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Multiobjective Mean-Risk Models for Optimization in Finance and Insurance

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Abstract

In this paper we propose some models for solving optimization problems which arise in finance and insurance. First the general framework for Mean-Risk models is introduced. Then several approaches for multiobjective programming, such as Mean-Value-at-Risk and Mean-Conditional Value-at-Risk are used for building the model Mean-Value-at-Risk-Conditional Value-at-Risk using both Value-at-Risk and Conditional Value-at-Risk simultaneously for risk assessment. A two stage portfolio optimization model is developed, using Value-at-Risk and also Conditional Value-at-Risk measures in two stages separately.

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1. Introduction

In order to solve portfolio selection problems, one of the most common approaches consists in the development and use of Mean-Risk models. They provide quantitative techniques for the comparison of return

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distributions employing two statistics: the mean value and a risk measure. The most important advantages of this approach are related to the easy and intuitive interpretation of the results and to the computational power. On the other hand, this approach has disadvantages since using only two parameters to describe a distribution lead to a considerable loss of information. The risk measure which is used has a key function in the decision making process. The first risk measure used in mean-risk models was variance, by Markowitz, 1952. Despite the criticism and the appearance of alternative ways of assessing risk, variance continues to be used for solving portfolio selection problems. Also, risk measures which evaluate the severity of the risk on the left tails of the return distributions, modeling the most unfavorable outcomes, are widely used by practitioners. The most common measure in this category is Value-at-Risk (VaR). VaR measure has not convenient theoretical properties, because it is not subadditive and consequently it does not encourage diversification. In the same time, this measure does not take into account the severity of the losses greater than the Value-at-Risk threshold. Conditional Value-at-Risk (CVaR) measure was proposed in order to overcome some of these shortcomings. CVaR measures the expected value of the losses greater than VaR, so it models more realistically the risk of the portfolio. Also, CVaR can be successfully used as objective function in optimization problems, as it is a convex measure.

The risk-based approach provides realistic and efficient methods for risk management. A lot of quantitative techniques are used for economic modeling in finance and insurance. Here can be mentioned the contributions of Ştefănoiu et al., 2014, Moinescu and Costea, 2014, Toma and Leoni-Aubin, 2013, Barik et al., 2012, Tudor and Dedu, 2012, Filip, 2012, Costea and Bleotu, 2012, Toma, 2012, Şerban et al., 2011, Nastac et al., 2009, Costea et al., 2009, Ştefănescu et al., 2008, Şerban and Dedu, 2007 and Filip, 2002. Modeling the trend of financial indices have caught the interest of the researchers, see, for example, Georgescu, 2014, Tudor, 2012, Tudor and Popescu-Duţă, 2012, Şerban et al., 2011, Lupu and Tudor, 2008 and Dedu and Şerban, 2007. Recently, Toma, 2014, Preda et al., 2014 and Toma, 2013 investigate different methods for financial data modeling using divergences and entropy measures. Other papers use risk measures as objective functions for building optimization problems applied to insurance and finance, see, for example, Toma and Dedu, 2014, Şerban et al., 2013 and Dedu and Ciumara, 2010.

Mean-risk models represent widely employed techniques for solving portfolio optimization problems. Recently, many research papers have been written about the issue of decision making using a variety of risk measures, see for example, Ogryczak, 2002, Dedu, 2012 and Şerban et al., 2011.

This paper develops a two-stage portfolio optimization approach, which retains all the advantages of Mean-Value-at-Risk and Mean-Conditional Value-at-Risk models, while simultaneously overcoming their disadvantages, because the two stages of the proposed algorithm complement each other. At the same time, this new approach uses an increased amount of information regarding the distribution of the portfolio return. In this model, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) are separately used as risk measures in each stage, using a priority order of the two risk measures. In the first stage, the primary risk measure is used to find all the efficient portfolios. In stage two, a secondary risk measure is used to optimize the efficient portfolios obtained in the first stage. This approach provides better results than the Mean-VaR and the Mean-CVaR models considered separately. Instead of using one single risk measure, we also propose a general Mean-VaR-CVaR approach using VaR and CVaR measures simultaneously. We note that the Mean-VaR and the Mean-CVaR approaches are special cases of the Mean-VaR-CVaR approach.

