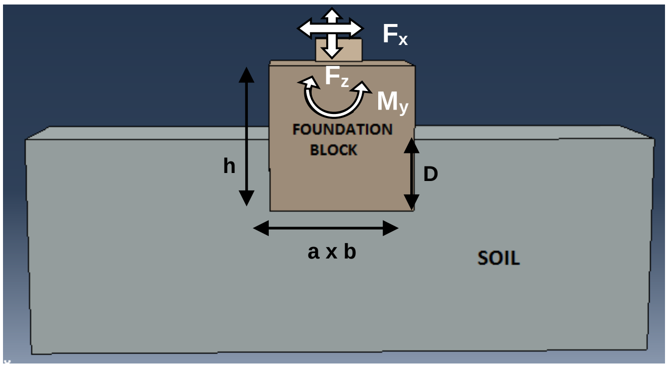

Figure – 1 : General Arrangement of Problem

The dynamic soil property, in the form of Co-efficient of Elastic Uniform Compression (C_u) is varied from 30,000 kN/m^3 to 1,00,000 kN/m^3 (at intervals of 10000 kN/m^3). The variation of range of C_u has been carefully selected on basis of chart proposed by Barkan (1962) [8], to include all possible forms of soil types usually encountered at site level. The position of Water Table has been kept at greater depths, so as to neglect it effect. The weight of the machine is assumed to be negligible compared with that of the block foundation.

The frequency ratio (Ƒ) (defined as the ratio of machine frequency and natural frequency of machine-soil combination $= \frac{\omega}{\omega_n}$) and amplitude ratio (Ϫ) (defined as the amplitude of vibration at a particular embedment ratio to the amplitude of vibration at zero embedment) variation is plotted against variation of embedment ratio, as stated earlier. The plots are repeated for variation in block size and Co-efficient of Elastic Uniform Compression (C_u). The calculations are repeated for Pure Vertical Vibration, Pure Sliding and Pure Rocking Vibrations with results and observations given in upcoming sections.

SAMPLE CALCULATIONS

The sample calculations have been shown separately for Vertical vibration, Sliding and Rocking vibration. The calculations have been shown for 3 m by 3 m by 3 m size of block at ძ = 0.5 (i.e D = 1.5 m) and C_u = 60,000 kN/m^3 . The values of C_{UD}, $C_{\Phi D}$, $C_{\tau D}$ at embedment depths of 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0 m have been assumed to have increased at 5, 10, 15, 20, 25 and 30 % respectively, than their surface values (Swamisaran) [7].

a = b = h = 3 m
C_U = 60000 kN/m^3
D = 1.5 m , i.e ძ = (1.5 / 3) = 0.5
Area of block = 9 sqm
Weight of block (W) = (3 x 3 x 3) x 25 = 675 kN
Mass of block (m) = 67500 kg
L = 1.5 m

$$I = \frac{3\,X\,3^3}{12} = 6.75\,m^4\,, I_o = \frac{3\,X\,1.5^3}{3} = 3.375\,m^4$$

Mass Moment of Inertia about a line passing through CG & in the direction of axis of rotation - Y-axis (M_m) $= \frac{m}{12}(a^2 + h^2) = \frac{67500}{12}(3^2 + 3^2) = 101250$ kgm^2
$M_{mo} = M_m + mL^2 =$ (101250 + 67500 x 1.5^2 = 253125 kgm^2
The weight of the machine is assumed to be negligible compared with that of the block foundation.

Operating frequency (ω) = (2ΠN/60) = (2Π*150/60) = 15.7 rad/sec

From equation.(10) , $C_\tau = 0.5C_u$ = 30000 kN/m^3
From equation.(11) , $C_\Phi = 1.73C_u$ = 103800 kN/m^3
C_{UD} = 69000 kN/m^3, $C_{\tau D}$ = 34500 kN/m^3, $C_{\Phi D}$ = 119370 kN/m^3 (assuming 15 % increase from surface level C_U)
$C_{\tau av}$ = 0.5(C_τ +$C_{\tau D}$) = 0.5(30000+34500) = 32250 kN/m^3. Similarly, C_{uav} = 64500 kN/m^3, $C_{\Phi av}$ = 111585 kN/m^3.

