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Many aerospace, civil and mechanical systems continue to be used despite ageing and the associated potential for damage accumulation. Therefore, the ability to monitor the structural health of these systems is becoming increasingly important. A wide variety of highly effective local non-destructive evaluation tools is available. However, damage identification based upon changes in vibration characteristics is one of the few methods that monitor changes in the structure on a global basis. A summary of developments in the field of global structural health monitoring that have taken place over the last thirty years is first presented. Vibration-based damage detection is a primary tool that is employed for this monitoring. Next, the process of vibration-based damage detection will be described as a problem in statistical pattern recognition. This process is composed of three portions: (i) data acquisition and cleansing; (ii) feature selection and data compression; and (iii) statistical model development. Current research regarding feature selection and statistical model development will be emphasized with the application of this technology to a large-scale laboratory structure.

Keywords: damage detection; structural health monitoring; statistical pattern recognition

1. Introduction

In the most general terms, damage can be defined as changes introduced into a system that adversely affect the current or future performance of that system. Implicit in this definition is the concept that damage is not meaningful without a comparison between two different states of the system, one of which is assumed to represent the initial, and often undamaged, state. This discussion is focused on the study of damage identification in structural and mechanical systems. Therefore, the definition of damage will be limited to changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the current or future performance of the systems.

The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical and aerospace engineering communities. Current damage-detection methods are either visual or localized experimental methods such as acoustic or ultrasonic methods, magnetic field methods, radiograph, eddy-current methods and thermal field methods (Doherty 1987). All of these experimental techniques require that the vicinity of the damage is known *a priori* and that the portion of the structure being inspected is readily accessible. Subjected to these limitations, these experimental methods can detect damage on or near the surface of the structure. The need for quantitative global damage-detection methods that can be applied to complex structures has led to the development of, and continued research into, methods that examine changes in the vibration characteristics of the structure.

The basic premise of vibration-based damage detection is that the damage will alter the stiffness, mass or energy dissipation properties of a system, which, in turn, will alter the measured dynamic response of the system. Although the basis for vibration-based damage detection appears intuitive, its actual application poses many significant technical challenges. The most fundamental challenge is the fact that damage is typically a local phenomenon and may not significantly influence the lower-frequency global response of a structure that is typically measured during vibration tests. This challenge is supplemented by many practical issues associated with making accurate and repeatable vibration measurements at a limited number of locations on structures often operating in adverse environments.

In an effort to emphasize the extent of the research efforts in vibration-based damage detection, a brief summary of applications that have driven developments in this field over the last thirty years is first presented. Recent research has begun to recognize that the vibration-based damage-detection problem is fundamentally one of statistical pattern recognition and this paradigm is described in detail. Current damage-detection methods are then summarized in the context of this paradigm and an application of the statistical pattern recognition methodology is presented.

2. Historical perspective

It is the authors' speculation that damage or fault detection, as determined by changes in the dynamic properties or response of systems, has been practised in a qualitative manner, using acoustic techniques, since modern man has used tools. More recently, this subject has received considerable attention in the technical literature and a brief summary of the developments in this technology over the last thirty years is presented below. Specific references are not cited; instead the reader is referred to Doebling *et al*. (1998) for a review of literature on this subject.

The development of vibration-based damage-detection technology has been closely coupled with the evolution, miniaturization and cost reductions of fast Fourier transform (FFT) analyser hardware and computing hardware. To date, the most successful application of vibration-based damage-detection technology has been for monitoring rotating machinery. The rotating machinery application has taken an almost exclusive non-model based approach to damage detection. The detection process is based on pattern recognition applied to time histories or spectra generally measured on the housing of the machinery during normal operating conditions. Databases have been developed that allow specific types of damage to be identified from particular features of the vibration signature. For these systems, the approximate location of the damage is generally known making a single-channel FFT analyser sufficient for most periodic monitoring activities. Today, commercial software integrated with measurement hardware is marketed to help the user systematically apply this technology to operating equipment.

During the 1970s and 1980s, the oil industry made considerable efforts to develop vibration-based damage-detection methods for offshore platforms. This damage-detection problem is fundamentally different from that of rotating machinery because the damage location is unknown and because the majority of the structure is not readily accessible for measurement. To circumvent these difficulties, a common methodology adopted by this industry was to simulate candidate damage scenarios with numerical models, examine the changes in resonant frequencies that were produced by these simulated changes, and correlate these changes with those measured on a platform. A number of very practical problems were encountered including measurement difficulties caused by platform machine noise, instrumentation difficulties in hostile environments, changing mass caused by marine growth and varying fluid storage levels, temporal variability of foundation conditions, and the inability of wave motion to excite higher modes. These issues prevented adaptation of this technology, and efforts at further developing this technology for offshore platforms were largely abandoned in the early 1980s.

The aerospace community began to study the use of vibration-based damage detection during the late 1970s and early 1980s in conjunction with the development of the space shuttle. This work has continued with current applications being investigated for the National Aeronautics and Space Administration's space station and reusable launch vehicle. The Shuttle Modal Inspection System (SMIS) was developed to identify fatigue damage in components such as control surfaces, fuselage panels and lifting surfaces. These areas were covered with a thermal protection system making these portions of the shuttle inaccessible and hence impractical for conventional local non-destructive examination methods. This system has been successful in locating damaged components that are covered by the thermal protection system. All orbiter vehicles have been periodically subjected to SMIS testing since 1987. Space station applications have primarily driven the development of experimental/analytical damage-detection methods. These approaches are based on correlating analytical models of the undamaged structure with measured modal properties from both the undamaged and damaged structure. Changes in stiffness indices as assessed from the two model updates are used to locate and quantify the damage. Since the mid 1990s, studies of damage detection for composite materials have been motivated by the development of composite fuel tanks for a reusable launch vehicle.

The civil engineering community has studied vibration-based damage assessment of bridge structures since the early 1980s. Modal properties and quantities derived from these properties such as mode-shape curvature and dynamic flexibility matrix indices have been the primary features used to identify damage in bridge structures. Environmental and operating condition variability present significant challenges to the bridge monitoring application. Regulatory requirements in eastern Asian countries, which mandate the companies that construct the bridges to periodically certify their structural health, are driving current research and development of vibration-based bridge monitoring systems.

