Voltage Stability Assessment in the Presence of Optimally Placed D-FACTS Devices

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***Abstract*—Distributed Flexible AC Transmission System (D-FACTS) devices offer many potential benefits to power system operations. This paper presents a novel strategy for the application of D-FACTS devices in controlling system voltage. The impact of installing D-FACTS devices is examined by studying the sensitivities of voltage magnitude with respect to line impedance. Sensitivities enable us to determine the potential benefits of the D-FACTS Devices offered to the system. Most appropriate locations to install D-FACTS devices for controlling system voltages are also determined. In this paper, steady state model of recently introduced D-FACTS device DSSC is incorporated in voltage stability assessment of an interconnected power system in terms of its reduced equivalent two-bus integrated system. The participation of a particular bus in global voltage instability is assessed in terms of global voltage stability index (GVSI). It has also been used to assess the global voltage stable state of the network. The proposed methodology has been applied under simulated condition on IEEE 30-bus test system.**

Keywords—Distributed FACTS; Voltage control; Reactive power control; Line impedance sensitivity

I. INTRODUCTION

Due to increase in power demand, modern power system networks are being operated under highly stressed conditions. This has resulted into the difficulty in meeting reactive power requirement, especially under contingencies and hence maintaining the bus voltage within acceptable limits. Voltage instability is one of the major problems associated with modern power systems [1]. Reports of the occurrence of voltage collapse are becoming more frequent and this problem has been an area of great interest to power system researchers [2-4].

Voltage collapse is a local phenomenon and occurs at a bus within an area of high loads and low voltage profile. The voltage problem of the affected bus may cause a series of line outages and resulting in system blackout. It is well recognized that voltage collapse normally occurs when there exists a large demand of reactive power [5] but at exactly what load level the failure will occur, is not easily predicted.

Flexible AC Transmission System (FACTS) was launched to solve the emerging power system problems [6,7]. It identifies alternating current transmission systems incorporating power electronic based controllers to enhance the controllability to increase power transfer capability. These controllers are used to regulate power flow, transmission voltage and can mitigate dynamic disturbances through rapid control action. Thyristor Controlled Series capacitor (TCSC) and Static Synchronous Series Capacitor (SSSC) are used to control the power flow through transmission lines. Other devices such as Static Var compensators (SVC) and static synchronous compensator (STATCOM) are widely used for shunt reactive compensation in order to maintain a flat voltage profile. To analyze the effect of these controllers, steady state models have been developed over the decade [7-9]. Power flow analysis of systems using such models would provide data necessary to calculate voltage collapse indicators in order to evaluate the response of the system.

Although FACTS devices are well-understood from a technical perspective but they have not experienced the massive deployment that their theory may warrant because of the huge investment costs, poor return on investment as well as reliability concerns. Improvements in available electrical technology allow us to revisit FACTS concepts from a fresh perspective and recently introduced distributed flexible AC transmission system (D-FACTS) devices offer such an opportunity.

More recently, Distributed Flexible AC Transmission System (D-FACTS) device, Distributed Static Series Compensator (DSSC) has been designed to address power control types of problems [10-12]. From power system perspective, D-FACTS devices have many potential benefits. D-FACTS devices can be attached directly to transmission lines and can be used to dynamically control effective line impedance. D-FACTS devices are smaller and less expensive than traditional FACTS devices which make them better candidates for wide scale deployment. D-FACTS devices can act inductive as well as capacitive, so both raising and lowering system voltage are important potential applications. In particular, this paper analyzes effects of changing transmission line impedances and the use of D-FACTS devices for voltage control.

Several incidences of voltage collapse have been observed in past few decades. With the concept of network equivalence [13-16], an attempt is made in this paper to describe a method of equivalence a multi-bus power network to an equivalent two-bus system [15] developed from the Newton-Raphson power flow considering D-FACTS controllers and thereby

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voltage stable states of the entire system following the load changes in 'weak' load buses investigated for a typical power system network. Here we examine the use of D-FACTS devices as a means to improve voltages in the IEEE 30-bus test system. Voltage stability enhancement using these D-FACTS controllers is compared in the test system considered. The simulation also includes the detection of the 'weak' load bus/buses [17,18] and identification of the global voltage stable states of the system following the derived two-bus equivalent system simulation.