For Uncoupled Vertical Vibration, Putting the values in equation. (1) for D = 1.5 m (ȡ = 0.5),
$K_{ze} =$ 1201500 kN/m
Putting the values in equation. (2) , (3) ,
$\omega_{nze} =$ 133.417 rad/sec, $A_{ze} = 2.954$ microns.
Similarly at Surface conditions (ȡ = 0),
$\omega_{nze} =$ 89.443 rad/sec , $A_{ze} = 6.688$ microns.
Hence, at ȡ = 0, frequency ratio, Ғ ($= \frac{\omega}{\omega_n}$) = (15.7 / 89.443) = 0.1755 , Amplitude ratio Ϫ$_z$ = 1.0.
At ȡ = 0.5 , Ғ = (15.7 / 133.417) = 0.1177
Amplitude ratio, Ϫ$_z$ = (2.954 / 6.688) = 0.4417

For Pure Sliding Vibration, Putting the values in equation. (4) for D = 1.5 m (ȡ = 0.5),
$K_{xe} =$ 1181250 kN/m
Putting the values in equation. (5), (6),
$\omega_{nxe} =$ 132.288 rad/sec, $A_{xe} = 0.859$ microns.
Similarly at Surface conditions (ȡ = 0),
$\omega_{nxe} = 63.246$ rad/sec , $A_{xe} = 3.947$microns.
Hence, at ȡ = 0, frequency ratio, Ғ = (15.7 / 63.246) = 0.248, Amplitude ratio Ϫ$_x$ = 1.0.
At ȡ = 0.5, Ғ = (15.7 / 132.288) = 0.119
Amplitude ratio, Ϫ$_x$ = (0.859 / 3.947) = 0.218

For Pure Rocking Vibration, Putting the values in equation. (7) for D = 1.5 m (ȡ = 0.5),
$K_{\Phi e} =$ 1834396.875 kN/m
Putting the values in equation. (8), (9),
$\omega_{n\Phi e} =$ 85.129 rad/sec, $A_\Phi = 1.129$ radians
Similarly at Surface conditions (ȡ = 0),
$\omega_{n\Phi e} = 52.574$ rad/sec , $A_\Phi = 3.139$ radians.
Hence, at ȡ = 0, frequency ratio, Ғ = (15.7 / 52.574) = 0.299, Amplitude ratio Ϫ$_\Phi$ = 1.0.
At ȡ = 0.5, Ғ = (15.7 / 85.129) = 0.184
Amplitude ratio, Ϫ$_\Phi$ = (1.129 / 3.139) = 0.360.

RESULTS & DISCUSSION

The results have been presented and discussed separately for the two controlling parameters – Ғ, Ϫ for vertical vibration, pure sliding and pure rocking vibration.

Frequency Ratio (Ғ)

For pure vertical vibration, it has been observed that, the C_U parameter kept constant, frequency ratio decreases with increase in embedment ratio and the percentage decrease is predominantly higher for blocks of smaller sizes (in our study – 3m square block). The frequency ratio is also seen to increase with block size, keeping ȡ & C_U constant. Moreover at ȡ = 0 (i.e surface condition), the frequency ratio is independent of block size (Figure -2) .Curves indicating variations with C_U are shown in Figure -3. With increase in ȡ, the Ғ is seen to decrease with higher values of C_U. The percentage change is significantly high for lower block sizes and lower values of C_U respectively.

For pure sliding vibration, similar trend of observations have been seen just as in pure vertical vibration. These have been shown in Figure – 4 & 5.

For pure rocking vibration, similar to the above cases, the frequency ratio decreases with increase in embedment ratio, with the smaller sizes having significantly higher percentage changes. An interesting observation in this case (Figure – 6), is that for a ȡ range of (0-0.667) , the smaller block size has higher values of Ғ (unlike in Vertical Vibration and Pure Sliding) , at ȡ = 0.667 all three sizes record almost same Ғ , for ȡ range of (0.667 – 1.0) , the smaller block records lower value of Ғ compared to higher sizes. All the above observations were based on constant C_U parameter. With variation of C_U, the Ғ records maximum percentage variation for lower C_U values , at a particular value of ȡ (Figure 7).

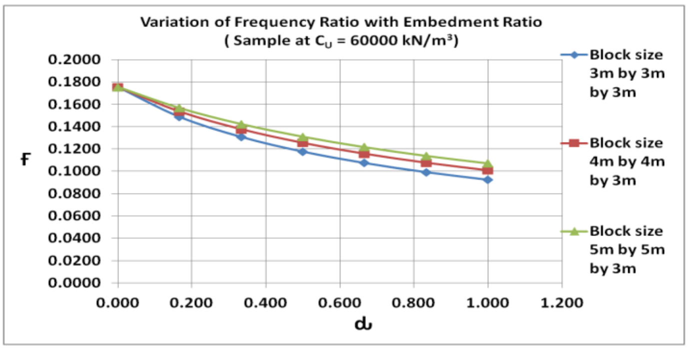

Figure – 2: Sample Ƒ v/s ძ curves at C_u = 60000 kN/m^3 for different sample block sizes in Pure Vertical Vibration.