In summary, the review of the technical literature presented by Doebling *et al.* (1998) shows an increasing number of research studies related to vibration-based damage detection. These studies identify many technical challenges to the adaptation of vibration-based damage detection that are common to all applications of this technology. These challenges include better use of the nonlinear response characteristics of the damaged system, development of methods to optimally define the number and location of the sensors, identification of the features sensitive to small damage levels, the ability to discriminate changes in features cause by damage from those caused by changing environmental and/or test conditions, the development of statistical methods to discriminate features from undamaged and damaged structures, and performance of comparative studies of different damage-detection methods applied to common datasets. These topics are currently the focus of various research efforts by many industries, including defence, automotive and semiconductor manufacturing, where multi-disciplinary approaches are being used to advance the current capabilities of vibration-based damage detection.

3. Vibration-based damage detection and structural health monitoring

The process of implementing a damage-detection strategy is referred to as *structural health monitoring*. This process involves the observation of a structure over a period of time using periodically spaced measurements, the extraction of features from these measurements, and the analysis of these features to determine the current state of health of the system. The output of this process is periodically updated information regarding the ability of the structure to continue to perform its desired function in light of the inevitable ageing and degradation resulting from the operational environments. Figure 1 shows a chart summarizing the structural health-monitoring process. The topics summarized in this figure are discussed below.

(*a*) *Operational evaluation*

Operational evaluation answers two questions in the implementation of a structural health-monitoring system.

(1) What are the conditions, both operational and environmental, under which the system to be monitored functions?

(2) What are the limitations on acquiring data in the operational environment?

Operational evaluation begins to set the limitations on what will be monitored and how the monitoring will be accomplished. This evaluation starts to tailor the damage-detection process to features that are unique to the system being monitored and tries to take advantage of unique features of the postulated damage that is to be detected.

(*b*) *Data acquisition and cleansing*

The data-acquisition portion of the structural health-monitoring process involves selecting the types of sensors to be used, the location where the sensors should be placed, the number of sensors to be used, and the data-acquisition/storage/transmittal hardware. This process will be application specific. Economic considerations will play a major role in making these decisions. Another consideration is how often the data should be collected. In some cases, it may be adequate to collect data immediately before and at periodic intervals after a severe event. However, if fatigue crack growth is the failure mode of concern, it may be necessary to collect data almost continuously at relatively short time intervals.

Because data can be measured under varying conditions, the ability to normalize the data becomes very important to the damage-detection process. One of the most common procedures is to normalize the measured responses by the measured inputs. When environmental or operating condition variability is an issue, the need can arise to normalize the data in some temporal fashion to facilitate the comparison

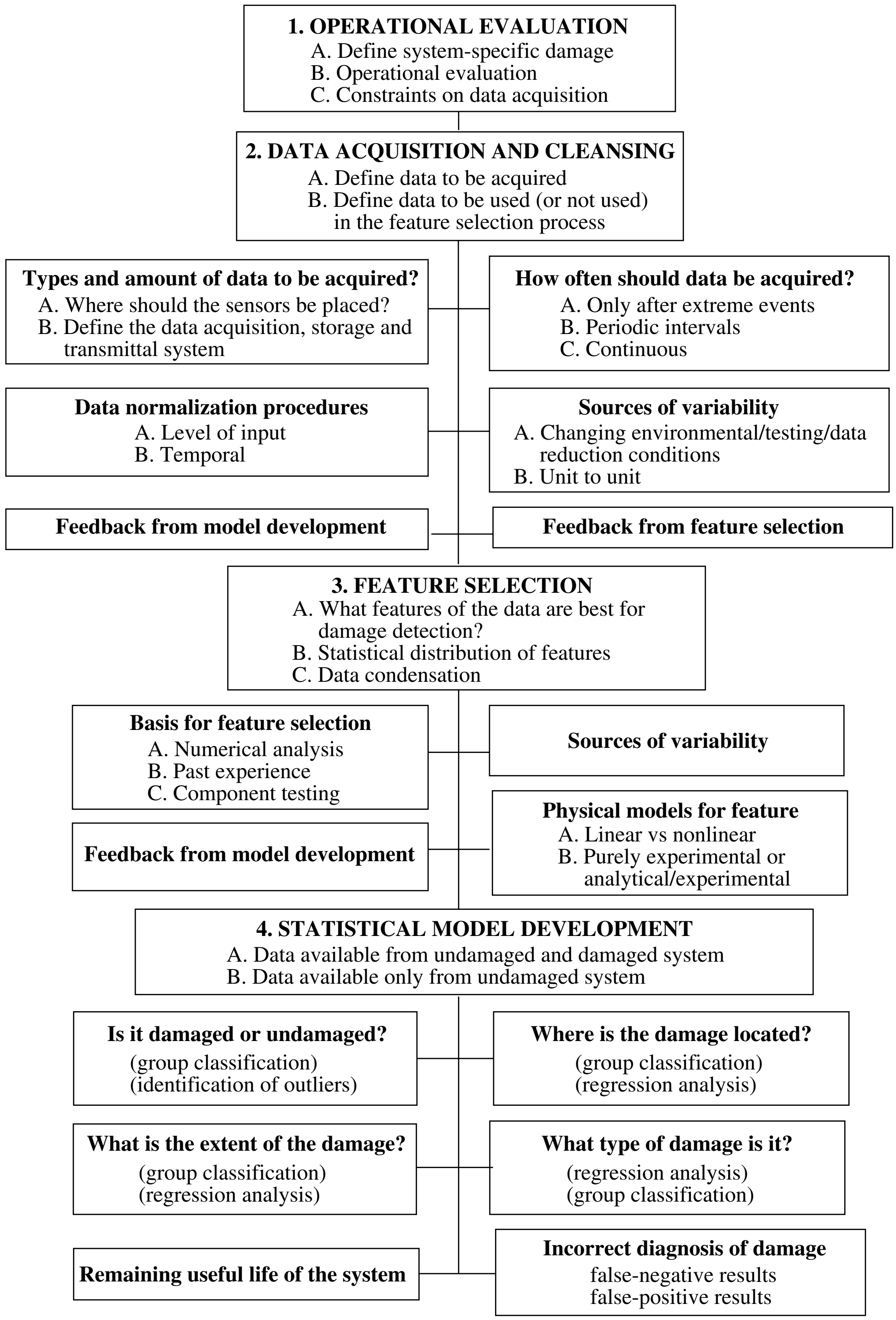

Figure 1. Flow chart for implementing a structural health-monitoring program.

of data measured at similar times of an environmental or operational cycle. Sources of variability in the data-acquisition process and with the system being monitored need to be identified and minimized to the extent possible. In general, all sources of variability can not be eliminated. Therefore, it is necessary to make the appropriate measurements such that these sources can be statistically quantified.

