Monte Carlo Simulation for Reliability Analysis of Emergency and Standby Power Systems

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Abstract

This paper describes a sequential Monte Carlo simulation method for the reliability analysis of standby and emergency power systems. The results obtained from this method are compared with those from the Markov cut-set approach. It is shown that the Monte Carlo simulation can yield additional useful information on the probability distribution of indices in addition to obtaining the estimates of the mean values.

1 Introduction

Different facilities have varying requirements for reliability of electric power supply. Even at the same facility various loads may have different reliability requirements. Some loads, such as medical facilities, emergency lighting, data processing and chemical process industries are very sensitive to interruptions in electric supply. Standby and emergency power systems [1, 2, 3] are installed at such premises to provide electric power of acceptable quality.

Reliability and cost considerations play an important role in the choice of various alternatives. These alternatives include less expensive utility supply enhanced by standby power, more expensive utility supply, and various configurations of standby systems. The analysis of these alternatives may become more important as the reliability differentiated power becomes available.

Reliability analysis of various options is important for the proper selection of standby power systems. Although this is an important problem, it has not been adequately addressed in the available literature. References [4] and [5] describe methods based on *Markov* and *Markov cut set* approaches. This paper describes a *Monte Carlo* [6] approach for this problem.

2 Background

Reference [5] outlines an approach based on a combination of the cut-set method and the Markov models. This approach consists in first identifying cut-sets which are basically components or events whose failure or occurrence would cause system failure. The equations for calculating the failure frequency and duration of these cut-sets are described in [3, 5]. Some of the events which involve dependent failures are analyzed using Markov processes [5, 6, 7]. The methodology for combining the frequency and duration of cut-sets to obtain system indices is described in [6, 7]. This approach is quite powerful, but may run into problems of dimensionality where large and complex configurations are involved.

3 Monte Carlo Approach

The reliability indices of an actual physical system can be estimated by collecting data on the occurrence of failures and the durations of repair. The Monte Carlo method mimics the failure and repair history of the components and the system by using the probability distributions of the component state durations. Statistics are then collected and indices estimated using statistical inference.

There are two basic approaches for Monte Carlo simulation, (1) sequential simulation, and (2) random sampling. The sequential simulation proceeds by generating a sequence of events using random numbers and probability distributions of random variables representing component state durations. In random sampling, states are drawn based on the probability distributions of component states and random numbers. Further, there are two methods for representing the passage of time in sequential simulation: (1) the fixed interval method, also called synchronous timing, and (2) the next event or asynchronous timing method. In the fixed interval method, time is advanced in steps of fixed length and the system state is updated. In the next event method, time is advanced to the occurrence of the next event. In actual implementations, it is likely that combinations of the timing controls may be used.

The sampling method is generally faster than the sequential technique, but is suitable when component failures and repairs are independent. This paper presents the sequential method for reliability analysis.

3.1 Description of the Method

The flowchart for this method is shown in FIGURE 1. The whole procedure consists of the following steps.

3.1.1 Data Input and Initialization

The input data consists of the *failure rate* (λ) and *duration* (r) of every component. The failure rate is the reciprocal of the *mean up time*. The *failure duration* or *mean down time* is the reciprocal of the *repair rate* (μ). The failure and repair rates, λ and μ, of a component will be used to determine how long the component will remain in the "UP" state and the "DOWN" state.

Simulation could be started from any system state, but it is customary to begin simulation with all the components in the "UP" state.

