

## Technical Note

# Comparison of 2D and 3D models for numerical simulation of vibration reduction by periodic pile barriers

Xinnan Liu<sup>a</sup>, Zhifei Shi<sup>a,\*</sup>, Y.L. Mo<sup>b</sup><sup>a</sup> School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, PR China<sup>b</sup> Department of Civil and Environmental Engineering, University of Houston, Houston, TX 77004, USA

## ARTICLE INFO

## Article history:

Received 6 September 2015

Received in revised form

10 September 2015

Accepted 11 September 2015

## Keywords:

Periodic structure

Pile barriers

2D model

3D model

Quadrature element method

## ABSTRACT

With the assumption of infinite pile length, frequency attenuation zones of periodic pile barriers for plane waves are analyzed by the 2D periodic structure theory in the previous studies. 2D finite element models are used to validate the frequency attenuation zones obtained by the periodic structure theory, although the piles in practice are of finite length. This note aims to compare the 2D and 3D models for simulation of vibration reduction by periodic pile barriers. Based on the weak form quadrature element method, frequency attenuation zones are calculated by using the 2D periodic structure theory. Moreover, it is found that the 3D numerical results converge to the 2D numerical results as the pile length increases. Therefore, the 2D model can be used to consider the key properties that dominate the response of periodic pile barriers.

© 2015 Elsevier Ltd. All rights reserved.

## 1. Introduction

Multi-row pile barriers have been widely used for ambient vibration isolation for a long time. It is noticed that great vibration reduction may take place for some frequency ranges, while amplification can be observed for some other frequency ranges [1–4]. In view of the periodically spatial configuration, multi-row pile barriers can be recognized as periodic structures. With the periodic structure theory, Huang and Shi [5] proposed the concept of frequency attenuation zones to gain a better understanding of this phenomenon. Their results show that amplitude will be reduced when frequencies of the waves fall in the attenuation zones.

Some assumptions are adopted in the periodic structure theory, such as the linear elastic materials, infinite unit cells in the  $x$ - $y$  plane and infinite pile length in the  $z$ -direction. With these assumptions, the wave propagation in periodic piles reduces to 2D plane strain problems. To date, several methods are proposed to study the attenuation zones that include the plane wave expansion method [6] and the weak form quadrature element method (WFQEM) [7]. To verify the attenuation zones predicted by the periodic structure theory, dynamic responses of 2D finite element models with finite unit cells are usually conducted. However, the lengths of pile barriers in practice are finite in the  $z$ -direction. It is noted that the effect of pile length is very important in design

[8,9]. For high operational frequencies, the screening effectiveness by short piles is acceptable due to the short wave length of the vibration. As the operational frequency decreases, the wave length increases, thus, larger pile length is required for effective screening, which can only be studied by the 3D model [10,11]. In the 2D model, the pile length is assumed to be infinite, where the effect of pile length cannot be taken into consideration. Hence, a comparison of the 3D and 2D models for numerical simulation of vibration attenuation by using periodic pile barriers is of significance to examine the convergence of the numerical results as the pile length increases.

In this technical note, the frequency attenuation zones are obtained by combining the 2D periodic structure theory with the WFQEM, as presented in Section 2. In Section 3, the frequency response functions (FRFs) of 3D models with different pile lengths ( $L=10/20/30/40/50$  m) are studied. In addition, a comparison of the 3D and 2D numerical results is made. Finally, the conclusions are given in Section 4.

## 2. Frequency attenuation zones by the 2D periodic structure theory

Fig. 1 shows a group of pile barriers with finite unit cells. The typical unit cell can be defined, whose periodic constant is  $a$ , as shown in Fig. 2(a). Consider an out-of-plane wave with an angular frequency  $\omega$  and a Bloch wave vector  $\mathbf{k}$  in the periodic pile

\* Corresponding author. Tel.: +86 10 51688367.

E-mail address: [zfshi178@bjtu.edu.cn](mailto:zfshi178@bjtu.edu.cn) (Z. Shi).