Data cleansing is the process of selectively choosing data to accept for, or reject from, the feature selection process. The data-cleansing process is usually based on knowledge gained by individuals directly involved with the data acquisition. Finally, it should be noted that the data-acquisition and cleansing portion of a structural health-monitoring process should not be static. Insight gained from the feature selection process and the statistical model development process will provide information regarding changes that can improve the data-acquisition process.

(*c*) *Feature selection*

The area of the structural damage-detection process that receives the most attention in the technical literature is the identification of data features that allow one to distinguish between the undamaged and damaged structure. Inherent in this feature selection process is the condensation of the data. The operational implementation and diagnostic measurement technologies needed to perform structural health monitoring typically produce a large amount of data. A condensation of the data is advantageous and necessary, particularly if comparisons of many datasets over the lifetime of the structure are envisioned. Also, because data may be acquired from a structure over an extended period of time and in an operational environment, robust data-reduction techniques must retain sensitivity of the chosen features to the structural changes of interest in the presence of environmental noise.

The best features for damage detection are typically application specific. Numerous features are often identified for a structure and assembled into a feature vector. In general, it is desirable to develop feature vectors that are of low dimension. It is also desirable to obtain many samples of the feature vectors. There are no restrictions on the types or combinations of data contained in the feature vector. As an example, a feature vector may contain the first three resonant frequencies of the system, a time when the measurements were made, and a temperature reading from the system. A variety of methods are employed to identify features for damage detection. Past experience with measured data from a system, particularly if damaging events have been previously observed for that system, is often the basis for feature selection. Numerical simulation of the damaged system's response to simulated inputs is another means of identifying features for damage detection. The application of engineered flaws, similar to ones expected in actual operating conditions, to specimens can identify parameters that are sensitive to the expected damage. Damage accumulation testing, during which significant structural components of the system under study are subjected to a realistic accumulation of damage, can also be used to identify appropriate features. Fitting linear or nonlinear, physical-based or non-physical-based models of the structural response to measured data can also help identify damage-sensitive features. Common features used in vibration-based damage-detection studies are briefly summarized below. A more detailed summary can be found in Doebling *et al*. (1998).

(i) *Basic modal properties*

The most common features that are used in vibration-based damage detection, and that represent a significant amount of data condensation from the actual measured quantities, are the common modal properties of resonant frequencies and mode-shape vectors. These features are identified from measured response time histories, most often absolute acceleration, or spectra of these time histories. The technology required to accurately make these measurements is summarized in McConnell (1995). Often these spectra are normalized by spectra of the measured force input to form frequency response functions. Well-developed experimental modal analysis procedures are applied to these functions or to the measured-response spectra to estimate the system's modal properties (Ewins 1995; Maia & Silva 1997).

The amount of literature that uses resonant frequency shifts as a data feature for damage detection is quite large. The observation that changes in structural properties cause changes in vibration frequencies was a primary impetus for developing vibration-based damage identification technology. In general, changes in frequencies cannot provide spatial information about structural changes. For applications to large civil engineering structures, the somewhat low sensitivity of frequency shifts to damage requires either very precise measurements of frequency change or large levels of damage. An exception to this limitation occurs at higher modal frequencies, where the modes are associated with local responses. However, the practical limitations involved with the excitation and identification of the resonant frequencies associated with these local modes, caused in part by high modal density and low participation factors, can make these resonant frequencies difficult to identify.

Damage detection methods using mode-shape vectors as a feature generally analyse differences between the measured modal vectors before and after damage. Mode-shape vectors are spatially distributed quantities; therefore, they provide information that can be used to locate damage. However, a large number of measurement locations can be required to accurately characterize mode-shape vectors and provide sufficient resolution for determining the damage location.

(ii) *Mode-shape curvature changes*

An alternative to using mode shapes to obtain spatially distributed features sensitive to damage is to use mode-shape derivatives, such as curvature. Mode-shape curvature can be computed by numerically differentiating the identified mode-shape vectors twice to obtain an estimate of the curvature. These methods are motivated by the fact that the second derivative of the mode shape is much more sensitive to small perturbations in the system than is the mode shape itself. Also, for beam- and plate-like structures, changes in curvature can be related to changes in strain energy, which has been shown to be a sensitive indicator of damage. A comparison of the relative statistical uncertainty associated with estimates of mode-shape curvature, mode-shape vectors and resonant frequencies showed that the largest variability is associated with estimates of mode-shape curvature followed by estimates of the mode-shape vector. Resonant frequencies could be estimated with least uncertainty (Doebling *et al*. 1997).

(iii) *Dynamically measured flexibility*

Changes in the dynamically measured flexibility matrix indices have also been used as damage sensitive features. The dynamically measured flexibility matrix, $[G]$, is estimated from the mass-normalized measured mode shapes, $[\Phi]$, and measured eigenvalue matrix (diagonal matrix of squared modal frequencies), $[\Lambda]$, as

$$[G] \approx [\Phi][\Lambda]^{-1}[\Phi]^{\mathrm{T}}. \tag{3.1}$$

The formulation of the flexibility matrix is approximate because in most cases all of the structure's modes are not measured. Typically, damage is detected using flexibility matrices by comparing the flexibility matrix indices computed using the modes of the damaged structure to the flexibility matrix indices computed using the modes of the undamaged structure. Because of the inverse relationship to the square of the modal frequencies, the measured flexibility matrix is most sensitive to changes in the lower-frequency modes of the structure.

(iv) *Updating structural model parameters*

Another class of damage-identification methods is based on features related to changes in mass, stiffness and damping matrix indices that have been correlated such that the numerical model predicts, as closely as possible, the identified dynamic properties (resonant frequencies, modal damping and mode-shape vectors) of the undamaged and damaged structures, respectively. These methods solve for the updated matrices (or perturbations to the nominal model that produce the updated matrices) by forming a constrained optimization problem based on the structural equations of motion, the nominal model and the identified modal properties (Friswell & Mottershead 1995). Comparisons of the matrix indices that have been correlated with modal properties identified from the damaged structure to the original correlated matrix indices provide an indication of damage that can be used to quantify the location and extent of damage. Degree of freedom mismatch between the numerical model and the measurement locations can be a severe limitation for performing the required matrix updates.

