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2013 Laser Phys. 23 105401

(http://iopscience.iop.org/1555-6611/23/10/105401)

Laser Phys. **23** (2013) 105401 (6pp) doi:10.1088/1054-660X/23/10/105401

Describing the propagation of intense laser pulses in nonlinear Kerr media using the ducting model

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Received 1 July 2013, in final form 21 August 2013
Accepted for publication 24 August 2013
Published 17 September 2013
Online at stacks.iop.org/LP/23/105401

Abstract

In this paper, the ducting model has been generalized by considering the temporal pulse shape function used to formulate it. This generalized form of the ducting model can be used to describe the propagation of laser pulses with various temporal shapes. In particular, propagation of Gaussian shaped laser pulses has been simulated. The model results have been compared with the results of the multi-focus structure and the moving nonlinear foci models, and it is shown that good correspondence exists between the results of our ducting model and those of the other two models. Our simulation results indicate that the Gaussian laser pulse creates a multi-focus structure inside the nonlinear Kerr medium which moves inside it at a very fast velocity. It is shown that this fast movement of focal points can create filaments in the form of conical frustums during pulse propagation.

(Some figures may appear in colour only in the online journal)

1. Introduction

A growing interest in self-focusing phenomena and the formation of filaments of nanosecond laser pulses occurred during the early development of nonlinear optics [1–3]. It is still a subject of intensive investigation, since self-focusing of laser pulses can initiate damage in transparent media and limits the attainable intensity of high power lasers [4]. Self-structuring and self-spectral broadening can also be visible in the output pulse of these lasers due to self-focusing phenomena.

Over the past four decades, a great number of publications have been devoted to various aspects of the propagation of laser beams in nonlinear optical media. Several theoretical models of self-focusing were proposed and investigated (a complete list of models can be found in various chapters of [5]). Following the first paper of Chiao *et al* [6], most of the models describing the propagation of light in nonlinear media were concerned with Kerr nonlinearity, wherein the refractive index varies linearly with the light intensity. Each one of these models has its own advantages and disadvantages and can describe some of the experimentally observed features of pulse propagation in nonlinear Kerr media. Among these models, the multi-focus structure (MFS) [7] and moving nonlinear foci (MNLF) [8] models have been the most developed theoretically and have been confirmed in many experiments, especially for laser pulses with nanosecond durations [9, 10]. Indeed, Loy *et al* [11] investigated the self-focusing of a single-mode ruby laser beam of 8 ns pulse duration in toluene and CS_2 and confirmed the MNLF prediction that the observed filaments are traces of the nonlinear focal locations which move along the propagation axis during that time [9].

Recently, a ducting model has been developed for predicting the Gaussian beam parameters during propagation in nonlinear Kerr media [12]. This is a stationary model which is based on the aberration-free theory and the assumption that nonlinear Kerr media are of a ducting nature. However, one must note that in real experiments that are designed for observing the self-focusing phenomenon, the incident beam is not often stationary. These experiments are mostly set up using pulsed lasers wherein the beam power varies with time in accordance with the pulse envelope. In order to increase the conformity of the ducting model with real

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experimental conditions, we have further developed the model by considering the time dependence of the laser beam intensity that appears in the refractive index of the Kerr medium. The model results have been obtained assuming a Gaussian shape laser pulse with nanosecond duration. It is shown that the model results are in good accord with the results of the MFS and MNLF models and are capable of providing more useful information about the characteristics of pulsed beam propagation in nonlinear Kerr media.

