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Impact of product proliferation on the reverse supply chain

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ABSTRACT

Product variety is one of the most important advantages in highly competitive markets. However, excessive product proliferation's reducing the profit margin has caused increased focus on developing a management method for maximal profit. In a closed-loop supply chain, product proliferation affects the reverse supply chain as well as the forward supply chain. Although increasing the number of product types can better satisfy diverse customer needs, complexity in the product recycling, remanufacturing, and resale processes may erode a firm's overall profits. In this study, we develop a mathematical model for analyzing a capacitated reverse supply chain consisting of a single manufacturer and multiple retailers. We reveal closed-form solutions for the optimal batch size and maximal profit, and discuss managerial insights into how the number of products and other factors can affect both batch size and profit. Finally, we investigate the relationship between product proliferation and the choice of logistics strategy.

1. Introduction

Rapidly evolving technologies, global competition, and changes in customer needs have contributed to an increase in product variety. According to Lee [1], product proliferation has been one of the most important market trends, and is very common in many industries [2,3,4,5]. For example, in 1992, over 2000 different PC models were available on the market, and between 1990 and 2004, the number of stock-keeping units in supermarkets increased from 16,500 to 25,153 [6].

Product variety can be defined in two ways: the breadth of products that a firm offers at any given time, and the rate at which a firm replaces existing products with new ones. Each of these parameters has steadily increased in many industries [7,8,9]. Firms regard product variety as an important tool of competition as it can better serve heterogeneous market segments and better satisfy diverse consumer preferences, enabling companies to increase or maintain their market share and enjoy higher profits. However, high product variety could also imply increased manufacturing complexity and cost [10,11]. In the past, firms relied solely on experience or intuition to determine the number of products to offer, and consequently tended to underestimate the operational inefficiencies and costs inherent in product variety. Kim and Chhajed [12] indicated that product proliferation may reduce manufacturing/logistics performance. Ramdas and Sawhney [13] claimed that simply increasing product variety does not guarantee an increase in long-term profits, and can, in fact, worsen competitiveness. Therefore, many firms have considered reducing the number of products they offer as a means of improving their supply chain performance. Raleigh [14] noted that Unilever uses its product logic framework to simplify its global home and personal care product portfolio. Yunes et al. [15] studied how the number of configurations for John Deere can be reduced to maximize profit. In both industry and academia, there is an ongoing debate about the cost–benefit tradeoff of product variety [9,16,17,18,19,20]. This uncertainty indicates the importance of carefully managing the number of products a company releases to maximize its profits.

Recent years have also seen increased research on the reverse supply chain due to the rising awareness of environmental protection issues. The reverse supply chain is defined as the series of activities required to retrieve a product from a customer in order to dispose of it or recover its remaining value [21]. The reverse supply chain process can be organized as five sequential key steps: collection of returned product or product acquisition, reverse logistics, inspection and disposition, remanufacturing or reconditioning, and selling and distribution [22]. Product return can be divided into two major types in a reverse supply chain: the return of a commercial product, and end of use (EOU) or end of life (EOL) product returns. In this paper, we focus on commercial product returns that Guide et al. [23] defined as products returned for any reason within 90 day of sale. The overall value of commercial product returns exceeds $100 billion annually in the United States [24], and therefore, management of the flow of

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product returns to maximize profit is a significant concern for many manufacturers.

Product proliferation affects not only the forward supply chain but also the reverse chain. Although increasing the number of products will satisfy diverse customer demands, the increased complexity of product recycling, remanufacturing, and resale may reduce a firm's profitability. Therefore, we developed a mathematical model to analyze the effects of product proliferation on a capacitated reverse supply chain consisting of a single remanufacturer and multiple retailers. We find the closed form solution for the optimal batch size and profit, and discuss managerial insights derived from the closed form solutions. We also explore the relationship between the number of products that a system offers and the logistic strategies in the reverse supply chain.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, we describe our research problem and assumptions in detail. In Section 4, we develop a mathematical model and derive the closed-form solutions. In Section 5, we present our managerial insights based on sensitivity analyses. The relationship between product variety and choice of logistics strategy is discussed in Section 6. Finally, in Section 7, we summarize our findings and discuss promising areas for future research.

2. Literature review

Three streams of literature are relevant to our research. The first examines the effects of batch size on manufacturing lead time. Several researchers have used queuing models to study the effects of batch size on manufacturing lead time and work-in-process costs for a system offering multiple products with setup times [25,26,27,28,29]. These models focus on determining batch sizes that minimize the manufacturing lead time and work-in-process costs. Tielemans and Kuik [30] investigated the relationship between batch size and the mean and variance of time in system. They found that the batch size that minimizes the average time in system will not minimize its variance when the utilization is high. Kuik and Tielemans [31] studied the relationship between batch size and lead time when the utilization is low. Koo et al. [32] explored the batch sizes of a bottleneck machine for maximal profit, and introduced a linear search algorithm for finding the optimal production rate and batch size. All the above-mentioned studies concentrated on batch size impact on the manufacturing lead time and work-in-process inventory in different manufacturing environments for a forward supply chain. In this paper, we focus on the relationship between batch size and reverse supply chain factors such as aggregate return rate, discount rate, number of products, and variance of unit remanufacturing time. Several distinct differences exist between forward and reverse supply chain models. First, in a forward supply chain, if the production line is automated, the process times required for the same product are relatively stable. In fact, many of the aforementioned papers have assumed a constant process time [33–35]. In a reverse supply chain, however, because the conditions of a return product tend to vary significantly, it is not reasonable to assume that the remanufacturing time is constant. Therefore, we assume that the remanufacturing time follows a general distribution. Second, because the conditions of returned products tend to vary greatly, remanufacturing is usually a more flexible process. Therefore, we ignore the setup times between product types.

The second stream studies the effects of product variety on inventory costs. Literature has demonstrated that product variety is an effective strategy for increasing market share because it enables a firm to serve heterogeneous market segments and satisfy various consumer needs. However, product proliferation induces certain operational challenges as well. For example, increasing the number of product types often results in higher inventory costs [9,20,36–38]. Eppen [36] and Zipkin [37] studied the effects of product variety on the performance of a two-level supply chain. Their model demonstrated that performance degrades in proportion to the square root of product variety. Fisher and Ittner [9] and Randall and Ulrich [20] used empirical data to examine the impact of product variety on the automobile and bicycle industries. Benjaafar et al. [38] introduced a multi-item production-inventory system model with finite capacity, and analyzed the effects of product variety on inventory-related costs. Su et al. [39] compared the time postponement (TP) strategy with the form postponement (FP) strategy using both time and cost as their performance matrices. They found that when the number of products increases above a certain threshold, TP is the preferable strategy in both performance matrices. Su et al. [40] compared two supply chain strategies, make-to-order (MTO) and configure-to-order (CTO), to address the challenges of product proliferation. They derived the conditions under which one is better than the other in terms of time and cost. All the above-mentioned papers explored the effects of product variety on forward supply chains using cost as their objective. In this paper, however, we explore the impact of product proliferation on the reverse supply chain. The average time in system for a return product is estimated using queuing theory and optimal batch size, and the best logistics strategy is determined as one that maximizes the time-discounted profit.