II. ANALYSIS OF LINE IMPEDANCE SENSITIVITIES

Sensitivities are linearized relationships between variables and are often used in power systems analysis. Linearized relationships can reveal the impact of a small change in a particular variable on the rest of the system. Linear approximations in nonlinear systems are useful because they can provide insight into how variables depend on other variables when such relationships may otherwise be difficult to characterize. Since D-FACTS devices change effective line impedance, line impedance sensitivities [19] are useful to determine potential benefits of D-FACTS devices.

A. Equations and Notation

The AC power injection equations for real power P and reactive power Q at a bus i are stated in (1a) and (1b),

$$P_{i,calc} = V_i \sum_{j=1}^{n} V_j \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right] \tag{1a}$$

$$Q_{i,calc} = V_i \sum_{j=1}^{n} V_j \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right] \tag{1b}$$

Where n is the number of buses.

Real and reactive power balance is expressed by the concatenated vector $f_{(p,q)}(s_{(\theta,V)})$ of Δp and Δq which must equal to zero,

$$\Delta p_i = P_{i,calc} - (P_{i,gen} - P_{i,load}) \tag{2a}$$

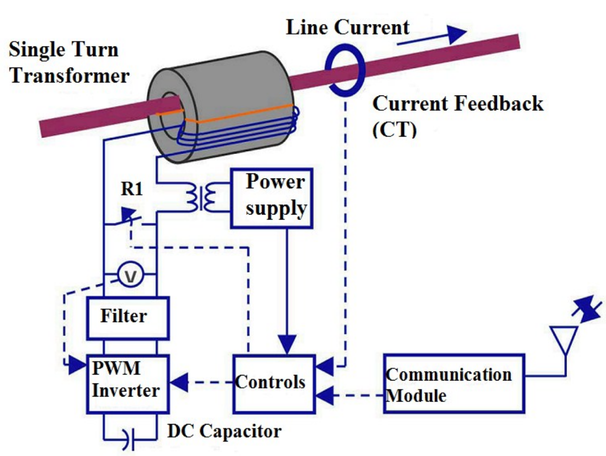

Fig. 1. Schematic Diagram of DSSC

$$\Delta q_i = Q_{i,calc} - (Q_{i,gen} - Q_{i,load}) \tag{2b}$$

$$f_{(p,q)}(s_{(\theta,V)}) = \left[\Delta p, \Delta q \right]^T \tag{3}$$

Where $s_{(\theta,V)}$ is a vector of bus voltage states represented in polar coordinates by magnitudes V and angles θ,

$$(s_{(\theta,V)}) = \left[\theta, V \right]^T \tag{4}$$

$G+jB$ is the system admittance matrix. Admittance matrix elements depend explicitly on reactive line impedances x as well as resistances r.

$$G_{ij} = -\frac{r_{ij}}{r_{ij}^{\,2} + x_{ij}^{\,2}} \, i \neq j \tag{5a}$$

$$B_{ij} = -\frac{x_{ij}}{r_{ij}^{\,2} + x_{ij}^{\,2}} \, i \neq j \tag{5b}$$

The above equations are used to analyze the impact of D-FACTS devices in power system.

B. Admittance Matrix Sensitivities

Since D-FACTS devices change the effective reactive impedance x of a line, it is useful to consider the power flow equations in terms of x (and r) instead of G and B. Expressing individual elements of G and B in terms of impedances as in (5a) and (5b) and taking the derivative of each term with respect to its reactive line impedance yields the following:

$$\frac{\partial G_{ij}}{\partial x_{ij}} = \frac{2 r_{ij} . x_{ij}}{(r_{ij}^{\,2} + x_{ij}^{\,2})^2} \tag{6a}$$

$$\frac{\partial B_{ij}}{\partial x_{ij}} = -\frac{2 x_{ij}^{\,2}}{(r_{ij}^{\,2} + x_{ij}^{\,2})^2} + \frac{1}{r_{ij}^{\,2} + x_{ij}^{\,2}} \tag{6b}$$

C. Power Injection and State Variable Sensitivities

The relationships between the power injection equations $f_{(p,q)}$ and state variables $s_{(\theta,V)}$ are given by the power flow Jacobian J.