2. Description of the model

The index of refraction of the Kerr medium changes with intensity as

$$n = n_0 + n_2 I \tag{1}$$

where n_0 is the linear refractive index, n_2 is the nonlinear refraction coefficient and I denotes the intensity of the laser beam within the sample [13]. Considering the propagation of a spatially Gaussian laser pulse in the z direction, in the aberration-free theory the complex amplitude of the electric field within the nonlinear Kerr sample is written as [14]

$$E(r, z_0 + z) = E_0 \frac{w_0}{w(z)} \exp\left(-\frac{\alpha z}{2}\right) \exp\left(\frac{-r^2}{w^2(z)}\right) f(t) \tag{2}$$

where r is the radial coordinate, z_0 is the position of the medium entrance plane (assuming that the origin of the coordinate system is set at the position of the Gaussian beam waist), E_0 is the value of the electric field at the center of the beam waist plane, α is the linear absorption coefficient and w_0 and $w(z)$ are the beam waist and the beam spot size at the z position, respectively. The function $f(t)$ has been introduced to describe the temporal shape of the laser pulse. This function can have any appropriate shape to describe the different kinds of laser pulses, like Gaussian, super-Gaussian, rectangular, hyperbolic secant, etc. From this point forward, the model will closely follow the ducting model relations [12], apart from the fact that now all the relations should contain the pulse shape function $f(t)$. Therefore, the time-dependent Gaussian beam parameters, namely its spot size and radius of curvature, can be found at each z position inside the Kerr medium using the following relations [12]:

$$w(z, t) = w_1 \sqrt{\left(A(z, t) + \frac{B(z, t)}{R_1}\right)^2 + \left(\frac{\lambda_n B(z, t)}{\pi w_1^2}\right)^2} \tag{3}$$

and

$$R(z, t) = \left[\left(A(z, t) + \frac{B(z, t)}{R_1}\right)^2 + \left(\frac{\lambda_n B(z, t)}{\pi w_1^2}\right)^2\right] \Bigg/ \left[\left(C(z, t) + \frac{D(z, t)}{R_1}\right)\left(A(z, t) + \frac{B(z, t)}{R_1}\right) + B(z, t) D(z, t) \left(\frac{\lambda_n}{\pi w_1^2}\right)^2\right]^{-1} \tag{4}$$

where w_1 and R_1 are, respectively, the beam spot size and radius of curvature on the entrance plane of the medium and $\lambda_n = \lambda_0/n_0$ is the wavelength of the light inside the medium. $A(z, t)$, $B(z, t)$, $C(z, t)$ and $D(z, t)$ are the ABCD matrix elements of the Kerr medium that depend on time and the z distance according to the following relations:

$$\begin{bmatrix} A(z, t) & B(z, t) \\ C(z, t) & D(z, t) \end{bmatrix} = \begin{bmatrix} \cos(\gamma(z, t) z) & \dfrac{\sin(\gamma(z, t) z)}{n_0' \gamma(z, t)} \\ -n_0' \gamma(z, t) \sin(\gamma(z, t) z) & \cos(\gamma(z, t) z) \end{bmatrix} \tag{5}$$

with

$$\gamma(z, t) = \frac{2}{\sqrt{a} w(z, t)} \sqrt{\frac{1}{1 + \frac{\pi n_0 w^2(z, t) \exp(\alpha z)}{2 P n_2 f(t)}}} \tag{6}$$

where a is a correction factor that may take on values between 3.77 and 6.4 [12, 15] and P denotes the laser pulse peak power.

In order to be able to know the values of $w(z, t)$ and $R(z, t)$ of the spatially Gaussian laser pulse during its propagation in the Kerr medium, the same numerical procedure as described in our previous report [12] can be used. At each instant of time, $w(z, t)$ can be obtained by numerically solving the equation which is accessible by inserting the expressions $A(z, t)$ and $B(z, t)$ into equation (3). Knowing the $w(z, t)$ values, γ and hence the ABCD matrix elements can be calculated at each z position along the propagation direction by using equations (6) and (5), respectively. Finally, $R(z, t)$ values can be obtained at each position by using the calculated ABCD matrix elements in equation (4).

