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Minimum Fuel Circling Flight for Unmanned Aerial Vehicles in a Constant Wind

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This paper investigates characteristics of minimum-fuel trajectories for an Unmanned Aerial Vehicle (UAV) in high altitude, circling flight under a constant wind. Previous research has shown that periodic circling flight, consisting of a boost arc (maximum thrust) and a coast arc (minimum thrust), improves the fuel consumption when compared to steady-state circling. Since the periodic flight includes ascending flight at the boost arc and descending flight at the coast arc, it is naturally expected that the wind energy influences the trajectories. In this work, numerical simulations are used to investigate the effects of both wind speed and direction on a UAV flying around one loop enclosed in a cylindrical boundary area. The results show that the optimal wind direction manifests as a tail wind just at the coast arc. In addition, the results demonstrate that the optimal wind direction changes with the wind speed and, in some cases, the trajectory under high winds results in smaller fuel consumption than the zero wind case. Thus, the importance of these results is two fold. First, that the periodic flight reveals the existence of an optimal wind direction for the minimum fuel circling. Second, and probably more importantly, generating optimal trajectories without rejecting wind disturbances provides an autonomous capability of using wind to its advantage and therefore improving fuel consumption or perhaps other mission performance metrics.

Nomenclature

c	Thrust Specific Fuel Consumption, [kg/N/s]
C_L	Lift Coefficient
C_D	Drag Coefficient
C_{D0}	Zero-Lift Drag Coefficient
f_X, f_Y, f_Z	Force on Ground-Fixed Axis, [N]
κ	Induced Drag Coefficient Factor
m	Airplane Mass, [kg]
S	Wing Planform Area, [m^2]
u	Inertial Velocity, [m/s]
u_a	Airspeed, [m/s]
u_w	Wind Speed, [m/s]
x, y, h	Ground-Fixed Axis, [m]
γ	Flight Path Angle on Ground-Fixed Axis, [deg]
ρ	Density, [kg/m^3]
ψ	Flight Direction on Ground-Fixed Axis, [deg]
ψ_w	Relative Wind Direction to Circling Region [deg]

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I. Introduction

Until more efficient fuels and alternate propulsion systems are available for aircraft, both manned and unmanned, there will be a need for improved fuel management, or energy management, for both civil and defense applications. Since long-endurance flights require some form of fuel management, there is ongoing research in the area of minimizing fuel consumption. For example, recent combat operations have identified a significant gap in Intelligence, Surveillance, Reconnaissance, and Target Acquisition (ISR/TA) capability that has confirmed the need for real-time situational awareness throughout the battlespace in order to enhance timely decision making. This gap stems in part from a shortfall in long-endurance Unmanned Aircraft Systems (UAS) needed for persistent surveillance in support of combat operations and planning. To improve this capability, an obvious area of improvement is that of vehicle fuel management. For minimizing fuel use, an optimal steady-state flight is not always sufficient. To improve the fuel consumption, consideration must be given to a periodic flight that switches between maximum and minimum thrust levels.[1,2]

Typically, optimal fuel consumption flights are modeled as long range trajectories, but since UAV missions usually involve some form of circling flight in a prescribed area, such as loitering over a target, then circling trajectories should also be considered. Given the growing need for longer-endurance UAV missions, this is exactly the focus of this research work-circling flight with constant radius as if loitering over an area of interest. Recent research work has shown that periodic circling flight consisting of a boost arc (maximum thrust) and a coast arc (minimum thrust) improves the fuel consumption more than that of steady-state circling.[3–7] However, these works did not consider wind effects. Other work involving UAV periodic flight has addressed wind effects, but only for long-range flights.[8,9]

Considering the influence of the wind on the circling flight could potentially improve the fuel consumption. If permissible to vary the circling radius within a prescribed tolerance, the optimal controller attempts to reduce the fuel consumption while in level flight.[10] However, if the circling radius is constrained and since the relative wind direction rotates 360 degrees during the constant-radius circling, then the total amount of energy from the wind would be zero for the steady-state circling. On the other hand, there would be an optimal relative wind direction to improve the periodic flight even if the circling radius is constrained.[11] The reason for this is that the trajectory includes ascending flight at the boost arc and descending flight at the coast arc.

It is the purpose of this paper to analyze the influence of a constant wind during periodic circling flight and ultimately to determine the optimal relative wind direction for reducing fuel consumption. To do so, an optimal control problem is formulated and solved using a pseudospectral-based method. The numerical results of both a calm wind and a strong wind are compared.