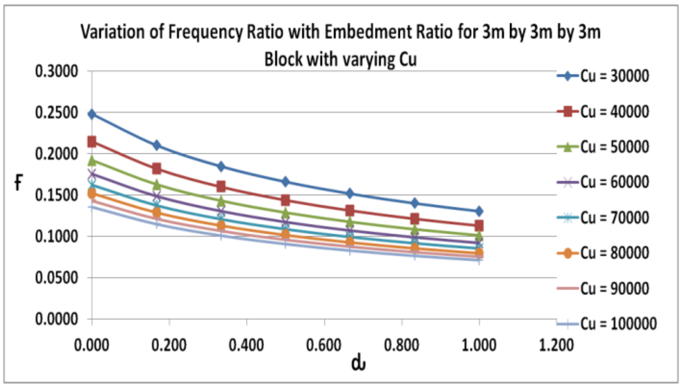

Figure – 3: Sample Ƒ v/s ძ curves for varying C_u = 30000 - 100000 kN/m^3 for 3m square block in Pure Vertical Vibration

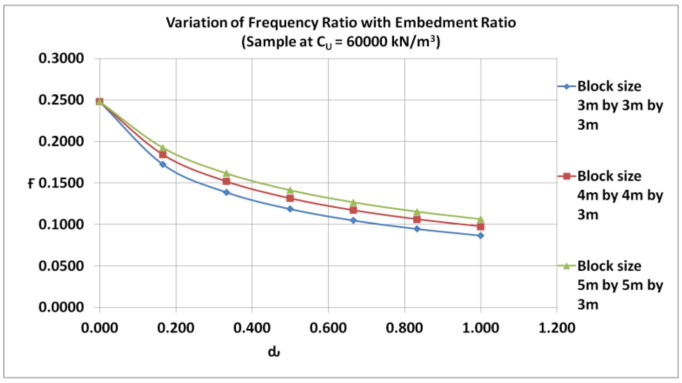

Figure – 4: Sample Ƒ v/s ძ curves at C_u = 60000 kN/m^3 for different sample block sizes in Pure Sliding Vibration.

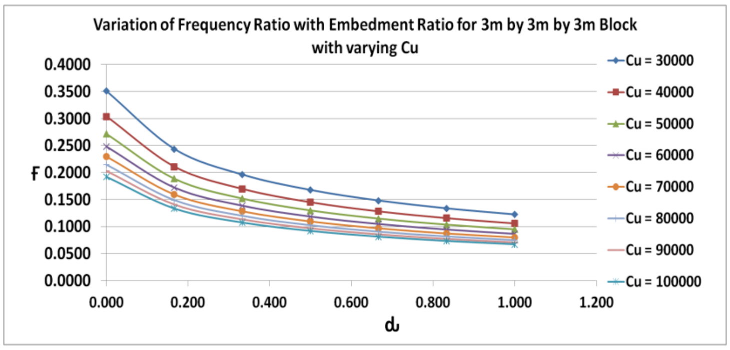

Figure – 5: Sample Ƒ v/s ძ curves for varying C_u = 30000 - 100000 kN/m^3 for 3m square block in Pure Sliding Vibration

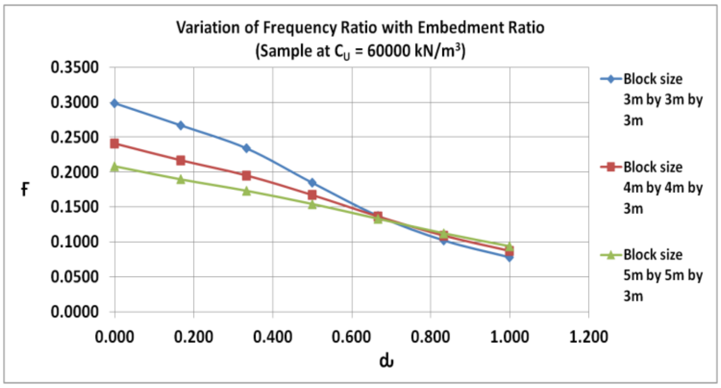

Figure – 6: Sample Ƒ v/s ძ curves at C_u = 60000 kN/m^3 for different sample block sizes in Pure Rocking Vibration

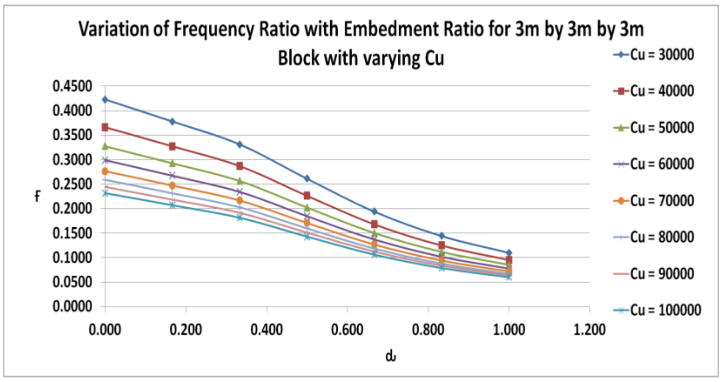

Figure – 7: Sample Ƒ v/s ძ curves for varying C_u = 30000 - 100000 kN/m^3 for 3m square block in Pure Rocking Vibration

Amplitude Ratio (Ϫ)

For pure vertical vibration, it is observed that the C_U parameter kept constant, amplitude ratio decreases with increase in embedment ratio and the percentage decrease is predominantly higher for blocks of smaller sizes – (Figure .8). Moreover, for a particular block size at a particular value of ძ, the Ϫ is almost independent of varying C_U (i.e the percentage change (w.r.t zero embedment ratio) recorded is almost same for all variations of C_U)- (Figure -9).