3. Results and discussion

Considering the propagation of a Gaussian shape laser pulse in a nonlinear Kerr medium, the pulse shape function $f(t)$ can be written as [16]

$$f(t) = \exp\left(\frac{-2 \ln 2 (t - t_0)^2}{\tau^2}\right)$$

where τ is the full width at half maximum (FWHM) of the pulse and it is assumed that the pulse is centered around time t_0. In our simulations, we have assumed that the laser pulse has a FWHM of 20 ns centered around 25 ns (inset of figure 1). Since the laser pulse is very short, the striction or thermal nonlinearity mechanisms cannot appear because redistribution of the density of matter by the striction forces or by nonuniform heating requires relatively longer times [17]. Furthermore, the temporal response of the electronic Kerr effect does not usually exceed 10^{-15} s and the electron nonlinearity mechanism cannot appear until we reach picosecond pulse durations. Therefore, for laser pulses with nanosecond durations, the main contribution to the nonlinearity of the medium can only be made by the orientational Kerr effect for, which the characteristic establishment time is 10^{-10}–10^{-12} s [17]. Hence, in our calculations we have used a value for n_2 of

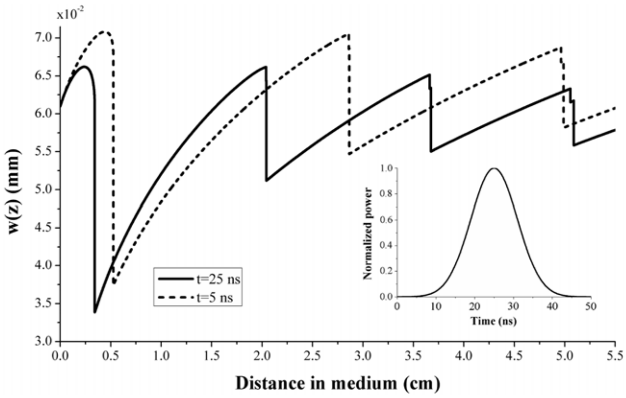

Figure 1. Spot size variations of the pulsed laser beam at two different instants of time. A moving multi-focus structure has been formed inside the nonlinear Kerr medium. The inset shows the temporal shape of the Gaussian laser pulse with 20 ns FWHM.

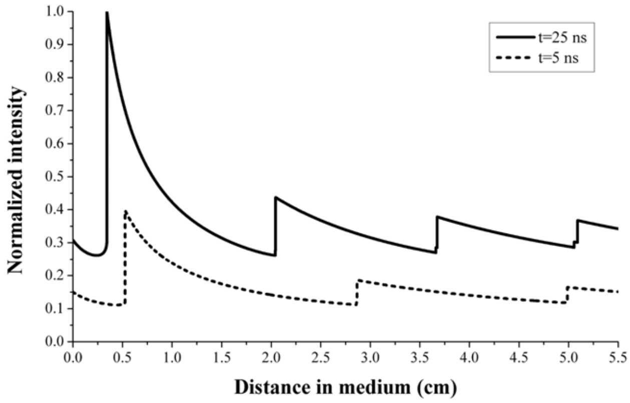

Figure 2. The normalized on-axis distributions of the light intensity along the propagation z axis at the two instants of time.

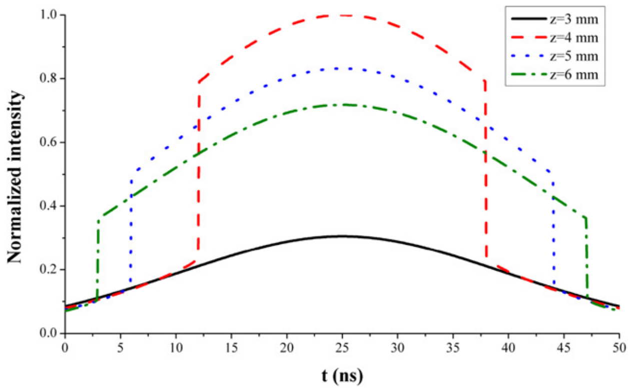

Figure 3. Temporal variation of the normalized intensity at different positions near the first self-focusing point.