II. Problem Formulation

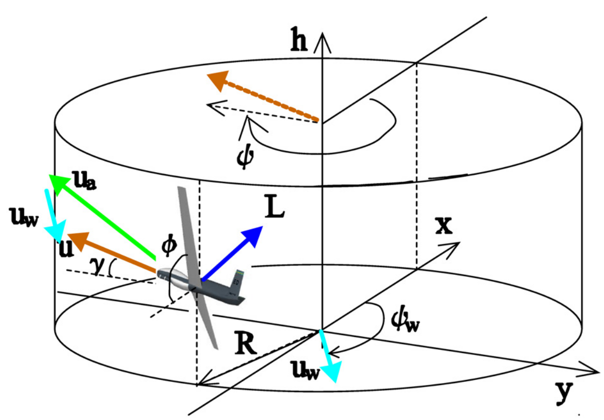

Figure 1. Coordinate System and Reference Frames of the UAV in Circling

The point-mass equations of motion for a UAV in circling flight with respect to Fig.1 are written below as Eq.(1)-(5).The inertial reference frame is defined by a vertical plane over a flat Earth with the coordinate system fixed to the ground. Only a wind speed component parallel to the ground surface u_w is considered in this paper. The inertial velocity u is with respect to the ground-fixed axes and airspeed u_a to the air axes. The lift force L and the drag force D are transposed to the ground-fixed axes.

$$\begin{bmatrix} m\dot{u} \\ mu\dot{\gamma} \\ mu\cos\gamma\dot{\psi} \end{bmatrix} = \begin{bmatrix} T - mg\sin\gamma \\ -mg\cos\gamma \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\gamma\cos\psi & \cos\gamma\sin\psi & \sin\gamma \\ -\sin\gamma\cos\psi & -\sin\gamma\sin\psi & \cos\gamma \\ -\sin\psi & \cos\psi & 0 \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \tag{1}$$

$$\dot{x} = u\cos\gamma\cos\psi \tag{2}$$
$$\dot{y} = u\cos\gamma\sin\psi \tag{3}$$
$$\dot{h} = u\sin\gamma \tag{4}$$

$$\dot{m} = -cT \tag{5}$$

Here

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \frac{1}{2}\rho u_a^2 \left\{ C_L \begin{bmatrix} -\sin\phi\sin\zeta - \cos\phi\sin\xi\cos\zeta \\ \sin\phi\cos\zeta - \cos\phi\sin\xi\sin\zeta \\ \cos\phi\cos\xi \end{bmatrix} + C_D \begin{bmatrix} -\cos\xi\cos\zeta \\ -\cos\xi\sin\zeta \\ -\sin\xi \end{bmatrix} \right\} \tag{6}$$

$$C_D = C_{D0} + \kappa C_L^2 \tag{7}$$

$$\begin{aligned} u_{ax} &= u\cos\gamma\cos\psi - u_w\cos\psi_w \\ u_{ay} &= u\cos\gamma\sin\psi - u_w\sin\psi_w \\ u_{az} &= u\sin\gamma \\ u_{axy} &= \sqrt{u_{ax}^2 + u_{ay}^2} \\ u_a &= \sqrt{u_{ax}^2 + u_{ay}^2 + u_{az}^2} \\ \sin\xi &= u_{az}/u_a \\ \cos\xi &= u_{axy}/u_a \\ \sin\zeta &= u_{ay}/u_{axy} \\ \cos\zeta &= u_{ax}/u_{axy}. \end{aligned} \tag{8}$$

The circling flight with constant radius R must satisfy Eq.(9) at any time and the periodic circling must satisfy Eq.(10)-(12).

$$(-f_x\sin\psi + f_y\cos\psi) - m\frac{u^2\cos\gamma^2}{R} = 0 \tag{9}$$

$$u(0) = u(t_f) \tag{10}$$
$$\gamma(0) = \gamma(t_f) \tag{11}$$
$$h(0) = h(t_f) \tag{12}$$

From these assumptions, the optimal control problem to minimize the fuel consumption is stated as follows, Eq.(16).