The last stream of research explores a closed-loop supply chain, especially for remanufacturing. In recent decades, various strategic and operational aspects of remanufacturing have been investigated, such as inventory control systems [41–43] and reverse channel/network design [44–46]. Several remanufacturing models focus on time value of product return [23] and limited durability and finite life cycles [47]. The primary objective of our study is to optimize the profit of the reverse supply chain by integrating two elements, reverse logistics and time value of product return. The literature most closely related to ours is reviewed as follows. Krumwiede and Sheu [48] developed a decision-making model for guiding an examination of reverse logistics feasibility and for determining whether to involve third-party logistics providers. Savaskan et al. [44] investigated a manufacturer's reverse channel choice in a single-manufacturer, single-retailer supply chain structure. Savaskan and Van Wassenhove [45] extended their own research by considering a competitive retailing environment. We explore the effects of product variety on the logistics strategies of returned products and the number of products as the logistics strategy switching point for determining whether to outsource. Guide et al. [23] claimed that a large proportion of the commercial product value erodes because of long processing delays in the reverse supply chain. They presented a network flow with delay models that includes the marginal value of time required to identify the drivers of a reverse supply chain design. They demonstrated that responsive decentralized return networks must be considered when the discount rate is high. In our model, we focus on commercial returns of multiple products and take into account the time value of the return product to maximize profit.

Two previous papers relate strongly with our study. First, Thonemann and Bradley [35] presented a mathematical model to analyze the effects of product variety on cycle time for a forward supply chain with a single manufacturer and multiple retailers. They demonstrated that the expected replenishment lead time and retailer costs are concave increasing in product variety. The present paper differs from that of Thonemann and Bradley [35] in three ways. First, we formulate our model to maximize profit, whereas Thonemann and Bradley's [35] objective was to minimize total cost. Second, we assume that the remanufacturing time follows a general distribution rather than remaining constant to more closely relate to the reality of a reverse supply chain. Third, we discuss the relationship between the choice of logistics strategy and product proliferation. In the second closely related study, Atasu and Cetinkaya [49] developed minimal cost models for efficient use of returns in making production, inventory, and remanufacturing decisions. They assumed that the value of a returned product decreases rapidly over time. They derived closed-form solutions for the optimal shipment frequency and observed that the fastest reverse supply chain may not always be the most efficient. Our research differs from Atasu and Cetinkaya [49] in four ways. First, we explore a reverse supply chain consisting of multiple products, a single remanufacturer, and multiple retailers, whereas Atasu and Cetinkaya [49] explored a reverse supply chain consisting of a single product, collector, and manufacturer. Second, we assume that the product demand and return rate follow a Poisson distribution rather than remaining constant in order to more authentically represent the reality of a reverse supply chain. Third, we formulate our model to maximize profit, whereas Atasu and Cetinkaya's [49] objective was to minimize the total cost. Finally, we discuss the relationship between the choice of logistics strategy and product proliferation.

3. Problem description

We consider a reverse supply chain in which N retailers retrieve I types of commercial products from consumers to supply a single remanufacturer. The remanufacturer remakes the products using shared resources with a limited capacity. The remanufactured products are sold on the secondary market. To study the effects of product proliferation and prevent other factors from disturbing the effects of the number of products, we assume that all products are homogeneous, with identical prices, return rates, discount rates, shipping costs, holding costs, and remanufacturing costs. We also assume a throttle demand strategy: aggregate demand is independent of the number of products. These two assumptions are commonly found in literature studying the effects of product proliferation, such as the work of Gupta and Srinivasan [50], Thonemann and Bradley [35], and Su et al. [39].

The return rate for each product i, $i=1,2,\ldots I$, at each retailer n, $n=1,2,\ldots N$, is independent and identically distributed. As the decision to return a product is generally randomly and independently made by individual consumers, we assume that the return rates follow a Poisson distribution with mean rate $\lambda_{in}=\lambda_{11}=\lambda_{12}=\ldots=\lambda_{1N}=\ldots=\lambda_{IN}=\lambda$. The aggregate return rate is $\Lambda=\sum_{i=1}^{I}\sum_{n=1}^{N}\lambda_{in}=IN\lambda$.

In addition, we assume that the value of the returned product decreases over time. This phenomenon can be found in computers, communication devices, and consumer electronics, known as the 3C products. These products have short lifecycles due to the progress of technology, producing newer and better products on the market in one or two years, and the value of these products erodes rapidly. Consequently, a remanufacturer must shorten the lead time for a product to return for sale in the secondary market. The discount rate is denoted as β. The higher the value of β, the faster this value erodes.

For this reason, a remanufacturer retrieves products from retailers more frequently to shorten its lead time. However, because the remanufacturer must pay a fixed shipping cost, S_F, which may be the expense of dispatching a truck or salesperson to retrieve a product from the retailer, the remanufacturer cannot afford to collect returned products too frequently. Therefore, the remanufacturer must wait for the quantity of returning products to reach a certain batch size, q, before retrieving them. In our model, the optimal batch size q to maximize discounted profit is used as a decision variable.

S_V is the variable shipping cost, and h is the unit inventory holding cost. A first-come, first-serve policy is used as the priority rule when the returned products arrive at the remanufacturing site. According to Guide et al. [51], the variance in remanufacturing times is higher than in new product manufacturing times. We assume that unit processing time is a random variable following a general distribution with mean processing time t and standard deviation σ. We also assume that the remanufacturing process is flexible. Therefore, product change-over time is so small that we can ignore it. This assumption is suitable in many business scenarios when there is very little tooling change such as the remanufacturing of personal computer or cellular phone.

Given this assumption, the expected remanufacturing time for a batch of q units is qt. The unit remanufacturing variable cost and unit selling price of the remanufactured products are denoted by C_V and P, respectively. The remanufacturer will sell the remanufactured products on the secondary market.

We do not treat price as a decision variable because the remanufacturer does not have power to set the price in many business scenarios. It can only take the price the market dictates. This is the scenario our model represents. The reverse supply chain structure is illustrated in Fig. 1.

4. Model formulation

According to the structure shown in Fig. 1, a mathematical model can be developed using queuing theory.

To develop the model, we first evaluate the expected lead time between the point at which a product is returned to the retailer and that at which the product is sold on the secondary market. This lead time consists of three elements.

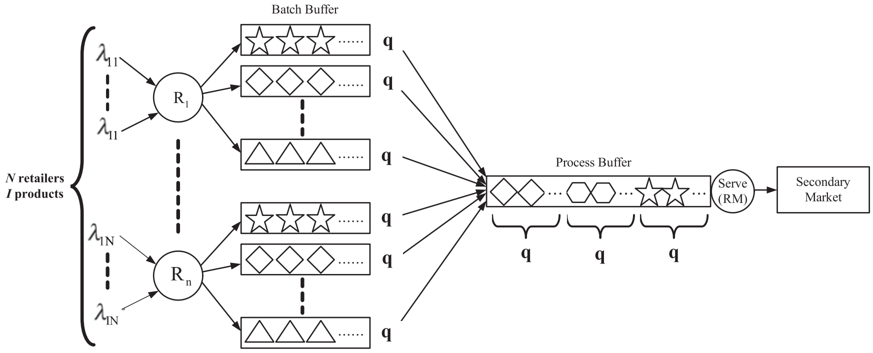

Fig. 1. Reverse supply chain structure.

The first element, E(W_B), is the expected waiting time for q units of a product to be returned to a retailer. The second element, E(W_Q), is the expected waiting time in a queue at the remanufacturing site. The third element, E(W_{RM}), is the expected remanufacturing time for a batch of q units.