10^{-14} cm^2 W^{-1} [18]. The laser wavelength λ_0 and its peak power P are also selected as 532 nm and 9 MW, respectively. In addition, for the linear refractive index n_0 and absorption coefficient α of the Kerr medium, typical values of 1 [19] and 10 m^{-1} [20] are set in our simulations. It is assumed that the laser beam waist is equal to 0.033 mm and the medium is placed 1 cm after the beam waist position where R_1 is positive and equal to 0.014 m and w_1 has a value of 0.061 mm [12].

In figure 1, the spot size variations of the pulsed laser beam has been shown at two different instants of time. The two different instants of time have been selected at the leading edge of the pulse, where the laser power is rising, and at the pulse peak, where the laser power is maximum. At each instant of time, a multi-focus structure is formed inside the medium which is consistent with the predictions of the MFS model. As was previously pointed out [12], this type of spot size variation is to be physically expected since for laser beams with Gaussian spatial intensity profiles propagating in media with a positive sign for the nonlinear index of refractions, the same effect as a convex lens occurs which counteracts the normal effect of diffraction. The interplay between these two effects leads to the depicted periodic profiles in figure 1. In the MFS model, which is completely based on wave optics, another plausible interpretation of the observed periodic structure in figure 1 has been suggested. According to this model, the light beam propagating in the nonlinear medium splits into ring zones in the transverse direction, and each zone is focused at different distances along the propagation z axis. Only a portion of the initial light beam power flows into the first nonlinear focus. A similar process occurs for other ring zones of the beam, and hence a multi-focus periodic structure will be obtained inside the nonlinear medium [9, 10, 17].

It should be noted that some models predict that propagation of a beam with power higher than the critical power of self-focusing leads to collapse of the beam to zero diameter [21]. However, as figure 1 shows, our model results indicate approximately a two-fold decrease of the beam spot size at the first self-focusing points. Furthermore, the beam diameter varies with the time slice. Indeed, since the focusing power of the Kerr medium is determined by the $n_2 I$ term in equation (1), the transverse dimensions of the nonlinear foci are smaller for $t = 25$ ns in figure 1, which corresponds to the highest intensity of the pulsed laser beam.

To clarify the temporal dynamics of the pulse in the case of self-focusing, the spatio-temporal distributions of intensity have been calculated. Figure 2 shows the on-axis distributions of the light intensity along the propagation axis at the two instants of time. It is obvious that at each instant of time, when the light reaches the self-focusing points its intensity greatly increases. The form of intensity variations is quite similar to that shown in the MNLF model [8].

Figure 3 shows the temporal shapes of the laser pulse at different positions near the first self-focusing point (see figure 1). As the laser power increases in the leading edge of the pulse, a sharp increase occurs in intensity when the first self-focusing point passes through the specified z positions in figure 3. A sharp decrease also occurs when the laser power decreases in the trailing edge of the pulse. These two sharp changes deform the initial Gaussian shape of the laser pulse (compare figure 3 with the inset of figure 1). To clearly see why these sharp changes in intensity occur, the variations in spot size of the pulsed laser beam are shown for different instants of time in figure 4. The variations in spot size at z equal to 4 and 5 mm reveal that for the two instants of time

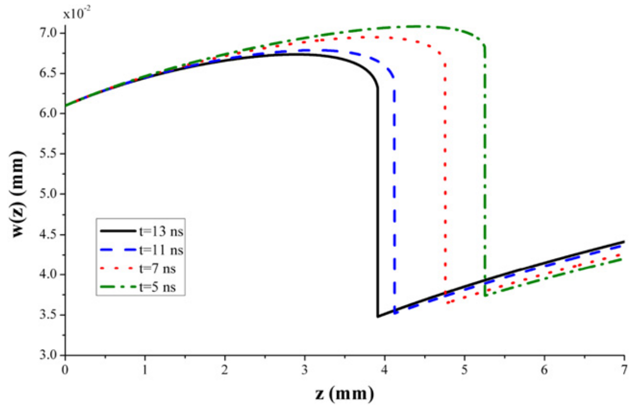

Figure 4. Variations in spot size of the pulsed laser beam at four instants of time that illustrate movement of the first self-focusing point inside the nonlinear sample.

