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Hybrid fuzzy MADM ranking procedure for better alternative discrimination

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ABSTRACT

In this paper, we propose a hybrid fuzzy decision making approach, combining elements of fuzzy-ELECTRE and Fuzzy-TOPSIS, towards a new ranking procedure. The main objective of FETOPSIS is to offer rankings with good alternative discriminatory power to decision makers (DMs). This research work was motivated by a real case study in which multiple attribute decision making techniques were used to select the best set of investment projects for the industrial restructuring of a small oil company in Brazil. After the application of Fuzzy-TOPSIS and ELECTRE II, the obtained rankings were quite deceptive from the DMs' point of view, either to very close scores or by the excess of indifferences among alternatives. Our developed approach uses the closeness coefficients to rank the alternatives, following Fuzzy-TOPSIS, however they are computed over the normalized fuzzy concordance and discordance indexes based on the ELECTRE family. Extensive computational experiments were performed to evaluate our method. The good results obtained by FETOPSIS in the experiments, both in terms of alternative discriminatory power of rankings, and eliminating ranking reversal cases, gave us the confidence to apply the method in the real case. The DMs praised the developed approach, since the obtained rankings were more discriminatory in the alternatives than both Fuzzy-TOPSIS and ELECTRE II, making it possible to select with confidence a set of suited alternatives.

1. Introduction

This research work was motivated by the difficulties of some multiple-attribute decision-making (MADM) methods to find a suitable ranking in a real world decision making process. The decision consisted of defining an investment portfolio for the industrial restructuring of a small oil refinery in Brazil. As investment project selection is a classic MADM problem (Greiner et al., 2003), the board committee members decided to apply an MADM method to find a suitable portfolio, considering different attributes or criteria. The decision-makers (DMs) defined a decision model consisting of nine alternatives and ten criteria. Details are given in Section 5. Initially, Fuzzy-TOPSIS (Chen, 2000), a fuzzy variant of the well-known technique for order preference by similarity to ideal solution (TOPSIS), was applied. Scores in the interval [0.41,0.48] were assigned to six alternatives. Considering that this decision making process involved many incomplete, unqualified, vague, and unquantifiable information, these scores were considered by the DMs too close for a good selection. Next, we applied ELECTRE II (Roy, 1990), a non-compensatory MADM method, in an attempt to facilitate the decision process for the DMs. However, several indifferences occurred in the first and second positions, making it even harder for the DMs to take a final decision. Further sensitivity analysis in the application of both methods had no impact in this lack of alternative discrimination. There were consensus among the DMs that although the methods were intuitive and mathematically sound, their final rankings were lacking alternative discrimination power. As a consequence, we decided to investigate this important issue for DMs in real-world decisions, developing a new method whose main objective is to offer simultaneously consistent and alternative discriminatory rankings.

Several methods, techniques and models were developed in order to support decision-makers to analyze a set of alternatives by their composite scores or values in a ratio scale (Yoon and Hwang, 1995). However, the application of some methods may cause problems in the effective selection/ranking of alternatives by assigning narrow gaps between the scores of two or more alternatives. For instance, Awasthi et al. (2011) reported scores of 0.554, 0.549, 0.545 for alternatives A_1, A_2, A_3, respectively, applying Fuzzy-TOPSIS for evaluating sustainable transportation systems. Although a decision-maker can assume that $A_1 \succ A_2 \succ A_3$ (A_1 is

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preferred to A_2, and A_2 is preferred to A_3), the narrow gap between the scores (0.005 and 0.004, respectively) makes this ranking meaningless in a real world-decision. Similar situation was reported by Byun (2001), describing the application of analytical hierarchy process for selecting an automobile purchase model. The final scores of the three alternatives were 0.340, 0.338, and 0.322, respectively. Such a phenomenon has been identified by researchers in the MADM field (Cook and Kress, 1991; Zanakis et al., 1998) and in the data envelopment analysis area (Adler et al., 2002). Particularly, we identified several applications of Fuzzy-TOPSIS in which the difference between the first and second best alternatives is less than 10% (Aydogan, 2011; Chen et al., 2006; Sun, 2010; Singh and Benyoucef, 2011; Yu et al., 2011). In three applications (Awasthi et al., 2011; Aydogan, 2011; Yu et al., 2011), the difference is less than 2%. In some cases (Awasthi et al., 2011; Yu et al., 2011), the difference between the best and the worst alternatives is less than 9%. In the MADM field, the lack of discrimination of alternatives is consequence both by biased rankings due to relative values or ratio scale used in the evaluation or the operation of normalized composite values, between zero and one (Shih, 2008). As pointed out by Tzeng et al. (2005), this is a common occurrence in choice and ranking problems, and may lead to ineffective decision-making processes or inappropriate decisions, as DMs always wish to know which option is the best. As a consequence, MADM methods must not only be mathematically sound, but also be able to discriminate the alternatives in order to effectively support DMs in complex real-world problems. Sensitivity analysis has been used as an attempt to overcome the effects of the lack of discrimination. However, the major aim of this analysis is to provide additional information about the range of parameters of alternatives and criteria so that DMs can be cautious in making decisions. There is no approach in the literature directed to choice or ranking alternative (Shih, 2008).

Both ELECTRE and TOPSIS, the initially applied methods in the real case problem, including their fuzzy extensions, have been criticized in the literature. The following drawbacks were cited for the ELECTRE family (Siskos, 1982; Wang and Triantaphyllou, 2008; Zandi and Roghanian, 2013): (i) the incomparability between two alternatives can occur either by lack of information, or by inability of the DM to compare the alternatives; (ii) the distillation procedure, used in these methods to rank the alternatives, does not provide a score to each alternative; (iii) the required set of parameters by some variants leads decision models that are very difficult to validate in real-world problems; and (iv) ELECTRE based methods have presented the worst ranking reversal performance among the traditional multi-attribute methods. TOPSIS for group decision-making has also been criticized due to its lack of alternative discrimination (Olson, 2004; Tzeng et al., 2005; Shih, 2008). The results presented in many Fuzzy-TOPSIS applications have hampered the interpretation and identification of the best alternatives or the possibility of ranking them in several real-world cases as cited above.

The aim of this paper is to propose a new ranking procedure, called FETOPSIS, that simultaneously minimizes the relations of incomparability and indifference among alternatives presented by the ELECTRE family, and reduces the lack of discrimination of Fuzzy-TOPSIS based methods, making the decision-making aid process more reliable and robust for the DM. The method innovates the way the positive and negative ideal solutions are computed, using fuzzy concordance and discordance indexes based on the ELECTRE family, rather than using the weighted normalized fuzzy-decision matrix as originally designed by Chen (2000). The final ranking is obtained using a modified closeness coefficient, in which the best alternative is the one that is simultaneously closest to the maximum concordance and minimum discordance indexes and farthest from the minimum concordance and maximum discordance indexes. The developed approach combines two widely known methods in the literature. Although Kabak and Ruan (Kabak et al., 2012) combined TOPSIS and ELECTRE methods, their approach applied the two methods sequentially, without a real intertwining of their principles or steps.

In order to evaluate the performance of the developed method, we have designed and conducted extensive computational experiments, comparing our approach with ELECTRE II, a ranking method belonging to the ELECTRE family, and Fuzzy-TOPSIS. First, the performance of the methods was compared based on ranking related measures, suggested by Zanakis et al. (1998), using different parameters such as the number of alternatives, criteria, and different weight distributions. We also compared our developed method with Fuzzy-TOPSIS based on ranking measures directly related with alternative discrimination. Next, a ranking reversal experiment was carried out based on the experimental settings defined in Wang and Triantaphyllou (2008). Finally, we compared the three methods using case studies from the literature. FETOPSIS significantly reduced or eliminated both ranking indifferences and reversals in a comparison with ELECTRE II, and increased the discrimination of alternatives when compared with Fuzzy-TOPSIS's rankings. Given the results obtained in the computational experiments, we felt confident to apply FETOPSIS in the industrial restructuring decision problem without causing a new disappointment for the DMs. The ranking obtained by FETOPSIS showed a much better alternative discrimination than the two previous applied methods. There was a consensus among the DMs that the developed method offered a better support towards the final selection, demanding from them, almost the same effort.

The contribution of this paper is fourfold: (i) we proposed an innovative method that combines elements of Fuzzy-ELECTRE and Fuzzy-TOPSIS, taking advantage of the best characteristics each method has to offer; (ii) we performed an unprecedented in-depth analysis of FETOPSIS, using simulation experiments. To the best of our knowledge, no such experiments have been conducted for any Fuzzy-ELECTRE based method in the literature; (iii) we provided rankings with better alternative discrimination than both ELECTRE II and Fuzzy-TOPSIS; and (iv) we addressed the ELECTRE family ranking reversal issue.

The remainder of this paper is organized as follows: Section 2 describes the fuzzy notation and definitions to be used in the paper, as well as summarizes descriptions of Fuzzy-ELECTRE and Fuzzy-TOPSIS methods. Section 3 describes the proposed method in detail. Section 4 reports the computational experiments conducted to evaluate the developed method. The application of FETOPSIS in the restructuring industrial problem is discussed in details in Section 5. Section 6 presents final remarks and indicates future work.

2. Background

2.1. Fuzzy set theory

In this section, some basic definitions concerning fuzzy set theory related to our developed method are reviewed.