Because $\Lambda = \sum_{i=1}^{I}\sum_{n=1}^{N}\lambda_{in} = IN\lambda$, the average return rate is $\lambda = \Lambda/IN$. According to Thonemann and Bradley [35],

$$E(W_B) = \frac{q-1}{2\lambda} = \frac{q-1}{2\Lambda}\times IN.$$

The process of arriving at the remanufacturing site is the superposition of IN identical q-Erlang renewal processes [35]. If IN is large enough, the arrival process can be approximated by the Poisson distribution [52]. Thonemann and Bradley [35] thoroughly analyzed the accuracy of using the Poisson process to approximate the arrival process of superposed Erlang processes by conducting a number of simulation experiments. They observed that improvement in accuracy of the expected replenishment lead time and the expected cost as I increases is obvious. The magnitudes of the errors were less than 20% in the majority of the cases.

On the basis of the above description, the arrival process follows the Poisson distribution, whereas the remanufacturing time follows the general distribution; we can use the M/G/1 model to estimate E(W_Q) using qt as the expected processing time and $q\sigma^2$ as the variance.

$$E(W_Q) = W_{M/G/1} = \frac{L_q}{\lambda} = \frac{\lambda^2(q\sigma^2)+\rho^2}{2\lambda(1-\rho)} = \frac{\lambda(q\mu^2\sigma^2+1)}{2\mu(\mu-\lambda)},$$

where $\lambda=\Lambda/q$ and $\mu=1/qt$. We can obtain

$$E(W_Q) = \frac{\Lambda(\sigma^2+qt^2)}{2(1-\Lambda t)}.$$

The expected remanufacturing time for q units of a product is simply

$$E(W_{RM}) = qt.$$

Therefore, the expected lead time is

$$E(LT) = E(W_B)+E(W_Q)+E(W_{RM}) = \frac{q-1}{2\Lambda}(IN)+\frac{\Lambda(\sigma^2+qt^2)}{2(1-\Lambda t)}+qt. \quad (1)$$

Next, we derive the profit function. Revenue is the unit selling price times the aggregate return rate discounted by the lead time. The revenue is written as

$$P\times\Lambda\times e^{-\beta[(q-1/2\Lambda)(IN)+(\Lambda(\sigma^2+qt^2)/2(1-\Lambda t))+qt]}.$$

Cost can be written as

$$\left(\frac{\Lambda}{q}\times S_F+\Lambda\times S_V\right)+\left(\frac{q-1}{2}\right)\times h\times I\times N+(\Lambda\times C_V).$$

The first term is the shipping cost of a returning product to be sent from the retailer to the remanufacturer. The second term is the holding cost and the last term is the remanufacturing cost.

The profit is equal to revenue minus cost. Let π be the profit function.

$$\pi = \left\{P\times\Lambda\times e^{-\beta\left[\frac{q-1}{2\Lambda}(IN)+\frac{\Lambda(\sigma^2+qt^2)}{2(1-\Lambda t)}+qt\right]}\right\} - \left(\frac{\Lambda}{q}\times S_F+\Lambda\times S_V\right)-\left[\left(\frac{q-1}{2}\right)\times h\times I\times N\right]-(\Lambda\times C_V)$$

We can use a simple numerical approach such as Newton's method to find the optimal batch size given all the parameter values. However, because the primary purpose of our study is to derive managerial insight, we attempt to develop an approximation to find a closed-form solution.

A Taylor expansion is used to eliminate nature exponent e in the profit function. Substitute $e^x=1+x+(x^2/2!)+(x^3/3!)+\ldots$ into the profit function, and the equation can be rewritten as

$$e^{-\beta[(q-1/2\Lambda)(IN)+(\Lambda(\sigma^2+qt^2)/2(1-\Lambda t))+qt]} = 1-\beta\left[\frac{q-1}{2\Lambda}(IN)+\frac{\Lambda(\sigma^2+qt^2)}{2(1-\Lambda t)}+qt\right] + \frac{\beta^2\left[\frac{q-1}{2\Lambda}(IN)+\frac{\Lambda(\sigma^2+qt^2)}{2(1-\Lambda t)}+qt\right]^2}{2!}-\ldots$$

Including more terms will definitely give us a more accurate approximation. However, because annual discount rate β is usually less than 0.2 [53–56], and the expected waiting time E(LT) is usually a value much lower than 1 when the time unit is in years, $\beta[IN(q-1)/2\Lambda+\Lambda(\sigma^2+qt^2)/2(1-\Lambda t)+qt]$ is much smaller than 1. Thus, we include only the first two terms in our approximation. This approach can be found in a number of studies such as Guide et al. [23].

The profit function can therefore be rewritten as follows:

$$\pi = \left\langle P\times\Lambda\times\left\{1-\beta\left[\frac{q-1}{2\Lambda}(IN)+\frac{\Lambda(\sigma^2+qt^2)}{2(1-\Lambda t)}+qt\right]\right\}\right\rangle - \left(\frac{\Lambda}{q}\times S_F+\Lambda\times S_V\right)-\left[\left(\frac{q-1}{2}\right)\times h\times I\times N\right]-(\Lambda\times C_V) \quad (2)$$

Because Eq. (2), the profit function, is concave in q, we can derive optimal batch size and optimal profit.

The optimal batch size and optimal profit are

$$q^* = \sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)}} \quad (3)$$

$$\pi^* = \Lambda(P-S_V-C_V)+\frac{(P\beta+h)IN}{2}-\frac{P\beta\Lambda^2\sigma^2}{2(1-\Lambda t)} - \frac{\sqrt{2S_F}\sqrt{P\Lambda\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN\Lambda(1-\Lambda t)}}{\sqrt{(1-\Lambda t)}}. \quad (4)$$

Note that $S_F \ge \{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}/2\Lambda(1-\Lambda t)$ must be held to ensure that the optimal batch size $q^*\ge 1$. Appendix A contains the proof.

Regarding the accuracy of the Taylor expansion, we verified the accuracy of this approximation by conducting a series of numerical analyses. We used the parameter values P=1000, 1500, 2000; Λ=10,000, 20,000, 30,000; β=0.05, 0.125, 0.2; I=1, 50, 100; and N=1, 50, 100; $\sigma^2=2.6\times10^{-9}$, t=0.000025, h=300, S_F=2000, S_V=70, C_V=75 (units forΛ, β, t, and T are in years). We tested 243 combinations, six of which yielded negative profit. For the remaining 237 combinations, the approximated profit under the Taylor approximation in 94% of the cases deviated less than 2% from actual profit. When P and Λ are lower and β, I, and N are higher, the errors are higher. The average and standard deviation of these errors are −0.59% and 1.81%, respectively. These results indicate that our approximations are adequate.

To illustrate the effect of number of retailers (N) and batch size (q) on the accuracy of the Taylor expansion approximation, we use P=1500, Λ=20000, β=0.125, I=50, N=50, $\sigma^2=2.6\times10^{-9}$, t=0.000025, h=300, S_F=2000, S_V=70, and C_V=75 as our base scenario. In Fig. 2, the actual profits are compared with the approximations. The results demonstrate that the Taylor approximation remains accurate for a broad range of N. For batch size, the Taylor expansion approximation is very accurate around optimal q, the batch size that maximizes the profit. The error becomes larger when the batch size deviates from the optimal q.