Where J is formed as follows:

$$J = \begin{array}{r} nq+nv\{ \\ nq\{ \end{array} \begin{pmatrix} \dfrac{\partial \Delta p}{\partial \theta} & \dfrac{\partial \Delta p}{\partial V} \\ \dfrac{\partial \Delta q}{\partial \theta} & \dfrac{\partial \Delta q}{\partial V} \end{pmatrix} \tag{7}$$

nq is the number of PQ buses and nv is the number of PV buses.

The negative inverse of the power flow Jacobian describes the way the state variables change in a solution of the power flow due to real and reactive bus power injection mismatch.

$$\Delta s_{(\theta,V)} = \left[-J \right]^{-1} . f_{(p,q)} \tag{8}$$

In addition to the relationship between power injections and state variables, we also need to define the relationship between power injections and line impedance. The power injection to impedance sensitivity matrix [PII] is found by taking the derivative of each entry in (3) with respect to line impedance.

$$\Delta f_{(p,q)} = [PII] \cdot \Delta x \tag{9}$$

Each row of [PII] contains the sensitivities of a real or reactive power injection at bus i to line impedances. The only nonzero elements in a row for bus i correspond to the lines connected to bus i. For a bus i, elements of [PII] are given by:

$$\frac{\partial P_i}{\partial x_{ij}} = V_i^2\left[-\frac{\partial G_{ij}}{\partial x_{ij}}\right] + V_iV_j\left(\left[\frac{\partial G_{ij}}{\partial x_{ij}}\right]\cos(\theta_i-\theta_j) + \left[\frac{\partial B_{ij}}{\partial x_{ij}}\right]\sin(\theta_i-\theta_j)\right) \tag{10a}$$

$$\frac{\partial Q_i}{\partial x_{ij}} = -V_i^2\left[-\frac{\partial B_{ij}}{\partial x_{ij}}\right] + V_iV_j\left(\left[\frac{\partial G_{ij}}{\partial x_{ij}}\right]\sin(\theta_i-\theta_j) - \left[\frac{\partial B_{ij}}{\partial x_{ij}}\right]\cos(\theta_i-\theta_j)\right) \tag{10b}$$

Where [PII] is structured as follows:

$$PII = \begin{array}{r} np+nv\{ \\ nq\{ \end{array} \overset{k}{\begin{pmatrix} \dfrac{\partial P}{\partial x} \\ \dfrac{\partial Q}{\partial x} \end{pmatrix}} \tag{11}$$

k is the number of lines with D-FACTS devices.

The product of the two matrices $-J^{-1}$ and [PII] describes how bus power injections change due to a change in line impedance and then how states change due to the change in bus power injections. The resulting state to impedance sensitivity matrix [SI] describes how the state variables V and θ change after a solution of the power flow due to a small change in line impedance.

$$[SI] = -J^{-1} \cdot [PII] \tag{12}$$

$$\Delta s_{(\theta,V)} = [SI] \cdot \Delta x \tag{13}$$

The matrix [SI] is the only full matrix involved in this work and its computation involves [PII] and the inverse of J. The dimension of the columns of [PII] is the number of lines equipped with D-FACTS devices, k. The rows of [PII] are sparse since not every bus is connected to each of the k lines. Thus, each column of [PII] is a sparse vector and sparse vector methods may be used to compute [SI] using the fast-forward and full back schemes as described in [20].

The relationship between state variables and line impedances is fundamental to the analysis; otherwise a D-FACTS device would not be able to exercise control over any variable other than those on its own line.

III. Identification of Effective D-FACTS Locations

D-FACTS devices are unique among power flow control devices in that they are well-suited to be placed at multiple locations in the system where their use could be the most beneficial. FACTS devices are often installed to provide reactive power support but reactive power support is most effective locally.

Once appropriate line flows have been targeted for control, we need to identify lines on which D-FACTS devices should be placed to achieve this control. Linear approximations of nonlinear relationships provide a useful local picture. Sensitivities can be used to identify lines that have a high impact for particular applications. Lines with higher sensitivities are able to provide more control, whereas lines with sensitivities near zero do not have much impact. For controlling multiple line flows, the best locations for D-FACTS devices depend on the desired control objective.