In pure sliding, similar observation pattern has been seen as in pure vertical vibration. The figures 10 and 11 are self-explanatory, in this context.

In pure rocking condition, the amplitude ratio is seen to decrease with increase in embedment ratio, at a particular C_U, with smaller size blocks recording significant higher percentage changes especially at higher embedment ratios. For 3m square block (C_U = 60000 kN/m^3) at ძ = 1 the percentage decrease in $Ϫ_\Phi$ was 93.81 % (w.r.t. ძ = 0) - Figure 12. The effect of variation in C_U on, $Ϫ_\Phi$ with ძ, is shown in Figure-13. It has been observed that the variation is negligible. Moreover, upto ძ = 0.1, the $Ϫ_\Phi$ observed for all three block sizes is found to almost equal.

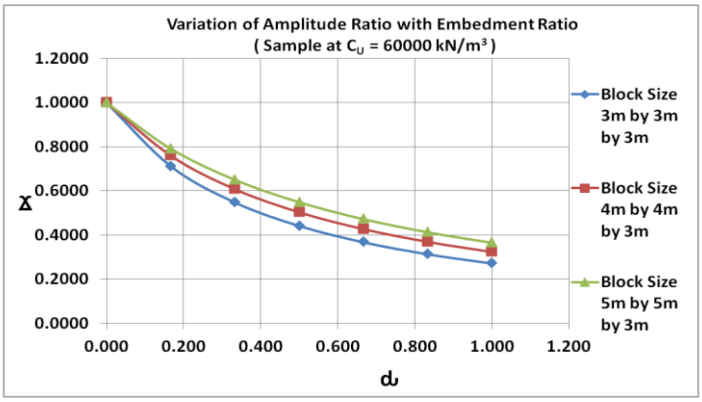

Figure – 8: Sample $Ϫ_Z$ v/s ძ curves at C_u = 60000 kN/m^3 for different sample block sizes in Pure Vertical Vibration

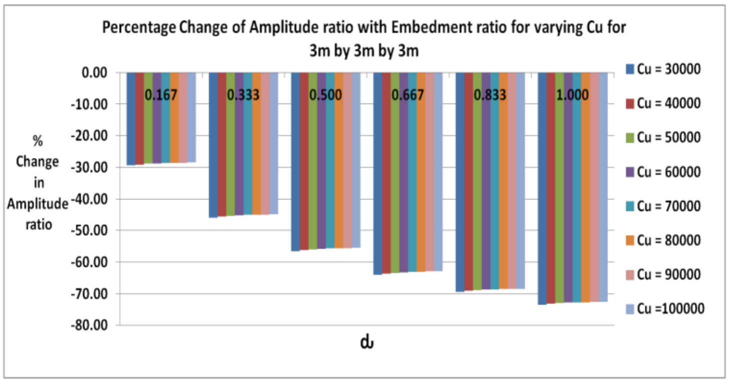

Figure – 9: Sample Percentage change in $Ϫ_Z$ v/s ძ curves at varying C_u for 3m square block size in Pure Vertical Vibration.

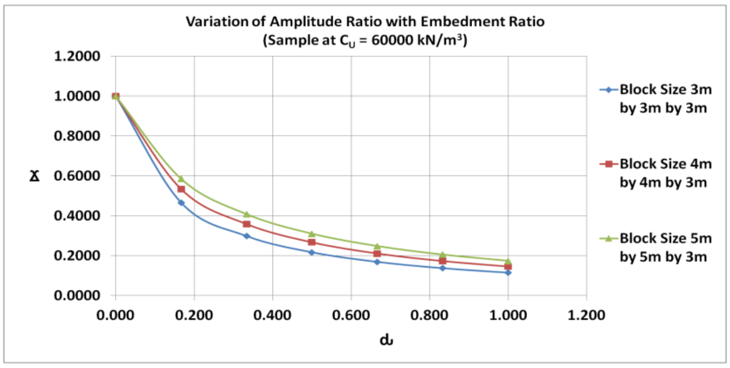

Figure – 10: Sample $Ϫ_X$ v/s ძ curves at C_u = 60000 kN/m^3 for different sample block sizes in Pure Sliding Vibration