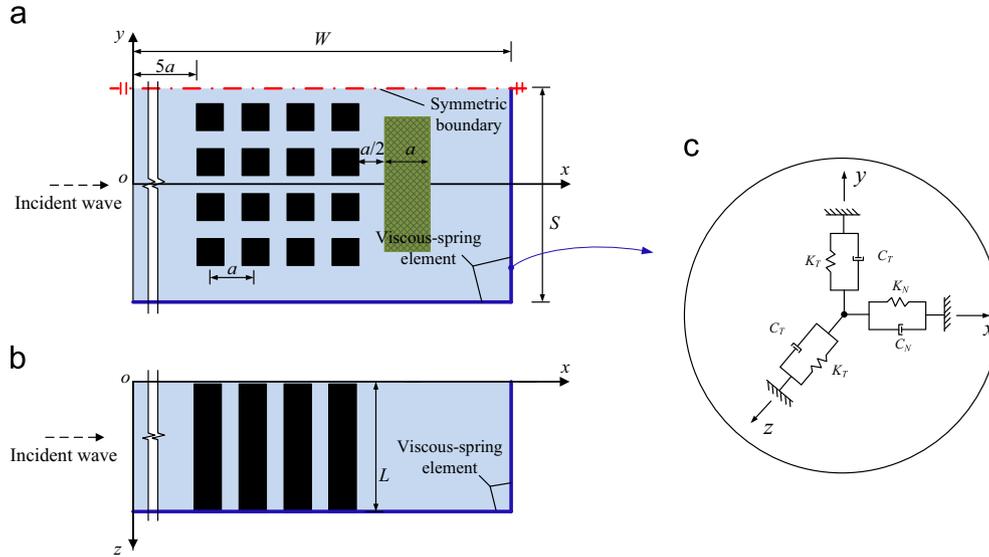


Fig. 1. (a) Top view and (b) side view of a finite-unit cell pile barriers, and (c) viscous-spring boundary element.

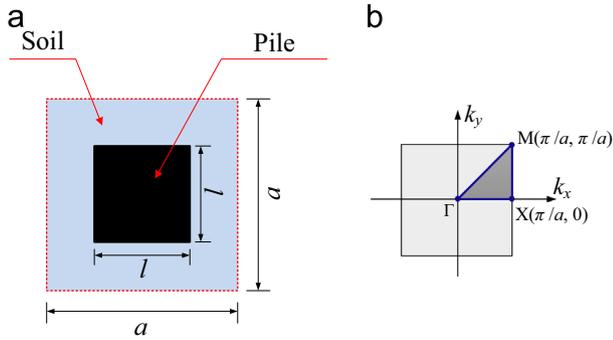


Fig. 2. (a) Cross section of a typical unit cell and (b) the first irreducible Brillouin zone.

Table 1  
Material properties and geometry parameters of the periodic piles.

Parameters	Value
Young modulus of concrete (GPa)	30
Mass density of concrete (kg/m <sup>3</sup> )	2500
Poisson ratio of concrete	0.2
Young modulus of soil (GPa)	$2 \times 10^{-2}$
Mass density of soil (kg/m <sup>3</sup> )	1800
Poisson ratio of soil	0.35
Length of the piles, $L$ (m)	10/20/30/40/50
Side length of the piles, $l$ (m)	1.2
Periodic constant of the unit cell, $a$ (m)	2
Length of the 3D model, $W$ (m)	28
Width of the 3D model, $S$ (m)	12

barriers. With the assumption of infinite pile length, the dynamic equation of the typical unit cell is given by [7]

$$[\mathbf{K}(\mathbf{k}) - \omega^2 \mathbf{M}(\mathbf{k})] \mathbf{d} = \mathbf{0}, \quad (1)$$

where  $\mathbf{K}(\mathbf{k})$ ,  $\mathbf{M}(\mathbf{k})$  and  $\mathbf{d}$  denote the stiffness matrix, the mass matrix and the nodal displacement vector, respectively. For a given wave vector  $\mathbf{k}$ , the corresponding eigen-frequencies  $\omega$  can be obtained from Eq. (1). The entire dispersion curves of out-of-plane waves in periodic pile barriers can be drawn by varying the wave vector  $\mathbf{k}$  along the edges of the first irreducible Brillouin zone (see the shaded triangle in Fig. 2(b)).

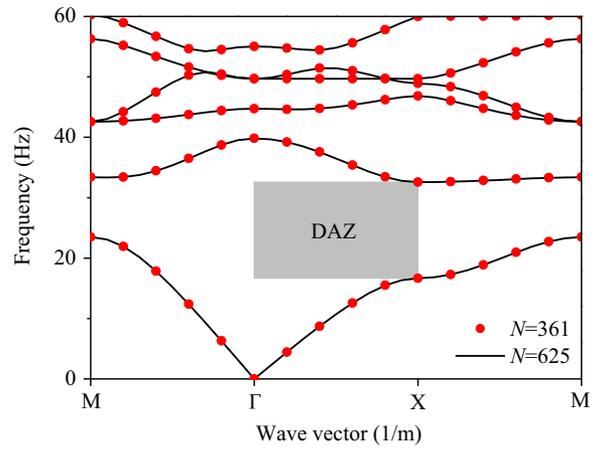


Fig. 3. The directional attenuation zone (DAZ) based on the 2D periodic structure theory.