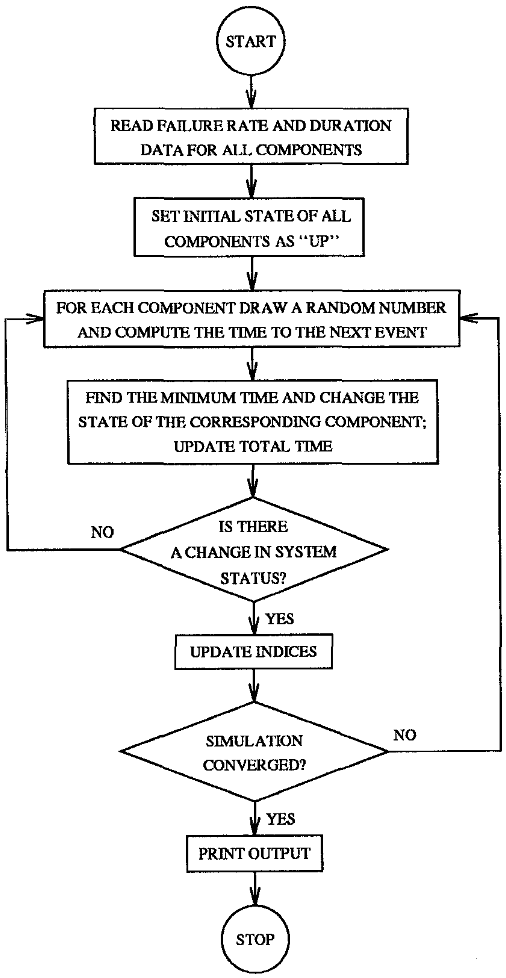

FIGURE 1: FLOWCHART FOR NEXT-EVENT SIMULATION

3.1.2 Random Number Generation

Simulation is performed by generating realizations of the underlying stochastic process, by using random numbers. These numbers constitute a sequence in which each number has an equal probability of assuming any one of the possible values, and is statistically independent of the other numbers in the sequence. Random numbers, therefore, basically constitute a uniform distribution over a suitably selected range of values. This distribution may be constructed using any suitable means. The method used in this work is a multiplicative congruential method [6] which obtains the $(n+1)$th random number R_{n+1} from the nth random number R_n using the following recurrence relation due to Lehmer

$$R_{n+1} = (aR_n)(\text{modulo } m) \tag{1}$$

where a and m are positive integers, $a < m$. The above notation signifies that R_{n+1} is the remainder when aR_n is divided by m. The first random number R_0 (called the seed) is assumed, and the subsequent numbers can be generated by the above recurrence relation. Now the sequence thus generated is periodic, so R_0, a and m should be carefully chosen so that the sequence cycle is larger than the number of random numbers required.

3.1.3 Computation of Time to the Next Event

The time to the next event is generated by using the *inverse of probability distribution* method. This method can be understood by considering the probability mass function of a random variable shown in FIGURE 2. The first step is to convert this mass function into the corresponding distribution function, as shown. Now a random number z between 0 and 1 is generated and $F(x)$ is set equal to z. The corresponding value of X gives the value of the random variable. An example is shown in FIGURE 2, with $z = 0.55$, for which $X = 2$.

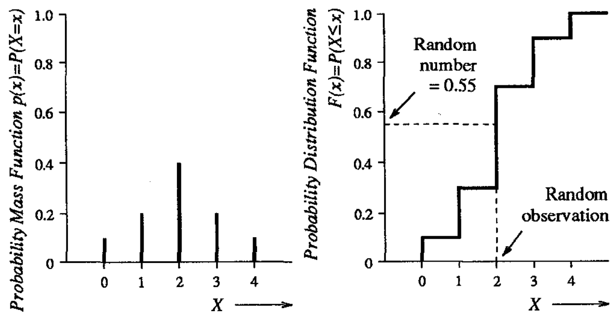

FIGURE 2: PMF AND PDF OF A RANDOM VARIABLE

It should be noticed that $F(x_i) - F(x_{i-1})$ is equal to $P(X = x_i)$, and if the random number falls in the interval $(F(x_i), F(x_{i-1}))$, the value of $X = x_i$ will be selected. The procedure therefore essentially allocates the random numbers to the random variables in the proportion of their probabilities of occurrence.

This procedure can also be used for continuous distributions. Continuous distributions are approximated by discrete distributions whose irregularly spaced points have equal probabilities. The accuracy can be increased by increasing the number of intervals into which (0,1) is divided. This requires additional data in the form of tables. Although the method is quite general, its disadvantages are the great amount of work required to develop tables and possible computer storage problems. The following analytic inversion approach is simpler.

Let z be a random number in the range 0 to 1 with a uniform probability density function, i.e., a triangular distribution function:

$$f(z) = \begin{cases} 0 & Z < 0 \\ 1 & 0 \leq Z \leq 1 \\ 0 & Z > 1 \end{cases} \tag{2}$$

Similarly

$$F(z) = \begin{cases} 0 & Z < 0 \\ z & 0 \leq Z \leq 1 \\ 1 & Z > 1 \end{cases} \quad (3)$$

Let $F(x)$ be the distribution function from which the random observations are to be generated. Let

$$z = F(x)$$

Solving the equation for x gives a random observation of X. That the observations so generated do have $F(x)$ as the probability distribution can be shown as follows.