The sensitivities of voltages with respect to line impedance are given by the lower section of [SI], denoted by $[SI_v]$:

$$[SI] = [SI_\Theta \quad SI_v] \tag{14}$$

Voltage control coupling indices can be determined from the row vectors of $[SI_V]$. These coupling indices can be used to determine which bus voltages are independently controllable [19].

IV. Development Of Equivalent Two Bus System

Two-bus equivalent network model for any multi-bus power system is obtained using the total active and reactive load and loss available from load flow analysis for a particular operating condition where none of the two buses are actually present in the system. The power loss of entire network being the algebraic sum of all line flows in the system, it is given by:

$$P_{loss,multibus} = \sum_{i=1}^{N}\sum_{k=1}^{N}(P_{ik}+P_{ki}) \tag{15a}$$

$$Q_{loss,multibus} = \sum_{i=1}^{N}\sum_{k=1}^{N}(Q_{ik}+Q_{ki}) \tag{15b}$$

where N is the total number of buses in the system

Let us consider a two bus network connected by a line-impedance representing the equivalent of the entire multi-bus network. In order to derive such a two bus network we consider the sending end quantities having source power P_g and Q_g while receiving end loads are P_{load} and Q_{load}. The power flow equation for such an equivalent network can be represented as:

$$P_g = P_{loss} + P_{load} \tag{16a}$$

$$Q_g = Q_{loss} + Q_{load} \tag{16b}$$

Here we assume that total transmission line loss is same both in multi-bus system and equivalent two bus system. The real and reactive power losses (P_{loss} *and* Q_{loss}) for this equivalent system are then given by:

$$P_{loss} = r_{eq}(P_g^2 + Q_g^2)/E^2 \tag{17a}$$

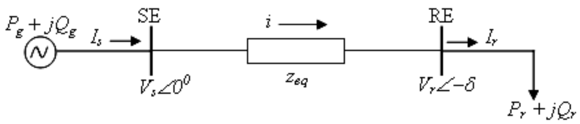

Fig. 2. A simple two bus network

$$Q_{loss} = x_{eq}(P_g^2 + Q_g^2)/E^2 \tag{17b}$$

where 'E' is the sending end voltage, r_{eq} and x_{eq} represent the equivalent resistance and reactance of the two bus network respectively. The equivalent impedance of the two bus network is then given by:

$$z_{eq} = r_{eq} + jx_{eq} \tag{18}$$

where,

$$r_{eq} = P_{loss}/(P_g^2 + Q_g^2) = (P_g - P_{load})/(P_g^2 + Q_g^2) \tag{19a}$$

$$x_{eq} = Q_{loss}/(P_g^2 + Q_g^2) = (Q_g - Q_{load})/(P_g^2 + Q_g^2) \tag{19b}$$

The sending end voltage 'E' is being assumed to be at nominal value (E = 1.0 p.u.). P_{loss} and Q_{loss} in (19a) and (19b) can be obtained from expression of P_{loss} and Q_{loss} in (17a) and (17b) respectively. The receiving end voltage V can easily be calculated as shown below:

$$V = E - z_{eq}(P_g - jQ_g)/E \tag{20}$$

The new values of system losses are given by

$$P_{loss,eq} = i^2 r_{eq} \tag{21a}$$

$$Q_{loss,eq} = i^2 x_{eq} \tag{21b}$$

In order to check the validity of the two bus equivalent at any particular load level, we compute the difference in total transmission line loss between multi-bus system and equivalent two bus system:

If $dP(=P_{loss,multibus} - P_{loss,eq})$ and $dQ(=Q_{loss,multibus} - Q_{loss,eq}) \leq \varepsilon$ a tolerance, then it can be reasonably concluded that the proposed model represents an equivalent two bus model of the multi-bus system.

Thus, the two-bus system described above becomes the equivalent model of a multi-bus network at any particular network and load configuration where the total interconnected system has been replaced by a single line two bus system with same generation, load and loss. The parameters of the equivalent model will obviously vary with change in load pattern or with change in any system configuration.

