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Portfolio Optimization Analysis with Markowitz Quadratic Mean-Variance Model

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Abstract
In this study, Markowitz mean-variance approach is tested on Istanbul Stock Exchange (BIST). 252 days of data belonging a year of 2015 are analyzed. First, a hypothetical portfolio is created. It involves ten securities with equal weights. They are chosen from three different industries to minimize risk of portfolio. However, the number of securities is not adequate for a well diversified portfolio alone. Markowitz model takes into account a relation between return on financial assets investing in portfolio. In empiricial analysis, I followed mean-variance model and created many portfolios. The model adjusted them as a minimum variance for a given expected return. Investors choose any of them as their risk preferences. Because they are all efficient. My optimal portfolio is constructed by eight assets with different weights. It provides more return comparing with a portfolio with equal shares of ten stocks.
Keywords: Markowitz, mean-variance approach, modern portfolio theory, efficient frontier

1. Introduction

Risk and the expected return are the main parameters of any kind of investment. After emergence of capital markets, individuals have a choice to earn money from different investment area. Investors are generally accepted as risk averse. They are trying to minimize risk while maximizing their return as well. In that sense, investment strategies of individuals heavily depend on how much risk they can take to achieve expected return.

Risk is measured by deviation from expected return and when the investor takes more risk, its profit or loss is larger as well. To avoid risk, diversification of financial assets is the main investment strategy until Modern Portfolio Theory of Harry Markowitz. Investors try to select securities with small variance and to add number of securities in their portfolios. However, this kind of diversification ignores a relation between return on financial assets. Modern Portfolio Theory asserts that an individual asset risk itself may be different when it is a part of a portfolio. This model takes into account co-movement of stocks. Portfolio risk is measured by covariance between return on financial assets. In that sense optimal portfolio is selected by minimizing variance at given expected return or maximizing expected return at given variance. This minimum variance values of given expected return constructs an efficient set and individuals can choose any of optimal portfolios as their risk preferences.

The aim of this paper is briefly to test mean-variance model on Istanbul Stock Exchange (BIST) and to select optimal portfolios. The rest of the paper is organized as follows. In section 2, Modern Portfolio Theory is briefly introduced. In section 3, mean – variance approach of Markowitz is used in empirical analysis to choose optimal portfolios from efficient frontier. Many statistical techniques are used in this section. Finally, section 4 concludes the paper.

2. Modern Portfolio Theory

Which portfolio is the best? This question has arisen since from the beginning of stock-market itself (Marling and Emanuelsson, 2012: 2). All investors have a difficulty to choose best assets among many alternatives. This problem is subject to portfolio theory (Elton and Gruber, 1997: 1743). However, Markowitz transformed portfolio selection process as a mathematical optimization problem (Marling and Emanuelsson, 2012: 2).

Markowitz (1952, 1959) can be accepted as a father of Modern Portfolio Theory. He solved portfolio optimization problem as taking into account the mean and variance of assets. Markowitz formulated theory as holding constant variance, maximize expected return, and holding constant expected return minimize variance. This two principles formulation give an efficient frontier which contains all possible optimal portfolio under given expected return and variance combinations. Then the investors make a choice according to his preference depending on risk behavior of himself. Markowitz dwelled on portfolio risks rather than individual assets' risk. In that sense, stocks could be selected on how much contribute the portfolio risk rather than individual risk value itself. These interactions between the returns on financial assets are key parameters to create a well diversified portfolio (Elton and Gruber, 1997: 1746).

Mean-variance model proposes not only diversification but also the "right kind" of diversification for the "right reason". Adding many more stocks are not enough for adequate diversification. For example, a portfolio with same industries' stocks would not be as well diversified as the same size portfolio with different industries' stocks. Similarly to find small variance stocks is not enough to construct diversified portfolio. It is crucial to avoid investing in securities with high covariances among themselves. Generally, companies in different industries have smaller covariances than firms within same industry (Markowitz, 1952, 89). In that sense, Markowitz advanced portfolio diversification a step forward to take into account not only number of assets but also their covariance relation (Mangram, 2012: 60). Because of the correlation between stocks, diversification reduces a risk of portfolio but not to eliminate it entirely. Markowitz was the first who considered the covariances between return on financial assets (Müller, 1988: 128). Modern Portfolio Theory contains many assumptions about markets and investors. Some of them are as such(Mangram, 2012: 61):