(v) *Nonlinear methods*

Identification of the previously described features is based on the assumption that a linear model can be used to represent the structural response before and after damage. However, in many cases, the damage will cause the structure to exhibit nonlinear response. Therefore, the identification of features indicative of nonlinear response can be a very effective means of identifying damage in a structure that originally exhibited linear response. The specific features that indicate that a system is responding in a nonlinear manner vary widely. Examples include the generation of resonant frequency harmonics in a cracked beam excited in a manner such that the crack opens and closes (Prime & Shevitz 1996). For extreme events, such as an earthquake, the normalized Arias intensity provides an estimate of kinetic energy of the structure and has been successfully used to identify the onset of nonlinear response of buildings subject to damaging earthquake excitations (Straser 1998). Deviations from a Gaussian probability distribution function of acceleration response amplitudes for a system subjected to a Gaussian input have been used successfully to identify that loose parts are present in a system. Temporal variation in resonant frequencies, as identified with canonical variate analysis, is another method to identify the onset of damage (Hunter 1999). In general, features based on the nonlinear response of a system have only been used to identify that damage has occurred. Few methods have been described that locate the source of the nonlinearity. Because all systems will exhibit some degree of nonlinearity, it becomes a challenge to establish a threshold at which changes in the nonlinear response features are indicative of damage. The statistical model building portion of the structural health-monitoring process is essential for establishing such thresholds.

4. Statistical model development

The portion of the structural health-monitoring process that has received the least attention in the technical literature is the development of statistical models to enhance damage detection. Almost none of the hundreds of studies summarized in Doebling *et al*. (1998) make use of any statistical methods to assess if the changes in the selected features used to identify damage are statistically significant. Statistical model development is concerned with the implementation of the algorithms that operate on the extracted features and unambiguously determine the damage state of the structure. The algorithms used in statistical model development usually fall into three categories and will depend on the availability of data from both an undamaged and a damaged structure. The first category is group classification, that is, placement of the features into respective 'undamaged' or 'damaged' categories. Analysis of outliers is the second type of algorithm. When data from a damaged structure are not available for comparison, do the observed features indicate a significant change from the previously observed features that can not be explained by extrapolation of the feature distribution? The third category is regression analysis. This analysis refers to the process of correlating data features with particular types, locations or extents of damage. All three algorithm categories analyse statistical distributions of the measured or derived features to enhance the damage-detection process.

The statistical models are used to answer the following questions regarding the damage state of the structure (Rytter 1993).

(1) Is there damage in the structure (existence)?

(2) Where is the damage in the structure (location)?

(3) How severe is the damage (extent)?

Successively answering these questions in the order presented requires increasing knowledge of the structure's damage state. Experimental structural dynamics techniques can be used to address the first two questions. Analytical models are usually needed to answer the third question unless examples of data are available from the system (or a similar system) when it exhibits varying levels of the damage. Statistical model development can also determine the type of damage that is present. To identify the type of damage, data from structures with the specific types of damage must be available for correlation with the measured features.

Finally, an important part of the statistical model development process is the testing of these models on actual data to establish the sensitivity of the selected features to damage and to study the possibility of false indications of damage. False indications of damage fall into two categories: (i) false-positive damage indication (indication of damage when none is present); and (ii) false-negative damage indications (no indication of damage when damage is present). Although the second category is usually very detrimental to the damage detection process and can have serious life-safety implications, false-positive readings can also erode confidence in the damage-detection process.

This paper will now summarize the application of methods from statistical pattern recognition and machine learning to the vibration-based damage-detection problem. A damage-detection experiment performed on concrete bridge columns will be described in terms of the statistical-pattern-recognition damage-detection paradigm that has just been summarized.

5. Application of vibration-based damage detection to concrete bridge column

A damage-detection study was conducted on two concrete columns that were quasi-statically loaded to failure in an incremental manner. The focus of this study was to establish a relatively simple feature vector coupled with a simple statistical model that would unambiguously identify that the columns had been damaged.

(a) *Test structure geometry*

The test structures consisted of two 24 in (61 cm) diameter concrete bridge columns that were subsequently retrofitted to 36 in (91 cm) diameter columns. Figure 2 shows the test structure geometry. The first column tested, labelled column 3, was retrofitted by placing forms around the existing column and placing additional concrete within the form. The second column, labelled column 2, was retrofitted to the 36 in diameter by spraying concrete in a process referred to as shotcreting. Column 2 was then finished with a trowel to obtain the circular cross-section.

The 36 in diameter portion of both columns was 136 in (345 cm) in length. The columns were cast on top of a 56 in^2 (142 cm^2) concrete foundation that was 25 in (63.5 cm) high. A 24 in^2 concrete block that had been cast integrally with the column extends 18 in (46 cm) above the top of the 36 in diameter portion of the column. This block was used to attach the hydraulic actuator to the columns for quasi-static cyclic testing and to attach the electro-magnetic shaker used for the experimental modal analyses. As is typical of actual retrofits in the field, a 1.5 in (3.8 cm) gap was left between the top of the foundation and the bottom of retrofit jacket. Therefore, the longitudinal reinforcement in the retrofitted portion of the column did not extend into the foundation. The concrete foundation was bolted to the 2 ft (0.6 m) thick testing floor in the University of California at Irvine structural-testing laboratory during both the static cyclic tests and the experimental modal analyses. The structures were not moved once testing was initiated.

(b) *Quasi-static loading*

Prior to applying lateral loads, an axial load of 90 000 lbf (400 kN) was applied to simulate gravitational loads that an actual column would experience. Next, a hydraulic actuator was used to apply lateral load to the top of the column in a