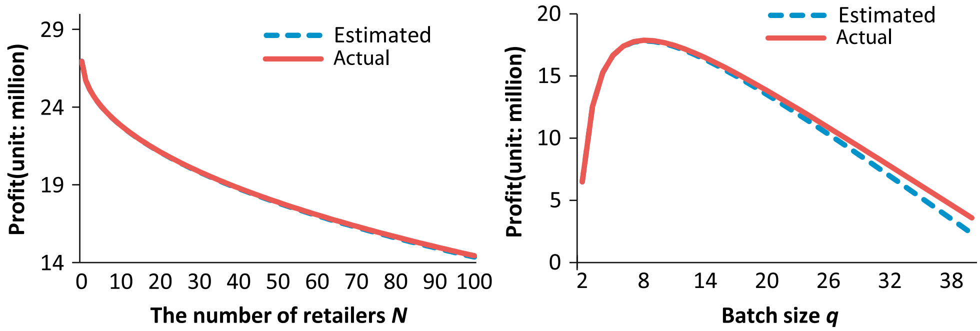

Fig. 2. Approximation accuracy.

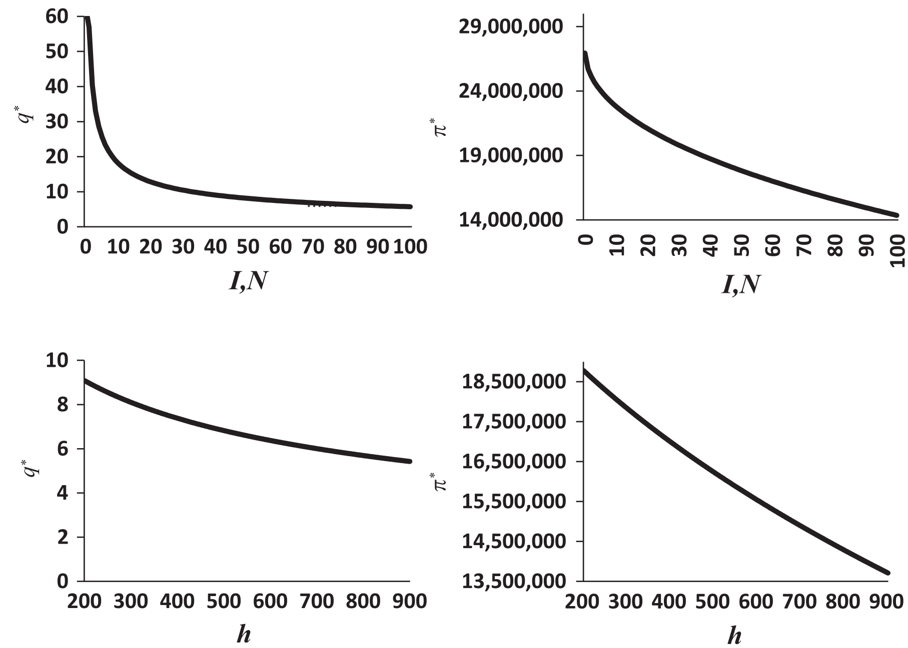

Fig. 3. Optimal batch size and profit as a function of the number of products, number of retailers, and holding cost.

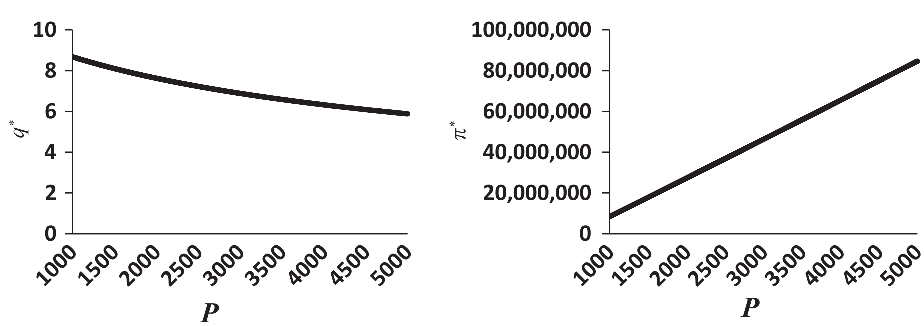

Fig. 4. Optimal batch size and profit as a function of unit selling price of remanufactured products.

5. Sensitivity analysis

In this section, on the basis of a sensitivity analysis of the closed-form solutions, we discuss managerial insights regarding how various factors affect the optimal batch size q^* and profit π^*. Appendix B contains all proofs. We also create one realistic scenario ($P=1500$, $\Lambda=20000$, $\beta=0.125$, $I=50$, $N=50$, $\sigma^2=2.6\times10^{-9}$, $t=0.000025$, $h=300$, $S_F=2000$, $S_V=70$, and $C_V=75$). The results illustrated in Figs. 3–7 are based on the realistic scenario in this section.

Proposition 1. *Both q^* and π^* are convex decreasing with I, N, h.*

In our model, there is no direct product change-over delay or cost. However, product proliferation still erodes a firm's profit margin because when the number of products I increases, the demand for each product decreases because of the throttle demand rate assumption. Therefore, a remanufacturer tends to reduce batch size q^* in order to shorten the waiting time at the retailer. Reducing the batch

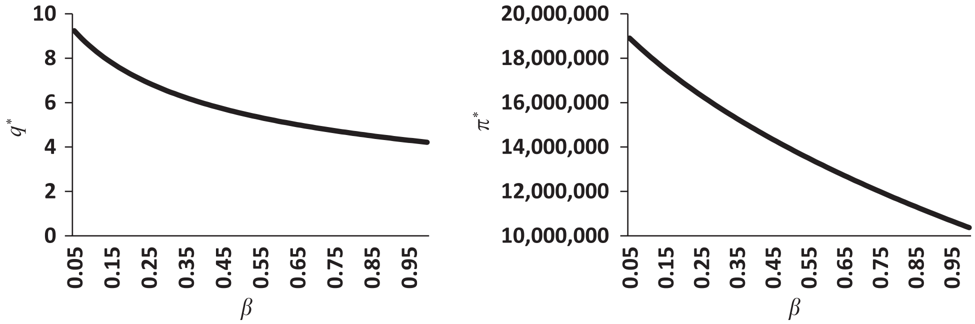

Fig. 5. Optimal batch size and profit as a function of the discount rate.

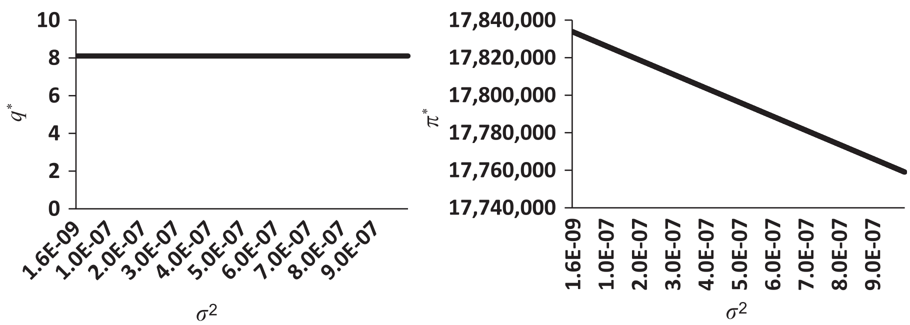

Fig. 6. Optimal batch size and profit as a function of variance of unit processing time.