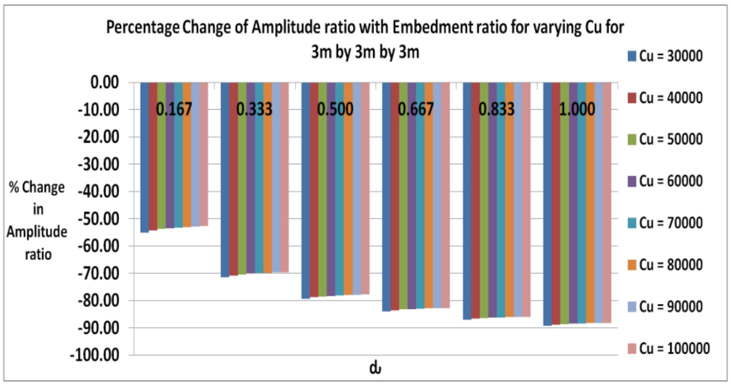

Figure – 11: Sample Percentage change in $Ϫ_X$ v/s ძ curves at varying C_u for 3m square block size in Pure Sliding Vibration.

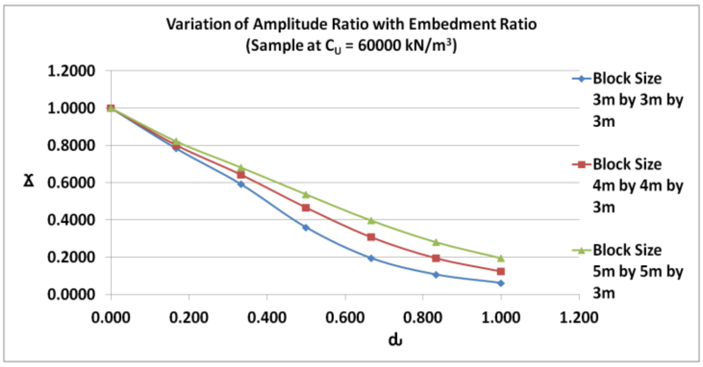

Figure – 12: Sample $Ӿ_Φ$ v/s ȡ curves at C_u = 60000 kN/m^3 for different sample block sizes in Pure Rocking Vibration

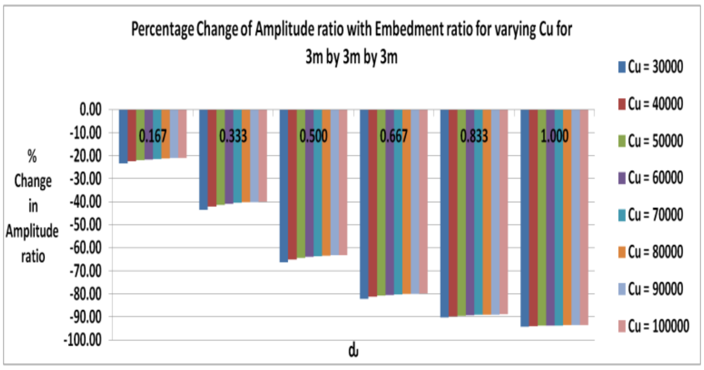

Figure – 13: Sample Percentage change in $Ӿ_Φ$ v/s ȡ curves at varying C_u for 3m square block size in Pure Rocking Vibration

STATISTICAL APPROACH

Based on the calculations, a statistical regression analysis was done to obtain simplified equations between Ƒ and ȡ, Ӿ and ȡ, in all three modes referred above. The discussions have been made separately for frequency ratio and amplitude ratio in the following sections. The error involved has been separately calculated to give an indication of variation.

Frequency Ratio (Ƒ)

An regression analysis, using Method of Least Squares, led to the development of equation of the form

$$Ƒ = a + bȡ + cȡ^2 \qquad (12)$$

Where a, b, c are variables which have been considered as function of C_U and can be obtained from Figure - 14 (Vertical Vibration), 15 (Pure Sliding Vibration) and 16 (Pure Rocking Vibration). In case of pure vertical vibration and rocking the values of a, b, c are distinct, hence the equation.12 which represents a parabola is satisfied .The best fit curve has been taken amongst the three sizes, with $R^2 = 0.996$ (Vertical Vibration), $R^2 = 0.996$ (for Pure Sliding) .In case of pure rocking, the value of c is very small (close to zero), hence it has been neglected in calculations, making equation .12 take a linear form. The plot of c has been shown in Figure-16, but significant variation in frequency ratio has not been observed with its inclusion, hence not considered. The sets of a, b have $R^2 = 0.996$.

The percentage variation obtained by using the above methodology with original data are given in Table –1 (Vertical Vibration), Table-2 (Pure Sliding Vibration) and Table-3 (Pure Rocking Vibration), for a sample value of C_U = 50000 kN/m^3, with additional square sizes just for comparison (2 m by 2 m by 3 m, 4.5 m by 4.5 m by 3 m, 6 m by 6 m by 3 m) and to get an indicative idea about formulae application. It has been observed that, the percentage variation is higher for lower block sizes, as discussed in earlier sections.