1) Investors are rational which means that they seek to maximize returns while minimizing risk,
2) Investors are risk averse which means that they desire to accept higher amounts of risk only if they are compensated by higher expected returns,
3) Investors reach all investment related information timely,
4) Investors are able to borrow or lend an unlimited amount of capital at a risk free rate of interest,
5) Investors have a one single period investment horizon,
6) There is no transaction costs or taxes in the market,
7) Markets are perfectly efficient

Consequently, Markowitz optimization methodology calculates mean-variance efficient portfolios. "It is based on mean-variance analysis, where the variance of the overall rate of return is taken as a risk measure and the expected value measures profitability" (Müller, 1988: 128).

3. Empirical Analysis

3.1. Data and Formulas

A hypothetical portfolio is created at first. Ten companies are selected arbitrarily from three different industries to create a diversified portfolio. They have traded in Istanbul Stock Exchange (BIST). They are operating in a petroleum, pharmaceutical and banking industry.

Table 1. Name and Operations of Companies

	Stocks	Company Name	Industry	Sector
1	LKMNH	Lokman Hekim A.Ş.	Pharmaceutical	Hospital
2	RTALB	RTA Laboratuvarları A.Ş.	Pharmaceutical	Laboratory
3	ATPET	Atlantik Petrol Ürünleri A.Ş.	Petroleum	Industrial oil
4	TRCAS	Turcas Petrol A.Ş.	Petroleum	Petroleum and electricity
5	MEPET	Mepet Petrol A.Ş.	Petroleum	Service station
6	PETKM	Petkim PetroKimya Holding A.Ş.	Petroleum	Petrochemistry
7	ISCTR	İş Bankası A.Ş.	Banking	Deposit and credit
8	KLNMA	Kalkınma Bankası A.Ş.	Banking	Deposit and credit
9	SKBNK	Şekerbank A.Ş.	Banking	Deposit and credit
10	VAKBN	Vakifbank A.Ş.	Banking	Deposit and credit

In this study, market return of companies are estimated by using daily returns (adjusted price for US dollar). They are achieved from the Isyatirim database[1]. One year period (252 working days) data is used. It belongs to a year of 2015.

To calculate for stocks daily return; the formula is applied as follows:

$$R_i = \frac{R_{it} - R_{it-1}}{R_{it-1}} \qquad (1)$$

where "R_i" is a daily return of stock i, "R_{it}" is a closing price of stock i in t date and "R_{it-1}" is a closing price of stock i in t - 1 date

To calculate average return of stocks; the formula is applied as follows:

$$E(Ri) = \frac{1}{N} \sum_{t=1}^{N} Rit \qquad (2)$$

Where "E (R_i)" is a average return for stock i, "R_{it}" is a market return in t date, "N" is a number of dates.

The general formulas of expected return for n assets is as belowed:

$$E(r_P) = \sum_{i=1}^{n} w_i E(r_i) \qquad (3)$$

Where "Σ w_i = 1", "n" is the number of stocks, "w_i" is the proportion of the funds invested in stock *i*, "r_i, r_p" is the return on *i*th stock and portfolio *p*, and "E (R_p)"the expectation of the variable in the parentheses

To calculate variance of stocks daily return and index return, following historical volatility formula is used:

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Ri - Raverage)^2 \qquad (4)$$

[1] http://www.isyatirim.com.tr/LT_isadata2.aspx. (22.02.2016)

