



# Multi-objective reactive power optimization strategy for distribution system with penetration of distributed generation



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## ABSTRACT

The study investigates multi-objective reactive power optimization (MORPO) of distribution system penetrated with distributed generation (DG). Integrating the reactive power of DG as one type of decision variables, a multi-objective model for RPO has been established to decrease the system active power loss, reduce voltage deviation and minimize the total capacity of reactive power compensation (RPC) devices (or minimize investments on RPC devices). Instead of converting the multiple objectives into a single one, a dynamically adaptive multi-objective particle swarm optimization (DAMOPSO) algorithm with introduction of special adaptive techniques has been proposed and validated and then applied to the MORPO problem with continuous and discrete variables. In order to the proposed MORPO model and the application of DAMOPSO, and to obtain a deep insight into MORPO with different objectives, a series of simulations on IEEE 33-bus system along with analysis and discussion are carried out. The results verified the feasibility and effectiveness of the proposed strategy.

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## Introduction

With energy and environmental challenges, distributed generation (DG, such as fuel-cells, biomass, micro-turbines, small hydro-electric and other forms of renewable energy technologies [1,2]) has attracted special attention all over the world, and its estimated share will increase significantly in electric power systems in the near future. In general, DG can be installed within distribution systems or on the customer side of the network [3]. However, the traditional distribution systems have been constructed without considering DG's penetration. The impact of DG on the distribution of the reactive power is significant in a distribution system with radial configuration and small X/R ratio [4–8]. Thus, RPO of the distribution system with DG is fundamental to ensure the economic operation of the system without violating technical and operational limits and to provide consumers with sufficient power of high quality.

RPO of the distribution system with integrated DG has been investigated, and a large number of optimization models and methods for this problem have been proposed. Firstly, different kinds of single-objective optimization model have been studied

and various single-objective optimization algorithms have been applied [8,9]. Afterwards, researchers advocated that a wide range of objective functions should be considered and a multi-objective formulation should be formed to effectively replicate different perspectives of RPO problem. The commonality of such researches is that a multi-objective problem (MOP) is converted to a single-objective one using a weighted aggregation approach [4,5,10] or fuzzy optimization method [6,7]. It have to be pointed that fuzzy method and weighted aggregation approach are inherently single-objective optimization techniques, and the only one best solution fails to provide the designer with alternative options [11,12], though they simplify the optimization process of MORPO problem. Then, single-objective optimization cannot accurately reflect the relationship between the various objectives, especially when the involved objectives are conflict with each other. Furthermore, fuzzy optimization turns out to be weighted aggregation approach with a set of stationary weights (preference factors [13]). If such a relative preference factor among the objectives is known for a specific problem, weighted aggregation approach is a simple and adequate method to deal with the MOPs. While, it's important to realize that the solution obtained by this strategy is largely sensitive to the relative vector used in forming the composite function [13,14].

Therefore, some multi-objective optimization (MOO) techniques, which have been proved to be efficient in solving MOPs

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by tackling multiple competing objectives simultaneously, should be applied to the MORPO problem. The objectives involved in MORPO are usually non-commensurable or even conflict with each other, and it is always impossible to find a solution which can optimize all the objectives at the same time. Instead of a single optimal solution, the solution to a MOP is a set of different solutions (so-called Pareto optimal set [13]). Via MOO algorithms, we aim to find Pareto solutions that represent the best possible compromises among the objectives. Deb [13] argued that the MOO technique can provide flexibility with a variety of diverse choices and it is more methodical, more practical and less subjective, compared with the method converting the multi-objective into a single objective. Such MOO technique has been applied to the MORPO problem, which made the MORPO come into true. Especially, the MOO technique has been applied to MORPO of distribution system with DG. Considering the voltage stability of the grid-connected wind farm, Zhao and Lv [15] addressed the MORPO model for power system with wind farms and indicated that the identified Pareto solutions can provide decision maker with alternative choices. To maintain the diversity of Pareto solutions, the niching technique was adopted, while one of its drawbacks is difficult to define the involved parameters.

This study aims to investigate the MORPO strategy for distribution system penetrated with DG. Integrate the reactive power of the DG to be the decision variables together with the traditional RPO decision variables and establish the MOO model, in which the objective functions include minimization of the system active power loss, voltage deviation and total capacity of the RPC devices (or investments on the RPC devices). To provide the decision maker with alternatives and to allow to analyze correlations between optimization objectives, a dynamically adaptive multi-objective optimization (DAMOPSO) algorithm has been presented to deal with the non-linear MORPO problem with continuous and discrete variables. For demonstrating advantages of the proposed model and effectiveness of the application of DAMOPSO, comparison and discussion have been carried out along with a series of simulations on modified IEEE 33-bus system.