The remainder of the paper is organized as follows. In Section 2 the Mean-Risk models are introduced. In Section 3 we focus on the general concepts of mean-risk models. In Section 4 we propose a more general portfolio optimization model: the Mean-VaR-CVaR, based on combining two portfolio optimization models: the Mean-VaR and the Mean-CVaR model. Thus, the usual Mean-VaR model and the Mean-CVaR model can be regarded as special cases of the integrated model. VaR and CVaR are alternatively used as risk measures during the optimization stage. Then a combined approach to portfolio selection is presented, which is given by a two-stage portfolio optimization strategy. The conclusions of the paper are provided in Section 5.

2. Mean-Risk models

Mean-risk models were designed specifically to address the problem of portfolio optimization, using different risk measures. In 1952, Markowitz proposed variance as a measure of risk for solving portfolio selection problems. Since then, many other risk measures have been approached in the literature. Choosing the most appropriate risk measure represents a very wide studied topic. In the mean-risk approach, two scalars are attached to each random variable: the expected value and a risk measure. A preference relation is defined using a trade-off between the expected value, where a larger value is the objective to be reached, and a risk measure, where a minimal value is desirable.

In the mean-risk approach, if risk is evaluated using the risk measure denoted by ρ, we say that the random variable R_x dominates or is preferred to the random variable R_y if and only if $E(R_x) \geq E(R_y)$ and $\rho(R_x) \leq \rho(R_y)$ with the condition that at least one inequality is strict. In this case it is said that the portfolio *x* dominates the portfolio *y*.

We consider a set composed by *n* assets and we denote by R_j the return of the asset *j* at the end of the entire investment period. We will model R_j using a random variable, as the price corresponding to the asset asset *j* in the future is unknown. We will denote by x_j be the proportion of the total capital which will be invested in the asset *j*. Consequently, we have: $x_j = w_j / w$, where w_j represents the proportion of capital invested in the asset *j* and *w* represents the total capital which will be invested. We denote by $x = (x_1, ..., x_n)$ the portfolio resulted from this choice. This return of the portfolio is modeled using the random variable $R_x = x_1 R_1 + ... + x_n R_n$ with cumulative distribution function $F(r) = \mathrm{P}(R_x \leq r)$, which depends on the choice $x = (x_1, ..., x_n)$.

The condition for the weights vector $(x_1, ..., x_n)$ in order to model a portfolio requires that the weights have to satisfy a set of constrains which constitutes a feasible set of decision vectors, denoted by *M*. The simplest way for defining a feasible set is by requiring the sum of weights to be equal to 1 and short selling to be not allowed. In this case, the set of feasible decision vectors is given by: $M = \left\{ (x_1, ..., x_n) \in \mathbf{R}^n \mid \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall j \in \overline{1,n} \right\}$.

The following problem is to take into account a realistic representation of the random variables which describe the asset returns and portfolio returns. These corresponding random variables will be modeled in the discrete case and they will be described by realizations under a number of *T* states of the world, generated by using a scenario generation procedure of finite samples of historical data. We consider that state $i \in \{1, ..., T\}$ occurs with the probability p_i, $\sum_{i=1}^{T} p_i = 1$. We denote by r_{ij} be the return of the asset *j* under scenario *i*, $i \in \{1, ..., T\}$, $j \in \{1, ..., n\}$.

The random variable denoted by R_j, which models the return of the asset *j*, is finitely distributed over the set $\{r_{1j}, ..., r_{Tj}\}$, with the corresponding probabilities $p_1, ..., p_T$. The random variable denoted by R_x, which models the return of the portfolio $x = (x_1, ..., x_n)$, is a finitely distributed random variable over the set of scenarios $\{R_{x1}, ..., R_{xT}\}$, where $R_{xi} = x_1 r_{i1} + ... + x_n r_{in}, \forall i \in \{1, ..., T\}$. The framework of this approach can be represented as follows:

- We consider a set composed by n assets, denoted by $S_j, j=1,2,...,n$ and we denote by R_j the random variables which models the rate of return of the asset S_j.
- We denote by $x_j \geq 0$ the proportion of capital which will be invested in the asset S_j.
- The portfolio resulted by this choice is given by the vector $x=(x_1,x_2,...,x_n)$, which must satisfy the condition: $\sum_{j=1}^{n} x_j = 1, \quad 0 \leq x_j \leq \alpha_j, \quad j=1,2,...,n$. (1)
- We denote by $R(x)$ the portfolio rate of return, given by:

$$R(x) = \sum_{j=1}^{n} x_j \cdot R_j \,. \tag{2}$$

- We will denote by $r(x)$ and $\rho(x)$ the mean and the risk of the return of the portfolio.

