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Does individual-stock skewness/coskewness reflect portfolio risk?

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A B S T R A C T

Many asset pricing studies assume that a stock's coskewness or idiosyncratic skewness is priced because of the characteristic's influence on portfolio skewness. From empirical returns, we show that the number of stocks in a portfolio is the most important determinant of portfolio skewness, while component stocks' coskewness or idiosyncratic skewness has marginal effects. This result indicates that individual stock skewness does not well represent portfolio skewness.

1. Introduction

Asset pricing literature shows that a rational investor prefers positive skewness in portfolio returns (Arrow, 1971; Eisdorfer 2010). As skewness indicates a tendency of experiencing infrequent but large shocks, it is better to have a few large positive shocks (positive skewness in returns) rather than negative shocks (negative skewness).[1] Empirically, market portfolios have negatively-skewed return distributions (Christie, 1982; French et al. 1987; Hong and Stein, 2003; Lehnert and Wolff 2004). Investors demand

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[1] Arrow (1971) argues that one of desirable properties for an investor's utility function is non-increasing absolute risk aversion. Non-increasing absolute risk aversion is an intuitive assumption that an investor does not reduce his investment in risky assets as he becomes wealthier. Arditti (1967) shows that non-increasing absolute risk aversion indicates preference for positive skewness.

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compensation for this risk (Kraus and Litzenberger, 1972; Harvey and Siddique, 1999, 2000; Mitton and Vorkink, 2007; Carmichael and Cöen 2013; Conrad et al., 2013; Langlois 2013). Many studies argue that the skewness of an individual stock return is a determinant of stock price, based on the assumption that the stock characteristic is related to portfolio skewness (e.g. Kraus and Litzenberger 1972; Harvey and Siddiqe 1999, 2000). Recently, Albuquerque (2012) finds an interesting contrast. Individual stock return skewness is in general positive, while market-level skewness is negative.

The contrast poses a challenge to the traditional asset pricing theories that assume portfolio skewness risk as a reflection of an individual-stock characteristic. If portfolio skewness is not a simple aggregate of individual-stock skewnesses, it is difficult to justify the pricing of individual-stock characteristics, as what matters to investors is the skewness of their portfolios.

To better understand the mechanism that converts positive individual-stock skewness to negative market skewness, we examine what happens in the middle by forming portfolios of different sizes from actual stock returns and track how portfolio skewness changes. In addition, we explore individual stock characteristics that are assumed to be associated with the skewness of various sized portfolios. The empirical relationship between a stock characteristic and portfolio skewness has not been thoroughly tested, especially for smaller size portfolios. Kraus and Litzenberger (1972) and Harvey and Siddique (1999, 2000) argue that a stock's idiosyncratic skewness does not affect prices, while Mitton and Vorkink (2007), Brunnermeier et al. (2007), Barberis and Huang (2008) find that idiosyncratic skewness may influence prices. Langlois (2013) derives an asset pricing model incorporating both coskewness and idiosyncratic skewness. The study also empirically finds that both skewness measures influence the prices of equities, government bonds, currencies, and commodities. Other asset pricing factors, such as size or book-to-market, may be related to a stock's contribution to portfolio skewness as well (Chung et al., 2006).

Since there are few theories that provide a priori knowledge regarding the skewness of smaller sized portfolios, we measure portfolio skewness using a bootstrap method. From the CRSP database, we randomly select stocks and form certain sized portfolios. This bootstrap method can reveal the true distribution of a test statistic, portfolio skewness in this case, without imposing certain prerequisites (Efron, 1979).

The empirically measured portfolio skewness exhibits the following. The most important determinant of portfolio skewness is number of stocks. While a stock's co-skewness or idiosyncratic skewness is argued to be the main determinant of portfolio skewness by the literature, these two characteristics have marginal or inconsistent explanatory powers on portfolio skewness when compared to number of stocks.

Individual stock characteristics appear to affect the level of portfolio skewness in univariate settings. Portfolios formed with only low co-skewness stocks or only low idiosyncratic skewness stocks have lower levels of portfolio skewness when number of stocks is the same. However, while studies on co-skewness argue that diversification eliminates the effect of idiosyncratic skewness, we find the effect of idiosyncratic skewness persists even in a portfolio with 100 stocks, indicating that idiosyncratic skewness is not diversified away.