V. Calculation of Global Voltage Stability Index

Once the global two-bus power network equivalent to multi-bus power system is obtained and then the global voltage stability index (GVSI) could be formulated in a straight forward manner from parameters of the global network as described below:

$$P_s = R_L \frac{(P_s^2 + Q_s^2)}{|\overline{Vs}|^2} + P_r \quad \text{and} \quad Q_s = X_L \frac{(P_s^2 + Q_s^2)}{|\overline{Vs}|^2} + Q_r$$

or

$$Q_s = \frac{(P_s - P_r)X_L + R_L Q_r)}{R_L} \tag{22}$$

Replacing Q_s in equation of P_s,

$$P_s = R_L \frac{\left[P_s^2 + \left\{\frac{(P_s - P_r)X_L + R_L Q_r)}{R_L}\right\}^2\right]}{|\overline{Vs}|^2} + P_r$$

Which, for $|\overline{Vs}|=1$ becomes

$$(R_L^2 + X_L^2)P_s^2 - (2X_L^2 P_r - 2R_L X_L Q_r + R_L)P_s$$
$$+ (X_L^2 P_r^2 + R_L^2 Q_r^2 - 2R_L X_L P_r Q_r + R_L P_r) = 0$$

i.e.

$$P_s = \frac{(2X_L^2 P_r - 2R_L X_L Q_r + R_L)}{2(R_L^2 + X_L^2)} \pm$$
$$\frac{\sqrt{(2X_L^2 P_r - 2R_L X_L Q_r + R_L)^2 - 4(R_L^2 + X_L^2)(X_L^2 P_r^2 + R_L^2 Q_r^2 - 2R_L X_L P_r Q_r + R_L P_r)}}{2(R_L^2 + X_L^2)}$$

Now P_s must have a real value, hence the discriminant of above equation must be greater than or equal to zero. This yields the following relation:

$$(2X_L^2 P_r - 2R_L X_L Q_r + R_L)^2 -$$
$$4(R_L^2 + X_L^2)(X_L^2 P_r^2 + R_L^2 Q_r^2 - 2R_L X_L P_r Q_r + R_L P_r) \geq 0$$

or

$$4\left[(X_L P_r - R_L Q_r)^2 + X_L Q_r + R_L P_r)\right] \leq 1 \tag{23}$$

Above expression is termed as Global Voltage Stability Index (GVSI) which gradually increases with increasing load in the actual power system and reach the value '1' at critical point of voltage instability (when load flow matrix Jacobian becomes singular). Therefore the value of GVSI is sufficient to assess the overall voltage stability status of a multi-bus power system at a particular operating point.

VI. Algorithm

An algorithm for system simulation is given below:

1) Solve Newton-Raphson load flow for base load case and determine the weakest bus of the multi bus system.

2) Find State to Impedance sensitivity matrix (SI) and Coupling Indices D-FACTS Devices and select the best k lines to place D-FACTS devices.

3) Make necessary changes in the admittance matrix for incorporating contingency and D-FACTS.

4) Increase the load of weakest bus by a small step at a constant power factor.

5) Solve load flow problem to obtain the system states. Go to step 7 if the load flow iterative process does not converge.

6) Calculate the total generation, load and transmission losses of the system. Calculated equivalent resistance (r_{eq}) and reactance (x_{eq}) for the two-bus equivalent model and hence Global Voltage Stability Index (GVSI). Go to step 4.

7) Stop

VII. Simulation And Discussion

A MATLAB program has been developed to perform the Newton-Raphson load flow analysis with above discussed models of DSSC and tested on the standard IEEE 30-bus system with base load equals 100MVA. The reactive power sensitivity analysis [13] reveals that bus no. 26 as the weakest bus of the system.

First, the power flow problem of the systems are successively solved for uniformly increasing load conditions (at an increment of 1% of base value keeping the load power factor constant) at the weakest bus until the power flow algorithm fails to converge. The Power flow problems are then similarly solved for application of DSSC at the most sensitive lines obtained from State to Impedance sensitivity matrix (SI) for voltage control at weakest bus. For each case and each load set, the two bus series-equivalent model parameters have been calculated and have been used to calculate the GVSI. It should be clear that the load increase is possible with any one or more bus in the system. Weakest bus of system is considered here only to describe the methodology in better way as weakest bus in term of reactive power sensitivity is most vulnerable to voltage collapse.