All material properties and geometric parameters used in the following analyses are listed in Table 1. Fig. 3 presents the dispersion curves of out-of-plane waves obtained by the WFQEM, where parameter  $N$  is the total number of discrete nodes used in the WFQEM. The results by the WFQEM are more accurate with a larger  $N$  [7]. It can be seen from Fig. 3 that the results for  $N=625$  are the same as those for  $N=361$ , which means the results are converged with sufficient accuracy. The shaded area in Fig. 3 is the directional attenuation zone (DAZ) in the  $x$ -direction, which ranges from 16.7 Hz to 32.6 Hz. Theoretically speaking, the out-of-plane waves in the  $x$ -direction with a frequency in the DAZ will be attenuated by the periodic pile barriers.

### 3. Result comparison of 2D and 3D models

2D and 3D models are built in the finite element software ANSYS. Five 3D models with various pile lengths ( $L=10/20/30/40/50$  m) are considered. In each 3D model, the pile length of all the piles is identical. Element SOLID45 is employed to model the piles and soil. Element COMBIN14 is used to apply the viscous-spring boundary condition, which will eliminate the reflection of outgoing waves at the boundaries. The degrees of freedom in the  $x$ - $y$

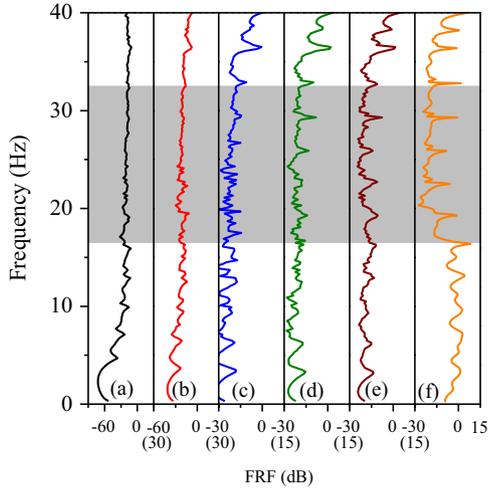


Fig. 4. Area-averaged FRF ( $z=0$ ) for the 3D models with  $L=10$  m (a),  $L=20$  m (b),  $L=30$  m (c),  $L=40$  m (d),  $L=50$  m (e) and the 2D model (f).

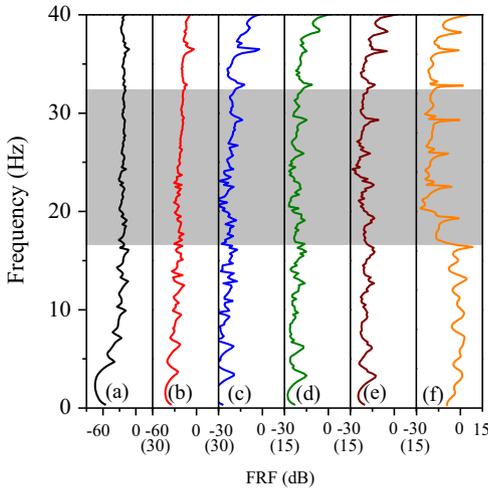


Fig. 5. Area-averaged FRF ( $z=-L/2$ ) for the 3D models with  $L=10$  m (a),  $L=20$  m (b),  $L=30$  m (c),  $L=40$  m (d),  $L=50$  m (e) and the 2D model (f).

plane are fixed to simulate the pure out-of-plane problem. Harmonic displacement excitations with a unit amplitude in the  $z$ -direction are applied to all the nodes on the area  $x=0$ . In addition, for the 2D model, both the pile length and the depth of the whole model are taken as unit length [12].

In order to give an objective evaluation of the vibration reduction by the periodic pile barriers, the area-averaged frequency response function (FRF) is introduced:

$$\text{FRF} = 20 \log_{10} \left( \frac{\int_{A_s} \delta_o / \delta_i dA}{A_s} \right), \quad (2)$$

where  $\delta_o$  and  $\delta_i$  are the amplitude of response nodes and the amplitude of the input excitation, respectively, and  $A_s$  is the rectangular area behind the pile barriers, as shown by the shaded part in Fig. 1. Note that the FRF is negative when  $\delta_o$  is less than  $\delta_i$ , which means the vibration is attenuated after transmission through the pile barriers.