Where "σ^2" is a variance of daily stock return, "R_i" is a daily return of stock i, "$R_{average}$" is average daily return, "n" is a sample size (252 days)
To measure how stocks vary together, standard formula for covariance can be used:

$$\text{Cov (X,Y)} = \frac{1}{n-1}\sum_{i=1}^{n}[(X_i - \bar{X}).(Y_i - \bar{Y})] \quad (5)$$

where the sum of the distance of each value X and Y from the mean is divided by the number of observations minus one. The covariance calculation enables us to calculate the correlation coefficient, shown as:

$$\text{Correlation Coefficient} = \frac{Cov(X,Y)}{\sigma_X.\sigma_Y} \quad (6)$$

where σ the standard deviation of each asset. However, if there are more than two financial assets in the portfolio, then correlation and covariance matrices are needed to solve equations.
To calculate standard deviation of portfolio (position), the following formula is applied:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(w_i^2.\sigma_i^2) + 2(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i.\sigma_i.w_j.\sigma_J.\rho ij)} \quad (7)$$

Where "σ_p" is a standard deviation of portfolio, "σ_i" is a standard deviation of stocks, "w_i" is a weight of stocks in a portfolio and "ρ_{ij}" is a correlation coefficient between stocks i and j.

3.2. Empirical Results

In this study, excel functions and data solver are used for all calculation. The calculation of mean –variance optimal portfolio involves the following steps:
Step 1 – Determining return and standard deviation of stocks (table 2)
Step 2 – Creating correlations matrice (table 3)
Step 3 – Creating covariance matrice (table 4)
Step 4 – Creating variance - covariance matrices with equal weights (table 5)
Step 5 – Calculating the volatility and return of the portfolio (table 6)
Step 6 – Creating variance - covariance matrices with different weights (table 7)
Step 7 – Calculating the volatility and return of the new portfolio (table 8)
Step 8 – Creating efficient frontier (table 9 and figure 1)

Table 2. Risk and return of stocks (2015)

Stocks	Average Return (daily)	Average Return (annualy)	Standard Deviation (daily)	Standard Deviation (annualy)	Variance (daily)	Variance (annualy)
LKMNH	0,001295	0,32634	0,023909	0,379538	0,000572	0,009074
RTALB	0,000389	0,098028	0,029844	0,473757	0,000891	0,014139
ATPET	0,006905	1,74006	0,053654	0,85173	0,002879	0,045699
TRCAS	0,003184	0,802368	0,02569	0,407813	0,00066	0,010477
MEPET	0,006855	1,72746	0,060377	0,958456	0,003645	0,057869
PETKM	0,000481	0,121212	0,01948	0,30924	0,000379	0,006024
ISCTR	0,002517	0,634284	0,025382	0,402922	0,000644	0,010227
KLNMA	-0,00029	-0,07308	0,044999	0,714342	0,002025	0,032145
SKBNK	0,001527	0,384804	0,023822	0,378169	0,000568	0,009009
VAKBN	0,002315	0,58338	0,027858	0,44223	0,000776	0,01232

We know from the modern portfolio theory that volatility of portfolio is less than sum of individual's because of correlation between assets. The relation between assets determines the degree of portfolio risk. When there is negative (small) correlation between assets, risk of portfolio is smaller and when there is positive (big) correlation between assets, risk of portfolio is greater. The degree of correlation coefficients value takes + 1 and – 1.

Table 3. Correlation matrice

	LKMNH	RTALB	ATPET	TRCAS	MEPET	PETKM	ISCTR	KLNMA	SKBNK	VAKBN
LKMNH	1,000									
RTALB	0,401	1,000								
ATPET	0,196	0,130	1,000							
TRCAS	0,525	0,425	0,190	1,000						
MEPET	0,342	0,271	0,074	0,372	1,000					
PETKM	0,520	0,385	0,137	0,535	0,368	1,000				
ISCTR	0,615	0,488	0,215	0,649	0,412	0,717	1,000			
KLNMA	0,192	0,077	0,041	0,162	0,102	0,173	0,142	1,000		
SKBNK	0,466	0,371	0,096	0,549	0,388	0,443	0,634	-0,026	1,000	
VAKBN	0,630	0,471	0,189	0,639	0,442	0,731	0,897	0,143	0,620	1,000

As seen from Table 3, all correlations except SKBNK-KLNMA between stocks are positive and not low enough. Nearly all of them are greater than zero. In that sense, the portfolio is not well diversified. Covariance measures similarity of assets. It shows us that how stocks move together. As seen from Table 4, securities move together in certain degree except SKBNK-KLNMA.