Fig. 3 and 4 exhibits the profile of GVSI and weak bus voltage for IEEE 30-bus system indicating that the system gradually moves towards voltage instability with increase in load as well as it is clear that with the application of DSSC, the GVSI have been improved with better load catering capability. Fig. 4 also suggests an improvement in voltage profile with the incorporation of DSSC though its actual significance lies in its capability of handling increased power flow and hence increased stability of the system even under stressed condition.

Beneficial choices of lines for D-FACTS placement may be determined from [SI_V]. Lines that have higher sensitivities are better choices because changing the impedance on that line has a higher impact on the system voltages. In a first case, D-FACTS devices are installed on the first best line and impedances are allowed to change by + 30%. As D-FACTS devices are able to change line impedance by any amount within their limits but here it is assumed that these devices are set at their limits [19].

To illustrate the importance of device placement, in a next scenario, D-FACTS devices are placed on the 2 and then 5 best lines determined by [SI_V]: (25,26), (27,28), (12,13), (9,11), (9,10). An interesting conclusion is that the weakest bus voltage for placing D-FACTS devices on all lines (which is unrealistic) and placing D-FACTS devices on only the 5 best lines are very similar. Thus, these 5 lines are good choices for use in voltage control. Conversely, there is little benefit to be gained by placing devices on ineffective lines. So for economic reason, it is better to employ the D-FACTS devices on the most beneficial lines rather than employing it at all the lines.

VIII. Conclusion

Seeing the results, it can be concluded that the developed equivalent two bus system can be applied to any multi-bus power network to assess the global voltage stability of the system in terms of GVSI. By inserting a series compensating voltage in the line, which can effectively change transmission line impedances, D-FACTS devices can be applied to problems such as controlling system voltages. The importance of choosing effective locations is clear from the results. After installing D-FACTS devices on the first several lines, no significant improvement is obtained by installing devices on the other lines. Effective D-FACTS device locations and independently controllable flows can be identified from sensitivities.

Although the benefits of D-FACTS devices discussed in this paper provide strong arguments for their use yet there is work to be done to understand the effects of D-FACTS devices on system stability by developing a transient stability model of D-FACTS devices.

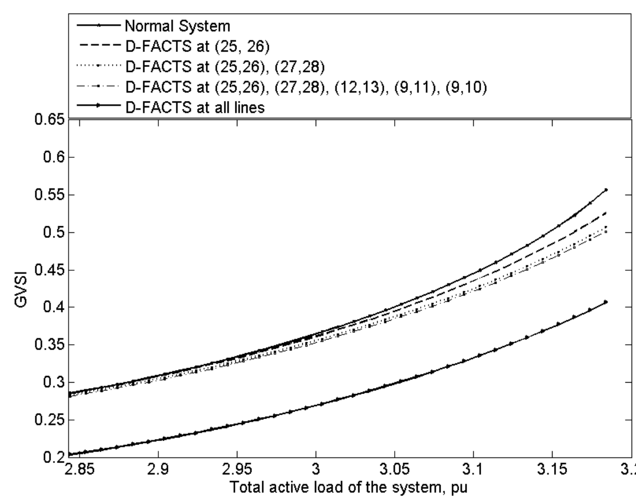

Fig. 3. Variation of GVSI for different operating conditions

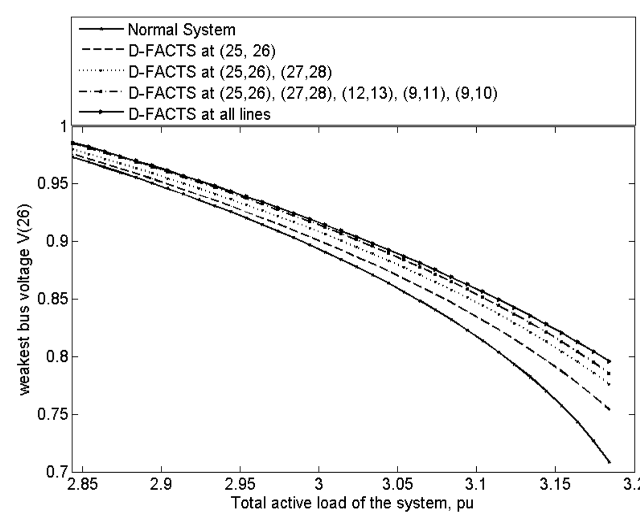

Fig. 4. Variation in weakest bus voltage for different operating conditions

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