Definition 2. (Kaufmann and Gupta, 1991) A trapezoidal fuzzy number (TFN) $\tilde{a}$ can be defined as $\tilde{a} = (a_1, a_2, a_3, a_4)$. The membership function $\mu_{\tilde{a}}(x)$ is defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} f^L_{\tilde{a}}(x), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \ge a_3 \\ f^R_{\tilde{a}}(x), & a_3 \le x \ge a_4 \\ 0, & \text{otherwise} \end{cases}$$

where $f_{\tilde{a}}^{L}(x):[a_1,a_2]\rightarrow[0,1]$ is a strictly increasing function and $f_{\tilde{a}}^{R}(x):[a_3,a_4]\rightarrow[0,1]$ is a strictly decreasing function.

Definition 2. (Sadi-Nezhad and Damghani, 2010) $\tilde{a}=(a_1,a_2,a_3,a_4)$ is called a positive TFN (PTFN) if a_1,a_2,a_3, and a_4 are positive and not identical.

Definition 3. (Chen et al., 2006) The fuzzy sum $\oplus$ and fuzzy subtraction $\ominus$ of any two TFNs are also TFNs by the extension principle. However, the multiplication of any two TFNs $\otimes$ is only an approximate TFN. Given two PTFNs, $\tilde{a}=(a_1,a_2,a_3,a_4)$ and $\tilde{b}=(b_1,b_2,b_3,b_4)$, and a non-fuzzy number $r\geq 0$, where $0\leq a_1\leq a_2\leq a_3\leq a_4$ and $0\leq b_1\leq b_2\leq b_3\leq b_4$, then the fuzzy operations of sum, subtraction, multiplication and multiplication by a scalar can be expressed by, respectively:

$$\tilde{a}\oplus\tilde{b}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$$
$$\tilde{a}\ominus\tilde{b}=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1)$$
$$\tilde{a}\otimes\tilde{b}\approx(a_1\times b_1,a_2\times b_2,a_3\times b_3,a_4\times b_4)$$
$$\tilde{a}\otimes r\approx(a_1\times r,a_2\times r,a_3\times r,a_4\times r)$$

Definition 4. (Chen, 2000) The vertex distance $\delta(\tilde{a},\tilde{b})$ between two TFNs $\tilde{a}=(a_1,a_2,a_3,a_4)$ and $\tilde{b}=(b_1,b_2,b_3,b_4)$ is defined as follows:

$$\delta(\tilde{a},\tilde{b})=\sqrt{\tfrac{1}{4}[(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2+(a_4-b_4)^2]}$$

Definition 5. (Sadi-Nezhad and Damghani, 2010) A matrix $\tilde{M}$ is a fuzzy matrix if, at least, one of its elements is a fuzzy number. A fuzzy matrix $\tilde{M}=[\tilde{m}_{ij}]_{m\times n}$, fully composed of trapezoidal fuzzy numbers, can be normalized to interval [0,1] as follows:

$$\tilde{m}_{ij}=\left(\frac{(m_{ij1}+|d^-|)}{(|d^-|+d^+)},\frac{(m_{ij2}+|d^-|)}{(|d^-|+d^+)},\frac{(m_{ij3}+|d^-|)}{(|d^-|+d^+)},\frac{(m_{ij4}+|d^-|)}{(|d^-|+d^+)}\right)$$

where d^+ is the maximum value of m_{ij4} and $|d^-|$ is the absolute value of the minimum value of m_{ij1}. In order to normalize non-positive TFNs, d^- is a necessary parameter. If the matrix is fully composed by PTFNs, $d^-=0$.

2.2. The Fuzzy-ELECTRE method

This section presents the key concepts related to the Fuzzy-ELECTRE method, as presented in Hatami-Marbini and Tavana (2011) and Rouyendegh and Erol (2012). Let us consider that there is a set A of alternatives to be ranked according to a set J of evaluation criteria. The MADM problem under consideration in this research can be expressed by vector $W=[w_1,w_2,\ldots,w_m]$, where w_j is the weight of criteria j, and the following matrix of preferences for m alternatives ranked on n criteria:

$$M=\begin{bmatrix} r_{11} & \cdots & r_{1n}\\ \vdots & \ddots & \vdots\\ r_{m1} & \cdots & r_{mn}\end{bmatrix}$$

where r_{ij} is the performance rating of alternative i in relation to criterion j. In fuzzy environments, both $\tilde{r}_{ij}$ and $\tilde{w}_j$ are linguistic variables that can be represented by PTFN.

In order to define an outranking relation between two alternatives $a\in A$ and $b\in A$, it is necessary to calculate two main measures called concordance index (c_{ab}) and discordance index (d_{ab}) among the set of alternatives. In fuzzy environments, the fuzzy concordance index $\tilde{c}_{ab}$ can be calculated as $\tilde{c}_{ab}=\sum_{j\in J^+}\tilde{w}_j$, where, $J^+=\{j|\tilde{r}_{aj}\geq\tilde{r}_{bj}\}$ is a set that contains the index of all criteria *in favor* of the assertion "a is at least as good b". On one hand, the concordance index measures the strength of the hypothesis of a given alternative a, which is at least as good as alternative b. On the other hand, the discordance index measures the strength of the evidence against this first hypothesis (Wang and Triantaphyllou, 2008). The fuzzy discordance index can be calculated as follows:

$$\tilde{d}_{ab}=\begin{cases}\dfrac{\max_{j\in J^-}(\tilde{r}_{bj}\ominus\tilde{r}_{aj})}{\max_{j\in J}\delta(\tilde{r}_{bj},\tilde{r}_{aj})}, & \text{if}\quad \tilde{r}_{aj}<\tilde{r}_{bj},\quad\forall j\in J\\ 0, & \text{otherwise}\end{cases}$$

where $J^-=\{j|\tilde{r}_{bj}\geq\tilde{r}_{aj}\}$ is a set that contains the index of all criteria against the assertion "a is at least as good b". Once these two measures were calculated, a fuzzy binary outranking relation S can be obtained using $a\,\tilde{S}\,b$ if and only if $\tilde{c}_{ab}\geq\tilde{c}^*$ and $\tilde{d}_{ab}\leq\tilde{d}^*$, where $\overline{c}^*$ and $\overline{d}^*$ are fuzzy thresholds defined by the decision-makers. Next, a directed graph, called outranking graph, can be drawn with the purpose of representing the outranking relations between all alternatives. The sensitivity of the solution can be analyzed by varying the values of $\tilde{c}^*$ and $\tilde{d}^*$ in order to select a reduced set of non-dominated alternatives.

When a full ranking of the alternatives is required, an extension of the seminal ELECTRE I must be used. The ELECTRE II method (Roy, 1990) is a well-known approach for ranking based on two outranking relations, called strong outranking relation (S_s) and weak outranking relation (S_w), which can be defined in a fuzzy environment as follows:

$$a\,\tilde{S_s}\,b\quad\text{if and only if }\tilde{c}_{ab}\geq\tilde{c}_1^*\quad\text{and}\quad\tilde{d}_{ab}\leq\tilde{d}_1^*\tag{1}$$

$$a\,\tilde{S_w}\,b\quad\text{if and only if }\tilde{c}_{ab}\geq\tilde{c}_2^*\quad\text{and}\quad\tilde{d}_{ab}\leq\tilde{d}_2^*\tag{2}$$

where $\tilde{c}_1^*$, $\tilde{c}_2^*$, $\tilde{d}_1^*$, and $\tilde{d}_2^*$ are fuzzy thresholds defined by the DMs, and $\tilde{c}_1^*>\tilde{c}_2^*$ and $\tilde{d}_1^*>\tilde{d}_2^*$. The final ranking of the alternatives is obtained according to ELECTRE's distillation algorithm (Roy, 1990).

Particularly, Chatterjee et al. (2010) proposed a very interesting outranking extension to ELECTRE II, taking into consideration two measures, called pure concordance (Ψ) and pure discordance (Ω) indexes, which provide numerical measures for ranking the alternatives from the best to the worst. These indexes are respectively computed as follows:

$$\Psi_a=\sum_{b=1}^{m}c(a,b)-\sum_{b=1}^{m}c(b,a),\quad\forall a=1,2,\ldots,m,\ \ a\neq b\tag{3}$$

$$\Omega_a=\sum_{b=1}^{m}d(a,b)-\sum_{b=1}^{m}d(b,a),\quad\forall a=1,2,\ldots,m,\ \ a\neq b\tag{4}$$

The set of alternatives is then sorted in descending order by Ψ and in ascending order by Ω. The final position of each alternative is given by the average position of each partial ranking.

3. Proposed method

This section presents the developed hybrid method, combining Fuzzy-ELECTRE and Fuzzy-TOPSIS elements. TOPSIS is a compensatory decision-making method based on an intuitive idea, where the best choice in relation to a set of evaluation criteria is the alternative closest to a positive ideal solution (A^*) and farthest from a negative ideal solution (A^-). Chen (2000) extended TOPSIS for solving group decision-making problems under fuzzy environments, which can be summarized as follows:

Step 1. Form a committee of decision-makers to identify the evaluation criteria.
Step 2. Define the importance weight of the criteria for each decision-maker using appropriated linguistic terms.
Step 3. Determine the performance ratings for each alternatives and criteria with respect to each decision-maker.
Step 4. Aggregated the weight of criteria and the ratings of the alternatives.

Step 5. Construct the normalized fuzzy decision matrix.
Step 6. Construct the weighted normalized fuzzy decision matrix.
Step 7. Determine the positive ideal solution (A^*) and the negative ideal solution (A^-).
Step 8. Calculate the distances of each alternative in relation to the ideal solutions.
Step 9. Calculate the closeness coefficient of each alternative.
Step 10. Rank the alternatives in descending order by the closeness coefficient.