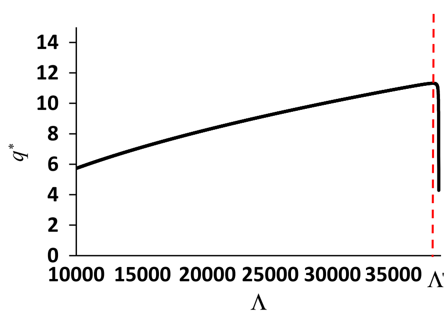

Fig. 7. Optimal batch size as a function of aggregate return rate.

size increases the delivery frequency, consequently increasing the shipping cost and decreasing the optimal profit π^*. The number of retailers N and unit holding cost h have similar effects.

This finding is important because firms formerly determined how many products to offer based solely on experience or intuition, and they underestimated the cost of product variety in the reverse supply chain. From Proposition 1, product proliferation erodes profits severely when high product variety cannot lead to higher demand. In contrast, when product variety is high and a firm does not reduce the batch size to shorten the waiting time at the retailer, a significant portion of the value erodes. For example, according to Guide et al. [23], the return process for HP inkjet printers takes at least 74 day on average. This impact is tremendous when the product lifecycle is short. Thus, firms must carefully manage their product proliferation in order to extract value from the commercial returns.

Proposition 2. *q^* and π^* are convex decreasing and increasing with P, respectively.*

When the price is higher, the remanufacturer has a higher incentive to accelerate the return process in order to prevent a large portion of the value from eroding. Therefore, the batch size is convex decreasing with P. In contrast, a higher unit selling price leads to a higher profit.

Proposition 3. *Both q^* and π^* are convex decreasing with β.*

Similar to Proposition 2, when the discount rate is higher, the remanufacturer tends to reduce the batch size in order to accelerate the return process and prevent a large portion of the value from eroding. Considering optimal profit π^* a high discount rate means a larger portion of the value is eroded, which consequently leads to a lower profit.

The implications suggested by Propositions 2 and 3 are particularly valuable for fast-clockspeed industries such as the 3C or fashion industries. Managers in these industries must consider the time effect in order to prevent a large portion of their value from eroding. In general, a smaller batch size should be used to accelerate the return process. Propositions 2 and 3 also suggest that a firm must assign higher return priorities for those products with higher prices, and/or higher discount rates, in order to recover more of their value.

Proposition 4. *q^* is independent of σ^2, and π^* linearly decreases with σ^2.*

The optimal batch size q^* is independent of the variance of unit processing time σ^2. This result is driven by the use of a Taylor expansion to eliminate nature exponent e in the profit function. However, it does demonstrate that in most practical scenarios, a variation of unit processing time does not significantly affect the batch size. For π^*, a higher variation leads to a linear decrease in optimal profit because of the longer lead time.

Proposition 5. *q^* is first increasing at a decreasing rate, then decreasing at an increasing rate with Λ.*

Optimal batch size increases at a decreasing rate up to threshold aggregate return level Λ', and then decrease at an increasing rate.

By taking the first derivative of q^* given in Eq. (3) with respect to aggregate return rate Λ and setting it to zero, we can obtain

$$\Lambda' = \frac{\sqrt{IN(P\beta+h)/IP\beta}}{t\left[1+\sqrt{IN(P\beta+h)/IP\beta}\right]}.$$

When the aggregate return rate is relatively small, it might take a very long time to fill the buffer area. Therefore, remanufacturers tend to reduce the buffer size in order to shorten the lead time, and the batch size becomes larger as the demand increases; when the aggregated demand rate exceeds a threshold level, the remanufacturer no longer needs to worry about the time required to fill the buffer area. Instead, it can decrease the batch size and increase the delivery frequency, minimizing the probability of an idle remanufacturing facility.

6. Product variety and logistics strategy

In this section, we explore the effects of product variety on the logistics strategies of returned products. Assume that the logistics of a returned product can be handled either in-house or outsourced, for example, using a third-party service provider such as FedEx or UPS. How does the number of offered products affect the logistics strategy?

If a remanufacturer adopts an outsourcing strategy, the fixed shipping cost S_F will be zero. Let S_V^O be the unit variable shipping cost of the outsourcing strategy, and π_O^* be the optimal profit of adopting this strategy. The optimal profit of an in-house strategy is denoted by π_I^*, which is always better than π_O^* when $S_V^O > S_F + S_V$. Also, when $S_V^O < S_F/q + S_V$, π_O^* is always better than π_I^*. In both scenarios, the choice of logistics strategy is obvious. Thus, we assume that $S_F + S_V > S_V^O > S_F/q + S_V$. Using an approach similar to deriving Eq. (4), we obtain

$$\pi_O^* = \left\langle P\times\Lambda\times\left\{1-\beta\left[\frac{\Lambda(\sigma^2+t^2)}{2(1-\Lambda t)+t}\right]\right\}\right\rangle - (\Lambda\times S_V^O) - (\Lambda\times C_V). \quad (5)$$

Because Proposition 1 states that π_I^* is convex decreasing with I, and π_O^* is independent of I from Eq. (5), the remanufacturer is therefore more inclined to adopt an in-house strategy when the number of products offered is small and adopts an outsourcing strategy when the number of products offered is large. As illustrated in Fig. 8, the remanufacturer switches its logistic strategy from in-house to outsourcing when the number of products offered is larger than I^S, which we call the logistics strategy switching point.

Letting Eq. (4) equal Eq. (5), we can derive I^S as follows:

$$I^S = \frac{1}{N(P\beta+h)(1-\Lambda t)} \times \left\{ \begin{array}{c} [P\beta\Lambda t(2-\Lambda t) - 2\Lambda(1-\Lambda t)(S_V^O - S_V - 2S_F)] \\ \hline -4\Lambda\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O - S_V - S_F)]} \end{array} \right\}. \quad (6)$$

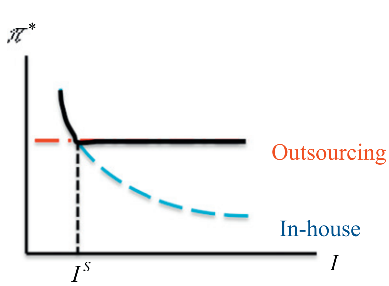

Fig. 8. Illustration of logistics strategy switching point.

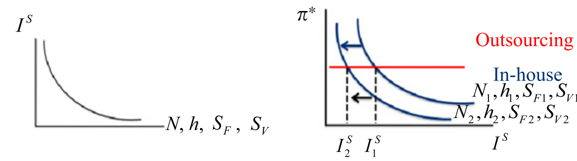

Fig. 9. Logistics strategy switching point based on the number of retailers and the holding, fixed shipping, and variable shipping costs.

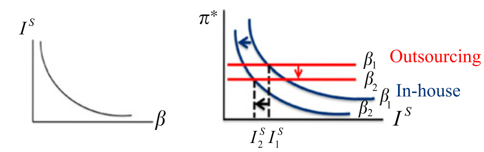

Fig. 10. Logistics strategy switching point based on discount rate.

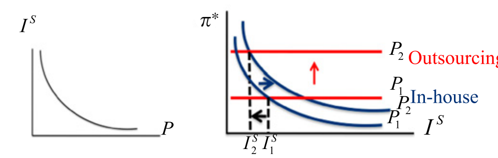

Fig. 11. Logistic strategy switching point based on the unit selling price of remanufactured products.

Next, we will discuss how the number of retailers, holding cost, shipping cost, discount rate, and unit selling price of remanufactured products affect switching point I^S. Appendix C contains all proofs.