## Multi-objective reactive power optimization model

### Objective function

Economic and safe operation of the electric power system is paramount to all others, which also results in great benefits to the society. Even though power loss cannot be completely removed, it can be brought down to an acceptable value. Moreover, reducing the power loss has a positive impact on relieving the feeders, decreasing the voltage drop and possessing other environmental and economical benefits [16]. Hence, power loss is a key and greatly concerned index regarding RPO.

Besides, stability of the system and quality of power supply become more and more important with the development of the society. Failure of power supply and lower power quality would produce faults to the terminal system even paralysis, the resulting effect is devastating and the disrupted productivity would cost billions of dollars in damages. Voltage deviation is one of indices to evaluate the stability of the system and the quality of power supply [17], and minimization of voltage deviation has been selected as one of the multiple objectives in [6,7,10].

Furthermore, the utmost aim of RPO is to reducing active power loss, improving voltage profile and promoting voltage stability with acceptable investment on RPC devices. Since the investment is related to the total capacity of the installed RPC devices, less investment means less total capacity of RPC devices while meeting reactive power demand. Under electric power market environment

with separation of power plant and power grid, researchers realized the importance and necessity of pricing the reactive power, and have investigated the RPO problem considering the cost of reactive power, such as decreasing investment of RPC equipments [4,5], minimizing reactive power injection costs [18].

Based on these considerations, this study propose MOO model for RPO of the distribution system penetrated with DG, in which the multiple objectives consist of minimizing the active power loss, the total voltage deviation and the total capacity of RPC devices (or investments on RPC devices). The mathematical formulations of the objectives can be expressed as follows.

$$\min f_{loss} = \sum_{k=1}^{N_{bra}} G_k [V_i^2 + V_j^2 - V_i V_j \cos \theta_{ij}] \quad (1)$$

$$\min f_{\Delta V} = \sum_{i=1}^{N_{bus}} (V_i - V_i^*)^2 \quad (2)$$

$$\min f_{cost/Q} = \sum_{s=1}^{N_Q} C_{CAPs} |Q_{qs}| \quad (3)$$

where  $f_{loss}$ ,  $f_{\Delta V}$  and  $f_{cost/Q}$  represent the total active power loss, the total voltage deviation and the investments on RPC devices (or total capacity of RPC devices), respectively.  $C_{CAP}$  indicates the investment for RPC devices per unit. If  $C_{CAP} = 1$ , Eq. (3) becomes a function of total capacity of RPC devices.  $N_{bra}$ ,  $N_{bus}$  and  $N_Q$  denote the total number of branches, buses and RPC devices in the system, respectively.  $G_k$  is the conductance of branch  $k$  which connects bus  $i$  and bus  $j$ .  $V$  and  $\theta$  are voltage magnitude and voltage angle, respectively.  $\theta_{ij} = \theta_i - \theta_j$ .  $Q_q$  represents the actual capacity of RPC device installed.

### Constraints

The conventional decision variables of RPO include generator terminal voltage magnitude  $V_G$ , reactive power of capacitors  $Q_C$  and tap of transformers  $T$ , while the state variables consist of reactive power of generator(s)  $Q_G$  and voltage magnitude of each load bus in the system. Considering the reactive power capability of DG, the decision variables also include the reactive power of the DG,  $Q_{DG}$ . Decision variables and state variables must keep within the pre-defined ranges to ensure the quality of power supply, economic and safe operation of the power system. According to the regulations of power system operation and technical and physical limits, constraints are defined as follows:

### Power balance constraints

The equality constraints are the power balance constraints with DG, which include two nonlinear recursive power flow equations. For bus  $i$ , it can be formulated as Eq. (4).

$$\begin{cases} P_{Gi} + P_{DG_i} - P_{Li} = V_i \sum_{j=1}^{N_{bus}} V_j (G_k \cos \theta_{ij} + B_k \sin \theta_{ij}) \\ Q_{Gi} + Q_{DG_i} + Q_{Ci} - Q_{Li} = V_i \sum_{j=1}^{N_{bus}} V_j (G_k \sin \theta_{ij} + B_k \cos \theta_{ij}) \end{cases} \quad (4)$$

where  $P_G$ ,  $P_{DG}$  and  $P_L$  represent active power of generator, DG and load at bus  $i$ , respectively.  $Q_G$ ,  $Q_{DG}$ ,  $Q_C$  and  $Q_L$  represent reactive power of generator, DG, RPC and load, respectively.  $B_k$  represent the susceptance of the branch  $k$ .

While, inequality constraints mainly include the following ones:

### Voltage limit for each bus

The voltage magnitude of each bus in the network reflects the quality of power supply and voltage stability is important for safe operation of the network. Hence, the voltage level at each bus is not allowed to fall outside the maximum and minimum values according to grid voltage regulations. For bus  $i$ , voltage limit can be expressed as Eq. (5).

$$V_{imin} \leq V_i \leq V_{imax}, \quad i = 1, 2, \dots, N_{bus} \quad (5)$$

where  $_{min}$  and  $_{max}$  represent minimum and maximum limits, respectively.