A mean-variance (MV) model can be formally represented by the means of the following optimization problem:

$$(MV_1)\begin{cases} \text{minimize} & \rho(x) \\ \text{subject to} & r(x) \geq r_0 \\ & x \in X \end{cases} \tag{3}$$

where $X \subset \mathbf{R}^n$ represents the set which is defined by $\sum_{j=1}^{n} x_j = 1, \quad 0 \leq x_j \leq \alpha_j, \quad j=1,2,...,n$.

This condition can contain supplementary linear constraints. The parameter r_0 stands for a constant which will be specified by the investor.

A mean-variance model can be represented in two different ways

$$(MV_2)\begin{cases} \text{maximize} & r(x) \\ \text{subject to} & \rho(x) \leq \rho_0 \\ & x \in X \end{cases} \tag{4}$$

$$(MV_3)\begin{cases} \text{maximize} & r(x) - \lambda\rho(x) \\ \text{subject to} & x \in X \end{cases} \tag{5}$$

There are a lot of risk measures which can be used for assessing risk, such as: variance, absolute deviation, Value-at-Risk measure, Conditional Tail Expectation and Conditional Value-at-Risk.

3. General concepts of Mean-Risk models

The mean-variance model represents the earliest approach to solving the portfolio selection problem. This method is based on the principle of diversification and it is widely used in portfolio management. There are some shortcomings of this approach, although it has gain widespread acceptance and it has been highly valued by practitioners and researchers for many years. Controlling the variance leads to low deviations from the expected return on the down side, but also on the up side and thus it may be disadvantageous with regard to possible gains. In this section, we draw upon the mean-variance approach. We consider a set composed by n

assets with rates of return given by $R_i, i = \overline{1,n}$.

- The means and covariances of the return rates are: $\mu_i = \mathrm{E}(R_i),\ \sigma_{ij} = \mathrm{cov}(R_i, R_j),\ i, j = \overline{1,n}$
- The portfolio vector is represented as: $x = (x_1, ..., x_n) \in \mathbf{R}^n$, with $\sum_{i=1}^{n} x_i = 1$.
- We define X as representing the set of all feasible portfolios: $X = \left\{ x \in \mathbf{R}^n \,\middle|\, \sum_{i=1}^{n} x_i = 1 \right\}$.
- The total return corresponding to the portfolio x is given by $R_x = \sum_{i=1}^{n} x_i R_i$.
- The mean and the variance corresponding to the portfolio return are given by:

$$\mu_x = E(R_x) = E\left(\sum_{i=1}^{n} x_i R_i\right) = \sum_{i=1}^{n} x_i \mu_i \ \text{ and } \ \sigma_x^2 = \mathrm{Var}\left(\sum_{i=1}^{n} x_i R_i\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} .$$

We will denote by $\rho(x)$ the risk measure corresponding to the return of the portfolio.

There exist two common models developed using the mean-risk principle. The objective of the one model is to select a portfolio x such that, for a fixed upper bound ρ_0 for the risk of the portfolio return ($\rho_x \leq \rho_0$), the mean value of the return μ_x to be maximal. The first type mean-risk model corresponding to the risk measure ρ is defined as follows:

$$\begin{aligned} &\text{maximize} && \mu_x \\ &\text{such that} && \rho_x \leq \rho_0 \\ & && x \in X \end{aligned} \tag{6}$$