FETOPSIS is based on the last four steps of the Fuzzy-TOPSIS method. Thus, assuming that all assessments were made in relation to the level of importance of the criteria, the performance ratings of alternatives and the preference thresholds were previously carried out, and the fuzzy concordance ($\tilde{C}$) and discordance ($\tilde{D}$) matrices were previously calculated as defined in Section 2.2, FETOPSIS can be described as follows:

Step 1. Normalize the fuzzy concordance matrix $\tilde{C} = [\tilde{c}_{ab}]_{m \times m}$ to a PTFN in the interval [0–1] as seen in Definition 5.
Step 2. Normalize the fuzzy discordance matrix $\tilde{D} = [\tilde{d}_{ab}]_{m \times m}$ to a PTFN in the interval [0,1] as seen in Definition 5.
Step 3. Compute the fuzzy pure concordance index of each alternative based on Eq. (3), as follows:

$$\tilde{\Psi}_a = \tilde{\sum}_{b=1}^{m} \tilde{c}(a,b) \ominus \tilde{\sum}_{b=1}^{m} \tilde{c}(b,a), \quad \forall a = 1,2,\ldots,m, \quad a \neq b$$

where $\tilde{\sum}$ refers to the TFNs summation operation and $\tilde{\Psi}_a = (\psi_{a1}, \psi_{a2}, \psi_{a3}, \psi_{a4})$.
Step 4. Compute the fuzzy pure discordance index based on Eq. (4), as follows:

$$\tilde{\Omega}_a = \tilde{\sum}_{b=1}^{m} \tilde{d}(a,b) \ominus \tilde{\sum}_{b=1}^{m} \tilde{d}(b,a), \quad \forall a = 1,2,\ldots,m, \quad a \neq b$$

where $\tilde{\Omega}_a = (\omega_{a1}, \omega_{a2}, \omega_{a3}, \omega_{a4})$.
Step 5. Normalize $\tilde{\Psi}_a$ and $\tilde{\Omega}_a$ indexes to a PTFN in the interval [0–1] as seen in Definition 5.
Step 6. Calculate the fuzzy positive ideal solution ($\tilde{A}^*$). According to the classic Fuzzy-TOPSIS (Chen, 2000), $\tilde{A}^*$ is formed by the highest possible values for each criterion obtained from the weighted decision matrix. In FETOPSIS, a new procedure for calculating $\tilde{A}^*$ was defined, simultaneously maximize the normalized fuzzy pure concordance index and minimize the fuzzy pure discordance index, as follows:

$$\tilde{A}^* = \{\tilde{v}_c^*, \tilde{v}_d^*\}$$

where $\tilde{v}_c^* = (v_{c1}^*, v_{c2}^*, v_{c3}^*, v_{c4}^*)$, $v_{c1}^* = v_{c2}^* = v_{c3}^* = v_{c4}^* = max_a \{\psi_{a4}\}$, $\tilde{v}_d^* = (v_{d1}^*, v_{d2}^*, v_{d3}^*, v_{d4}^*)$, and $v_{d1}^* = v_{d2}^* = v_{d3}^* = v_{d4}^* = min_a \{\omega_{a1}\}$. Therefore, the positive ideal solution is the one simultaneously closest to the maximum fuzzy concordance and the minimum fuzzy discordance indexes.
Step 7. Calculate the fuzzy negative ideal solution ($\tilde{A}^-$). $\tilde{A}^-$ is well known in the literature (Chen, 2000) as the minimum benefit solution, since it includes the worst values for each criterion. In our method, $\tilde{A}^-$ simultaneously minimize the normalized fuzzy pure concordance and maximize the normalized fuzzy pure discordance indexes as follows:

$$\tilde{A}^- = \{\tilde{v}_c^-, \tilde{v}_d^-\}$$

where $\tilde{v}_c^- = (v_{c1}^-, v_{c2}^-, v_{c3}^-, v_{c4}^-)$, $v_{c1}^- = v_{c2}^- = v_{c3}^- = v_{c4}^- = min_a \{\psi_{a1}\}$, $\tilde{v}_d^- = (v_{d1}^-, v_{d2}^-, v_{d3}^-, v_{d4}^-)$, and $v_{d1}^- = v_{d2}^- = v_{d3}^- = v_{d4}^- = max_a\{\omega_{a4}\}$. Thus, the negative ideal solution proposed in this work is the one simultaneously closest to the minimum fuzzy concordance and maximum fuzzy discordance indexes.
Step 8. Calculate the distance of each alternative (S_a^*) in relation to positive ideal solution $\tilde{A}^*$. In FETOPSIS, S_a^* is computed as the sum of the distance of each alternative in relation to the maximum fuzzy concordance ($\tilde{v}_c^*$)and the minimum fuzzy discordance ($\tilde{v}_d^*$) indexes as follows:

$$S_a^* = \delta(\tilde{\Psi}_a, \tilde{v}_c^*) + \delta(\tilde{\Omega}_a, \tilde{v}_d^*)$$

where $\delta(\tilde{\Psi}_a, \tilde{v}_c^*)$ and $\delta(\tilde{\Omega}_a, \tilde{v}_d^*)$ are computed as seen in Definition 4.
Step 9. Calculate the distance of each alternative (S_a^-) in relation to $\tilde{A}^-$ as the sum of the distance of each alternative in relation to the minimum fuzzy concordance ($\tilde{v}_c^-$) and the maximum fuzzy discordance ($\tilde{v}_d^-$) indexes as follows:

$$S_a^- = \delta(\tilde{\Psi}_a, \tilde{v}_c^-) + \delta(\tilde{\Omega}_a, \tilde{v}_d^-)$$

where $\delta(\tilde{\Psi}_a, \tilde{v}_c^-)$ and $\delta(\tilde{\Omega}_a, \tilde{v}_d^* -)$ are computed as seen in Definition 4.
Step 10. Calculate the closeness coefficient (CC_a) of each alternative as follows:

$$CC_a = \frac{S_a^-}{S_a^* + S_a^-}$$

Although the CC_a's formulation remains the same as the classic Fuzzy-TOPSIS, the meaning of the FETOPSIS' closeness coefficient is different, since the best alternative is the one that is simultaneously closest to the maximum concordance and minimum discordance indexes, and farthest from the minimum concordance and maximum discordance indexes.
Step 11. Rank the alternatives in the descending order of their respective CC_a.

4. Computational experiments

4.1. Simulation experiments

Simulation is a very flexible and adaptable method that allows the use of different samples, controlled and replicable experiments, that provides a large set of results, from which the pattern of solutions obtained by different methods can be studied (Chamodrakas et al., 2011). Through simulation, we can evaluate MADM methods, considering different sample sizes and parameters, in terms of the number of criteria, alternatives, and different ways of rating alternatives and weight criteria distribution. Based on this conclusion, we decided to perform a numerical simulation to evaluate FETOPSIS, comparing it with ELECTRE II, a well-known extension of ELECTRE that is used to rank alternatives, and Fuzzy-TOPSIS. Our objective was to get confidence in the method before applying in the industrial restructuring problem. The experiments were based on previous comparative analysis of MADM methods (Zanakis et al., 1998; Wang and Triantaphyllou, 2008; Chamodrakas et al., 2011). The following parameters were used in the simulation experiments: (i) Number of alternatives (Alt): 5, 7, 9 and 11. (ii) Number of criteria (Crit): 5, 10, 15 and 20. (iii) Ratings of the alternatives: randomly generated from a uniform distribution in interval [0,1]. (iv) Weights of the criteria (Type): three kinds of distribution were used to define the weights: (a) equal values to all criteria, representing a decision-making process where there is no preference among the criteria; (b) uniform distribution, to represent an unbiased, indecisive or uninformed decision-maker; and (c) a U-shaped distribution, to represent a biased decision-maker, clearly favoring some criteria in relation to others. (v) Number of replication: 300 replications were used for each combination (four criteria levels, four alternative levels and three different weight distributions), totaling 14,400 different decision problems.

The simulation and the three MADM methods were implemented in Java Language and all experiments were carried out in a personal computer with the following specifications, Intel Core i5-3450 3.1 GHz CPU with 8 GB of RAM memory. Our computational implementations of Fuzzy-TOPSIS and ELECTRE II were validated by running test cases and comparing our implementations' output with the output provided by Chen et al. (2006) and Chatterjee et al. (2010), respectively. The results obtained with our implementations were identical to the ones reported by these two research studies. Extensive validation tests were carried out to insure FETOPSIS' effectiveness. A series of test examples, with different number of alternatives and criteria, was created in order to validate and adjust the performance of the implementation. In addition, sensitivity analysis was performed by systematically changing the input variable values and parameters over a large range of interest and observing the effect upon the implementation's performance. This was done by running the models with extreme values of the parameters and comparing the solution set of decision variables and their values with manual calculations.

Three categories of performance measures were used to evaluate FETOPSIS. The first group is formed by similarity statistics (Chamodrakas et al., 2011; Zanakis et al., 1998), as follows: (i) Spearman's correlation for ranks (SCR), computed using the following formula $SRC = 1-[(6\sum_{i=1}^{n}(r_i^p - r_i^e)^2)/(n(n^2-1)]$, (ii) percent of matching alternatives in the three first positions (%TOP3), and (iii) percent of matching alternatives in the three worst positions (%BOT3). The goal of this group of measures is to assess the agreement among the three methods which will be significant if SCR, %TOP3 and %BOT3 are equal (or close) to one. The second group of measures assess the capabilities of the methods to offer alternative discriminatory rankings, as follows: (i) Number of indifferences produced by the methods (IND). Two scores with 2% or less of difference were considered as indifference criterion in FETOPSIS and Fuzzy-TOPSIS. (ii) The standard deviation of the rankings (SD). (iii) The score difference between the best and the worst ranked alternatives (DBW). (iv) The score difference between the first and second best ranked alternatives (DFS). Measures SD, DBW, and DFS were only computed for FETOPSIS and Fuzzy-TOPSIS, since ELECTRE II offers rankings in an ordinal scale. The last performance measure is related to the analysis of ranking reversal (RR). We measured RR by the fact that an effective MADM method should not change the indication of the best alternative when a non-optimal alternative is replaced by a worse one, given that the relative importance of each decision criterion remains the same, as defined by Wang and Triantaphyllou (2008). The number of decision problems without the same best alternative, when an irrelevant one was included, was used as a proxy variable to evaluate RR.