Fig. 4 shows the FRFs at  $z=0$  for the 3D models with  $L=10/20/30/40/50$  m and that for the 2D model, where the excitation frequency ranges from 0.1 Hz to 40 Hz with an interval  $\Delta f=0.1$  Hz. The shaded areas in Fig. 4 are the DAZ obtained by the 2D periodic structure theory. Great vibration reduction can be found for all the 3D and 2D models when the excitation frequency is inside the

Table 2

Averaged attenuation  $\overline{\text{FRF}} = \frac{\int_{f_s}^{f_e} \text{FRF} df}{f_e - f_s}$  in the DAZ for 3D models and 2D model.

Models		FRF ( $z=0$ )	FRF ( $z=-L/2$ )
Pile length for 3D model (m)	10	-20.43	-23.42
	20	-21.53	-22.72
	30	-21.39	-22.35
	40	-20.71	-21.91
	50	-19.18	-20.81
2D model		-18.50	-18.50

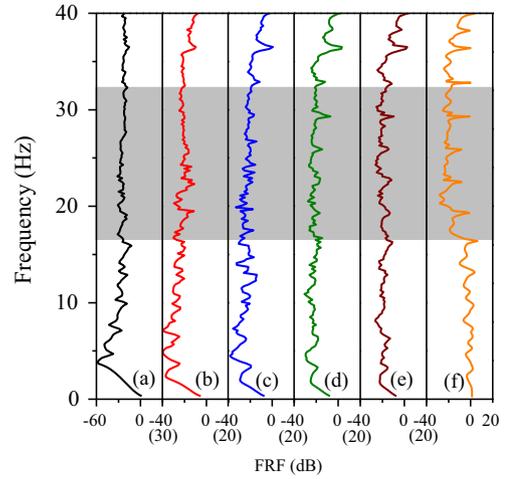


Fig. 6. Area-averaged relative (FRF)<sub>r</sub> ( $z=0$ ) for the 3D models with  $L=10$  m (a),  $L=20$  m (b),  $L=30$  m (c),  $L=40$  m (d),  $L=50$  m (e) and the 2D model (f).

DAZ. As the pile length increases, it can be seen that the 3D solutions converge to the 2D solution. This is expected because as the pile length approaches to the infinite value, the 3D problem reduces to a plane strain problem that the 2D model actually represents. Similar phenomenon can also be found for the FRFs at  $z=-L/2$ , as shown in Fig. 5. Table 2 shows the averaged attenuation in the DAZ for 3D models and 2D model which is defined by  $\overline{\text{FRF}} = \frac{\int_{f_s}^{f_e} \text{FRF} df}{f_e - f_s}$ , where  $f_s$  and  $f_e$  are the starting frequency and ending frequency of the DAZ, respectively. It can be clearly seen from Table 2 that the value of averaged attenuation in the 3D model converges to that in the 2D model as the pile length increases.

Further, the area-averaged relative frequency response function (FRF)<sub>r</sub> is introduced:

$$(\text{FRF})_r = 20 \log_{10} \left( \frac{\int_{A_s} \delta_p dA}{\int_{A_s} \delta_s dA} \right) \quad (3)$$

where  $\delta_p$  and  $\delta_s$  are the amplitude of response nodes in periodic pile barriers model and pure soil model, respectively. Figs. 6 and 7 show the (FRF)<sub>r</sub>s at  $z=0$  and  $z=-L/2$ , respectively. Again, it can be seen that the (FRF)<sub>r</sub> inside the DAZ by the 3D model is quite similar to that by the 2D model, especially as the pile length increases to  $L=30$  m. Table 3 shows the averaged relative attenuation in the DAZ for 3D models and 2D model which is defined  $(\overline{\text{FRF}})_r = \frac{\int_{f_s}^{f_e} (\text{FRF})_r df}{f_e - f_s}$ . The discrepancy between the 3D results with  $L=50$  m and the 2D outcomes is less than 10%. Hence, the results obtained based on the assumption of infinite pile length are acceptable when the pile length is sufficiently large.