Let ϕ be the inverse of F; then

$$x = \phi(z)$$

Now x is the random observation generated. We determine its probability distribution as follows

$$\begin{aligned} P(x \leq X) &= P(F(x) \leq F(X)) \\ &= P(z \leq F(X)) \\ &= F(X) \end{aligned} \quad (4)$$

Therefore the distribution function of x is $F(X)$, as required. In the case of several important distributions, special techniques have been developed for efficient random sampling.

In this paper, the distributions assumed for up and down times are exponential. The exponential distribution has the following probability distribution

$$P(X \leq x) = 1 - e^{-\rho x} \quad (5)$$

where $1/\rho$ is the mean of the random variable X. Setting this function equal to a random decimal number between 0 and 1,

$$z = 1 - e^{-\rho x}$$

Since the complement of such a random number is also a random number, the above equation can as well be written as

$$z = e^{-\rho x}$$

Taking the natural logarithm of both sides and simplifying, we get

$$x = -\frac{ln(z)}{\rho} \quad (6)$$

which is the desired random observation from the exponential distribution having $1/\rho$ as the mean.

This method is used to determine the time to the next transition for every component, using λ or μ for ρ, depending on whether the component is UP or DOWN. The smallest of these times indicates the most imminent event, and the corresponding component is assigned a change of state. If this event also results in a change of status, (i.e., failure or restoration) of the system, then the corresponding system indices are updated.

3.1.4 The Indices

At any time t, the mean failure frequency is given by

$$\lambda_t = \frac{1}{t}(\text{number of failures till time } t) \quad (7)$$

and the mean down time is given by

$$r_t = \frac{1}{t}(\text{total time spent in failed state}) \quad (8)$$

The values of λ and r at the instant the simulation converges (see § 3.1.5) are the reliability indices for the system as obtained from the Monte Carlo method.

3.1.5 Convergence

The simulation is said to have converged when the indices attain stable values. This "stabilization" of the value of an index i is measured by its standard error, defined as:

$$\eta = \frac{\sigma_i}{\sqrt{n_c}} \quad (9)$$

where σ_i = standard deviation of the index i
n_c = number of cycles simulated

Convergence is said to occur when the standard error drops below a prespecified fraction, ϵ, of the index i, i.e., when

$$\eta \leq \epsilon i \quad (10)$$

If, for instance, the mean down time r is chosen as the index to converge upon, then, after every system restoration simulated, the following relation is tested for validity:

$$\frac{\sigma_r}{\sqrt{n_c}} \leq \epsilon r \quad (11)$$

If this criterion is satisfied, the simulation is said to have converged.

3.2 Statistics Obtained from Simulation

Simulation is advantageous in that it not only allows the computation of indices at various points in the system, but also permits the accumulation of data pertaining to the distribution of these indices, thereby affording a better understanding of the system behavior.

For an emergency power system, for instance, statistics may be collected for failure frequency and duration at various points in the system, the annual incidence rates for failures, as well as for the variances of these indices.

This feature of the method is demonstrated in the following sections.

4 The Test System

This section describes the test system used in this study, and briefly discusses some modeling considerations.

4.1 System Description

The test system is shown in FIGURE 3. This system is the same as the one analyzed in [5]. The power is normally supplied from utility input power through the UPS. A synchronized bypass and static switch protect the critical load in the event of an inverter failure. If voltage is lost to the critical load, the STS reestablishes voltage in less than one quarter of a cycle. This is considered continuous power for most loads.

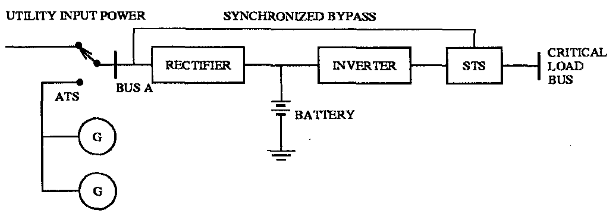

FIGURE 3: PARALLEL SUPPLIED NONREDUNDANT UNINTERRUPTIBLE POWER SUPPLY

When the generators are in standby mode, their failures remain undetected except during periodic inspections. Therefore, while starting, there is a probability p_s that a generator may fail to start.

Only one generating unit is taken out for planned maintenance. If a generator fails while the other is on planned maintenance, it is possible to accelerate the maintenance on the second generator by a factor of α.

If power fails at bus A, the battery can sustain the load for upto 4 hours.

TABLE I lists the data pertaining to the system.