$$\boldsymbol{X} = \begin{bmatrix} u(t) & \gamma(t) & \psi(t) & h(t) & m(t) \end{bmatrix}^T \in \mathbb{X} \subseteq \mathbb{R}^5 \tag{13}$$
$$\boldsymbol{U} = \begin{bmatrix} T(t) & C_L(t) & \phi(t) \end{bmatrix}^T \in \mathbb{U} \subseteq \mathbb{R}^3 \tag{14}$$
$$\boldsymbol{P} = \begin{bmatrix} \psi_w \end{bmatrix} \in \mathbb{P} \subseteq \mathbb{R} \tag{15}$$

Minimize :

$$J = \frac{1}{t_f}\int_0^{t_f} cT(t)dt \tag{16}$$

Subject to :

$$Eqs.(1),(4)-(12) \tag{17}$$

$$m(0) = m_0 \tag{18}$$
$$\psi(0) = 0 \tag{19}$$
$$\psi(t_f) = \psi_f \tag{20}$$

Now with the problem posed as a standard optimal control formulation, it is readily solvable employing a nonlinear optimization tool.

III. Numerical Results

The circling trajectory is partitioned into a boost arc (maximum thrust) and a coast arc (minimum thrust) with a time-axis folding method[2] and the periodic frequency for flight around one loop[5] is $f_p = 1$[Hz] as indicated in Fig.2. The data for the numerical simulations uses that of the Global Hawk RQ-4B, where $m_0 = 9100$[kg], $T_{max} = 37000$[N] at sea level, $C_{D0} = 0.017$, $\kappa = 0.016$ and $c = 1.8123 \times 10^{-5}$[kg/N/s]. The initial altitude is constrained at $h_0 = 17500$[m] and the circling radius at $R = 10$[km]. The maximum thrust would be 4000[N] at altitude h_0. The optimal control problem is solved by a modified method based on a Jacobi pseudospectral collocation technique.[12]

Numerical optimization is performed to find the optimal wind direction ψ_w with respect to each given constant wind speed($u_w = 0 \sim 20$[m/s]). For the purpose of preliminary analysis, wind shear is omitted in this work and only a constant wind speed is implemented in the operating region. To compare each case, the UAV ' s initial flight direction is aligned with the north heading($\psi(t_0) = 0$[deg]). As shown in Fig.3, the UAV immediately starts its boost arc from $\psi = 0$[deg], the initial alignment direction, and then transitions from the boost arc to the coast arc around $\psi = 270$[deg] for the typical optimal trajectory.

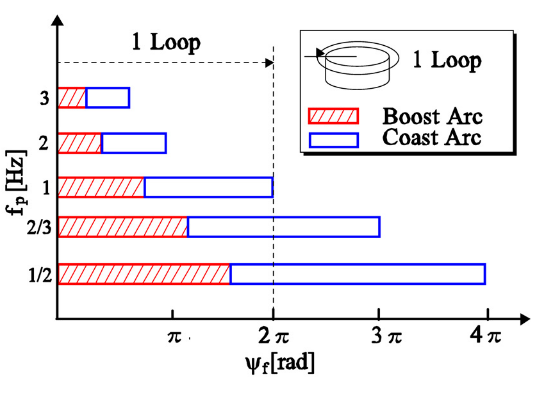

Figure 2. Periodic Circling Frequency f_p [Hz]

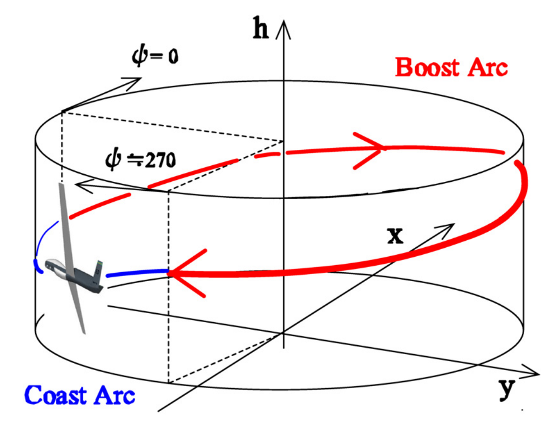

Figure 3. Outline of Boost Arc and Coast Arc Configuration for Optimal Trajectory

To compare the steady-state circling and the periodic circling, the fuel use with respect to the relative wind direction is shown in Figs.4 and 5, where the wind direction is specified during the numerical iteration in this case. It is clear that the fuel use has the minimum value for the periodic circling rather than the steady-state circling. As indicated in Fig.5, the optimal wind direction that provides the minimum fuel use occurs at around $\psi_w = 310$[deg]. The wind direction from $\psi_w = 270$[deg] to $\psi_w = 360$[deg] essentially acts as a tail wind on the UAV as it transitions from the boost arc to the coast arc.