Table 4. Covariation matrice

	LKMNH	RTALB	ATPET	TRCAS	MEPET	PETKM	ISCTR	KLNMA	SKBNK	VABN
LKMNH	0,00057	0,00028	0,00025	0,00032	0,00049	0,00024	0,00037	0,00021	0,00026	0,00042
RTALB	0,00028	0,00089	0,00021	0,00032	0,00049	0,00022	0,00037	0,00010	0,00026	0,00039
ATPET	0,00025	0,00021	0,00287	0,00026	0,00024	0,00014	0,00029	0,00010	0,00012	0,00028
TRCAS	0,00032	0,00032	0,00026	0,00066	0,00027	0,00027	0,00027	0,00027	0,00027	0,00027
MEPET	0,00049	0,00049	0,00024	0,00058	0,00363	0,00043	0,00063	0,00028	0,00056	0,00074
PETKM	0,00024	0,00022	0,00014	0,00027	0,00043	0,00038	0,00035	0,00015	0,00020	0,00040
ISCTR	0,00037	0,00037	0,00029	0,00042	0,00063	0,00035	0,00064	0,00016	0,00038	0,00063
KLNMA	0,00021	0,00010	0,00010	0,00019	0,00028	0,00015	0,00016	0,00202	-0,00003	0,00018
SKBNK	0,00026	0,00026	0,00012	0,00033	0,00056	0,00020	0,00038	-0,00003	0,00057	0,00041
VAKBN	0,00042	0,00039	0,00028	0,00046	0,00074	0,00040	0,00063	0,00018	0,00041	0,00077

It is very difficult to calculate volatility of portfolio which gets more than two assets for using formula 7. Variance-covariance matrice is a very simple way of measuring risk of portfolio. It is done by multiplied each assets' weights and covariance values for each cell.

For example, in Table 5, value of C3 cell is achieved by (0,1) * 0,00057 * (0,1). Value of C13 cell shows risk contribution of LKMNH stock to the portfolio. Volatility (risk) of portfolio is achieved by sum of each stocks' risk. F15 cell shows volatility of portfolio (=sum (C13: L13)).

As seen from the Table 5, our portfolio is created by ten stocks with equal weights (10 %). Sum of weights of stocks must be equal one.

Table 5. Variance-covariation matrice with equal weight

A	B	C	D	E	F	G	H	I	J	K	L
1		LKMNH	RTALB	ATPET	TRCAS	MEPET	PETKM	ISCTR	KLNMA	SKBNK	VABN
2	w	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
3	0,1	5,69E-06	1,62E-08	7,129E-09	8,03E-09	1,58E-08	1,18E-08	8,96E-09	7,65E-09	5,44E-09	1,1E-08
4	0,1	2,85E-06	2,53E-08	1,840E-08	6,74E-09	1,58E-08	1,08E-08	8,21E-09	3,78E-09	2,69E-09	1,02E-08
5	0,1	2,5E-06	5,19E-09	5,947E-08	7,47E-08	6,2E-09	3,41E-09	4,17E-09	2,88E-09	1,21E-09	3,44E-09
6	0,1	3,21E-06	1,04E-08	8,463E-09	1,71E-08	1,75E-08	7,12E-09	7,12E-09	7,12E-09	7,12E-09	7,12E-09
7	0,1	4,91E-06	2,39E-08	1,158E-08	1,37E-08	2,09E-07	1,56E-07	2,71E-08	1,73E-08	1,54E-08	4,12E-08
8	0,1	2,41E-06	5,37E-09	3,187E-09	3,82E-09	1,15E-08	1,63E-08	1,34E-08	5,32E-09	3,09E-09	8,09E-09
9	0,1	3,72E-06	1,37E-08	1,072E-08	1,23E-08	2,65E-08	2,22E-08	2,27E-08	1,03E-08	6,15E-09	2,41E-08
10	0,1	2,06E-06	2,11E-09	1,013E-09	1,85E-09	5,16E-09	4,16E-09	2,43E-09	3,25E-08	-5,7E-09	-5E-10
11	0,1	2,64E-06	6,94E-09	3,20E-09	4,09E-09	1,86E-08	1,14E-08	7,81E-09	-1,1E-09	-1,6E-09	2,32E-08
12	0,1	4,18E-06	1,63E-08	1,097E-08	1,28E-08	3,37E-08	2,93E-08	2,5E-08	1,12E-08	7,3E-09	3,17E-08
13	1	3,42E-05	1,25E-07	1,34172E-	1,55E-07	3,6E-07	2,73E-07	1,27E-07	9,71E-08	4,11E-08	1,6E-07
14			Daily								
15	Variance of portfolio = 0,000410672										
16	Standard deviation of portfolio = 0,0202650										
17	Return of portfolio = 0,002518										