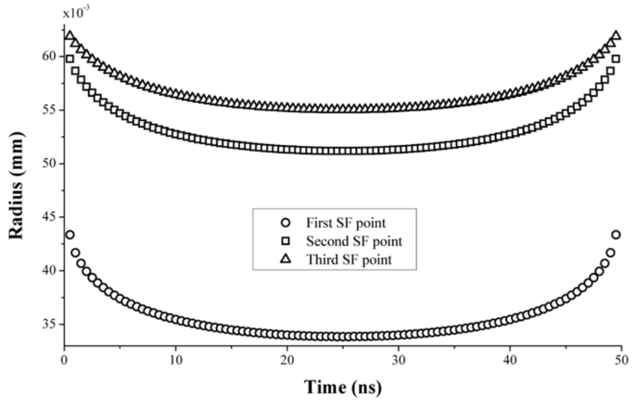

Figure 5. Variation of the transverse dimension of the first three focal points during the time interval of the laser pulse.

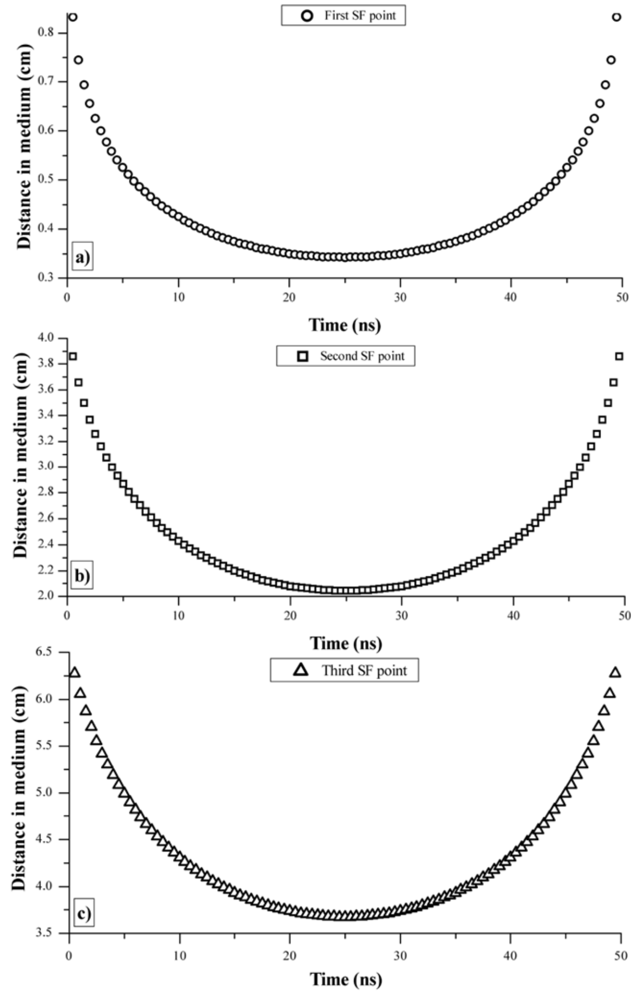

Figure 6. Distance of the first three self-focusing points from the entrance plane of the nonlinear sample at different instants of time.

Table 1. Maximum movement distances of the first four self-focusing points (SF) inside the nonlinear Kerr sample. Each point moves twice along the specified distances.

Maximum movement distance (cm)			
First SF	Second SF	Third SF	Fourth SF
0.49	1.82	2.60	3.03

there is a sharp change in which self-focusing points locate on either side of these positions. These sharp changes in spot size reveal themselves as sharp changes in the intensity profiles in figure 3. One must note that for the z positions far from the self-focusing points (e.g. $z = 3$ mm in figure 3) the intensity profile remains Gaussian since no sharp variations in spot size occur at these points.