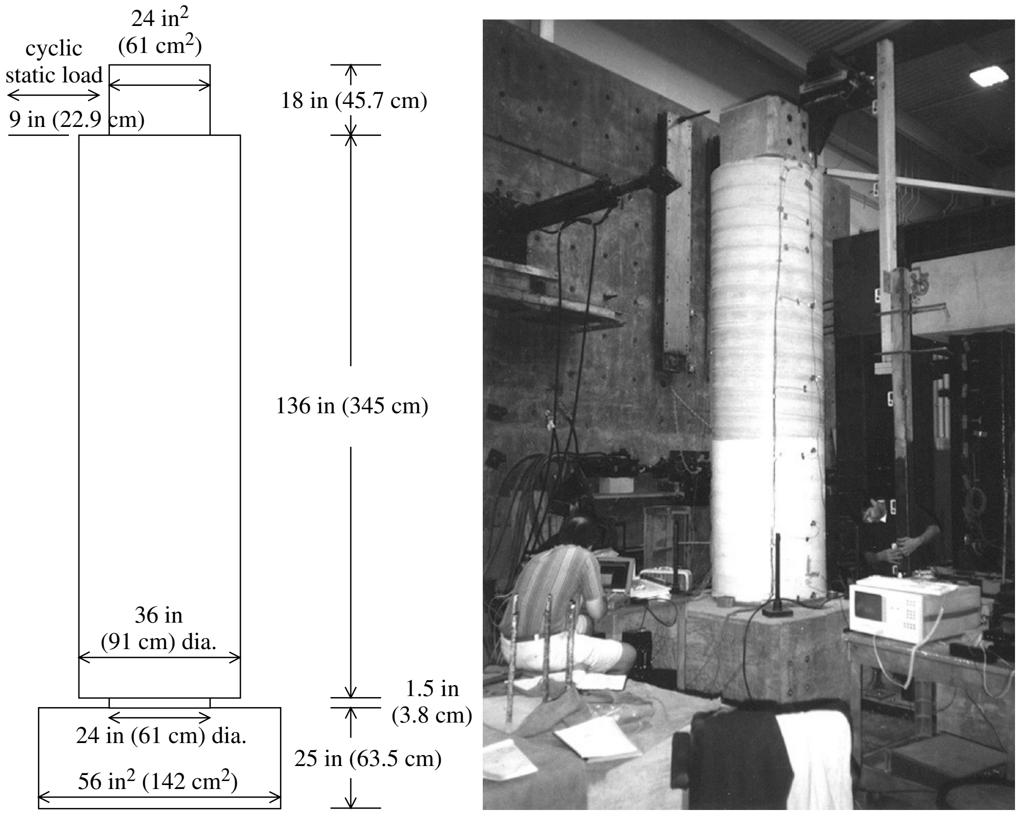

Figure 2. Column dimensions and photo of an actual test structure.

cyclic manner. The loads were first applied in a force-controlled manner to produce lateral deformations at the top of the column corresponding to $0.25\Delta y_{\mathrm{T}}$, $0.5\Delta y_{\mathrm{T}}$, $0.75\Delta y_{\mathrm{T}}$ and Δy_{T}. Here Δy_{T} is the lateral deformation at the top of the column corresponding to the theoretical first yield of the longitudinal reinforcement. The structure was cycled three times at each of these load levels.

Based on the observed response, a lateral deformation corresponding to the actual first yield, Δy, was calculated and the structure was cycled three times in a displacement-controlled manner to that deformation level. Next, the loading was applied in a displacement-controlled manner, again in sets of three cycles, at displacements corresponding to $1.5\Delta y$, $2.0\Delta y$, $2.5\Delta y$, etc., until the ultimate capacity of the column was reached. Load deformation curves for column 3 are shown in figure 3. This manner of loading put incremental and quantifiable damage into the structures. The axial load was applied during all static tests.

(*c*) *Dynamic excitation*

For the experimental modal analyses, the excitation was provided by an electromagnetic shaker mounted off-axis at the top of the structure. The shaker rested on a steel plate attached to the concrete column. Horizontal load was transferred from the shaker to the structure through a friction connection between the supports of the

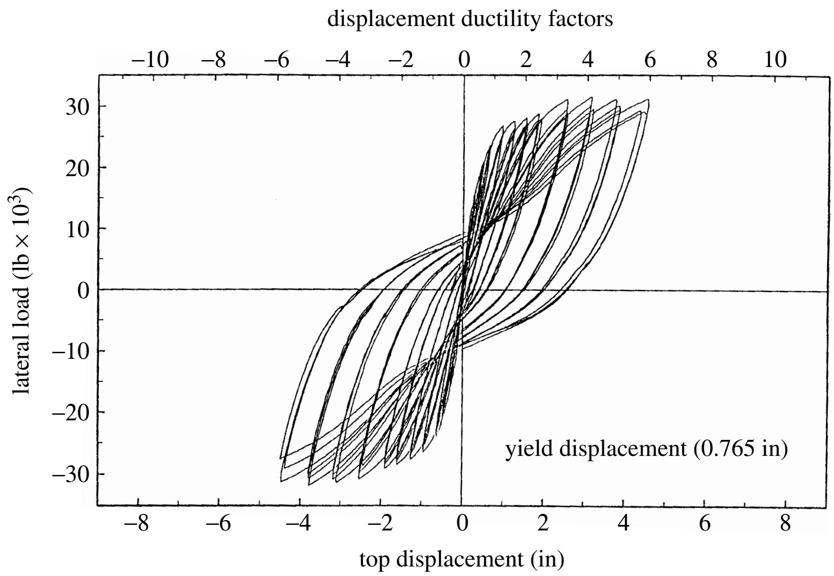

Figure 3. Load–displacement curves for the cast-in-place column.

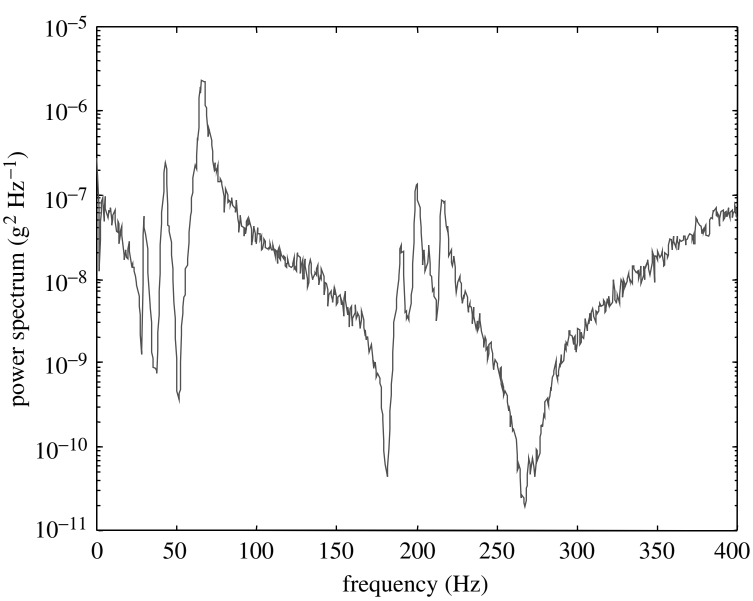

Figure 4. Input power spectra.

shaker and the steel plate. This force was measured with an accelerometer mounted to the sliding mass (0.18 lb s^2 in^{-1} (31 kg)) of the shaker. A 0–400 Hz uniform random signal was sent from a source module in the data-acquisition system to the shaker, but feedback from the column and the dynamics of the mounting plate produced an input signal that was not uniform over the specified frequency range. Figure 4 shows a typical input power spectrum.