Randomly generated matrices $\tilde{M}$ were used in all experiments. To evaluate RR, matrices $\tilde{M}_1$ were constructed from $\tilde{M}$ by replacing a non-optimal alternative by another randomly chosen, with values {10, 20, 30 or 40%} lower than the initial value. The three methods were applied considering the same criteria weights. ELECTRE II was solved by generating binary matrices, originated from applying Eqs. (1) and (2) on matrices $\tilde{C}$ and $\tilde{D}$, respectively. The final ranking obtained follows the ELECTRE's distillation algorithm as described in Roy (1990). Simulation results were analyzed with SPSS® package. Initially, the significance of the performance measures with a 95% confidence level were analyzed with the two-way ANOVA parametric test. Based on the ANOVA's results, we can affirm that Alt, Crit, Type and their interactions affect all performance measures of the first and second groups of measures, while the RR measure was affected by all parameters and their interactions. These results followed those reported by Zanakis et al. (1998) and Wang and Triantaphyllou (2008).

Table 1
Similarity comparison results.

Alt	Crit	FETOPSIS/ELECTRE II			FETOPSIS/FTOPSIS		
		SCR	%TOP3	%BOT3	SCR	%TOP3	%BOT3
5	5	0.986	0.992	1.000	0.957	0.934	0.986
5	10	0.974	0.999	1.000	0.849	0.889	0.999
5	15	0.972	0.999	1.000	0.826	0.883	0.999
5	20	0.967	0.999	1.000	0.808	0.870	0.999
7	5	0.988	0.999	1.000	0.891	0.900	0.996
7	10	0.982	0.998	1.000	0.824	0.861	0.996
7	15	0.975	0.997	1.000	0.814	0.842	0.996
7	20	0.974	0.998	1.000	0.805	0.843	0.998
9	5	0.989	0.998	1.000	0.883	0.888	0.995
9	10	0.986	0.996	1.000	0.820	0.858	0.993
9	15	0.981	0.994	1.000	0.816	0.821	0.988
9	20	0.977	0.986	1.000	0.821	0.792	0.984
11	5	0.993	0.998	1.000	0.871	0.891	0.995
11	10	0.987	0.995	1.000	0.815	0.837	0.992
11	15	0.984	0.988	1.000	0.823	0.830	0.982
11	20	0.980	0.974	1.000	s 0.836	0.824	0.967

Table 1 presents the average similarity measures among the rankings obtained by FETOPSIS compared with ELECTRE II, and FETOPSIS compared with Fuzzy-TOPSIS. The most important finding of the similarity comparison is to conclude that the rankings produced by FETOPSIS are closer to the rankings of ELECTRE II than with the ones obtained by Fuzzy-TOPSIS. In most of the decision problems, FETOPSIS and ELECTRE II choose the same set of alternatives to compose the best and the worst group of alternatives, as indicated by %TOP3 and %BOT3. The average value of SCR equal to 0.981 comparing the rankings of FETOPSIS and ELECTRE II ratify this conclusion. Measures SCR, %TOP3 and %BOT3 remained almost unchanged in regard to parameters Alt, Crit, and Type, when FETOPSIS is compared with ELECTRE II. When comparing FETOPSIS and Fuzzy-TOPSIS, it is possible to observe that as the number of alternatives and criteria increase, the two rankings became less congruent (see values of measures SCR and %TOP3). Moreover, the rankings of these methods are more similar with respect to the worst alternatives, than with the best, as values of %BOT3 are closer to one than %TOP3, in all analyzed decision scenarios. In summary, it seems that FETOPSIS maintains the non-compensatory nature of the ELECTRE family, but without being so far from Fuzzy-TOPSIS (average SCR of 0.841).

Table 2 shows the average results obtained by the three methods concerning alternative discrimination and ranking reversal measures for each combination of alternatives and criteria. The rankings obtained by FETOPSIS presented better results than Fuzzy-TOPSIS and ELECTRE II for all discrimination and RR measures. FETOPSIS significantly improved the alternative discrimination of Fuzzy-TOPSIS' rankings, increasing, on average, the differences between the best and worst, and the first and second ranked alternatives in 64.91% and 66.01%, respectively. The standard deviation of the scores was also increased, on average, 64.74%. On average, FETOPSIS decreased the number of indifferences in comparison with Fuzzy-TOPSIS in 53%. The number of indifferences decreased more significantly when the number of alternatives and criteria have smaller values. Both methods have similar behaviors concerning RR.

Comparing FETOPSIS and ELECTRE II, an important contribution of our approach is the complete elimination of RR in all analyzed decision scenarios. Since ELECTRE has been often cited as one of the MADM methods that presents the worst ranking reversal performance (Zanakis et al., 1998; Wang and Triantaphyllou, 2008), this a very important result towards improving the reliability of the ELECTRE family. On average, ranking reversal occurred in 26.6% of the decision problems analyzed with ELECTRE II, with a

Table 2
Discrimination and ranking reversal measures.

Alt	Crit	FETOPSIS					Fuzzy-TOPSIS					ELECTRE II	
		DBW	DFS	SD	RR	IND	DBW	DFS	SD	RR	IND	RR	IND
5	5	0.404	0.103	1.974	0	0.270	0.267	0.068	1.308	2	0.380	65	1.483
5	10	0.302	0.083	1.484	0	0.340	0.169	0.045	0.830	0	0.54700	96	1.263
5	15	0.266	0.079	1.290	0	0.500	0.149	0.043	0.721	1	0.60700	101	1.203
5	20	0.224	0.064	1.090	0	0.507	0.120	0.034	0.582	1	0.83000	120	1.147
7	5	0.462	0.098	4.079	0	0.720	0.328	0.069	2.922	0	0.79300	65	2.670
7	10	0.321	0.071	2.836	0	1.083	0.195	0.044	1.721	2	1.37700	80	2.603
7	15	0.278	0.061	2.442	0	1.160	0.161	0.036	1.416	0	1.53000	86	2.537
7	20	0.241	0.050	2.122	0	1.363	0.134	0.028	1.183	1	1.73700	105	2.400
9	5	0.440	0.072	6.101	0	1.197	0.306	0.052	4.294	0	1.41300	47	3.967
9	10	0.333	0.061	4.548	0	1.850	0.209	0.037	2.865	1	2.14000	65	3.763
9	15	0.298	0.056	4.115	0	2.263	0.179	0.031	2.479	0	2.64700	74	3.630
9	20	0.261	0.047	3.554	0	1.317	0.144	0.026	1.971	0	2.92700	106	2.400
11	5	0.462	0.067	9.166	0	2.150	0.325	0.045	6.558	0	2.66000	52	5.117
11	10	0.321	0.051	6.412	0	2.930	0.205	0.033	4.080	1	3.49300	68	4.893
11	15	0.300	0.047	5.919	0	3.380	0.178	0.031	3.486	0	4.32000	73	4.347
11	20	0.259	0.044	5.090	0	2.813	0.153	0.025	3.000	0	4.55300	74	3.813

Table 3
Case studies comparison I.

Zandi and Roghanian (2013)					Sun (2010)				
FETOPSIS		ELECTRE II	Fuzzy-TOPSIS		FETOPSIS		ELECTRE	Fuzzy-Topsis	
Ranking	CC_a	Ranking	Ranking	CC_a	Ranking	CC_a	Ranking	Ranking	CC_a
A_2 (1)	0.813	A_2 (1)	A_2 (1)	0.620	A_1 (1)	0.705	A_1 (1)	A_1 (1)	0.523
A_3 (2)	0.519	A_3 (2)	A_3 (2)	0.525	A_2 (2)	0.533	A_2 (2)	A_2 (2)	0.473
A_1 (3)	0.172	A_1 (3)	A_1 (3)	0.392	A_3 (3)	0.453	A_3 (3)	A_3 (3)	0.456
					A_4 (4)	0.275	A_4 (4)	A_4 (4)	0.353
SD	0.321			0.115		0.129			0.033
DFS	0.294			0.095		0.172			0.050
DBW	0.641			0.228		0.430			0.170

Table 4
Case studies comparison II.

Hatami-Marbini and Tavana (2011)					Kaya and Kahraman (2011)				
FETOPSIS		ELECTRE II	Fuzzy-TOPSIS		FETOPSIS		ELECTRE II	Fuzzy-TOPSIS	
Ranking	CC_a	Ranking	Ranking	CC_a	Ranking	CC_a	Ranking	Ranking	CC_a
A_2 (1)	0.727	A_2, A_3 (1)	A_2 (1)	0.642	A_4 (1)	0.808	A_4,A_5 (1)	A_4 (1)	0.456
A_3 (2)	0.692		A_3 (2)	0.622	A_5 (2)	0.690		A_3 (2)	0.441
A_4 (3)	0.439	A_4 (2)	A_4 (3)	0.511	A_3 (3)	0.583	A_3 (2)	A_6 (3)	0.399
A_1 (4)	0.397	A_1 (3)	A_1 (4)	0.495	A_6 (4)	0.498	A_6 (3)	A_5 (4)	0.358
A_5 (5)	0.195	A_5 (4)	A_5 (5)	0.403	A_2 (5)	0.331	A_2 (4)	A_2 (5)	0.336
					A_1 (6)	0.164	A_1 (5)	A_1 (6)	0.329
SD	0.157			0.071		0.113			0.029
DFS	0.035			0.020		0.118			0.014
DBW	0.535			0.239		0.644			0.1126

maximum average value of 40% for 5 alternatives and 20 criteria, and for a minimum average of 15.66% for 9 alternatives and 5 criteria. FETOPSIS also significantly decreased the number of indifferences in comparison with ELECTRE II, on average in 53%. This decrease is more expressive when the number of criteria is smaller.