In figure 5, the radii of the first three foci inside the nonlinear Kerr medium are shown at different instants of time. It is clear that the transverse dimensions of the focal points are minimum at the pulse peak ($t = 25$ ns) and become larger for other instants of time during the pulse period. Figure 5 also confirms the result of one of the first experimental works on self-focusing phenomena with nanosecond lasers [22], i.e. the focal points that are closer to the sample entrance plane have smaller transverse dimensions.

A closer inspection of figure 1 reveals that by increasing the laser pulse power, the locations of the foci move toward the sample entrance plane. The amounts of movement are smaller for the focal points nearer the sample entrance plane. In order to see this feature more clearly, figures 6(a)–(c) show the first three focal locations at different instants of time. It is easy to see that the higher order focal points travel a larger distance inside the nonlinear medium. The U-curves obtained in figure 6 have already been shown to correctly describe the motion of focal points inside a nonlinear Kerr medium [23].

As was previously pointed out in the MNLF model, since the positions of the foci along the beam axis depend on the initial power and the power itself varies with time, the foci move along the beam axis [17]. In fact, in this model the observed thin light filaments in experiments were proposed to be traces of the nonlinear focal locations which move along the propagation axis during time. In table 1 we show the maximum distances moved by the first four focal points during pulse propagation in the nonlinear Kerr medium. The obtained movement distances agree with the experimentally observed length of filaments of about some centimeters [17]. It must be emphasized that our ducting

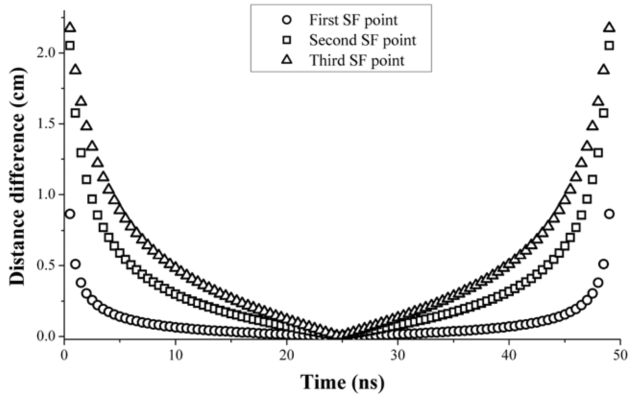

Figure 7. Time variations of the difference in distance between the locations of two consecutive focal points corresponding to two consecutive instants of time. The differences in distance have been depicted for the first three self-focusing points.

model also predicts that these filaments cannot have a perfect cylindrical shape, since the radii of the focal points change during their movements (see figure 5). Indeed, combination of the movement of the focal points along the optical axis with their radial increase in the transverse direction should result in the formation of conical frustums whose smaller bases are located toward the nonlinear sample entrance plane.

A close examination of figure 6 indicates that for each of the focal points, by decreasing the laser pulse power (compare figure 6 with the inset of figure 1), the difference in the distance between two consecutive locations increases. For the first three focal points, at each instant of time, the difference in distance between the location of the focal point at that time and its previous instant of time is been shown in figure 7.

As can be seen, the differences in distance are negligible when the laser pulse power is maximum ($t = 25$ ns) and rapidly increase with decreasing power; finally, they asymptotically approach infinity. This is in accord with the concept of critical beam power which states that below a finite laser power, the self-focusing disappears and the locations of the focal points move toward infinity. Indeed, for the parameters used in our simulations, we have found that for the laser powers below 1.95 kW the self-focusing structure in figure 1 disappears and the beam spot size continuously increases due to its natural diffraction.

The different distances traveled in equal time intervals in figure 7 indicate that the self-focusing points should have different velocities during pulse propagation inside the Kerr medium. In figure 8, the computed velocities of the focal points are shown. The obtained results indicate that, despite the prediction of the MNLF model [17], velocities of the focal points do not increase monotonically. Furthermore, for all instants of time, the movement velocities of the focal points remain well below the velocity of light. Hence, the albeit interesting but somewhat unrealizable result of moving faster than the light velocity of the focal points that is predicted by other models [3], does not occur here.