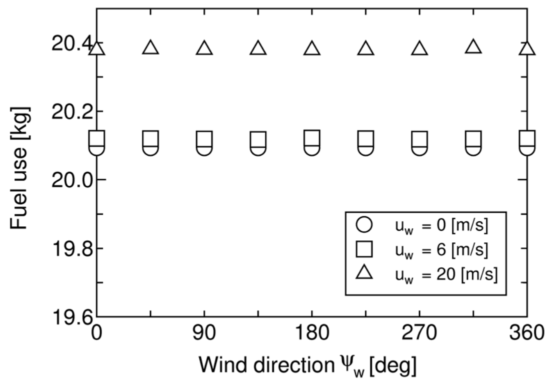

Figure 4. Fuel use with respect to wind direction for steady-state circling

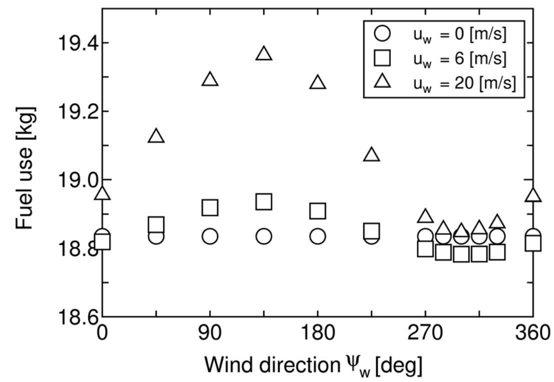

Figure 5. Fuel use with respect to wind direction for periodic circling

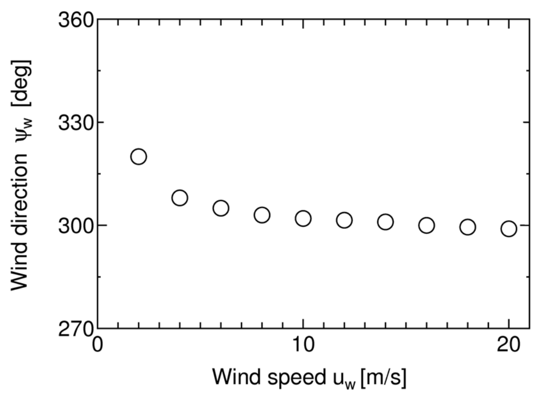

Figure 6. Optimal wind direction with respect to wind speed

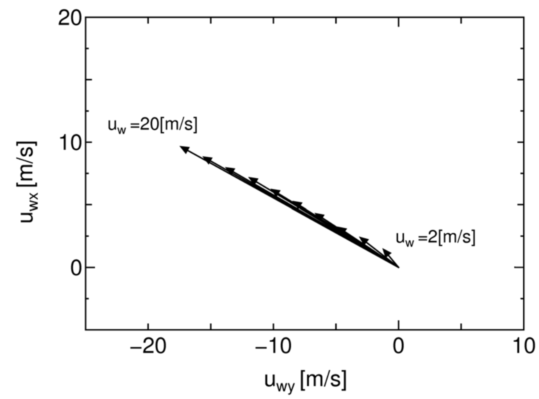

Figure 7. Optimal wind vector profile for periodic circling

In Fig.6, the numerically obtained optimal wind direction with respect to the wind speed shows that the optimal wind direction varies with the wind speed. The corresponding optimal wind vector profile is shown in Fig.7. It is evident that to align the wind direction in the circling area with the optimal wind direction, it is required to manage the relative direction of the periodic circling trajectory based on the wind speed.

The fuel use rate with respect to the wind speed is shown in Fig.8. Note that the fuel use rate is normalized by the zero wind case. As shown, from a wind speed of approximately 1 to 19[m/s], the fuel use rate is lower than the zero wind case. To help analyze this phenomenon, the time required for the boost arc with respect to the wind speed is shown in Fig.9. By comparing this plot with Fig.8, it is clear that this lower fuel consumption rate is caused by the decrease in boost arc time which is ultimately a consequence of the UAV using the wind to its advantage.

Figures 10 to 13 show the inertial speed, altitude, airspeed and bank angle, respectively, with respect to flight direction for the three different wind speeds, $u_w = 0, 6, 20$[m/s]. The resulting optimal trajectory is circling at a relatively constant airspeed through the middle of the arc; therefore, inertial speed varies with the wind vector variation during the circling. This speed variation is coordinated by the bank angle ϕ. Additionally, the altitude fluctuates from approximately 17 to 18.25 [km] for this analysis, but there would probably be more substantial changes if wind shear with respect to altitude is considered.