Proposition 6. *I^S is convex decreasing with N, h, S_F, S_V.*

I^S decreases when the number of retailers N increases because the optimal profit of outsourcing strategy π_O^* is unchanged and the optimal profit of in-house strategy π_I^* decreases as N increases. Thus, I^S shifts to the left. Unit holding cost h fixed shipping cost S_F and variable shipping cost S_V have a similar effect as shown in Fig. 9. On the basis of Proposition 6, firms switch to an outsourcing strategy when the number of products offered is small, as the number of retailers, unit holding cost, fixed shipping cost, and/or variable shipping cost increase. Managers can use this principle to determine whether to adopt an outsourcing strategy to maximize their firms' profit margin.

Proposition 7. *I^S is convex decreasing with β.*

I^S decreases when discount rate β increases. When the discount rate increases, both the optimal profit of in-house strategy π_I^* and the optimal profit of outsourcing strategy π_O^* decrease as shown in Fig. 10. On the basis of our proof, the optimal profit of an in-house strategy decreases more than that of an outsourcing strategy because the lead time of an in-house strategy is longer, thereby eroding more profit. Thus, an increase in discount rate shifts I^S to the left.

Proposition 8. *I^S is convex decreasing with P.*

I^S decreases when the unit selling price of remanufactured products P increases as shown in Fig. 11. When P increases, the optimal profits of both in-house and outsourcing strategies also increase. From our proof, the optimal profit of an outsourcing strategy increases more than that of an in-house strategy because the lead time of an outsourcing strategy is shorter, thereby eroding less profit. Thus, an increase in P shifts I^S to the left.

From Propositions 2 and 3, we demonstrate that managers in fast-clockspeed industries tend to use smaller batch sizes and assign higher return priorities for products with higher prices and larger discount rates. In Propositions 7 and 8, we find that managers in these industries tend to switch to an outsourcing strategy when the number of products offered is smaller for products with higher prices and larger discount rates.

7. Conclusions and future research

Firms regard product variety as an important tool for gaining a competitive advantage and enjoying greater profits, as such variety can expand their market share by satisfying diverse customer needs. However, because high product variety can increase their manufacturing complexity and costs, the number of products offered by a company must be managed carefully for maximal profit.

Because commercial product returns have significant value, manufacturers must focus on reverse supply chain management to extract most of this value. Therefore, in this paper, we explore the effects of product variety on the reverse supply chain. A mathematical model based on queuing theory is developed, and closed-form solutions of optimal profit and batch size are ascertained.

Our research findings offer several managerial implications. First, past literature has demonstrated that product variety reduces profits, primarily because of change-over delay or cost. We demonstrate that, in the reverse supply chain, even without product change-over delay or cost, product proliferation still erodes firm's profit due to the increase in shipping cost. If there is product change-over delay or cost, the firm should be even more cautious about product proliferation. That is, managers must carefully evaluate the cost of product variety. Second, managers tend to reduce their optimal batch size when the number of products, number of retailers, holding costs, price, and/or discount rates are larger. A variance of unit processing time does not significantly affect batch size. Batch size will first increase at a decreasing rate up to a threshold aggregated return level, and then decrease at an increasing rate with respect to the aggregated return rate. Third, we demonstrated that the number of products offered is an important determinant for a firm's logistics strategy. Managers dealing with products with higher prices and larger discount rates tend to switch to an outsourcing strategy when the number of products offered is smaller.

A number of areas can be investigated further on the basis of our study. First, our research is based on a theoretical model. A meaningful extension of our work would be to apply it to a real world case to confirm the managerial insights derived from this paper. Second, market competition has not been considered in our model. Our model can be extended to incorporate competition to study how such competition changes the dynamics of the reverse supply chain. Third, our model can be extended to use a multi-class queue if differentiation between product types is important. The analysis of multi-class queue has been discussed in Buzacott and Shanthikumar [57]. The M/G/1 multiple-class backlogged demand FCFS model can be used to estimate the number of type i jobs. Fourth, we do not take cannibalization between products into account, because the main reason we formulate our problem as profit maximization is to capture the impact of time value. Although cannibalization between products is an important issue in revenue management, this study does not focus on revenue management but on how product proliferation affects the logistics of the remanufacturing system. Even without cannibalization, product proliferation erodes profit because of increased shipping cost. Therefore, if there is cannibalization, the firm should be even more cautious about product proliferation. Nonetheless, developing a more sophisticated model that captures cannibalization's impact presents an opportunity for very challenging and interesting future research.

Appendix A

The procedures for deriving closed-form solutions of optimal batch size and maximal profit are described below. By taking the first and second derivatives of profit with respect to batch size q, we have the following:

$$\frac{\partial\pi}{\partial q} = -P\Lambda\beta\left[\frac{IN}{2\Lambda}+\frac{\Lambda t^2}{2(1-\Lambda t)}+t\right]-\frac{1}{2}hIN+\frac{\Lambda S_F}{q^2}, \tag{A.1}$$

$$\frac{\partial^2\pi}{\partial q^2} = \frac{-2\Lambda S_F}{q^3}. \tag{A.2}$$

Then, setting Eq. (A.1) equal to zero and solving for batch size q yields the following solutions:

$$q_1^* = \sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)}}, \tag{A.3}$$

$$q_2^* = -\sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)}}. \tag{A.4}$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, and both q_2^* and Eq. (A.2) are negative. Thus, q_1^* is the optimal batch size for maximal profit. Substituting Eq. (A.3) into Eq. (2), we obtain Eq. (4).

The fixed shipping cost, $S_F \geq \{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}/2\Lambda(1-\Lambda t)$, must be held to ensure that the optimal batch size $q^*\geq 1$.

Appendix B

Proposition 1. *Both q^* and π^* are convex decreasing with I, N, h.*

Proof. Taking the first and second derivatives of q^* and π^* given in Eqs. (3) and (4), each with respect to I, N, and h, we obtain the following:

$$\frac{\partial q^*}{\partial I} = -\frac{N(1-\Lambda t)(P\beta+h)\sqrt{2\Lambda(1-\Lambda t)S_F}}{2\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3},$$

$$\frac{\partial^2 q^*}{\partial I^2} = \frac{3\left[N(1-\Lambda t)(P\beta+h)\right]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^5},$$

$$\frac{\partial \pi^*}{\partial I} = \frac{(P\beta+h)N}{2}\left[1-\sqrt{\frac{2\Lambda(1\text{-}\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)}}\right],$$

$$\frac{\partial^2 \pi^*}{\partial I^2} = \frac{(P\beta+h)^2N^2(1-\Lambda t)\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3},$$

$$\frac{\partial q^*}{\partial N} = -\frac{I(1-\Lambda t)(P\beta+h)\sqrt{2\Lambda(1-\Lambda t)S_F}}{2\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3},$$

$$\frac{\partial^2 q^*}{\partial N^2} = \frac{3\left[I(1-\Lambda t)(P\beta+h)\right]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^5},$$

$$\frac{\partial \pi^*}{\partial N} = \frac{(P\beta+h)I}{2}\left[1-\sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)}}\right],$$

$$\frac{\partial^2 \pi^*}{\partial N^2} = \frac{(P\beta+h)^2I^2(1-\Lambda t)\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3},$$

$$\frac{\partial q^*}{\partial h} = -\frac{IN(1-\Lambda t)\sqrt{2\Lambda(1-\Lambda t)S_F}}{2\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3},$$

$$\frac{\partial^2 q^*}{\partial h^2} = \frac{3[IN(1-\Lambda t)]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^5},$$

$$\frac{\partial \pi^*}{\partial h} = \frac{IN}{2}\left[1-\sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta\left[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2\right]+hIN(1-\Lambda t)}}\right],$$

$$\frac{\partial^2 \pi^*}{\partial h^2} = \frac{I^2N^2(1-\Lambda t)\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3}.$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, and we thus have

$$\frac{\partial q^*}{\partial I}<0,\ \frac{\partial^2 q^*}{\partial I^2}>0,\ \frac{\partial q^*}{\partial N}<0,\ \frac{\partial^2 q^*}{\partial N^2}>0,\ \frac{\partial q^*}{\partial h}<0,\ \frac{\partial^2 q^*}{\partial h^2}>0.$$