The Mean-Variance model of Harry Markowitz (1952) asserts that investors are risk-averse and efficient portfolios must be satisfied at least two conditions as "*(1) minimize the variance of portfolio return, given the expected return, and (2) maximize the expected return, given the variance.*" (Miao, 2013: 6-7). In that sense, minimizing the risk or variance of the portfolio is one of the main aims of optimal portfolio analysis. This subject function of Markowitz mean – variance portfolio can be showed as belowed:

$$\text{Min } \sigma^2 \sum_{i=1}^{n} \sum_{j=1}^{n} WiWjCovij \qquad (8)$$

Where "W_i" and "W_j" are weights of stocks in the portfolio and "Cov_{ij}" is covariance value between stocks i and j.

There are mainly two constraints in Markowitz standard quadratic convex optimization problem. They are written as

$$\sum_{i=1}^{n} Wi \;\; E\,(Ri) \geq E^{*} \qquad (9)$$

where E^* is the target expected return, E (R_i) is an expected return and

$$\sum_{i=1}^{n} Wi = 1.0 \tag{10}$$

The portfolio manager determines a target return which must be equal to the expected return. The first constraint above (formula 9) provides this condition. The second constraint asserts that the weights of the stocks invested in the portfolio must sum to one (formula 10). A third constraint can be added if short sale restriction sets in variance problem (formula 11).

$$W_i \geq 0, \qquad i = 1, \ldots\ldots\ldots\ldots, N \tag{11}$$

If short sale is allowed, it is possible to sell these shares without owning it. In that sense, weight column of shares can take negative sign if optimal portfolio captures short sale shares. However, in all circumstances, sum of shares invested in portfolio must be 1.

Table 6. Parameters of Excel Solver

Target cell	F 15 (variance of portfolio)
Equal to	Minimum (variance of portfolio)
By changing cells	\$C\$2 : \$L\$2
Constraints	\$C\$2 : \$L\$2 ≥ 0 (short sale restriction)
	\$B\$13 = 1
	\$F\$17 = 0,027 (expected return)

In this study, all these three constraints are identified by an excel data solver. Optimal portfolio is achieved by following Markowitz mean – variance model steps. Solutionss are seen from Table 7 and Table 8.

Table 7. Variance- covariation matrice without short sales

A	B	C	D	E	F	G	H	I	J	K	L
1		LKMNH	RTALB	ATPET	TRCAS	MEPET	PETKM	ISCTR	KLNMA	SKBNK	VAB
2	W	0,053	0	0,143	0,275	0,044	0,196	0,021	0,048	0,221	0
3	0,053	1,59E-06	0	1,89E-06	4,62E-06	1,12E-06	2,51E-06	4,03E-07	5,18E-07	3,09E-06	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0,143	1,89E-06	0	5,84E-05	1,02E-05	1,48E-06	4,01E-06	8,54E-07	6,70E-07	3,84E-06	0
6	0,275	4,67E-06	0	1,024E-05	4,98E-05	3,20E-06	1,44E-05	1,51E-06	3,49E-06	1,62E-05	0
7	0,044	1,13E-06	0	1,483E-06	6,92E-06	6,92E-06	3,69E-06	5,64E-07	5,72E-07	5,36E-06	0
8	0,196	2,50E-06	0	4,010E-06	1,44E-05	3,69E-06	1,45E-05	1,42E-06	1,40E-06	8,881E-06	0
9	0,021	4,04E-07	0	8,542E-07	2,38E-06	5,64E-07	1,42E-06	2,71E-07	1,57E-07	1,73E-06	0
10	0,048	5,17E-07	0	6,70E-07	2,44E-06	5,72E-07	1,40E-06	1,57E-07	4,55E-06	-2,96E-07	0
11	0,221	3,08E-06	0	3,849E-06	2,03E-05	5,36E-06	8,88E-06	1,73E-06	-2,98E-07	2,75E-05	0
12	0	0	0	0	0	0	0	0	0	0	0
13	1										
14			Daily								
15	Variance of portfolio = 0,000366829										
16	Standard deviation of portfolio = 0,0191528										
17	Return of portfolio = 0,0027										