Fama-French (1992) asset pricing factors are not priced as compensation for portfolio skewness. Portfolios of small stocks or high book-to-market stocks have more positive skewness than otherwise. As such, small stocks or high book-to-market stocks outperform other stocks in terms of the first moment (average return) and the third moment (skewness).

Our contribution is to show how individual-stock characteristics are empirically related to portfolio skewness. While many of the individual-stock characteristics are argued to be a determinant of portfolio skewness and be priced as a result, we show that the importance of individual-stock characteristics is marginal compared to the interactions among stocks at the portfolio level. Our results can be useful for investment decisions or asset pricing models that aim to incorporate the risk of higher moments.

2. Data and method

We acquire stock returns data from the Center for Research in Stock Prices (CRSP) database. The sample period is from 1966 to 2012, as stock returns prior to the early 1960s are not very accurate according to Fama and French (1992). We merge previous year accounting data obtained from Compustat to the stock returns data. The results reported in this paper are based on monthly returns in excess of the risk free rate. Excess returns are obtained by subtracting the risk free rates from the CRSP returns, and the monthly risk free rates are from Ken French's Data Library. We also check to see whether daily returns show different patterns from the monthly returns and we find qualitatively similar results.

We form portfolios with different numbers of stocks. We construct portfolios with the following number of stocks: 4, 9, 16, 25, 36, 49, 64, 81, and 100. At the end of each calendar year, we randomly select stocks to form a portfolio and track the portfolio return for the next entire calendar year. The composition of a portfolio changes every year as a result. We obtain a series of monthly returns for the entire sample period (1966–2012). This process is repeated 100 times, generating 100 time-series returns for each number of stocks. The repeated process yields approximately (100 iterations $\times$ 9 different number of stocks $\times$ 46 years $\times$ 12 months $=$ 496,800) monthly portfolio returns.

From the equal-weighted portfolio returns, portfolio statistics, such as standard deviation and skewness, are calculated. Results from the value-weighted portfolio returns are similar. Portfolios betas are also estimated to examine the co-movements between a portfolio and the market. The beta of a portfolio is obtained by regressing the entire time-series return of the portfolio with the market return.

The coskewness of individual stock is calculated following the method of Kraus and Litzenberger (1972), using the previous 60 months of stock returns.

$$\text{Coskewness}_i = \left[\sum_{t=1}^{60} (r_{Mt} - \bar{r}_M)^2 (r_{it} - \bar{r}_i)\right] \Bigg/ \left[\sum_{t=1}^{60} (r_{Mt} - \bar{r}_M)^3\right] \quad (1)$$

where r_m is the market return, r_i is the individual stock return and overbar indicates time-series average.

Idiosyncratic skewness is derived according to the method of Boyer et al. (2010). The previous 60 months of stock returns are regressed by corresponding monthly Fama-French 3 factors, i.e. market factor, size factor, and book-to-market factor. The skewness of the residual is idiosyncratic skewness.

$$\text{Idiosyncratic Skewness}_i = \frac{1}{60} \frac{\sum_{i=1}^{60} \varepsilon_i^3}{\sigma_i^3} \quad (2)$$

where ε is the residual of the regression, and σ is the standard deviation of the residual.

Fama-French asset pricing factors, size and book-to-market, are calculated using the method in Fama and French (1992). Stocks are ranked into size or book-to-market quintiles every calendar year.

Finally, firm-level diversification is acquired from the Historical Segments Data in Compustat. The segments data begins in 1976, so relevant analyses are based on a shorter time series of stock returns from 1976 to 2012. There are four categories of segments in the database: business, geographic, operation, or state. For each category, we calculate the Herfindahl index of sales. A smaller Herfindahl index indicates greater diversification and many of the firms have an index value of one, implying no diversification. Since there are various avenues of diversification, we define a firm's diversification level as the minimum of four Herfindahl indices from each category.

3. Number of stocks and portfolio skewness

Our first analysis is to form portfolios with different numbers of stocks without imposing restrictions. The mean, standard deviation, and skewness of the returns are calculated from the whole time-series returns of the formed portfolios. As there are 100 iterations of the time-series returns, the average of 100 portfolio statistics is reported in Table 1. The standard errors of the averages are in parentheses. Columns 5 and 6 report the elasticity of the standard deviation or skewness to the number of stocks. The elasticity is calculated by dividing the percentage change of a portfolio statistic with the percentage change in the number of stocks. Column 7 provides the portfolio betas.