### Feeder transmission capacity constraints

Power flow through any distribution feeder must comply with the thermal capacity of the line, which can be expressed by Eq. (6).

$$S_{kmin} \leq S_k \leq S_{kmax}, \quad k = 1, 2, \dots, N_{bra} \quad (6)$$

where  $S_k$  represents the transmission capacity of branch  $k$ .

### Constraints for generators, transformers and RPC devices

Such constraints mainly include limits on terminal voltage of the generators, reactive power output of the generators and DGs, the ratio of load tap changer transformers and output of RPC equipment, which can be sequentially formulated as Eqs. (7)–(11).

$$V_{gmin} \leq V_g \leq V_{gmax}, \quad g = 1, 2, \dots, N_G \quad (7)$$

$$Q_{Ggmin} \leq Q_{Gg} \leq Q_{Ggmax}, \quad g = 1, 2, \dots, N_G \quad (8)$$

$$Q_{DGlmin} \leq Q_{DGl} \leq Q_{DGlmax}, \quad l = 1, 2, \dots, N_{DG} \quad (9)$$

$$T_{jmin} \leq T_j \leq T_{jmax}, \quad j = 1, 2, \dots, N_T \quad (10)$$

$$Q_{Csmin} \leq Q_{Cs} \leq Q_{Csmax}, \quad s = 1, 2, \dots, N_Q \quad (11)$$

where  $N_{DG}$  and  $N_T$  denote number of DG and transformers, respectively.

From equations described above, it can be seen that the objective functions with technical and operational constraints are formulated. The MORPO model has nonlinear equality constraints defined by power flow equations; it also has nonlinear optimization objective, minimization of the system loss. Appropriate MORPO model considering several objectives of interest is essential for RPO. Such a model is worthless when it is optimized with an inaccurate optimization method. Traditionally, the MORPO problem has been solved by linear programming, and usually one of the objectives is optimized and the others are included in the restrictions, or using fuzzy method and weighted aggregation approach where the MOP is converted into a mono-objective one. These methods simplify the optimization process of MORPO, but generate disadvantages [5,19]:

- Representation of the objectives by means of the restrictions in linear programming can lead to unfeasible problems.
- There is not a clear criterion for choosing the suitable objective function and in many cases the fulfillment of one single objective can be in conflict with others.
- Fuzzy optimization turns out to be a weighted aggregation approach with a set of stationary weights (preference factors).
- The weighted aggregation approach cannot accurately reflect the relationship between the various objectives, especially when the involved objectives are conflicted with each other.
- The only one best solution fails to provide the designer with alternative options.

Compared with mono-objective optimization techniques, the main advantage of MOO is that a set of diverse optimal solutions

are identified instead of one optimal solution, which gives more flexibility to the decision maker. To offer a set of solutions to do tradeoff analysis and provide a deep insight into the MORPO problem with DG, effective MOO algorithms should be applied to this non-convex, nonlinear problem with discrete and continuous variables.

### Dynamically adaptive multi-objective particle swarm optimization algorithm

Particle swarm optimization (PSO) is an evolutionary computation technique. In real number space each particle  $i$  is associated with its velocity  $\mathbf{v}_i = [v_{i1}, v_{i2}, \dots, v_{iD}]^T$  and position  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iD}]^T$ , where  $D$  stands for the dimensions of the decision variables. The best position ever found so far by particle  $i$  is recorded as  $\mathbf{p}_i = [p_{i1}, p_{i2}, \dots, p_{iD}]^T$ , whose fitness value is  $p_{best}$ . Moreover, the best position found by any particle is recorded as  $\mathbf{p}_g = [p_{g1}, p_{g2}, \dots, p_{gD}]^T$ , and its fitness value is  $g_{best}$ . During the evolutionary process, the velocity and position update formulae of particle  $i$  on the dimension  $d|_{d=1,2,\dots,D}$  are updated according to Eqs. (12) and (13), respectively.

$$v_{id}(t+1) = wv_{id}(t) + c_1r_1[p_{id} - x_{id}(t)] + c_2r_2[p_{gd} - x_{id}(t)] \quad (12)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (13)$$

where  $t$  is the current iteration,  $w$  is the inertia weight, and  $c_1$  and  $c_2$  are acceleration coefficients, and  $r_1$  and  $r_2$  are random numbers with uniform distribution between 0 and 1.