The optimal batch size is $q^*>1$, which implies that $1-q^*<0$. From Eq. (A.3), we can obtain

$$1-\sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)}}<0.$$

Thus, we have

$$\frac{\partial \pi^*}{\partial I}<0,\frac{\partial^2 \pi^*}{\partial I^2}>0,\ \frac{\partial \pi^*}{\partial N}<0,\ \frac{\partial^2 \pi^*}{\partial N^2}>0,\ \frac{\partial \pi^*}{\partial h}<0,\ \frac{\partial^2 \pi^*}{\partial h^2}>0.$$

That is, both q^* and π^* are convex decreasing with I, N, and h. Proposition 1 is thus proved.

Proposition 2. *q^* and π^* are convex decreasing and increasing with P, respectively.*

Proof. Taking the first and second derivatives of q^* and π^* given in Eqs. (3) and (4), each with respect to P, we obtain the following:

$$\frac{\partial q^*}{\partial P} = -\frac{\beta\left[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2\right]\sqrt{2\Lambda(1-\Lambda t)S_F}}{2\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}}^3},$$

$$\frac{\partial^2 q^*}{\partial P^2} = \frac{3\beta^2[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]+hIN(1-\Lambda t)\}^5}},$$

$$\frac{\partial \pi^*}{\partial P} = \Lambda \times \left\{1-\beta\left[\frac{q^*-1}{2\Lambda}(IN)+\frac{\Lambda(\sigma^2+q^* t^2)}{2(1-\Lambda t)}+q^* t\right]\right\},$$

$$\frac{\partial^2 \pi^*}{\partial P^2} = \frac{\beta^2[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4(1-\Lambda t)\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]+hIN(1-\Lambda t)\}^3}}. \quad \square$$

The utilization rate is $\rho < 1$, which implies that $(1-\Lambda t) > 0$. In addition, the other parameters are all greater than zero, and we thus have

$$\frac{\partial q^*}{\partial P} < 0, \frac{\partial^2 q^*}{\partial P^2} > 0.$$

Because $\beta[IN(q^*-1)/2\Lambda+\Lambda(\sigma^2+q^* t^2)/2(1-\Lambda t)+q^* t]$ is far less than 1, we have

$$\frac{\partial \pi^*}{\partial P} > 0, \ \frac{\partial^2 \pi^*}{\partial P^2} > 0$$

That is, q^* is convex decreasing with P, and π^* is convex increasing with P. Proposition 2 is thus proved.

Proposition 3. *Both q^* and π^* are convex decreasing with β.*

Proof. Taking the first and second derivatives of q^* and π^* given in Eqs. (3) and (4), each with respect to β, we obtain the following:

$$\frac{\partial q^*}{\partial \beta} = -\frac{P[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]\sqrt{2\Lambda(1-\Lambda t)S_F}}{2\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]+hIN(1-\Lambda t)\}^3}},$$

$$\frac{\partial^2 q^*}{\partial \beta^2} = \frac{3P^2[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]+hIN(1-\Lambda t)\}^5}},$$

$$\frac{\partial \pi^*}{\partial \beta} = \frac{PIN(1-\Lambda t)(1-q^*)-P\Lambda t q^*(2-\Lambda t)-P\Lambda^2\sigma^2}{2(1-\Lambda t)},$$

$$\frac{\partial^2 \pi^*}{\partial \beta^2} = \frac{P^2[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]^2\sqrt{2\Lambda(1-\Lambda t)S_F}}{4(1-\Lambda t)\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]+hIN(1-\Lambda t)\}^3}}.$$

The utilization rate is $\rho < 1$, which implies that $(1-\Lambda t) > 0$. In addition, all other parameters are greater than zero, giving us

$$\frac{\partial q^*}{\partial \beta} < 0, \ \frac{\partial^2 q^*}{\partial \beta^2} > 0.$$

The optimal batch size is $q^* > 1$, which implies that $1-q^* < 0$. In addition, all other parameters are greater than zero, and $(2-\Lambda t) > 0$. Thus, we obtain

$$\frac{\partial \pi^*}{\partial \beta} < 0, \ \frac{\partial^2 \pi^*}{\partial \beta^2} > 0.$$

That is, both q^* and π^* are convex decreasing with β. Proposition 3 is thus proved.

Proposition 4. *q^* is independent of σ^2, and π^* linearly decreases with σ^2.*

Proof. Because

$$q^* = \sqrt{\frac{2\Lambda(1-\Lambda t)S_F}{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2 t^2]+hIN(1-\Lambda t)}}$$

does not include σ^2, q^* is independent of σ^2. □

Taking the first and second derivatives of π^* given in Eq. (4) with respect to σ^2, we obtain

$$\frac{\partial \pi^*}{\partial \sigma^2} = -\frac{P\beta\Lambda^2}{2(1-\Lambda t)}, \text{ and}$$

$$\frac{\partial^2 \pi^*}{\partial \sigma^{22}} = 0.$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, giving us

$$\frac{\partial \pi^*}{\partial \sigma^2}<0$$

That is, q^* is independent of σ^2, and π^* linearly decreases with σ^2. Proposition 4 is thus proved.

Proposition 5. *q^* is first increasing at a decreasing rate, then decreasing at an increasing rate with Λ.*

Proof. Taking the first and second derivatives of q^*given in Eq. (3) with respect to Λ, we obtain

$$\frac{\partial q^*}{\partial \Lambda}=\frac{[IN(P\beta+h)(1-\Lambda t)^2-P\beta\Lambda^2t^2]\sqrt{S_F}}{\sqrt{2\Lambda(1-\Lambda t)}\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}^3}},$$

$$\begin{aligned}\frac{\partial^2 q^*}{\partial \Lambda^2}=&\frac{-I^2N^2(P\beta+h)^2(1-\Lambda t)^4\sqrt{S_F}-P^2\beta^2\Lambda^4t^4(5-4\Lambda t)\sqrt{S_F}}{\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}^5}\sqrt{[2\Lambda(1-\Lambda t)]^3}}\\&-\frac{2P\beta IN\Lambda t(P\beta+h)(1-\Lambda t)[2\Lambda^3t^3+5\Lambda t(1-\Lambda t)+4(1-\Lambda t)^3]\sqrt{S_F}}{\sqrt{\{P\beta[(IN+2\Lambda t)(1-\Lambda t)+\Lambda^2t^2]+hIN(1-\Lambda t)\}^5}\sqrt{[2\Lambda(1-\Lambda t)]^3}}.\end{aligned}$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, and we obtain $\partial q^*/\partial\Lambda>0$ or $\partial q^*/\partial\Lambda<0$, and $\partial^2q^*/\partial\Lambda^2<0$.