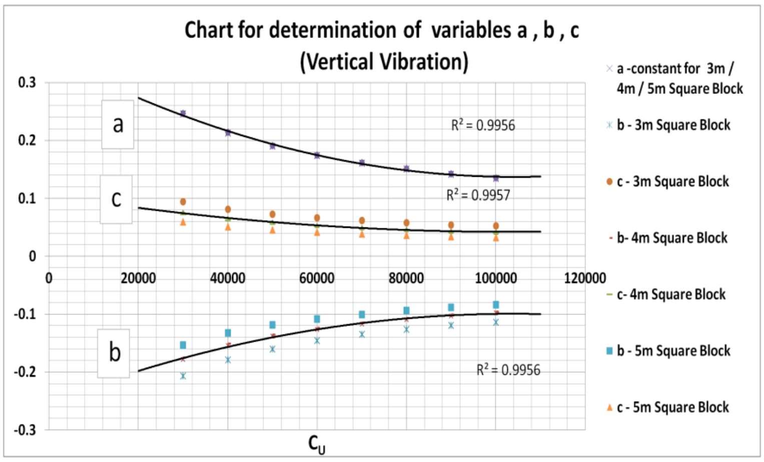

Figure – 14: Determination of variables a, b, c used in the computation of frequency ratio (For Vertical Vibration)

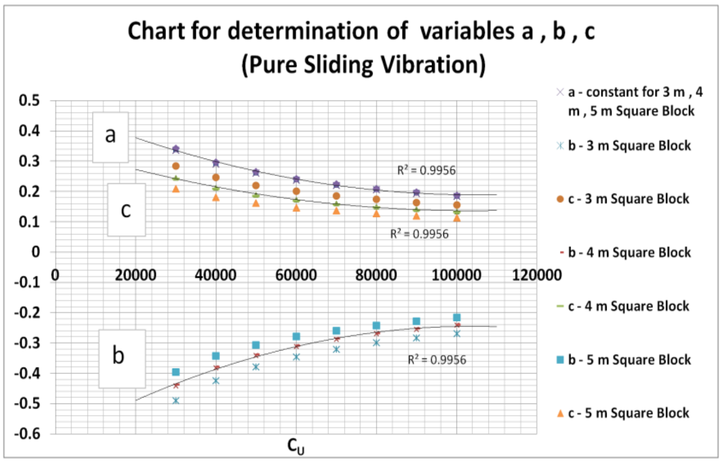

Figure – 15: Determination of variables a, b, c used in the computation of frequency ratio (Pure Sliding)

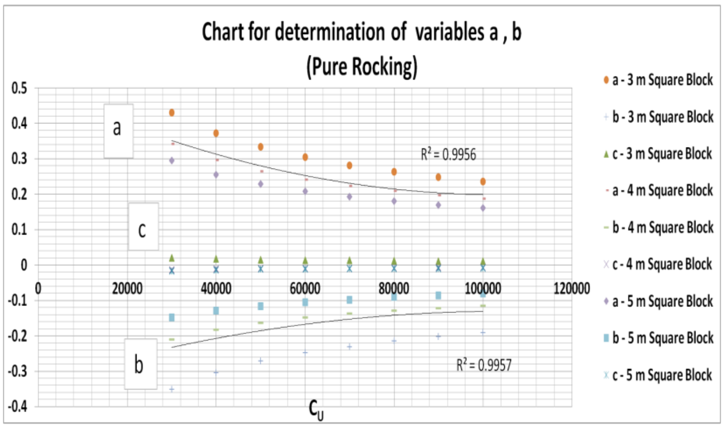

Figure – 16: Determination of variables a, b used in the computation of frequency ratio (Pure Rocking)

Table 1: Percentage Error Recorded Using Proposed Regression (Vertical Vibration) (C_U = 50000 kN/m^3)

Size (in meters)	Embedment Ratio , d_r	Percentage variation(%)
3 m X 3 m X 3 m	0	-1.188
	0.5	4.726
	1.0	8.542
4 m X 4 m X 3 m	0	-1.188
	0.5	-1.802
	1.0	-0.503
5 m X 5 m X 3 m	0	-1.188
	0.5	-5.936
	1.0	-6.348
2 m X 2 m X 3 m	0	-1.188
	0.5	16.681
	1.0	24.716
4.5 m X 4.5 m X 3 m	0	-1.188
	0.5	-4.051
	1.0	-3.678
6 m X 6 m X 3 m	0	-1.188
	0.5	-8.784
	1.0	-10.423

Table 2: Percentage Error Recorded Using Proposed Regression (Sliding Vibration) (C_U = 50000 kN/m^3)