TABLE I: SYSTEM DATA

Equipment/Supply	λ (f/y)	r (h/f)
Utility Supply, single circuit	0.53700	5.66
Generator (per hour of use)	0.00536	478.00
Inverter	1.25400	107.00
Rectifier	0.03800	39.00
ATS	0.00600	5.00
STS	0.08760	24.00
Battery	0.03130	24.00
Equipment Maintenance	**freq (/y)**	**dur (h)**
Generator	1.00	10.00
UPS	1.00	4.00
Other data:		
Battery can supply load for		4.0 h
Common mode failure of generators	(λ_{cm})	0.0
Acceleration factor for planned maintenance of generators	(α)	2.0
Probability of failure to start a generator	(p_s)	0.015

4.2 Simulation Model

The flowchart shown in FIGURE 1 was implemented. In computing the time to the next event, the method described in § 3.1.3 was used, but the following dependencies were also included:

1. A generator cannot fail while the utility supply is up
2. A generator cannot be taken out for maintenance if the utility is down or if the other generator is down or under maintenance
3. The UPS cannot be taken out for maintenance unless power is available at bus A

The failure of a generator to start was modeled as follows. A random number z was generated, $0 \leq z \leq 1$. If $z \leq p_s$, the generator was assumed to fail.

5 Results

In this section the indices obtained from simulation are compared with those obtained analytically, using cut-set analysis, in [5].

5.1 Cut-Set Analysis

The reliability indices of the test system have been analytically evaluated, using the cut-set approach, in [5]. A summary of the solution steps and the results will be presented here, to provide a basis for validation of the simulation technique.

First, the combination of the utility supply and the two generators is analyzed for failure modes. FIGURE 4 shows the possible states this combination can assume, and the transition rates between these states. Based on these transition rates, the probabilities and frequencies of occurrence of the failed states are determined. This enables computation of the failure rate and duration of the utility-generator subsystem, which are determined to be

$$\lambda_p = 0.001576/y \quad \text{and} \quad r_p = 5.443h$$

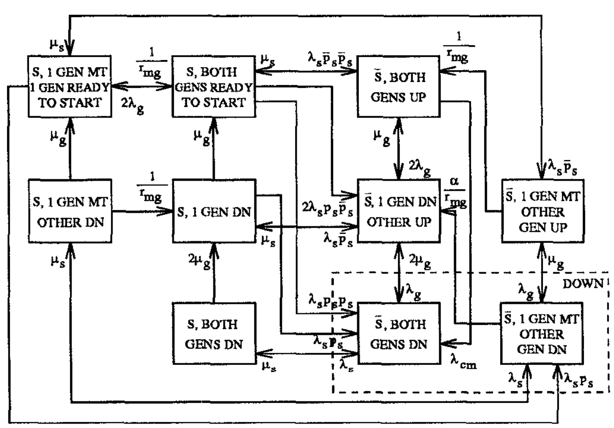

FIGURE 4: STATE TRANSITION DIAGRAM FOR UTILITY SUPPLY AND GENERATORS

The next step involves computation of the rate and duration of power failure at bus A:

$$\lambda_A = \lambda_p + \lambda_{ATS} = 0.007576/y$$
$$r_A = \frac{\lambda_p r_p + \lambda_{ATS} r_{ATS}}{\lambda_p + \lambda_{ATS}} = 5.092h$$

The remainder of the cut-set analysis is performed as shown in TABLE II. The system indices, i.e., rate and duration of power loss at the Critical Load Bus (CLB) are determined to be

$$\lambda_{CLB} = \sum_{\text{cut-sets}} \lambda = 0.005225/y$$

$$r_{CLB} = \frac{\sum \lambda r}{\sum \lambda} = 9.648h$$

If all distributions are assumed exponential, then the standard deviations of all the up times and down times would equal the corresponding mean up times and down times. This implies that the standard deviations of the failure rates would also equal the corresponding means computed.

TABLE II: FREQUENCY AND DURATION OF POWER LOSS AT CRITICAL LOAD BUS (CLB)

Cut Set	λ (f/y)	r (h/f)	λr
Power loss at bus A > 4 h	0.003454	5.092	0.017588
Power loss at bus A (0.007576, 5.092) and Failure of [Inverter or battery or STS] (1.3729, 99.812)	0.000125	4.845	0.000606
Maintenance on UPS (1.0, 4) and Power loss at bus A (0.007576, 5.092)	0.000003	2.240	0.000007
Inverter failure(1.254, 107.0) and STS failure(0.0876, 24.0)	0.001643	19.603	0.032208
$\sum$	0.005225		0.050409

5.2 Simulation Results

The simulation method described in **section 3** was used to compute the following statistics for the test system:

1. Frequency λ_p and duration r_p of failure of the utility-generator subsystem (FIGURE 4); the standard deviations of λ_p and r_p.