The first output that is immediately noticeable is that portfolio skewness becomes negative as the number of stocks increases. The result is consistent with an earlier result of Simkowitz and Beedles (1978), indicating that this phenomenon is not time specific. Investors who value skewness face a tradeoff as additional diversification makes skewness worse, while standard deviation makes it better.

Other than the standard deviation and skewness, the number of stocks has little effect on other portfolio properties. Column 2 in Table 1 indicates that portfolio mean returns are similar across different numbers of stocks. In Column 7, portfolio betas do not vary much either, indicating little material difference in the co-movements with the market.

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Table 1
Portfolio skewness by the number of stocks.

Number of stocks	Average return	Std. dev.	Skewness	Elasticity: std. dev.	Elasticity: skewness	Beta
4	0.95%	0.10	1.26			1.13
	(0.03%)	(0.0009)	(0.14)			(0.009)
9	0.97%	0.08	0.55	−0.29	−1.02	1.13
	(0.02%)	(0.0004)	(0.05)			(0.006)
16	0.95%	0.07	0.28	−0.18	−1.15	1.12
	(0.02%)	(0.0003)	(0.03)			(0.004)
25	0.98%	0.07	0.22	−0.11	−0.56	1.13
	(0.01%)	(0.0002)	(0.03)			(0.004)
36	0.98%	0.07	0.07	−0.11	−2.83	1.13
	(0.01%)	(0.0002)	(0.02)			(0.003)
49	0.96%	0.06	−0.01	−0.08	−6.54	1.12
	(0.01%)	(0.0001)	(0.02)			(0.003)
64	0.97%	0.06	−0.01	−0.04	−2.74	1.13
	(0.01%)	(0.0002)	(0.01)			(0.002)
81	0.97%	0.06	−0.05	−0.04	−4.78	1.13
	(0.01%)	(0.0001)	(0.01)			(0.002)
100	0.97%	0.06	−0.07	−0.04	−2.02	1.13
	(0.01%)	(0.0001)	(0.01)			(0.002)

Note: We construct portfolios from the CRSP database by randomly selecting stocks at the end of each calendar year. The return of the formed portfolio is tracked for the next calendar year, creating monthly equal-weighted or value-weighted portfolio returns. The yearly selection process is done for the entire sample period from 1966 to 2012. The whole process is repeated 100 times with replacements, yielding 100 time-series observations for each number of stocks category. Accordingly, the reported statistics in this table are the averages from the 100 time-series observations, except for the elasticity, which is calculated from the averages. The standard errors of the averages are in parentheses. Portfolio mean, standard deviation, and skewness are calculated from the whole time-series returns of each portfolio and are reported in Columns 2, 3, and 4, respectively. Columns 5 and 6 provide the elasticity of standard deviation or skewness to the number of stocks. The elasticity is calculated by dividing the percentage change of a portfolio statistic with the percentage change in number of stocks. Column 7 reports portfolio betas, calculated by regressing a portfolio's whole time-series return with the market return.

If diversification is an important factor on portfolio skewness, similar results would be noticeable at a more micro level. A firm can be thought of as a portfolio holding multiple assets, and a firm that holds diversified assets may exhibit return skewness similar to that of a diversified portfolio of stocks.[2] In unreported results, we also find that the portfolios of more diversified firms have more negative skewness than the portfolios of less diversified firms.

4. Does individual-stock skewness matter?: co-skewness and idiosyncratic skewness

We examine the effect of co-skewness and idiosyncratic skewness on portfolio skewness in this section. Stocks are ranked into quintiles (Ranks 1-5) every year by their co-skewness with market returns and idiosyncratic skewness using the past five years of returns. Portfolios are formed from the lowest ranked stocks only or from the highest ranked stocks only (Fig. 1).

Co-skewness affects the level of portfolio skewness, as a portfolio of high co-skewness stocks exhibits high portfolio skewness. However, the effect of diversification does not change with the stock characteristics. Additional diversification decreases portfolio skewness even if the portfolios are formed with only high co-skewness stocks (Fig. 2).

The cases of idiosyncratic skewness are similar. The relationship between diversification and portfolio skewness is robust to idiosyncratic skewness as well. Note that the effect of idiosyncratic skewness is not completely diversified away even when a portfolio contains 100 stocks. If the effect of idiosyncratic skewness is to be diversified, as argued by the papers emphasizing co-skewness, a 100-stock portfolio of high idiosyncratic skewness stocks should have a similar skewness to a low idiosyncratic skewness portfolio.