Figures 14 to 16 show the circling trajectories with the wind speed and direction overlaid (i.e. wind vector). The plots also show exaggerated UAV symbols at time intervals of 30[s]. From these plots, it is evident that the optimal wind direction coincides with the middle point of the coast arc. That is, the optimal wind direction is essentially a tail wind that adds energy to the vehicle at the coast phase.

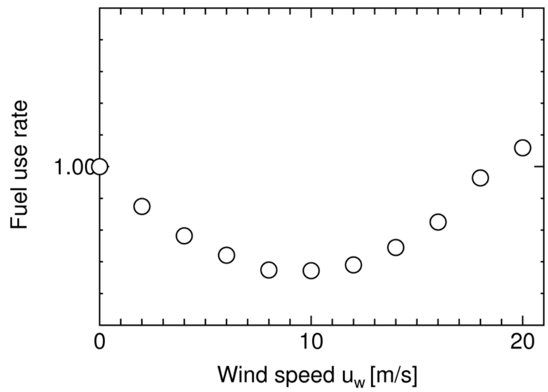

Figure 8. Fuel use rate with respect to wind speed

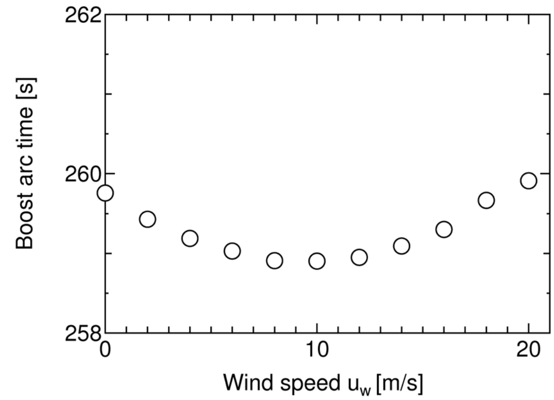

Figure 9. Boost arc time with respect to wind speed

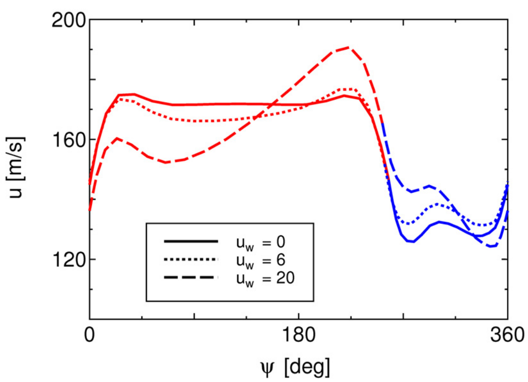

Figure 10. Inertial speed with respect to flight direction

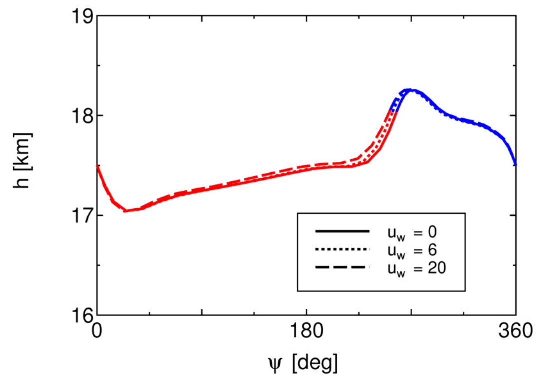

Figure 11. Altitude with respect to flight direction

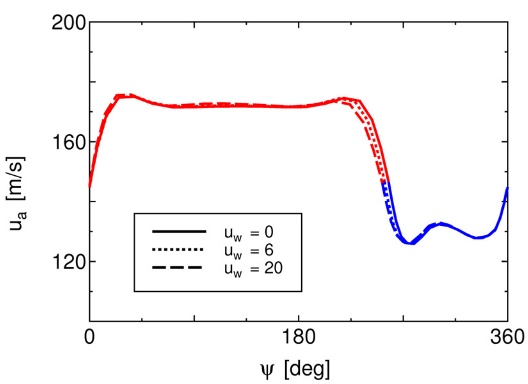

Figure 12. Airspeed with respect to flight direction

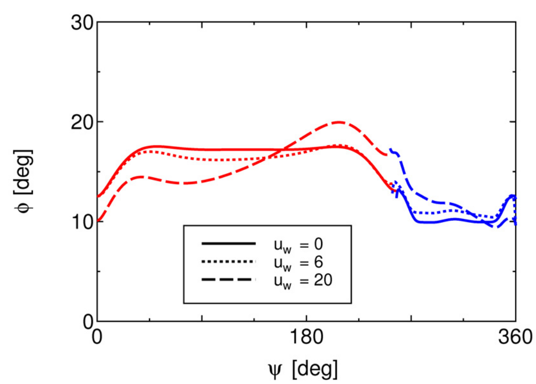

Figure 13. Bank angle with respect to flight direction

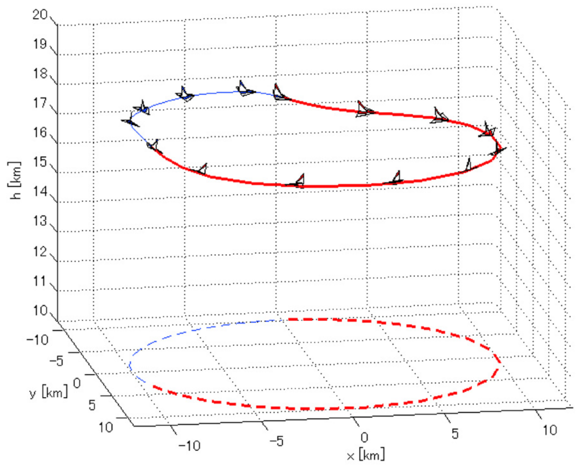

Figure 14. Trajectory of the Circling Flight $u_w = 0.0$**[m/s]**

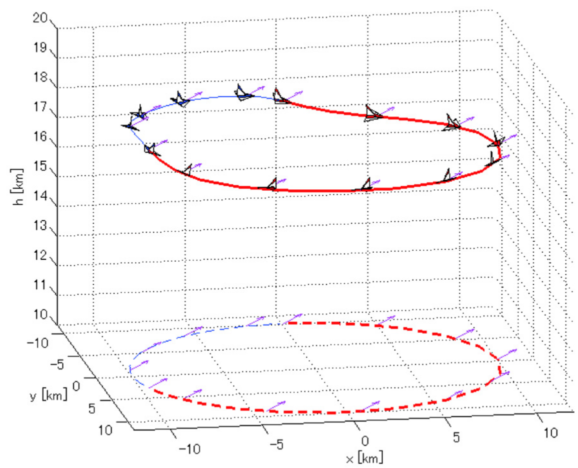