For different choices of stocks, the investor will get different combinations of expected return and risk. The set of all possible expected return and risk combinations is called the attainable set. Those risk and expected return with minimum variance for a given expected return or more and maximum expected return for a given variance or less are called the efficient set (or efficient frontier). Because an investor wants a high profit and a small risk, he wants to maximize expected return and minimize variance and therefore he should choose a portfolio which gives an expected return - risk combination in the efficient set (Marling and Emanuelsson, 2012: 2). While original portfolio contains ten stock with equal weights, under the three conditions shown in Table 7, new optimal portfolio contains just eight stocks with different weights. This new portfolio involves % 5,3 LKMNH, % 14,3 ATPET, % 27,5 TRCAS, % 4,4 MEPET, %19,9 PETKM, %2 ISCTR, %4,8 KLNMA and %22,1 SKBNK stocks. It is possible to create many portfolio under the different target expected return. All these portfolios are efficient and are located over the efficient frontier. I created 11 portfolios a range between 0,00001 and 0,006 expected return.

Table 8. Weight and risk of different expected return portfolios

			LKMNH	RTALB	ATPET	TRCAS	MEPET
	standard deviation	return	Weights of stocks in a portfolio				
Portfolio 1	0,028747169	0,00001	0,00000	0,00000	0,00000	0,00000	0,00000
Portfolio 2	0,017401218	0,0005	0,00000	0,14812	0,00000	0,00000	0,00000
Portfolio 3	0,016682763	0,001	0,09221	0,08072	0,04206	0,00000	0,00000
Portfolio 4	0,016910244	0,0015	0,08725	0,04695	0,07983	0,08823	0,00000
Portfolio 5	0,017625047	0,002	0,07747	0,01638	0,11004	0,17845	0,00936
Portfolio 6	0,018663879	0,0025	0,06394	0,00000	0,13456	0,24982	0,03502
Portfolio 7	0,0191528	0,002700	0,05291	0,00000	0,14274	0,27541	0,04366
Portfolio 8	0,019946118	0,003000	0,03648	0,00000	0,15458	0,31207	0,05604
Portfolio 9	0,023038145	0,004000	0,00000	0,00000	0,19951	0,43011	0,10377
Portfolio 10	0,027356428	0,005000	0,00000	0,00000	0,29370	0,48634	0,20052
Portfolio 11	0,033964434	0,006000	0,00000	0,00000	0,43739	0,23886	0,32374

Table 9. Weight and risk of different expected return portfolios (continue)

			PETKM	ISCTR	KLNMA	SKBNK	VAKBN
	standard deviation	return	Weights of stocks in a portfolio				
Portfolio 1	0,028747169	0,00001	0,00000	0,00000	0,00000	0,00000	0,00000
Portfolio 2	0,017401218	0,0005	0,59839	0,00000	0,12790	0,12559	0,00000
Portfolio 3	0,016682763	0,001	0,44863	0,00000	0,09389	0,24249	0,00000
Portfolio 4	0,016910244	0,0015	0,37016	0,00000	0,07792	0,24967	0,00000
Portfolio 5	0,017625047	0,002	0,29815	0,00000	0,06372	0,24643	0,00000
Portfolio 6	0,018663879	0,0025	0,23156	0,00000	0,05180	0,23330	0,00000
Portfolio 7	0,0191528	0,002700	0,19632	0,02056	0,04751	0,22088	0,00000
Portfolio 8	0,019946118	0,003000	0,14106	0,05741	0,04116	0,20121	0,00000
Portfolio 9	0,023038145	0,004000	0,00000	0,14992	0,00768	0,10901	0,00000
Portfolio 10	0,027356428	0,005000	0,00000	0,01944	0,00000	0,00000	0,00000
Portfolio 11	0,033964434	0,006000	0,00000	0,00000	0,00000	0,00000	0,00000

Markowitz solution asserts that an investor should choose a portfolio from the efficient frontier, depending on risk averse characteristics (Marling and Emanuelsson, 2012: 2).