Overall, FETOPSIS presented more discriminatory rankings than ELECTRE II and Fuzzy-TOPSIS in our experiments, simultaneously decreasing the number of ranking reversals in ELECTRE II. This better performance can be explained by the combination of compensatory and non-compensatory elements from Fuzzy-TOPSIS and fuzzy-ELECTRE, resulting in an improvement of alternative discriminatory power and stability of the ranking procedure, important attributes for a good decision making process in real world problems.

4.2. Comparison based on case studies from the literature

This section compares the three methods using reported cases in recently published papers on fuzzy-ELECTRE or Fuzzy-TOPSIS methods. These works were selected with the purpose of validating and demonstrating the effectiveness of FETOPSIS in realistic situations not created in laboratory. However, as these methods have different backgrounds (Fuzzy-TOPSIS is a compensatory MADM, while Fuzzy-ELETRE is a no-compensatory one), these comparisons must be viewed with cautions. These methods require different decision making preference structures and diverse ways of defining criteria weights.

Tables 3 and 4 present the comparison rankings when the three method were applied to the case studies conducted by Zandi and Roghanian (2013) and Sun (2010), and by Hatami-Marbini and Tavana (2011) and Kaya and Kahraman (2011), respectively. Zandi and Roghanian (2013) developed a hybrid method, combining fuzzy ELECTRE and VIKOR for a site selection problem, where three candidate locations were evaluated by six criteria. Sun (2010) combined AHP and Fuzzy-TOPSIS to evaluate notebook computer companies, where four alternatives were evaluated against four criteria. Hatami-Marbini and Tavana (2011) applied Fuzzy-ELECTRE to evaluate five candidates in relation to five criteria in a supplier selection context, while Kaya and Kahraman (2011) also applied fuzzy-ELECTRE to assess the environmental impact generated by six different industrial districts through eight evaluation criteria.

Overall, the results of these experiments followed the simulation experiments' conclusions. FETOPSIS demonstrated a better discriminatory power than Fuzzy-TOPSIS in all cases, increasing the values of SD, DFS, and DBW, on average, by factors of 2.9, 3.5, and 2.9, respectively. FETOPSIS has also eliminated all indifferences presented by ELECTRE II. There was a general agreement in the rankings of the three methods, with exception of case (Kaya and Kahraman, 2011), where we found a difference in the final ranking of the alternatives between FETOPSIS/ELECTRE II and Fuzzy-TOPSIS. Some differences were expected, since the introduction of the concordance and discordance indexes affects the way the distances to the positive and negative ideal solutions are computed in Fuzzy-TOPSIS. The results ratified that the ranking provided by FETOPSIS are closer to ELECTRE II than Fuzzy-TOPSIS, showing evidences of a non-compensatory behavior to our approach.

Table 5
Criteria and sub-criteria definitions.

Criteria	Sub-criteria
1. Economic (C1)	Expectation of growth (C11) Exposure to enterprise risks (C12) Alignment with controllers (C13) Profitability (C14)
2. Social (C2)	Income generation (C21) Tax generation (C22) Research and innovation (C23)
3. Environmental (C3)	Efficient industrial processes (C31) Environmentally sustainable products (C32) Exposure to environmental risks (C33)

Table 6
Linguistic variables for importance level of criteria and alternative ratings.

Weights	Fuzzy numbers	Ratings	Fuzzy numbers
Very low	(1, 1, 3)	Unimportant	(1, 1, 3)
Low	(1, 3, 5)	Moderately important	(1, 3, 5)
Average	(3, 5, 7)	Important	(3, 5, 7)
High	(5, 7, 9)	Very important	(5, 7, 9)
Very high	(7, 9, 9)	Extremely important	(7, 9, 9)

5. A real case study

Given the good results obtained in the previous experiments, this section succinctly describes the application of FETOPSIS in the selection of investment projects for the industrial restructuring of a small oil refinery located in the southern Brazil. A field test is generally accepted as a proper way of assessing the effectiveness of a decision-making method, allowing for users to capture the simplicity, trustworthiness, robustness, and quality of the method (Zanakis et al., 1998). As stated in Section 1, this real decision has motivated FETOPSIS development towards improving the discrimination of alternatives in the final rankings. Previous applications of consolidated methods, namely Fuzzy-TOPSIS and ELECTRE II, led to unsatisfactory rankings for the board committee members. The scores of the alternatives obtained by the former method were too close for a final decision, while the latter method originated excessive indifferences among alternatives (see Table 15). As the decision involved several uncertainties, vague, and incomplete information, the three members of the board, however very experienced in the petroleum business, felt insecure in taking a decision with the rankings provided by these two methods. It is under this context that the development and application of FETOPSIS occurred.

Although the company has shown positive economic results in the last four years, its medium-term prospects are not promising, since its facilities are incompatible with the new specifications of fuels that became effective in Brazil in 2014. Thus, the refinery will very soon lose its internal market supply function, seriously compromising the continuity of the refinery operations and, consequently, causing severe impacts on the region's economy. The configuration of a crisis scenario has motivated the company board committee to begin an industrial restructuring of the refinery. The discussion has necessarily involved the evaluation of industrial restructuring alternatives, composed of new business strategies and their respective technologies that ensure the company's sustainability in the long term.

Several workshops, coordinated by the board committee and facilitated by external consultants, were carried out to raise possible restructuring alternatives. The strategic review was

Table 7
Linguistic variable for rating of each alternative with respect to each criterion.

Alternative	C11	C12	C13	C14	C21	C22	C23	C31	C32	C33
REF	Low	Average	High	Low	Average	Average	Average	Very low	Low	Very high
UTC	Low	Low	Average	Very low	Low	Very low	High	Low	Very low	High
LOG	Very high	Very low	Very high	High	Very low	Low	High	High	High	Low
TREF	Very high	Very low	Very high	Very high	Very low	Low	Very high	Very high	High	Low
SOLV	Average	Low	High	High	Average	Average	Average	Low	Average	High
DSOL	Average	Low	Very low	Very high	Average	Average	Very low	High	High	Very low
LUB	High	Average	Average	Average	High	High	Average	Average	Average	Average
BIO	Average	High	High	Low	High	Very high	High	High	Very high	High
REC	Low	Very high	High	Low	Very high	Low	Very high	High	High	High

Table 8
Fuzzy decision matrix.

Alternative	C_{11}	C_{12}	C_{13}	C_{14}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}
REF	(1.00, 3.00, 3.00, 5.00)	(3.00, 5.00, 5.00, 7.00)	(5.00, 7.00, 7.00, 9.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(1.00, 1.00, 1.00, 3.00)	(1.00, 3.00, 3.00, 5.00)	(7.00, 9.00, 9.00, 9.00)	(1.00, 3.00, 3.00, 5.00)
UTC	(1.00, 3.00, 3.00, 5.00)	(1.00, 3.00, 3.00, 5.00)	(3.00, 5.00, 5.00, 7.00)	(1.00, 3.00, 3.00, 5.00)	(1.00, 1.00, 1.00, 3.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 3.00, 3.00, 5.00)	(1.00, 1.00, 1.00, 3.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 1.00, 1.00, 3.00)
LOG	(7.00, 9.00, 9.00, 9.00)	(1.00, 1.00, 1.00, 3.00)	(7.00, 9.00, 9.00, 9.00)	(1.00, 1.00, 1.00, 3.00)	(1.00, 3.00, 3.00, 5.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 3.00, 3.00, 5.00)	(5.00, 7.00, 7.00, 9.00)
TREF	(7.00, 9.00, 9.00, 9.00)	(1.00, 1.00, 1.00, 3.00)	(7.00, 9.00, 9.00, 9.00)	(1.00, 1.00, 1.00, 3.00)	(1.00, 3.00, 3.00, 5.00)	(7.00, 9.00, 9.00, 9.00)	(7.00, 9.00, 9.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 3.00, 3.00, 5.00)	(7.00, 9.00, 9.00, 9.00)
SOLV	(3.00, 5.00, 5.00, 7.00)	(1.00, 3.00, 3.00, 5.00)	(5.00, 7.00, 7.00, 9.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(1.00, 3.00, 3.00, 5.00)	(3.00, 5.00, 5.00, 7.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)
DSOLV	(3.00, 5.00, 5.00, 7.00)	(1.00, 3.00, 3.00, 5.00)	(1.00, 1.00, 1.00, 3.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(1.00, 1.00, 1.00, 3.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 1.00, 1.00, 3.00)	(7.00, 9.00, 9.00, 9.00)
LUB	(5.00, 7.00, 7.00, 9.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)	(3.00, 5.00, 5.00, 7.00)
BIO	(3.00, 5.00, 5.00, 7.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(7.00, 9.00, 9.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(7.00, 9.00, 9.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 3.00, 3.00, 5.00)
REC	(1.00, 3.00, 3.00, 5.00)	(7.00, 9.00, 9.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(7.00, 9.00, 9.00, 9.00)	(1.00, 3.00, 3.00, 5.00)	(7.00, 9.00, 9.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(5.00, 7.00, 7.00, 9.00)	(1.00, 3.00, 3.00, 5.00)
w_j	(3.00, 6.30, 6.30, 9.00)	(1.00, 3.00, 3.00, 7.00)	(1.00, 7.00, 7.00, 9.00)	(3.00, 7.00, 7.00, 9.00)	(1.00, 4.30, 4.30, 9.00)	(1.00, 3.00, 3.00, 7.00)	(1.00, 3.60, 3.60, 9.00)	(1.00, 5.60, 5.60, 9.00)	(1.00, 7.00, 7.00, 9.00)	(5.00, 7.60, 7.60, 9.00)

Table 9
Fuzzy concordance matrix.