(*d*) *Operational evaluation*

Because the structure being tested was a laboratory specimen, operational evaluation was not conducted in a manner that would typically be applied to an *in situ* structure. However, the vibration tests were not the primary purpose of this investigation. Therefore, compromises had to be made regarding the manner in which the vibration tests were conducted. The primary compromise was associated with the mounting of the shaker. These compromises are analogous to operational constraints that may occur with *in situ* structures. Environmental variability was not considered an issue because these tests were conducted in a laboratory setting. The available measurement hardware and software placed the only constraints on the data-acquisition process.

(*e*) *Data acquisition and cleansing*

Forty accelerometers were mounted on the structure, as shown in figure 5. These locations were selected based on the initial desire to measure the global bending, axial and torsional modes of the column. Note that the accelerometers at locations 2, 39 and 40 had a nominal sensitivity of $10\,\mathrm{mV\,g^{-1}}$ and were not sensitive enough for the measurements being made. As part of the data-cleansing process, data from these channels were not used in subsequent portions of the damage-detection process. Locations 33, 34, 35, 36, and 37 were accelerometers with a nominal sensitivity of $100\,\mathrm{mV\,g^{-1}}$. All other channels had accelerometers with a nominal sensitivity of $1\,\mathrm{V\,g^{-1}}$.

A commercial data-acquisition system was used to record and digitize all accelerometer signals. Data-acquisition parameters were specified such that frequency-response functions (FRFs), input and response power spectra, cross-power spectra and coherence functions in the 0–400 Hz range could be measured. Each spectrum was calculated from 30 averages of 2 s duration time histories discretized with 2048 points. These sampling parameters produced a frequency resolution of 0.5 Hz. Hanning windows were applied to all measured time histories prior to the calculation of spectral quantities. A second set of measurements was acquired from 8 s duration time histories discretized with 8192 points. Only one average was measured. A uniform window was specified for these data, as the intent was to measure a time history only.

(*f*) *Feature selection*

Typically, systematic differences between time-series from the undamaged and damaged structures are nearly impossible to detect by eye. Therefore, other features of the measured data must be examined for damage detection. Originally, damage-detection features were to be based on common modal properties as has been done in many previous studies. However, the feedback from the structure and mounting system to the shaker produced an input that did not have a uniform power spectrum over the frequency range of interest as previously discussed. This input form coupled with the nonlinear response observed at higher levels of damage made it extremely difficult to track changing modal properties through the various levels of damage. Therefore, other features were sought for the damage-detection process.

The alternative features were selected based on previous experience from speech pattern recognition where auto-regressive models have been used to estimate the

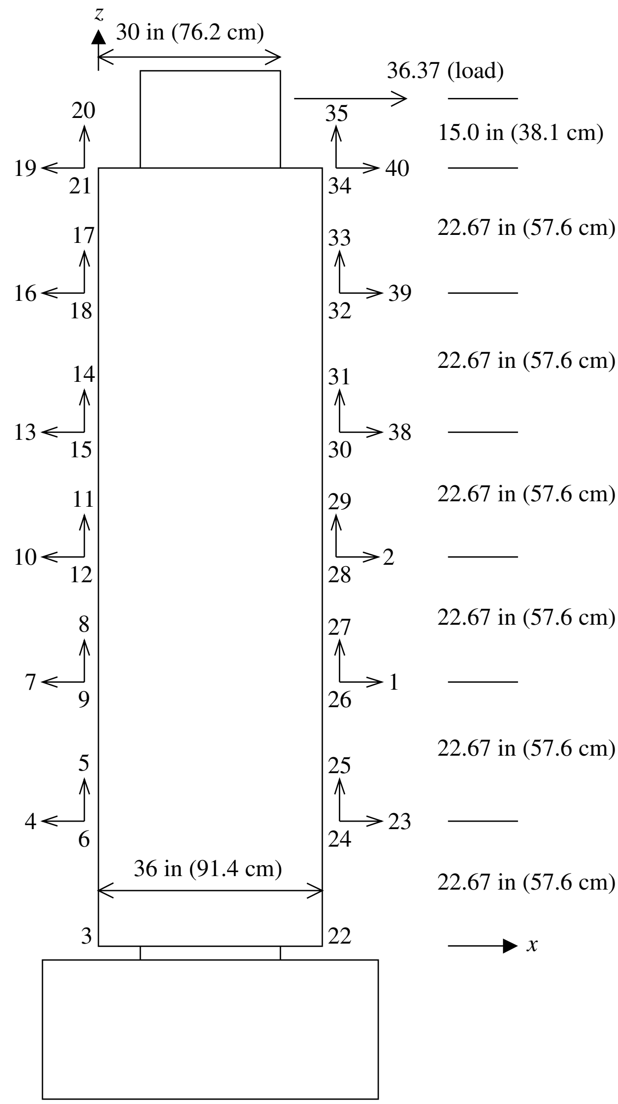

Figure 5. Accelerometer locations and coordinate system for modal testing. Accelerometers 3, 6, 9, 12, 15, 18, 21, 22, 24, 26, 28, 30, 32, and 34 are mounted in the $-y$-direction.

transfer function of the human vocal track (Morgan & Scofield 1992). The time-series were modelled using a common method of auto-regressive estimation referred to as linear predictive coding (LPC) (Rabiner & Shafe 1978). The LPC algorithm is an Nth-order model that attempts to model the current point in a time-series, $s'(n)$, as a linear combination of the previous N points. That is,

$$s'(n) = \sum_{i=1}^{N} a_i s(n-i). \tag{5.1}$$

Third-order LPC models were developed for each column using 512-point windows with 97% overlap resulting in 480 samples of the a_i. Over these segments of the time-series, the a_i that best model the time-series in a least-squares sense are used as features that are assumed to be representative of the system's dynamic response during those samples. Hanning windows were applied to these data prior to the estimate of the coefficients. These models were developed with data from sensors 3 and 21 (figure 5). Sensor 3 was located close to the damage, but because of the test configuration this sensor was not expected to experience large amplitude response, as it primarily measures torsional motion of the structure near its fixed end. Sensor 21 was located farther from the damage and experienced some of the largest amplitude response as it primarily measured the bending response at the free end of this cantilever structure.