[2] See Chung et al. (2014) for further development of this argument.

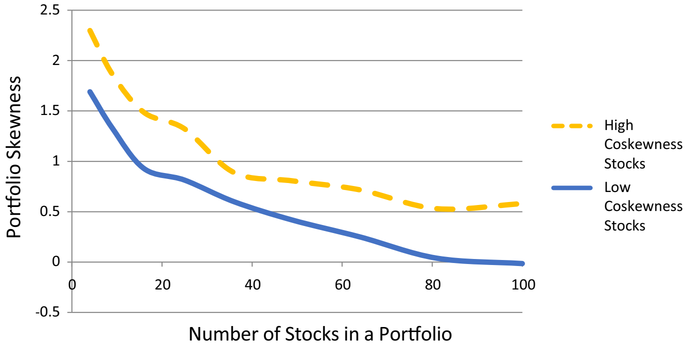

Fig. 1. Skewness of portfolios constructed from high (low) co-skewness stocks.

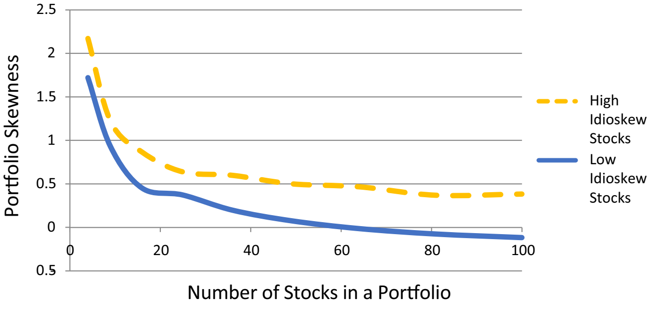

Fig. 2. Skewness of portfolios constructed from high (Low) idiosyncratic skewness stocks.

We further test a few potential determinants of portfolio skewness in the literature. Chung et al. (2006) attempts to explain the Fama-French (1992) asset pricing factors as compensation for the risk of higher moments. In addition, a factor, like size, may be mechanically related to skewness as small firms may experience large negative shocks more frequently. Accordingly, we examine whether Fama-French (1992) asset pricing factors are associated with portfolio skewness. We find that 1. A small-sized-stocks portfolio has more positive portfolio skewness, and 2. A high book-to-market stocks portfolio has more positive portfolio skewness. These results indicate that size premium or book-to-market premium are not compensation for negative skewness. The results are available upon request.

5. A test on the determinant of portfolio skewness

In this section, we investigate which characteristic has the most significant effect on portfolio skewness using a multivariate regression approach. We re-examine the returns of the randomly formed portfolios used in Table 1 for this purpose. Portfolio skewness is calculated every calendar year from the monthly returns of the portfolio, and the corresponding stock characteristics are the averages from the stocks in the portfolio. Our estimation equation is:

$$\begin{aligned}\text{Portfolio Skewness}_{i,t} = {} & a + b1 \cdot Log(Number\,of\,Stocks_{i,t}) + b2 \cdot Herfindhal_{i,t} \\ & + b3 \cdot Coskewness_{i,t} + b4 \cdot Idio\_skewness_{i,t} + b5 \cdot Log(Size_{i,t}) \\ & + b6 \cdot BTM_{i,t} + \varepsilon_{i,t} \end{aligned} \quad (3)$$

where the portfolio skewness of a portfolio i in year t is regressed by the number of stocks and the average of firm-level diversification measured by the Herfindahl index of segments, co-skewness, idiosyncratic skewness, market value, and the book-to-market value acquired from the stocks in a portfolio. Following Fama and French (1992), we take natural log of market value and book-to-market value in the

Table 2
Stock characteristics and portfolio skewness.