Alternative	*REF*	*UTC*	*LOG*	*TREF*	*SOLV*	*DSOLV*	*LUB*	*BIO*	*REC*
REF	(0.00, 0.00, 0.00, 0.00)	(12.50, 34.65, 34.65, 49.50)	(4.00, 11.30, 11.30, 18.00)	(4.00, 11.30, 11.30, 18.00)	(3.00, 10.65, 10.65, 17.00)	(4.00, 15.65, 15.65, 25.00)	(2.00, 10.00, 10.00, 16.00)	(4.00, 10.30, 10.30, 16.00)	(6.50, 17.75, 17.75, 29.50)
UTC	(5.50, 19.75, 19.75, 36.50)	(0.00, 0.00, 0.00, 0.00)	(3.50, 8.50, 8.50, 12.50)	(3.00, 7.00, 7.00, 9.00)	(2.50, 9.80, 9.80, 19.50)	(2.50, 11.50, 11.50, 19.50)	(2.50, 9.50, 9.50, 18.50)	(2.00, 8.00, 8.00, 15.00)	(3.00, 9.65, 9.65, 16.00)
LOG	(14.00, 43.10, 43.10, 68.00)	(14.50, 45.90, 45.90, 73.50)	(0.00, 0.00, 0.00, 0.00)	(5.50, 20.10, 20.10, 30.50)	(11.50, 39.30, 39.30, 63.50)	(7.00, 23.90, 23.90, 41.00)	(14.00, 43.10, 43.10, 68.00)	(12.00, 34.20, 34.20, 51.00)	(12.50, 37.65, 37.65, 56.50)
TREF	(14.00, 43.10, 43.10, 68.00)	(15.00, 47.40, 47.40, 77.00)	(12.50, 34.30, 34.30, 55.50)	(0.00, 0.00, 0.00, 0.00)	(14.00, 43.10, 43.10, 68.00)	(10.00, 29.50, 29.50, 50.00)	(14.00, 43.10, 43.10, 68.00)	(13.00, 37.50, 37.50, 59.00)	(13.50, 40.95, 40.95, 64.50)
SOLV	(15.00, 43.75, 43.75, 69.00)	(15.50, 44.60, 44.60, 66.50)	(6.50, 15.10, 15.10, 22.50)	(4.00, 11.30, 11.30, 18.00)	(0.00, 0.00, 0.00, 0.00)	(6.00, 20.30, 20.30, 33.00)	(8.00, 21.90, 21.90, 33.00)	(8.50, 20.75, 20.75, 29.50)	(11.00, 28.20, 28.20, 43.00)
DSOLV	(14.00, 38.75, 38.75, 61.00)	(15.50, 42.90, 42.90, 66.50)	(11.00, 30.50, 30.50, 45.00)	(8.00, 24.90, 24.90, 36.00)	(12.00, 34.10, 34.10, 53.00)	(0.00, 0.00, 0.00, 0.00)	(9.00, 26.80, 26.80, 43.00)	(9.00, 22.55, 22.55, 34.00)	(12.00, 32.80, 32.80, 52.00)
LUB	(16.00, 44.40, 44.40, 70.00)	(15.50, 44.90, 44.90, 67.50)	(4.00, 11.30, 11.30, 18.00)	(4.00, 11.30, 11.30, 18.00)	(10.00, 32.50, 32.50, 53.00)	(9.00, 27.60, 27.60, 43.00)	(0.00, 0.00, 0.00, 0.00)	(11.50, 27.40, 27.40, 38.50)	(11.00, 28.20, 28.20, 43.00)
BIO	(14.00, 44.10, 44.10, 70.00)	(16.00, 46.40, 46.40, 71.00)	(6.00, 20.20, 20.20, 35.00)	(5.00, 16.90, 16.90, 27.00)	(9.50, 33.65, 33.65, 56.50)	(9.00, 31.85, 31.85, 52.00)	(6.50, 27.00, 27.00, 47.50)	(0.00, 0.00, 0.00, 0.00)	(10.00, 31.80, 31.80, 52.00)
REC	(11.50, 36.65, 36.65, 56.50)	(15.00, 44.75, 44.75, 70.00)	(5.50, 16.75, 16.75, 29.50)	(4.50, 13.45, 13.45, 21.50)	(7.00, 26.20, 26.20, 43.00)	(6.00, 21.60, 21.60, 34.00)	(7.00, 26.20, 26.20, 43.00)	(8.00, 22.60, 22.60, 34.00)	(0.00, 0.00, 0.00, 0.00)

Table 10
Fuzzy discordance matrix.

Alternative	*REF*	*UTC*	*LOG*	*TREF*	*SOLV*	*DSOLV*	*LUB*	*BIO*	*REC*
REF	(0.00, 0.00, 0.00, 0.00)	(−0.38, 0.38, 0.38, 2.47)	(0.00, 1.21, 1.21, 1.29)	(0.00, 1.21, 1.21, 1.29)	(−0.38, 0.38, 0.38, 2.47)	(0.25, 1.16, 1.16, 1.16)	(0.00, 0.77, 0.77, 1.93)	(0.36, 1.08, 1.08, 1.44)	(0.28, 0.83, 0.83, 1.10)
UTC	(0.00, 0.18, 0.18, 0.27)	(0.00, 0.00, 0.00, 0.00)	(−1.40, 1.40, 1.40, 1.68)	(−1.40, 1.40, 1.40, 1.68)	(0.36, 1.08, 1.08, 1.44)	(0.18, 1.17, 1.17, 1.21)	(0.09, 0.27, 0.27, 0.37)	(0.15, 0.30, 0.30, 0.30)	(0.07, 0.21, 0.21, 0.28)
LOG	(0.00, 0.10, 0.10, 0.14)	(−0.07, 0.07, 0.07, 0.13)	(0.00, 0.00, 0.00, 0.00)	(−1.15, 1.15, 1.15, 2.31)	(0.00, 0.13, 0.13, 0.20)	(−1.00, 1.00, 1.00, 1.20)	(0.05, 0.14, 0.14, 0.19)	(0.04, 0.13, 0.13, 0.17)	(0.08, 0.17, 0.17, 0.17)
TREF	(0.00, 0.10, 0.10, 0.14)	(−0.07, 0.07, 0.07, 0.13)	(0.00, 0.00, 0.00, 0.00)	(0.00, 0.00, 0.00, 0.00)	(0.00, 0.13, 0.13, 0.20)	(−1.00, 1.00, 1.00, 1.20)	(0.05, 0.14, 0.14, 0.19)	(0.04, 0.13, 0.13, 0.17)	(0.08, 0.17, 0.17, 0.17)
SOLV	(0.00, 0.00, 0.00, 0.00)	(−0.36, 0.36, 0.36, 1.08)	(−1.40, 1.40, 1.40, 1.68)	(−1.40, 1.40, 1.40, 1.68)	(0.00, 0.00, 0.00, 0.00)	(0.18, 1.17, 1.17, 1.21)	(−0.12, 0.12, 0.12, 0.46)	(0.00, 0.15, 0.15, 0.30)	(0.00, 0.14, 0.14, 0.28)
DSOLV	(0.06, 0.17, 0.17, 0.23)	(0.06, 0.18, 0.18, 0.24)	(−1.40, 1.40, 1.40, 1.68)	(−1.40, 1.40, 1.40, 1.68)	(0.06, 0.18, 0.18, 0.24)	(0.00, 0.00, 0.00, 0.00)	(0.00, 0.13, 0.13, 0.20)	(0.06, 0.18, 0.18, 0.24)	(0.12, 0.24, 0.24, 0.24)
LUB	(−0.39, 0.39, 0.39, 1.16)	(−0.38, 0.38, 0.38, 2.47)	(0.00, 1.21, 1.21, 1.29)	(0.00, 1.21, 1.21, 1.29)	(−0.38, 0.38, 0.38, 2.47)	(0.00, 1.21, 1.21, 1.29)	(0.00, 0.00, 0.00, 0.00)	(0.00, 0.80, 0.80, 1.20)	(0.00, 0.55, 0.55, 0.83)
BIO	(−0.65, 0.65, 0.65, 2.51)	(0.00, 0.45, 0.45, 2.09)	(0.18, 1.17, 1.17, 1.21)	(0.18, 1.17, 1.17, 1.21)	(0.00, 0.45, 0.45, 2.09)	(0.18, 1.17, 1.17, 1.21)	(−0.72, 0.72, 0.72, 2.79)	(0.00, 0.00, 0.00, 0.00)	(−0.36, 0.36, 0.36, 0.72)
REC	(0.00, 0.77, 0.77, 1.93)	(0.12, 0.48, 0.48, 1.94)	(0.25, 1.16, 1.16, 1.16)	(0.25, 1.16, 1.16, 1.16)	(0.12, 0.48, 0.48, 1.94)	(0.18, 1.17, 1.17, 1.21)	(0.00, 0.77, 0.77, 1.93)	(0.36, 1.08, 1.08, 1.44)	(0.00, 0.00, 0.00, 0.00)

Table 11
Normalized fuzzy discordance matrix.