The objective of the second model is to select a portfolio x such that, for a fixed lower bound μ_0 of the expected value of the return of the portfolio ($\mu_x \geq \mu_0$), ρ_x to be minimal. The second type mean-risk model corresponding to the risk measure ρ is defined as follows:

$$\begin{aligned} &\text{minimize} && \rho_x \\ &\text{such that} && \mu_x \geq \mu_0 \\ & && x \in X \end{aligned} \tag{7}$$

4. Two stage optimization approach

In this section, we develop a combined portfolio optimization approach based on a two stage procedure. It incorporates the power of both mean-VaR and mean-CVaR approaches and solves the problems given by their weak points, because these strategies complement one each other. In this new approach, VaR and CVaR are used as risk measures in these two stages separately, using a priority ordering of the two measures of risk involved. In the initial stage, all the efficient portfolios based on the first risk measure are collected. In the second stage, the efficient portfolios obtained in the first stage are optimized using the second risk measure as objective function. Some versions of these two-stage portfolio optimization models are developed using the priority order of the two risk measures. This new approach produces results which are significantly better than those which are produced by the old model, which considers only a single risk measure. Instead of using only a single risk measure, in our approach the risk will be evaluated by assessing not only the value of the greatest losses which can occure with a certain probability level, but also severity of the losses above the Value-at-Risk threshold. We note that Mean-Value-at-Risk and Mean-Conditional Value-at-Risk models constitute special cases of our model. Next we will focus upon the concepts and procedures that will be drawn upon in the next section.

4.1. Mean-VaR model with minimal Conditional Value-at-Risk

In this subsection we propose two optimization models in which we consider that VaR is the first used portfolio risk measure. In the primary stage, the risk measure used is the Value-at-Risk of the portfolio return, in order to collect all the mean-VaR efficient portfolios. Then, the portfolios obtained in the first step will be optimized using Conditional Value-at-Risk as risk measure of the portfolio return in the second stage. We propose two optimization models, represented by (8) and (9), as follows.

- The Min-Max Model:

$$\begin{aligned} &\text{minimize} \quad \mathrm{CVaR}_x \\ &\text{such that} \quad x \in X_{opt} \end{aligned} \tag{8}$$

where X_{opt} is a solution set of the first type Mean-VaR model, defined by (6).

- The Min-Min Model:

$$\begin{aligned} &\text{minimize} \quad \mathrm{CVaR}_x \\ &\qquad\qquad x \in W_{opt} \end{aligned} \tag{9}$$

where X_{opt} is a solution set of the second type Mean-VaR model, defined by (7).

4.2. Mean-CVaR model with minimal Value-at-Risk

Now we propose two optimization models in which CVaR is the first used portfolio risk measure. In the first stage, all the Mean-VaR efficient portfolios are collected. In the second stage, the efficient portfolios obtained in the first stage are optimized using Conditional Value-at-Risk as a secondary risk measure. The models obtained are given by (10) and (11), as follows.

- The Min-Max Model:

$$\begin{aligned} &\text{minimize} \quad \mathrm{VaR}_x \\ &\text{such that} \quad x \in X_{opt} \end{aligned} \tag{10}$$

where X_{opt} is a solution set of the first type Mean-CVaR model, defined by (6).

- The Min-Min Model:

$$\begin{aligned} &\text{minimize } \mathrm{VaR}_x \\ &\qquad x \in X_{opt} \end{aligned} \tag{11}$$

where X_{opt} is a solution set of the second type Mean-CVaR model, defined by (7).

5. Conclusions

Mean-VaR and Mean-CVaR represent often used models in the framework of Mean-Risk approach. In this paper we proposed to combine these two models by using the Mean-VaR-CVaR approach and by defining a two stage optimization model. It incorporates the strong points of both Mean-VaR and Mean-CVaR approaches while avoiding their shortcomings, because these two strategies complement one another. In this new approach, VaR and CVaR risk measures are separately used in the two stages. In the first stage, all the efficient portfolios based on the primary risk measure are collected. In stage two, the efficient portfolios obtained in the first stage are optimized based on the second risk measure. Some different variants of the portfolio optimization model involving two stages models are proposed, based on the priority ordering of the risk measures. The approach proposed in this paper could be extended by combining other risk measures in the two stage approach, which can lead to models with better performances in the attempt to solve more complex problems from finance and insurance.

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