Over a time-series, many overlapping 'windows' give rise to LPC coefficient vectors, which become the multi-dimensional data samples to be analysed in the statistical model development portion of the damage-detection process. While the overlapping of windows provides a smoother estimate of the features' changes over time, samples that result from overlapping windows will not be independent.

Normalization of the data was not attempted because these tests were conducted in a laboratory environment, where the input could be applied in a very controlled manner. Other considerations that led to the decision not to normalize the data included the assumption that environmental and test-to-test variability was negligible, damage was introduced in discrete increments, and it was assumed that the vibration levels were such that the physical condition of the test structures did not change during the dynamic tests.

(*g*) *Statistical model development: Fisher's discriminant*

Consider two data generation processes A and B, with independent multi-dimensional samples $\{x\}$ being generated by both processes. Assuming A and B have some systematic difference in the samples that they generate, Fisher's discriminant (Fisher 1936; Bishop 1995) represents the optimal linear projection of the multi-dimensional sample space that maximally discriminates the $\{x_{\mathrm{A}}\}$ from the $\{x_{\mathrm{B}}\}$. That is, it defines a linear projection $\{w\}$ such that

$$y = \{w\}^{\mathrm{T}}\{x\} \tag{5.2}$$

produces a scalar projection, y, of the multi-dimensional space onto which the distribution of $\{x_{\mathrm{A}}\}$ is as distinct as possible from the distribution of $\{x_{\mathrm{B}}\}$. Once this projection is determined from previous samples of $\{x_{\mathrm{A}}\}$ and $\{x_{\mathrm{B}}\}$, it can be used to provide the relative probability that a novel sample $\{x\}$ was generated by process A or B. Thus the Fisher discriminant maximizes the function $F(\{w\})$, which is the distance between the means of the transformed distributions, μ_i, normalized by the total within-class covariance, s_k^2,

$$F(\{w\}) = \frac{(\mu_{\mathrm{A}} - \mu_{\mathrm{B}})^2}{s_{\mathrm{A}}^2 + s_{\mathrm{B}}^2}, \tag{5.3}$$

where

$$\mu_i = \{w\}^{\mathrm{T}}\{\mu_i\}, \tag{5.4}$$

$$\{\mu_i\} = \frac{1}{N_i}\sum\{x_i\} \tag{5.5}$$

and

$$s_k^2 = \sum (y_n - \mu_k)^2. \tag{5.6}$$

Here N_i is the number of samples and y_n are the samples of the scalar projection obtained by applying (5.2) to each sample $\{x_i\}$. Using (5.2) and (5.6), and the definition of a multi-dimensional sample mean given by (5.4), equation (5.3) can be rewritten explicitly in terms of $\{w\}$ as

$$F(\{w\}) = \frac{\{w\}^{\mathrm{T}}[S_{\mathrm{b}}]\{w\}}{\{w\}^{\mathrm{T}}[S_{\mathrm{w}}]\{w\}}, \tag{5.7}$$

where

$$[S_{\mathrm{b}}] = (\{\mu_{\mathrm{B}}\} - \{\mu_{\mathrm{A}}\})^{\mathrm{T}}(\{\mu_{\mathrm{B}}\} - \{\mu_{\mathrm{A}}\}) \tag{5.8}$$

is the between-class covariance matrix,

$$[s_{\mathrm{w}}] = \sum (\{x_{\mathrm{A}}\} - \{\mu_{\mathrm{A}}\})(\{x_{\mathrm{A}}\} - \{\mu_{\mathrm{A}}\})^{\mathrm{T}} + \sum (\{x_{\mathrm{B}}\} - \{\mu_{\mathrm{B}}\})(\{x_{\mathrm{B}}\} - \{\mu_{\mathrm{B}}\})^{\mathrm{T}} \tag{5.9}$$

is the total within-class covariance matrix and the summations in (5.9) are over the available samples of $\{x_{\mathrm{A}}\}$ and $\{x_{\mathrm{B}}\}$, respectively.

To maximize $F(\{w\})$, the derivative of F with respect to $\{w\}$ is set equal to zero, yielding

$$(\{w\}^{\mathrm{T}}[S_{\mathrm{b}}]\{w\})[S_{\mathrm{w}}]\{w\} = (\{w\}^{\mathrm{T}}[S_{\mathrm{w}}]\{w\})[S_{\mathrm{b}}]\{w\}. \tag{5.10}$$

The magnitude of $\{w\}$ is not of concern, only its direction is, and the scalar quantities

$$[(\{w\}^{\mathrm{T}}[S_{\mathrm{b}}]\{w\})] \quad \text{and} \quad (\{w\}^{\mathrm{T}}[S_{\mathrm{w}}]\{w\})$$

are therefore replaced with arbitrary α and β, respectively. After rearrangement and multiplication by $[S_{\mathrm{w}}]^{-1}$ (note that because $[S_{\mathrm{w}}]$ is a covariance matrix, $[S_{\mathrm{w}}]$ is invertible), the following relation is obtained:

$$[S_{\mathrm{w}}]^{-1}[S_{\mathrm{b}}]\{w\} = (\alpha/\beta)\{w\}. \tag{5.11}$$

Thus, with standard numerical methods, $\{w\}$ is found as an eigenvector of $[S_{\mathrm{w}}]^{-1}[S_{\mathrm{b}}]$.

Once the data have been projected down onto the scalar y dimension, the distribution of y_{A} and y_{B} points can be described by an appropriate probability density function. Since it was originally assumed that $\{x\}$ was a multi-dimensional random variable, then $y = \{w\}^{\mathrm{T}}\{x\}$ is a sum of random variables and the central limit theorem is invoked to justify modelling y_{A} and y_{B} with Gaussian density functions.

Novel data $\{x_{\mathrm{new}}\}$ can be projected to get $y_{\mathrm{new}} = \{w\}^{\mathrm{T}}\{x_{\mathrm{new}}\}$ and the likelihood, p, of y_{new} with respect to the Gaussian for class A and the Gaussian for class B can be determined. The probability that y_{new} was generated by class A can be obtained by integrating over a small region of the likelihood function:

$$\Pr(y_{\mathrm{new}} \mid \mathrm{A}) = \int_{\Delta y} p_{\mathrm{A}}(y_{\mathrm{new}})\,\mathrm{d}y. \tag{5.12}$$

Because $\Pr(\mathrm{A} \mid y_{\text{new}})$ is of interest, and

$$\Pr(\mathrm{B} \mid y_{\text{new}}) = 1 - \Pr(\mathrm{A} \mid y_{\text{new}}), \tag{5.13}$$

if A and B are mutually exclusive, Bayes's rule can be used to obtain

$$\Pr(\mathrm{A} \mid y_{\text{new}}) = \frac{\Pr(y_{\text{new}} \mid \mathrm{A}) \Pr(\mathrm{A})}{\Pr(y_{\text{new}})}, \tag{5.14}$$

where the denominator is typically ignored when $\Pr(y_{\text{new}})$ is uniform (or unknown) and $\Pr(\mathrm{A})$ is the prior probability (i.e. relative frequency) of class A versus class B. In the case where class A is 'undamaged' and class B is 'damaged', a probability of a damaged system having produced a given observed sample $\{x_{\text{new}}\}$ can now be estimated.