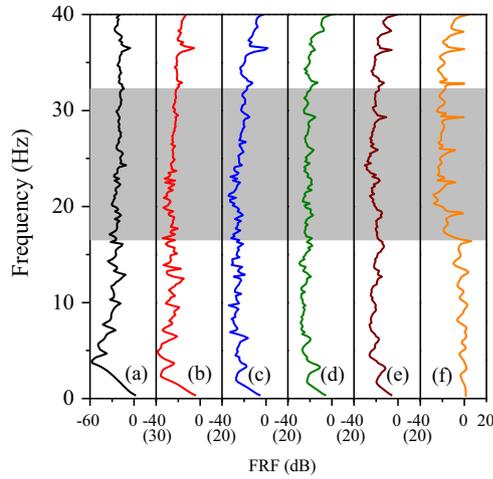


Fig. 7. Area-averaged relative (FRF)<sub>r</sub> (z = -L/2) for the 3D models with L = 10 m (a), L = 20 m (b), L = 30 m (c), L = 40 m (d), L = 50 m (e) and the 2D model (f).

Table 3

Averaged relative attenuation  $(\overline{FRF})_r = \frac{\int_{f_s}^{f_e} (FRF)_r df}{f_e - f_s}$  in the DAZ for 3D models and 2D model.

Models		$(\overline{FRF})_r (z=0)$	$(\overline{FRF})_r (z=-L/2)$
Pile length for 3D model (m)	10	-24.44	-21.77
	20	-21.20	-25.29
	30	-22.57	-24.18
	40	-21.43	-22.00
	50	-19.79	-21.34
2D model		-19.29	-19.29

4. Conclusions

Based on the 2D periodic structure theory, the attenuation zones for out-of-plane waves in periodic pile barriers are calculated by the WFQM. Both 2D and 3D models are employed to simulate the vibration reduction by periodic pile barriers. As the pile length increases, the FRF in the 3D models converges to that in the 2D model. Thus, the 2D model can be used to study the dynamic response of periodic pile barriers with sufficiently large pile length. On the other hand, the 2D model has its limitations as

it cannot take into account the effect of pile length, which is important for effective screening especially at low frequencies.

It should be noted that the conclusions mentioned above are for out-of-plane waves. Some more complicated problems such as the attenuation zones for in-plane waves and Rayleigh waves need to be studied further.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (51178036), the 111 Project (B13002), the Research Fund of Beijing Jiaotong University (2014YJS105), and U.S. Department of Energy (Proj. no. CFA-14-6446). The opinions expressed in this study are those of the authors and do not necessarily reflect the views of the sponsors.

References

- [1] Avilés J, Sánchez-Sesma FJ. Foundation isolation from vibrations using piles as barriers. *J Eng Mech* 1988;114(11):1854–70.
- [2] Cai YQ, Ding GY, Xu CJ. Amplitude reduction of elastic waves by a row of piles in poroelastic soil. *Comput Geotech* 2009;36(3):463–73.
- [3] Xia TD, Sun MM, Chen C, Chen WY, Ping X. Analysis on multiple scattering by an arbitrary configuration of piles as barriers for vibration isolation. *Soil Dyn Earthq Eng* 2011;31(3):535–45.
- [4] Xu B, Xu MQ. Numerical analysis of vibration isolation using pile rows against the vibration due to moving loads in a viscoelastic medium. *Int J Eng Math* 2014;2014:1–12.
- [5] Huang JK, Shi ZF. Attenuation zones of periodic pile barriers and its application in vibration reduction for plane waves. *J Sound Vib* 2013;332(19):4423–39.
- [6] Huang JK, Shi ZF. Vibration reduction of plane waves using periodic in-filled pile barriers. *J Geotech Geoenviron* 2015;141(6):04015018.
- [7] Liu XN, Shi ZF, Xiang HJ, Mo YL. Attenuation zones of periodic pile barriers with initial stress. *Soil Dyn Earthq Eng* 2015;77:381–90.
- [8] Avilés J, Sánchez-Sesma FJ. Piles as barriers for elastic waves. *J Geotech Eng* 1983;109(9):1133–46.
- [9] Kattis SE, Polyzos D, Beskos DE. Vibration isolation by a row of piles using a 3-D frequency domain BEM. *Int J Num Meth Eng* 1999;46(5):713–28.
- [10] Kattis SE, Polyzos D, Beskos DE. Modelling of pile wave barriers by effective trenches and their screening. *Soil Dyn Earthq Eng* 1999;18(1):1–10.
- [11] Lu JF, Xu B, Wang JH. Numerical analysis of isolation of the vibration due to moving loads using pile rows. *J Sound Vib* 2009;319(3):940–62.
- [12] Chen CH, Wang CL. A solution for an isotropic sector under anti-plane shear loadings. *Int J Solids Struct* 2009;46(11–12):2444–52.