That is, q^* is first increasing at a decreasing rate, then decreasing at an increasing rate with Λ. Set $\partial q^*/\partial\Lambda$ to zero, and we can obtain,

$$\Lambda'=\frac{\sqrt{IN(P\beta+h)/IP\beta}}{t[1+\sqrt{IN(P\beta+h)/IP\beta}]}$$

Proposition 5. is thus proved.

9. Appendix C

Proposition 6. *I^S is convex decreasing with N, h,S_F,S_V.*

Proof. Taking the first and second derivatives of I^S given in Eq. (6) with respect to N, h, S_F, and S_V, we obtain

$$\frac{\partial I^S}{\partial N}=\frac{4\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}-[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]}{N^2(P\beta+h)(1-\Lambda t)},$$

$$\frac{\partial^2 I^S}{\partial N^2}=\frac{2[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-8\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}}{N^3(P\beta+h)(1-\Lambda t)},$$

$$\frac{\partial I^S}{\partial h}=\frac{4\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}-[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]}{N(P\beta+h)^2(1-\Lambda t)},$$

$$\frac{\partial^2 I^S}{\partial h^2}=\frac{2[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-8\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}}{N(P\beta+h)^3(1-\Lambda t)},$$

$$\frac{\partial I^S}{\partial S_F}=\frac{2\Lambda\left\{\sqrt{4(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}-[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-2S_F)]\right\}}{N(P\beta+h)\sqrt{(1-\Lambda t)S_F\left[P\beta t(2-\Lambda t)-(1-\Lambda t)\left(S_V^O-S_V-S_F\right)\right]}},$$

$$\frac{\partial^2 I^S}{\partial {S_F}^2}=\frac{-\Lambda}{N(P\beta+h)\sqrt{(1-\Lambda t)}}\left\langle\frac{4(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]-[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-2S_F)]^2}{\sqrt{\{S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]\}^3}}\right\rangle,$$

$$\frac{\partial I^S}{\partial S_V}=\frac{2\Lambda\left\{\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}-(1-\Lambda t)S_F\right\}}{N(P\beta+h)\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}},$$

$$\frac{\partial^2 I^S}{\partial {S_V}^2}=\frac{\Lambda(1-\Lambda t)^3{S_F}^2}{N(P\beta+h)\sqrt{\{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]\}^3}}.\quad\square$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, and $S_F+S_V>S_V^O$. We therefore obtain

$$4\Lambda\sqrt{(1\text{-}\Lambda t)S_F[P\beta t(2-\Lambda t)\text{-}(1\text{-}\Lambda t)(S_V^O-S_V\text{-}S_F)]}>0,$$
$$[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1\text{-}\Lambda t)(S_V^O-S_V-2S_F)]>0.$$

Because $I^S>0$, we can see that

$$[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]>4\Lambda\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}.$$

Therefore, we have

$$\frac{\partial I^S}{\partial N}<0,\ \frac{\partial^2 I^S}{\partial N^2}>0,\ \frac{\partial I^S}{\partial h}<0,\ \frac{\partial^2 I^S}{\partial h^2}>0,\frac{\partial I^S}{\partial S_F}<0,\ \frac{\partial^2 I^S}{\partial {S_F}^2}>0,\ \frac{\partial I^S}{\partial S_V}<0,\ \frac{\partial^2 I^S}{\partial {S_V}^2}>0.$$

That is, I^S is convex decreasing with N, h, S_F, and S_V. Proposition 6 is thus proved.

Proposition 7. *I^S is convex decreasing with β.*

Proof. Taking the first and second derivatives of I^Sgiven in Eq. (6) with respect to β, we obtain the following:

$$\frac{\partial I^S}{\partial \beta}=\frac{P}{N(1-\Lambda t)(P\beta+h)^2}\times\left\langle\begin{array}{l}\Lambda t(2-\Lambda t)(P\beta+h)\left[1-\sqrt{\dfrac{4(1-\Lambda t)S_F}{P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)}}\right]\\-\{[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-4\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}\}\end{array}\right\rangle,$$

$$\frac{\partial^2 I^S}{\partial \beta^2}=\frac{-2P^2}{N(1-\Lambda t)(P\beta+h)^3}\times\left\langle\begin{array}{l}\Lambda t(2-\Lambda t)(P\beta+h)\left[1-\sqrt{\dfrac{4(1-\Lambda t)S_F}{P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)}}\right]\\-\{[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-4\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}\}\end{array}\right\rangle$$
$$+\frac{P}{N(1-\Lambda t)(P\beta+h)^2}\left\{P\Lambda t^2(2-\Lambda t)^2(P\beta+h)\times\sqrt{\frac{(1-\Lambda t)S_F}{P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)}}\right\}.\quad\square$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, and $4(1-\Lambda t)S_F\geq P\beta t(2-\Lambda t)-(1-\Lambda t)\left(S_V^O-S_V-S_F\right)$. Using a similar approach to that proving Proposition 6, we can obtain

$$4\Lambda\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}>0,$$
$$[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]>0,$$
$$[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-4\Lambda\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}>0.$$

Therefore, we have

$$\frac{\partial I^S}{\partial \beta}<0,\ \frac{\partial^2 I^S}{\partial \beta^2}>0$$

That is, I^S is convex decreasing with β. Proposition 7 is thus proved.

Proposition 8. *I^S is convex decreasing with P.*

Proof. Taking the first and second derivatives of I^S given in Eq. (6) with respect to P, we obtain the following:

$$\frac{\alpha I^S}{\alpha P}=\frac{\beta}{N(1-\Lambda t)(P\beta+h)^2}\times\left\langle\begin{array}{l}\Lambda t(2-\Lambda t)(P\beta+h)\left[1-\sqrt{\dfrac{4(1-\Lambda t)S_F}{P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)}}\right]\\-\{[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-4\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}\}\end{array}\right\rangle,$$

$$\frac{\alpha^2 I^S}{\alpha P^2}=\frac{-2\beta^2}{N(1-\Lambda t)(P\beta+h)^3}\times\left\langle\begin{array}{l}\Lambda t(2-\Lambda t)(P\beta+h)\left[1-\sqrt{\dfrac{4(1-\Lambda t)S_F}{P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)}}\right]\\-\{[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-4\Lambda\sqrt{S_F(1-\Lambda t)[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}\}\end{array}\right\rangle$$

$$+\frac{\beta}{N(1-\Lambda t)(P\beta+h)^2}\left\{\beta\Lambda t^2(2-\Lambda t)^2(P\beta+h)\times\sqrt{\frac{(1-\Lambda t)S_F}{P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)}}\right\}. \quad \square$$

The utilization rate is $\rho<1$, which implies that $(1-\Lambda t)>0$. In addition, all other parameters are greater than zero, and $4(1-\Lambda t)S_F \geq P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)$. Using a similar approach to that proving Proposition 6, we can obtain

$$4\Lambda\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}>0,$$

$$[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]>0,$$

$$[P\beta\Lambda t(2-\Lambda t)-2\Lambda(1-\Lambda t)(S_V^O-S_V-2S_F)]-4\Lambda\sqrt{(1-\Lambda t)S_F[P\beta t(2-\Lambda t)-(1-\Lambda t)(S_V^O-S_V-S_F)]}>0.$$

Therefore, we have

$$\frac{\partial I^S}{\partial P}<0, \quad \frac{\partial^2 I^S}{\partial P^2}>0$$

That is, I^S is convex decreasing with P. Proposition 8 is thus proved.

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