(*h*) *Application of Fisher's discriminant to concrete column data*

Fisher's discriminant was defined using data from the vibration tests conducted on the undamaged columns and from the vibration tests conducted after the first level of damage corresponding to initial yielding of the steel reinforcement. Subsequent damage levels were then identified based on this same Fisher projection. As illustrated in figure 6, when Fisher's discriminant is applied to data from both sensors on either column, there is statistically significant separation between the LPC coefficients for the undamaged cases and damage level 1 cases (solid and dashed Gaussian density functions). The results of using the previously determined Fisher projection to project many samples of data from increasingly greater levels of damage into this space are plotted as straight lines in figure 6. While increasing damage is not necessarily related to increasing Fisher coordinate, all damaged cases have a profile significantly different from that of the undamaged case. The discrimination between damaged and undamaged structures is obtained with data from both sensors. This result is significant because the response measured by sensor 3 was of relatively low amplitude with noise contributing significantly to the measured signal. Higher-order LPC models and different size data windows produced similar results.

6. Concluding comments

Recent work in structural health monitoring and vibration-based damage detection has been briefly reviewed to show that this subject is the focus of many active research efforts and to identify some of the technical challenges in this field. A major shortcoming associated with many of these efforts is that statistical models are not applied to identify when changes in the selected features are significant. Therefore, a statistical-pattern-recognition paradigm has been proposed for the general problem of structural health monitoring. This paradigm breaks the process of structural health monitoring into the four tasks of operational evaluation, data acquisition and cleansing, feature selection and statistical model development. A structural damage-detection study of concrete columns subjected to quasi-static cyclic loading to failure is then posed in terms of this paradigm.

The results of a damage-detection study applied to reinforced concrete bridge piers were then summarized. This study attempted to identify the relatively simple features of the measured data that were sensitive to damage. Other criteria for selecting

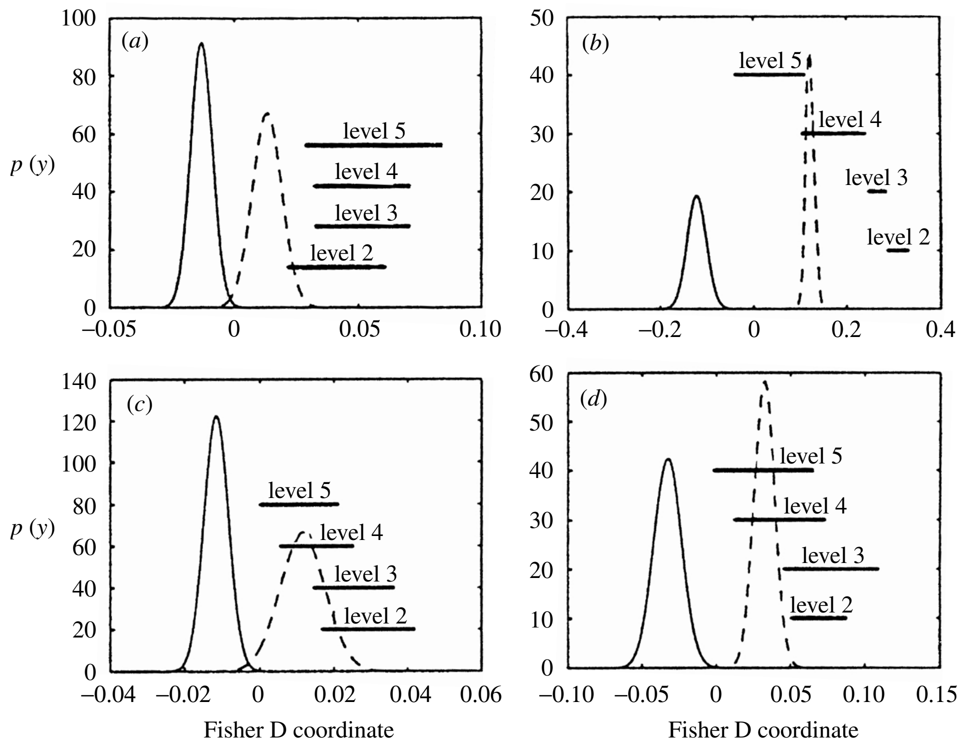

Figure 6. Distribution of LPC-generated feature vectors projected onto the Fisher-D coordinate. The horizontal lines represent widths of the distributions for higher damage levels. (a) Column 1, sensor 3; (b) column 2, sensor 3; (c) column 1, sensor 21; (d) column 2, sensor 21. Solid curves represent undamaged levels and dashed curves represent damage level 1.

the features were to keep the dimension of the feature vector small and have the number of samples of the vector large. The feature vectors used were the coefficients of a third-order linear predictive coding model. A well-developed procedure for group classification, the linear discriminant operator referred to as 'Fisher's discriminant', was introduced for application to this vibration-based damage-detection problem. This procedure requires data to be available from both the undamaged and damaged structures. The results of this study indicate a strong potential for using linear discriminant operators to identify the presence of damage. An attractive attribute of this statistical model is that it was applied to features obtained from response data only, implying that it is appropriate for structures subjected to ambient vibration from sources such as traffic or wind excitation.

The results of this study also suggest that if one or more common forms of damage occur, it may be possible not only to determine that a system is damaged, but to determine which form of damage has occurred. Additional data are required to explore this possibility. Another attractive feature of the linear discriminant operator that was not fully explored during this investigation is its ability to combine data from various types of sensors. This feature will become particularly attractive when monitoring structures that experience significant variations in their dynamic response resulting from changing environmental and operating conditions. Further analyses are also required to demonstrate the ability of the linear discriminant operator to avoid false-positive indications of damage. However, multiple samples of data from the undamaged columns were not measured.

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