Figure 15. Trajectory of the Circling Flight $u_w = 6.0$**[m/s]**

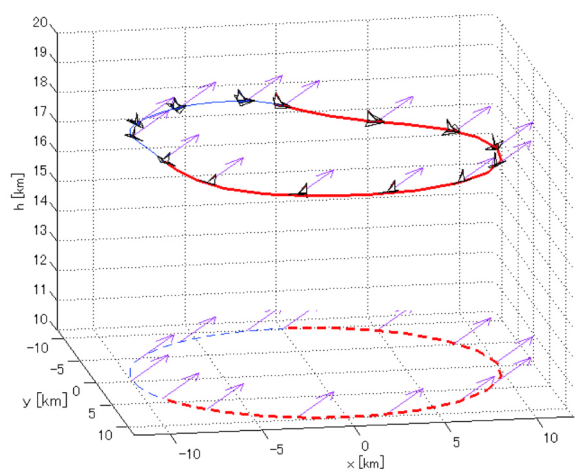

Figure 16. Trajectory of the Circling Flight $u_w = 20.0$**[m/s]**

IV. Conclusions

The optimal periodic circling for a UAV in a constant wind has been analyzed by numerical simulations. Preliminary results demonstrate that the optimal wind direction changes with the wind speed and in some cases yields smaller fuel consumption compared to a zero wind case. The conclusion of this investigation is that there exists an optimal wind direction for the minimum fuel circling under periodic flight. In addition, these results illustrate the power and relative simplicity of using optimal control techniques, such as the pseudospectral-based method employed in this work, to help investigate how to improve the operation and flight characteristics of aerial vehicles under the influence of wind effects. Overall, the approach used in this work for optimizing fuel utilization has proven to be a viable technique for applications requiring long-endurance flights.

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