2. Frequency λ_A and duration r_A of power failure at bus A; the standard deviations of λ_A and r_A.

3. Frequency λ_{CLB} and duration r_{CLB} of power failure at the critical load bus; the standard deviations of λ_{CLB} and r_{CLB}.

4. Data for the *Probability Mass Function* of the number of system failures per year, N_f.

5. Data for the *Probability Distribution Function* of the system down time, T_f.

TABLE III compares the indices obtained from simulation with those obtained analytically.

TABLE III: COMPARISON OF SIMULATED AND CALCULATED INDICES

Index	Simulated		Calculated	
	Mean	SD	Mean	SD
λ_p (f/y)	0.001352	0.001497	0.001576	0.001576
r_p (h/f)	5.028	5.503	5.443	5.443
λ_A (f/y)	0.007193	0.006851	0.007576	0.007576
r_A (h/f)	5.216	5.201	5.092	5.092
λ_{CLB} (f/y)	0.004868	0.004888	0.005225	0.005225
r_{CLB} (h/f)	9.7135	13.3312	9.648	9.648

TABLE IV compares the PMF of the number of system failures per year. Now for exponentially distributed up times the failures are Poisson distributed, i.e.,

$$P(N_f = k) = \frac{\lambda_{CLB}^k e^{-\lambda_{CLB}}}{k!} \tag{12}$$

This equation is used to generate the calculated data for the PMF of N_f, in TABLE IV.

TABLE IV: PMF OF NUMBER OF FAILURES PER YEAR

k	$P(N_f = k)$ Simulated	$P(N_f = k)$ Calculated
0	0.9952	0.9947
1	0.4825×10^{-2}	0.5198×10^{-2}
2	0.2167×10^{-4}	0.1358×10^{-4}
3	0.0000	0.2365×10^{-7}
4	0.0000	0.3089×10^{-10}

FIGURE 5 compares the PDF of the system down time. For exponentially distributed down time T_f, the PDF is given by

$$F(t) = P(T_f \le t) = 1 - e^{-t/r_{CLB}} \tag{13}$$

This equation is used to generate the calculated data for the PDF of T_f, for FIGURE 5.

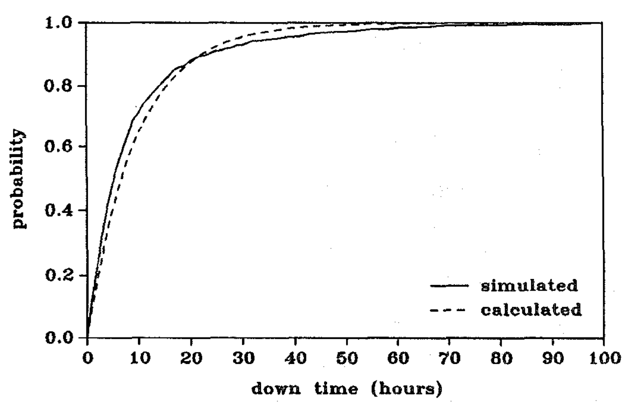

FIGURE 5: PROBABILITY DISTRIBUTION FUNCTION OF DOWN TIME

6 Discussion and Conclusion

This paper has demonstrated the application of the Monte Carlo method to the reliability analysis of emergency and standby power systems. A relatively small system consisting of two standby generators and one UPS was used for illustrating the method and comparing the results with those obtained by an analytical method. It is no surprise that the results obtained from the simulation are close to those obtained using cut-sets, since both methods are based on sound mathematical foundation.

In practice, both methods use approximations, but in different ways. The cut-set approach is exact only if all cut-sets are utilized. In practice, however, only cut-sets of upto a selected order, generally second order, are used. The other source of approximation comes from the use of equations for probability and frequency of system failure. Although exact equations are available [6], the equations used in [5] and many applications are approximate. The approximations do not have a significant effect so long as the component probabilities of success are close to unity.

The approximation in the Monte Carlo method comes from the fact that in this approach, the statistics are estimates of the true values, and therefore cannot be exact. It can be seen from equation (11) that if the error is to be reduced to half, the number of samples has to increase four times. Thus for systems with high reliability (or low probability of failure), the Monte Carlo simulation can take a very long time to converge. The main advantage of simulation is that it is very flexible for incorporating dependent failures and is very suitable for large systems. Also, it yields the probability distribution of indices in addition to estimating mean values. These probability distributions are useful for performing cost benefit analysis.

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