Size (in meters)	Embedment Ratio , d_r	Percentage variation(%)
3 m X 3 m X 3 m	0	-7.183
	0.5	-0.776
	1.0	8.990
4 m X 4 m X 3 m	0	-7.183
	0.5	-10.384
	1.0	-3.150
5 m X 5 m X 3 m	0	-7.183
	0.5	-16.679
	1.0	-11.227
2 m X 2 m X 3 m	0	-7.183
	0.5	16.081
	1.0	29.911
4.5 m X 4.5 m X 3 m	0	-7.183
	0.5	-13.822
	1.0	-7.548
6 m X 6 m X 3 m	0	-7.183
	0.5	-4.385
	1.0	-17.048

Table 3: Percentage Error Recorded Using Proposed Regression (Rocking Vibration) (C_U = 50000 kN/m^3)

Size (in meters)	Embedment Ratio , d_r	Percentage variation(%)
3 m X 3 m X 3 m	0	-15.980
	0.5	-8.632
	1.0	10.949
4 m X 4 m X 3 m	0	4.255
	0.5	0.893
	1.0	-1.060
5 m X 5 m X 3 m	0	20.343
	0.5	9.198
	1.0	-7.848
2 m X 2 m X 3 m	0	-40.652
	0.5	-16.637
	1.0	33.911
4.5 m X 4.5 m X 3 m	0	12.780
	0.5	5.251
	1.0	-4.911
6 m X 6 m X 3 m	0	32.936
	0.5	48.835
	1.0	-11.865

Note: (+) indicates proposed value higher than actual value

Amplitude Ratio (Ϫ)

From previous sections, it has been inferred that variation of amplitude ratio at a particular embedment ratio is negligible with variation of C_U. Utilising this property and using regression analysis from best fit curves, the following power equations were developed between Ϫ and d_r.

The vertical amplitude ratio, Ϫ_z , for all values of C_U, is given by equation (13) with $R^2 = 0.9725$

$$\text{Ϫ}_z = -0.6668\text{ȡ}^3 + 1.6594\text{ȡ}^2 - 1.6723\text{ȡ} + 0.9971 \qquad (13)$$

The sliding amplitude ratio, Ϫ_x , for all values of C_U, is given by equation (14) with $R^2 = 0.9853$

$$\text{Ϫ}_x = -5.1796\text{ȡ}^5 + 16.343\text{ȡ}^4 - 20.183\text{ȡ}^3 + 12.609\text{ȡ}^2 - 4.4442\text{ȡ} + 0.9999 \qquad (14)$$

The rocking amplitude ratio, Ϫ_Φ, for all values of C_U, is given by equations (15) & (16) with $R^2 = 0.9667$ & 0.9831 respectively. Since the variation recorded with a single formulae was quite high for smaller block sizes, equation (15) is suggested for square block sizes greater than equal to 4 m, equation (16) is suggested for square block sizes less than 4 m.

$$\text{Ϫ}_\Phi = 0.3414\text{ȡ}^4 - 0.2456\text{ȡ}^3 + 0.1985\text{ȡ}^2 - 1.1651\text{ȡ} + 0.9983 \qquad \text{(Size >= 4 m)} \qquad (15)$$

$$\text{Ϫ}_\Phi = -3.1823\text{ȡ}^4 + 7.48\text{ȡ}^3 - 4.7204\,\text{ȡ}^2 - 0.5347\text{ȡ} + 0.9979 \qquad \text{(Size < 4 m)} \qquad (16)$$

The variation of 'proposed Ϫ' and 'actual Ϫ' for different block sizes have been represented in Figure-17 (Vertical Vibration), Figure-18 (Pure Sliding) and Figure-19 (Pure Rocking), with additional square sizes just for comparison (2 m by 2 m by 3 m , 4.5 m by 4.5 m by 3 m , 6 m by 6 m by 3 m) and to get a indicative idea about practicability of formulae developed. The firm line represents the developed equation. It clearly shows that maximum variation from proposed expression is obtained for smaller block sizes.

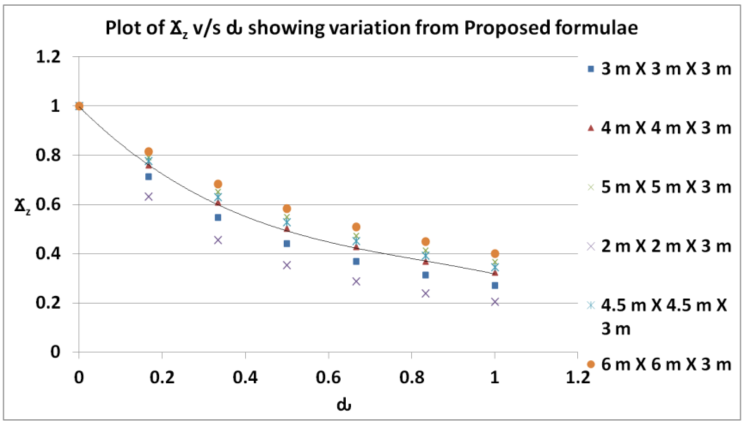

Figure – 17: Plot of Ϫ_z v/s ȡ showing variation from proposed formulae (For Vertical Vibration)