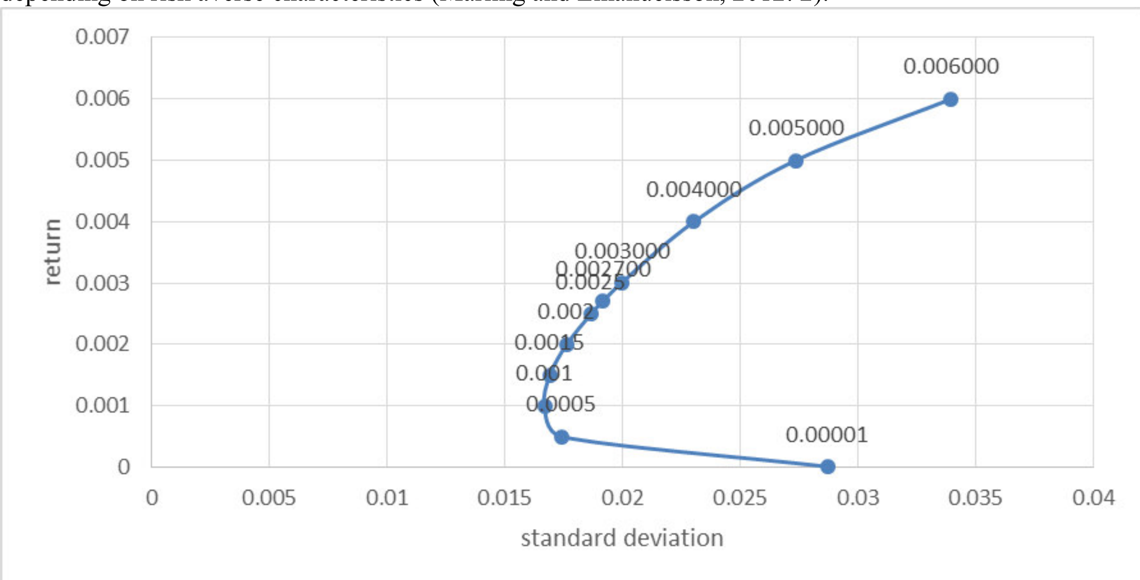

Figure 1. Efficient frontier and portfolios

As stated above, efficient frontier shows us minimum – variance portfolios. Given expected return, all efficient portfolios are located over efficient frontier curve. Investors or portfolio managers can choose any one of them as their risk preferences. As seen from Figure 1, many portfolios are existed on efficient frontier curve with different risk and return combinations.

Table 9. Comparison of portfolio return

Investment Alternatives	Daily Return (%)	Annual Return (%)	Daily standard deviation (%)	Annual standard deviation (%)
Portfolio with equal weights	0,002518	0,634536	0,0202650	0,321697
Optimal Portfolio 1	0,002700	0,680400	0,0191528	0,304041

4. Conclusion

Individuals are mainly two related diversification strategy as minimizing risk or maximize return. Investor aims to select best diversification. In this paper, I tested workability of portfolio optimization and diversification on Istanbul Stock Exchange (BIST). I created a portfolio which contains ten stocks from three different industry. One year daily stock return data is used for the analysis. I followed mean – variance approach of Markowitz involving best possible combination of expected return and risk. I believe that I constructed efficient portfolios in my analysis. I made my choice from this efficient set. My investment portfolio which did not contain short sale was achieved by excel data solver. I compared return and risk of optimal portfolio with an original portfolio with equal weights of ten stocks. This portfolio involves eight assets with different weights. The return of my optimal portfolio is greater than an original portfolio with equal weights of ten stocks as seen from Table 9. In that sense, Markowitz mean-variance approach provide best solutions in many alternatives.

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