Alternative	*REF*	*UTC*	*LOG*	*TREF*	*SOLV*	*DSOLV*	*LUB*	*BIO*	*REC*
REF	(0.00, 0.00, 0.00, 0.00)	(0.16, 0.45, 0.45, 0.64)	(0.05, 0.15, 0.15, 0.23)	(0.05, 0.15, 0.15, 0.23)	(0.04, 0.14, 0.14, 0.22)	(0.05, 0.20, 0.20, 0.32)	(0.03, 0.13, 0.13, 0.21)	(0.05, 0.13, 0.13, 0.21)	(0.08, 0.23, 0.23, 0.38)
UTC	(0.07, 0.26, 0.26, 0.47)	(0.00, 0.00, 0.00, 0.00)	(0.05, 0.11, 0.11, 0.16)	(0.04, 0.09, 0.09, 0.12)	(0.03, 0.13, 0.13, 0.25)	(0.03, 0.15, 0.15, 0.25)	(0.03, 0.12, 0.12, 0.24)	(0.03, 0.10, 0.10, 0.19)	(0.04, 0.13, 0.13, 0.21)
LOG	(0.18, 0.56, 0.56, 0.88)	(0.19, 0.60, 0.60, 0.95)	(0.00, 0.00, 0.00, 0.00)	(0.07, 0.26, 0.26, 0.40)	(0.15, 0.51, 0.51, 0.82)	(0.09, 0.31, 0.31, 0.53)	(0.18, 0.56, 0.56, 0.88)	(0.16, 0.44, 0.44, 0.66)	(0.16, 0.49, 0.49, 0.73)
TREF	(0.18, 0.56, 0.56, 0.88)	(0.19, 0.62, 0.62, 1.00)	(0.16, 0.45, 0.45, 0.72)	(0.00, 0.00, 0.00, 0.00)	(0.18, 0.56, 0.56, 0.88)	(0.13, 0.38, 0.38, 0.65)	(0.18, 0.56, 0.56, 0.88)	(0.17, 0.49, 0.49, 0.77)	(0.18, 0.53, 0.53, 0.84)
SOLV	(0.19, 0.57, 0.57, 0.90)	(0.20, 0.58, 0.58, 0.86)	(0.08, 0.20, 0.20, 0.29)	(0.05, 0.15, 0.15, 0.23)	(0.00, 0.00, 0.00, 0.00)	(0.08, 0.26, 0.26, 0.43)	(0.10, 0.28, 0.28, 0.43)	(0.11, 0.27, 0.27, 0.38)	(0.14, 0.37, 0.37, 0.56)
DSOLV	(0.18, 0.50, 0.50, 0.79)	(0.20, 0.56, 0.56, 0.86)	(0.14, 0.40, 0.40, 0.58)	(0.10, 0.32, 0.32, 0.47)	(0.16, 0.44, 0.44, 0.69)	(0.00, 0.00, 0.00, 0.00)	(0.12, 0.35, 0.35, 0.56)	(0.12, 0.29, 0.29, 0.44)	(0.16, 0.43, 0.43, 0.68)
LUB	(0.21, 0.58, 0.58, 0.91)	(0.20, 0.58, 0.58, 0.88)	(0.05, 0.15, 0.15, 0.23)	(0.05, 0.15, 0.15, 0.23)	(0.13, 0.42, 0.42, 0.69)	(0.12, 0.36, 0.36, 0.56)	(0.00, 0.00, 0.00, 0.00)	(0.15, 0.36, 0.36, 0.50)	(0.14, 0.37, 0.37, 0.56)
BIO	(0.18, 0.57, 0.57, 0.91)	(0.21, 0.60, 0.60, 0.92)	(0.08, 0.26, 0.26, 0.45)	(0.06, 0.22, 0.22, 0.35)	(0.12, 0.44, 0.44, 0.73)	(0.12, 0.41, 0.41, 0.68)	(0.08, 0.35, 0.35, 0.62)	(0.00, 0.00, 0.00, 0.00)	(0.13, 0.41, 0.41, 0.68)
REC	(0.15, 0.48, 0.48, 0.73)	(0.19, 0.58, 0.58, 0.91)	(0.07, 0.22, 0.22, 0.38)	(0.06, 0.17, 0.17, 0.28)	(0.09, 0.34, 0.34, 0.56)	(0.08, 0.28, 0.28, 0.44)	(0.09, 0.34, 0.34, 0.56)	(0.10, 0.29, 0.29, 0.44)	(0.00, 0.00, 0.00, 0.00)

Table 12
Normalized fuzzy discordance matrix.

Alternative	*REF*	*UTC*	*LOG*	*TREF*	*SOLV*	*DSOLV*	*LUB*	*BIO*	*REC*
REF	(0.33, 0.33, 0.33, 0.33)	(0.24, 0.43, 0.43, 0.92)	(0.33, 0.62, 0.62, 0.64)	(0.33, 0.62, 0.62, 0.64)	(0.24, 0.43, 0.43, 0.92)	(0.39, 0.61, 0.61, 0.61)	(0.33, 0.52, 0.52, 0.79)	(0.42, 0.59, 0.59, 0.68)	(0.40, 0.53, 0.53, 0.60)
UTC	(0.33, 0.38, 0.38, 0.40)	(0.33, 0.33, 0.33, 0.33)	(0.00, 0.67, 0.67, 0.73)	(0.00, 0.67, 0.67, 0.73)	(0.42, 0.59, 0.59, 0.68)	(0.38, 0.61, 0.61, 0.62)	(0.36, 0.40, 0.40, 0.42)	(0.37, 0.41, 0.41, 0.41)	(0.35, 0.38, 0.38, 0.40)
LOG	(0.33, 0.36, 0.36, 0.37)	(0.32, 0.35, 0.35, 0.37)	(0.33, 0.33, 0.33, 0.33)	(0.06, 0.61, 0.61, 0.88)	(0.33, 0.37, 0.37, 0.38)	(0.10, 0.57, 0.57, 0.62)	(0.35, 0.37, 0.37, 0.38)	(0.34, 0.36, 0.36, 0.38)	(0.35, 0.37, 0.37, 0.37)
TREF	(0.33, 0.36, 0.36, 0.37)	(0.32, 0.35, 0.35, 0.37)	(0.33, 0.33, 0.33, 0.33)	(0.33, 0.33, 0.33, 0.33)	(0.33, 0.37, 0.37, 0.38)	(0.10, 0.57, 0.57, 0.62)	(0.35, 0.37, 0.37, 0.38)	(0.34, 0.36, 0.36, 0.38)	(0.35, 0.37, 0.37, 0.37)
SOLV	(0.33, 0.33, 0.33, 0.33)	(0.25, 0.42, 0.42, 0.59)	(0.00, 0.67, 0.67, 0.73)	(0.00, 0.67, 0.67, 0.73)	(0.33, 0.33, 0.33, 0.33)	(0.38, 0.61, 0.61, 0.62)	(0.31, 0.36, 0.36, 0.44)	(0.33, 0.37, 0.37, 0.41)	(0.33, 0.37, 0.37, 0.40)
DSOLV	(0.35, 0.38, 0.38, 0.39)	(0.35, 0.38, 0.38, 0.39)	(0.00, 0.67, 0.67, 0.73)	(0.00, 0.67, 0.67, 0.73)	(0.35, 0.38, 0.38, 0.39)	(0.33, 0.33, 0.33, 0.33)	(0.33, 0.37, 0.37, 0.38)	(0.35, 0.38, 0.38, 0.39)	(0.36, 0.39, 0.39, 0.39)
LUB	(0.24, 0.43, 0.43, 0.61)	(0.24, 0.43, 0.43, 0.92)	(0.33, 0.62, 0.62, 0.64)	(0.33, 0.62, 0.62, 0.64)	(0.24, 0.43, 0.43, 0.92)	(0.33, 0.62, 0.62, 0.64)	(0.33, 0.33, 0.33, 0.33)	(0.33, 0.52, 0.52, 0.62)	(0.33, 0.47, 0.47, 0.53)
BIO	(0.18, 0.49, 0.49, 0.93)	(0.33, 0.44, 0.44, 0.83)	(0.38, 0.61, 0.61, 0.62)	(0.38, 0.61, 0.61, 0.62)	(0.33, 0.44, 0.44, 0.83)	(0.38, 0.61, 0.61, 0.62)	(0.16, 0.51, 0.51, 1.00)	(0.33, 0.33, 0.33, 0.33)	(0.25, 0.42, 0.42, 0.51)
REC	(0.33, 0.52, 0.52, 0.79)	(0.36, 0.45, 0.45, 0.80)	(0.39, 0.61, 0.61, 0.61)	(0.39, 0.61, 0.61, 0.61)	(0.36, 0.45, 0.45, 0.80)	(0.38, 0.61, 0.61, 0.62)	(0.33, 0.52, 0.52, 0.79)	(0.42, 0.59, 0.59, 0.68)	(0.33, 0.33, 0.33, 0.33)

consolidated in a SWOT matrix constructed for each possible business line and compatible with the potentialities identified in the refinery. As a result of these workshops, the members of the board identified the following business lines as potential alternatives: (i) petroleum refining in all units (REF), (ii) unit test for research center (UTC), (iii) logistic services (LOG), (iv) treatment of waste and effluent (TREF), (v) special solvents (SOLV), (vi) solvent distribution (DSOL), (vii) lubricant formulation (LUB), (viii) biofuels (BIO), and (ix) plastic recycling (REC).

As the main objective of the industrial restructuring was to ensure sustainable business continuity of the refinery, the set of criteria was defined from a hierarchical structure model based on three drivers: (i) create shareholder value; (ii) contribute to regional development; and (iii) act in an environmentally responsible way. Initially, 75 indicators applicable to the context of industrial enterprises were identified. Considering the decision-making stage and the availability of information, the decision-making group selected 10 criteria for assessing the corporate sustainability of the restructuring alternatives. Table 5 presents the final set of criteria interactively defined by the board committee after several meetings.