Explanatory variables	Model 1: all variables	Model 2: diversification variables	Model 3: skewness variables	Model 4: asset pricing variables	Model 5: reduced observations
Log (number of stocks in a portfolio)	−0.14* (−28.94)	−0.14* (−28.68)			−0.13* (−8.03)
Herfindahl index (under- diversification)	−0.24* (−9.34)	−0.19* (−12.96)			−0.26* (−3.28)
Co-skewness	0.11 (0.31)		0.23 (0.69)		−0.01 (−0.03)
Idiosyncratic skewness	0.15* (6.52)		−0.14* (−6.57)		0.26* (3.67)
Log (firm market value)	−0.03 (−2.91)			−0.06* (−12.11)	−0.02 (−0.77)
Log (book-to-market value)	−0.13* (−7.01)			0.34* (23.80)	−0.16 (−2.76)
Adjusted R^2	2.86%	2.56%	0.11%	3.29%	3.03%
Observations	31,611	31,611	40,890	40,896	3125

We construct portfolios from the CRSP database by randomly selecting stocks at the end of each calendar year. The return of the formed portfolio is tracked for the next calendar year, creating monthly equal-weighted or value-weighted portfolio returns. The yearly selection process is done for the entire sample period from 1966 to 2012. The process is repeated 100 times with replacements yielding 100 time-series observations for each number of stocks category. Portfolio skewness is calculated each calendar for each iteration by a different number of stocks. Our estimation equation is:

$$\text{Portfolio Skewness}_{i,t} = a + b1 \cdot Log(Number\ of\ Stocks_{i,t}) + b2 \cdot Herfindhal_{i,t} + b3 \cdot Coskewness_{i,t} + b4 \cdot Idio\_skewness_{i,t} + b5 \cdot Log(Size_{i,t}) + b6 \cdot BTM_{i,t} + \varepsilon_{i,t} \quad (3)$$

where the portfolio skewness of a portfolio i in year t is regressed by the natural log of number of stocks and the average of the Herfindahl index of segments, co-skewness, idiosyncratic skewness, the natural log of market value, and the natural log of book-to-market value acquired from the stocks in a portfolio. The estimation method is a Generalized Method of Moments (GMM) that corrects for heteroscedasticity and autocorrelation in error terms. The coefficient on co-skewness is multiplied by 10^3 for visual convenience. t-statistics are in parentheses and coefficients significant at the 1% level are marked with an asterisk (*).

regression. Also, since we vary the number of stocks in a non-linear way, we take the natural log of the number of stocks. The results are robust to log transformations in that we acquire the same results with non-transformed variables. The estimation method is a Generalized Method of Moments (GMM) that corrects for heteroscedasticity and autocorrelation in error terms.

Given that we have nearly 30,000 yearly observations of portfolio skewness, it is safe to assume that a *t*-stat less than three indicates insignificant explanatory power.[3] In Model 1, we use all the variables and in Model 2–4, we use subsets of variables. Model 1 and Model 2 have smaller number of observations because the Herfindahl index is only available from 1971, while our full sample begins from 1966. A model without the Herfindahl index yields similar results. In Model 5, we randomly select only 10% of observations to prevent large sample size from exaggerating statistical significance.

In Model 1, the log number of stocks has the most significant effect on portfolio skewness (*t*-stat: −28.94), trailed by the Herfindahl index of segments (*t*-stat: −9.34), book-to-market (*t*-stat: −7.01) and idiosyncratic skewness (*t*-stat: 6.52). Note that a higher Herfindahl index indicates less diversification, and so a negative coefficient means that more diversification is associated with positive portfolio skewness. Therefore, the coefficient on firm-level diversification is the opposite of the result in Table 2. Despite the significance of the *t*-statistics in Model 1 and Model 5, the sign of idiosyncratic skewness flips in Model 3, indicating that the effect is conditional upon other factors or that the effect is marginal. Book-to-market shows a similar flipping pattern, implying that this variable is not a good proxy of portfolio skewness. Coskewness has an insignificant coefficient, even when controlling for only idiosyncratic skewness in Model 3. In Model 5, where we only use 10% of observations, the t-statistics of size and book-to-market are not significant at the 1% level anymore. Overall, the results show that the number of stocks is the most important determinant of portfolio skewness while characteristics of individual stocks have marginal effects.

6. Conclusion

We analyze the return skewness of smaller portfolios using empirical returns from CRSP. We find that the variables that are traditionally rendered as a main determinant of portfolio skewness, such as coskewness or idiosyncratic skewness, have marginal or no correlation with the empirical portfolio skewness as compared to the number of stocks in a portfolio.

The relationship between diversification and portfolio risk profiles warrants further studies on this topic. Other than Albuquerque (2012), there are few theories that explain the contrast between positive individual stock skewness and negative portfolio skewness. Our findings indicate that the empirical mechanism behind portfolio skewness is more complex than assumed by many of the existing studies.

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[3] See Kim and Stoll (2014) for a discussion regarding the relationship between the number of observations and *t*-statistics.

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