Different traveling speeds of the focal points in figure 8 suggest that the distance between the self-focusing points, for instance the first and the second self-focusing points in

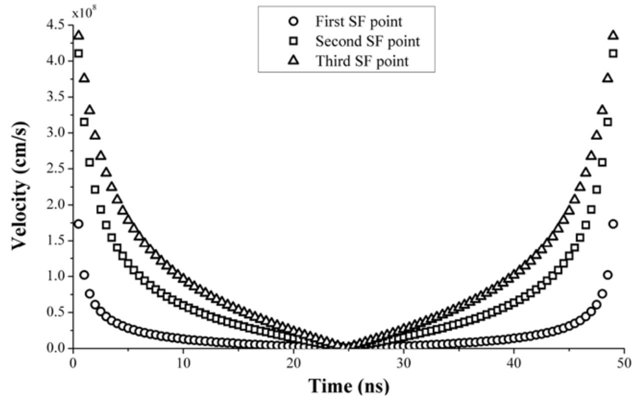

Figure 8. The movement velocity of the first three self-focusing points inside the nonlinear Kerr medium.

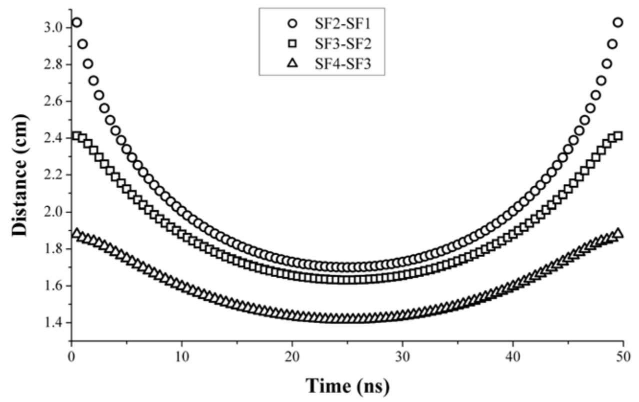

Figure 9. Distance between the first four self-focusing points at different instants of time during pulse propagation.

figure 1, should also vary in time. In figure 9, the distance between the self-focusing points is shown during the time interval of the laser pulse. Inspection of this figure shows that for the initial stages of the laser pulse, the distances between the focal points are large. They gradually decrease up to the time of the pulse peak and again gradually increase thereafter.

4. Conclusion

In this work, the ducting model [12] for describing the propagation of spatially Gaussian laser beams in nonlinear Kerr media has been further developed by considering the temporal pulse shape function in its relations. A general form of the pulse shape function has been considered in relations which increases the capability of the model to describe the propagation of laser pulses with various temporal shapes in nonlinear media. In this way it will be possible to analyze the propagation of various types of laser pulses, like super-Gaussian, rectangular, hyperbolic secant, etc, in future works. In particular, propagation of Gaussian shaped laser pulses has been simulated in this work. It is shown that the obtained self-focusing structure and the movement of the focal points inside nonlinear Kerr media are in good agreement with the results of the well-developed MFS [7] and MNLF [8] models, as well as the results of some experimental works.

As a case in point, it is shown that the very fast movement of the focal points can be the possible reason for the observation of filaments in self-focusing experiments. Our generalized ducting model predicts that since the transverse dimensions of the self-focusing points gradually change during Gaussian laser pulse propagation, these filaments cannot have a perfect cylindrical shape. Indeed, they should be in the form of conical frustums whose smaller bases are located toward the nonlinear sample entrance plane. Experimental verification of these conical filaments needs more precise spatial analysis, since our simulations indicate that their transverse dimensions change by around hundreds of micrometers during the laser pulse propagation (see figure 5).

Because of the simple formulation of the ducting model for obtaining the laser beam parameters during propagation in nonlinear Kerr media, we are of the opinion that this model can be used instead of the more complex models of nonlinear wave propagation.

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