The linguistics terms used to assess the alternatives are presented in Table 6. Table 7 shows the evaluation of the alternatives for each criterion carried out by the DMs. In order to avoid bias resulting from the central tendency, we asked the DMs to classify at least one alternative as “very high” and another as “very low”, distributing the other alternatives between these two ends.

In order to apply FETOPSIS, the following matrices must be defined: (i) fuzzy decision matrix (Table 8) – computed using the evaluation of the decision makers concerning the criteria weights and the ratings of each alternative with respect to each criterion and (ii) fuzzy concordance (Table 9) and fuzzy discordance matrices (Table 10) – computed as in Section 2.2. After obtaining the above matrices, FETOPSIS' ranking procedure generates the following results in each step:

1. Normalized fuzzy concordance matrix, see Table 11.
2. Normalized fuzzy discordance matrix, see Table 12.
3. Fuzzy pure concordance index ($\tilde{\Psi}_a$), see Table 13.
4. Fuzzy pure discordance index ($\tilde{\Omega}_a$), see Table 13.
5. $\tilde{\Psi}_a$ and $\tilde{\Omega}_a$ normalized, see Table 13.
6. Fuzzy positive ideal solution $\tilde{A}^* = \{(1,1,1,1),(0,0,0,0)\}$, given that $\max_a\{\psi_{a4}\} = 1$ and $\min_a\{\{\omega_{a1}\}\} = 0$.
7. Fuzzy negative ideal solution $\tilde{A}^- = \{(0,0,0,0),(1,1,1,1)\}$, given that $\min_a\{\psi_{a1}\} = 0$ and $\max_a\{\omega_{a4}\} = 1$.
8. Distance of each alternative in relation to positive ideal solution (S_a^*), see Table 14.
9. Distance of each alternative in relation to negative ideal solution (S_a^-), see Table 14.
10. Closeness coefficient (CC_a), see Table 14. Based on the results presented in this table, alternative TREF has ranked as the best alternative.

Table 15 presents a comparison of the ranking obtained by Fuzzy-TOPSIS, ELECTRE II, and FETOPSIS for this real case study. In this table, some dispersion measures concerning the rankings of Fuzzy-TOPSIS and FETOPSIS are also presented. The scores of Fuzzy-TOPSIS and FETOPSIS led to significant different rankings of the alternatives. In the former method, six alternatives presented scores in the interval [0.41,0.48]. The DMs felt extremely uncomfortable with this situation. Given the narrow difference between the scores, they were reluctant in defining which are the best ones for further consideration. FETOPSIS clearly indicated two best alternatives, TREF and LOG, being the former the best ranked one. The dispersion measures SD, DFS, and DBW presented by FETOPSIS were improved in around 93.67%, 75.80%, and 68.32%, respectively,

Table 13
Fuzzy pure concordance and discordance indexes.

Alternative	$\tilde{\Psi}_a$	$\tilde{\Omega}_a$	$\tilde{\Psi}_a$	$\tilde{\Omega}_a$
REF	(−5.96, −2.49, −2.49, 1.10)	(−1.50, 1.11, 1.11, 3.37)	(0.06, 0.33, 0.33, 0.61)	(0.28, 0.67, 0.67, 1.00)
UTC	(−6.71, −3.48, −3.48, 0.35)	(−2.98, 0.87, 0.87, 1.98)	(0.00, 0.25, 0.25, 0.55)	(0.06, 0.63, 0.63, 0.79)
LOG	(−1.88, 1.81, 1.81, 5.18)	(−2.87, −1.44, −1.44, 1.98)	(0.38, 0.66, 0.66, 0.93)	(0.07, 0.29, 0.29, 0.79)
TREF	(−0.94, 2.63, 2.63, 6.13)	(−3.15, −1.99, −1.99, 1.70)	(0.45, 0.73, 0.73, 1.00)	(0.03, 0.20, 0.20, 0.75)
SOLV	(−3.88, −0.30, −0.30, 3.18)	(−3.37, 0.36, 0.36, 1.65)	(0.22, 0.50, 0.50, 0.77)	(0.00, 0.55, 0.55, 0.74)
DSOLV	(−2.69, 0.93, 0.93, 4.38)	(−2.89, −1.22, −1.22, 1.38)	(0.31, 0.59, 0.59, 0.86)	(0.07, 0.32, 0.32, 0.71)
LUB	(−3.32, 0.26, 0.26, 3.74)	(−2.20, 0.73, 0.73, 3.01)	(0.26, 0.54, 0.54, 0.81)	(0.17, 0.61, 0.61, 0.95)
BIO	(−2.61, 0.89, 0.89, 4.45)	(−1.53, 0.54, 0.54, 3.06)	(0.32, 0.59, 0.59, 0.87)	(0.27, 0.58, 0.58, 0.95)
REC	(−3.79, −0.24, −0.24, 3.27)	(−0.59, 1.05, 1.05, 2.96)	(0.23, 0.50, 0.50, 0.78)	(0.41, 0.66, 0.66, 0.94)

Table 14
FETOPSIS final ranking.

Alternative	$\delta(\tilde{\Psi}_a, \tilde{v}_c^*)$	$\delta(\tilde{\Omega}_a, \tilde{v}_d^*)$	S_a^*	$\delta(\tilde{\Psi}_a, \tilde{v}_c^-)$	$\delta(\tilde{\Omega}_a, \tilde{v}_d^*-)$	S_a^-	CC_a
REF	0.70	0.70	1.40	0.38	0.43	0.82	0.37
UTC	0.76	0.60	1.36	0.33	0.55	0.88	0.39
LOG	0.39	0.45	0.84	0.69	0.69	1.38	0.62
TREF	0.34	0.40	0.74	0.75	0.75	1.50	0.67
SOLV	0.54	0.54	1.08	0.53	0.60	1.14	0.51
DSOLV	0.45	0.42	0.87	0.62	0.69	1.31	0.60
LUB	0.50	0.65	1.14	0.57	0.50	1.07	0.48
BIO	0.45	0.64	1.10	0.62	0.47	1.09	0.50
REC	0.53	0.69	1.23	0.54	0.38	0.92	0.43

Table 15
Comparison of ranking and scores of the applied methods.

Alternative	Fuzzy-TOPSIS		ELECTRE II	FETOPSIS	
	CC	Ranking	Ranking	CC	Ranking
REF	0.373	7	3	0.368	9
UTC	0.299	9	3	0.392	8
LOG	0.419	6	1	0.620	2
TREF	0.436	5	1	0.670	1
SOLV	0.424	4	2	0.513	4
DSOL	0.370	8	1	0.599	3
LUB	0.438	3	2	0.483	6
BIO	0.479	1	2	0.499	5
REC	0.451	2	2	0.429	7
SD	0.053			0.104	
DFS	0.028			0.049	
DBW	0.179			0.301	

when compared with the same values obtained by Fuzzy-TOPSIS. Comparing ELECTRE II and FETOPSIS, it is possible to note that their rankings were compatible, placing the same alternatives in TOP3 and BOT3. But again, the DMs felt much more confident with FETOPSIS ranking, given the high number of indifferences in ELECTRE II ranking. In summary, FETOPSIS improved the discrimination of alternatives, increasing the DMs' confidence in selecting the best set of alternatives for further evaluation.

6. Conclusions

In this paper, we proposed a new approach to rank alternatives based on elements from Fuzzy-TOPSIS and Fuzzy-ELECTRE. Basically, our developed approach uses the closeness coefficient, as in the Fuzzy-TOPSIS method, to rank the alternatives. However, this coefficient is computed over the developed normalized fuzzy concordance and discordance indexes. Based on the results obtained from extensive simulation experimentation, it is possible to affirm that the proposed ranking approach improves the final ranking of the alternatives obtained by ELECTRE II and Fuzzy-TOPSIS in terms of a better discrimination of alternatives, taking into consideration the preferences and values of the DMs. Our proposed method also significantly reduced, for all case studies (regardless of the combination of the number of alternatives, criteria, and weight distribution), the number of ranking reversal cases in comparison with ELECTRE II, using the criteria introduced by Wang and Triantaphyllou (2008).

Given the results obtained by the computational experiments, we applied FETOPSIS in an industrial restructuring decision making problem of a small oil refinery in Brazil. Actually, the development of FETOPSIS was motivated by the difficulty of the DMs to take a decision given the lack of alternative discrimination on the rankings obtained by the previous application of ELECTRE II and Fuzzy-TOPSIS (Chen et al., 2006). As a whole, the DMs delivered an overall positive evaluation for the application of FETOPSIS for the problem, given the better discrimination of alternatives provided by this ranking procedure. Moreover, the members of the board committee praised the use of fuzzy MADM techniques by the use of linguistic terms both for the evaluation of the alternatives and for the elicitation of preferences. The use of these methods also introduced an important governance tool that surmounted the limitations imposed by traditional analysis and evaluation tools that prioritize only operational and financial issues.

Although we cannot affirm that our approach obtains the most appropriate rankings for all MADM problems, the rankings provided are much less susceptible to ranking irregularities and have a higher power of alternative discrimination than the ones obtained by ELECTRE II and Fuzzy-TOPSIS. This fact is a direct consequence of a well balanced combination of compensatory and non-compensatory elements from both these methods.

Future work will be directed towards additional comparisons with other MADM methods (AHP, VIKOR, and Promethee), using both simulations and field test cases. These additional experiments will provide the range of situations where our developed ranking procedure can be applied with confidence by DMs.

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