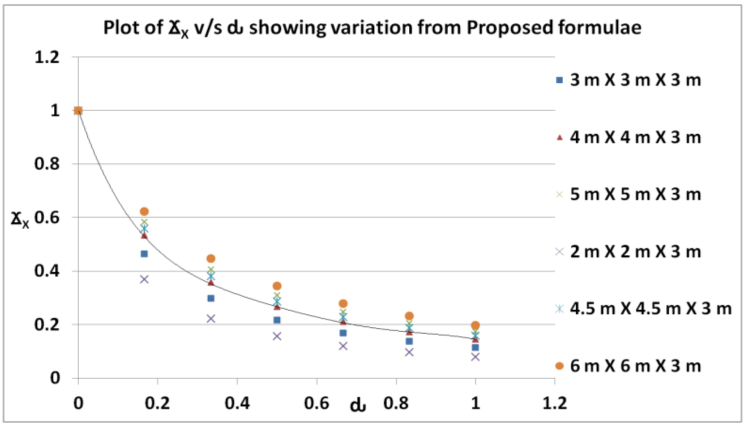

Figure – 18: Plot of Ϫ_x v/s ȡ showing variation from proposed formulae (For Sliding Vibration)

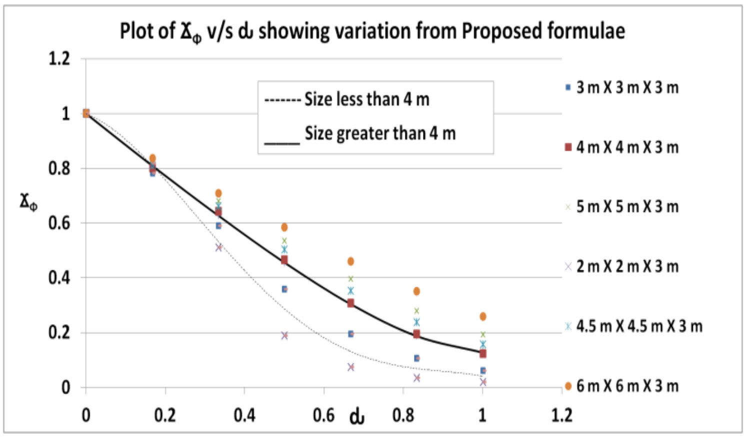

Figure – 19: Plot of Ϫ_Φ v/s ȡ showing variation from proposed formulae (Pure Rocking Vibration)

CONCLUSIONS

The theoretical study conducted above was an effort to establish simplified relations between Frequency ratio (Ғ) and Embedment ratio (ȡ), Amplitude ratio (Ꭓ) and Embedment ratio (ȡ). The important findings are –(i) both (Ғ) and (Ꭓ) was found to decrease with increase in (ȡ), (ii) the percentage decrease in both (Ғ) and (Ꭓ) was significantly high for lower block sizes (iii) the effect of dynamic soil parameter was significant on variation of (Ғ) but negligible on variation of (Ꭓ),at a particular (ȡ) for a particular block size. The above observations were made for uncoupled vertical, pure sliding and pure rocking vibrations. These observations have been utilised to develop the regression equations, which shall give simplified indicative response of both (Ғ) and (Ꭓ) with variation of (ȡ), with varying dynamic soil parameter (C_u), whose range has been carefully selected to include the spectrum of all available soil types encountered at site level (Table-4). It has been observed that maximum variation in proposed findings are for smaller size blocks compared to higher sizes. In this case, the maximum variation was found for 2 m square block, compared with 3 m, 4 m, 4.5 m, 6 m square blocks with same height of 3 m. The proposed graphs and equations presented in simplified manner may be used at site levels. Future scope may include refining the above equations to minimise the percentages of error especially for smaller size blocks. The same may also be developed for Yawing, Coupled Rocking and Sliding vibration. Similar study with development of simplified relations may also be done for other shapes (e.g: rectangular) of block foundations.

Table 4: Recommended values of C_U for A =10 m^2 (Barkan 1962) [8[, Swamisaran (2006) [7]

Soil Group	C_U (kN/m^3)
Weak soils (clays and silty clays with sand in plastic state; clayey and silty sands; soils of categories II & III with laminae of organic silt and of peat)	upto 30,000
Soils of medium strength (clays; silty clays with sand with water content close to P.L; Sand)	30,000 – 50,000
Strong Soils (clays and silty clays with sand of hard consistency; gravels and gravelly sands; loess & loessial soils)	50,000 - 1,00,000
Rocks	>1,00,000

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