Because of fast convergence, fewer parameters to adjust, robust adaptability, and relative simplicity of implement, PSO has been extended for multi-objective particle swarm optimization (MOPSO). MOPSO is the process of identifying well-spread solutions as diverse as possible and as close to the real Pareto front as possible. To make PSO suitable for MOPSO, techniques must be introduced to strengthen the global search ability and prevent the premature. Besides, how to promote the diversity of Pareto solutions is also to be considered. In the study a dynamically adaptive MOPSO (DAMOPSO) is proposed, which introduces the following strategies to ensure its performance.

### Strategy for inertia weight and acceleration factor

The inertia weight controls a balance between global and local exploration. To make the swarm converge to global optimum and ensure an effective exploitation, the balance between exploration and exploitation adjusts dynamically. The inertia weight can adjust adaptively according to Eq. (14).

$$w(t) = w_0 + r_3(1 - w_0) \quad (14)$$

where  $r_3$  is a random number with uniform distribution in the range of  $[0, 1]$ ;  $w_0 \in [0, 1]$  is a positive constant and the suggested range for  $w_0$  is  $[0, 0.5]$  [20].

The velocity of each particle updates according to a modified formula described as Eq. (15).

$$v_{id}(t+1) = wv_{id}(t) + c[r_1(p_{id} - x_{id}(t)) + r_2(p_{gd} - x_{id}(t))] \quad (15)$$

where  $c$  is called acceleration factor. To enhance the global search ability of the swarm at the end of iteration and help to jump out of the local optimum, especially for MOPs, it linearly changes with iteration according to Eq. (16) [20,21].

$$c(t) = c_0 + t/M_t \quad (16)$$

where  $M_t$  indicates the max number of iterations;  $c_0$  is a constant and the suggested range is  $[0.5, 1]$ .

### Selection of leader particle for each individual

Selection of leader(s) is a key component in designing a MOPSO algorithm and appropriate selection technique would be effective in promoting diversity of the swarm and Pareto solutions. Unlike PSO, instead of only one global best (leader), there are a set of non-nominated solutions to be select as leader(s) to guide the flight of the swarm. The conventional way is to select a leader for all particles in the swarm using random method, roulette-wheel, tournament or crowding distance sorting. While, in the study, a leader for each individual is selected from the current non-dominated solutions, and the solution with max *fitness* value is selected as leader  $\mathbf{p}_g$  according to Eq. (17). As a result, each individual with a different leader and all the non-dominated solutions have the opportunity to be selected as leader.

$$fitness = 1 / \sum_{i=1}^M w_i f_i, \quad w_i = \lambda_i / \sum_{i=1}^M \lambda_i, \quad \lambda_i = U(0, 1) \quad (17)$$

where  $M$  is the number of objectives, and  $f_i$  is the  $i$ th objective-function. The function  $U(0, 1)$  generates a uniformly distributed random number within the interval  $[0, 1]$ .

### Strategy for diversity promotion of Pareto solutions

NSGA-II [22] crowding distance sorting (NCDS) is popularly used to enhance the diversity of Pareto solutions. However, this technique ignores the effect on the CD(s) of the selected non-nominated solutions caused by the nearby eliminated one(s), which results in the selected solutions being too sparse. Improve NCDS and introduce dynamic elimination strategy for selection of particles for next iteration. Let  $N$  and  $N_{ND}$  denote the population size and number of current non-dominated solutions, respectively. Store the current non-dominated solutions in  $\mathbf{list}_{ND}$ . Firstly, compute the CDs of the non-nominated solutions. Then eliminate the one with the least CD. Thirdly, re-compute the CDs of the rest non-nominated solutions and also eliminate the one with the least CD. Repeat the operations until  $N$  solutions remain and store them in  $\mathbf{NewP}$  for next iteration. In case of  $N > N_{ND}$ , first of all, copy all individuals in  $\mathbf{ND}$  to  $\mathbf{NewP}$ . Then identify the non-nominated solutions from the dominated solutions and then do the similar operation.

Finally, in order to avoid premature caused by diversity loss of the swarm, mutation operation are also incorporated.

Table 1 summaries a comparison between mean values of convergence (GD) and diversity ( $\Delta$ ) on ZDT1 ~ ZDT4, performed by DAMOPSO and those performed by NSGA-II [22], MOPSO [23], AEPSo [21], CDMOPSO [24], AIPSo [25] and LH-MOPSO [26]. Results indicated that DAMOPSO has resulted in best convergence on all test problems in terms of GD measure, especially on the most difficult problems ZDT3 and ZDT4, markedly better than the other algorithms. Hence, DAMOPSO is a highly competitive MOO

method, it can provide diverse solutions well spreading along the Pareto front with good global convergence performance.

After validating the effective of DAMOPSO on a set of benchmark functions ZDT1 ~ ZDT4, it will be applied to MORPO.

### MORPO problem based on DAMOPSO

The decision variables can be represented as

$$X = [V_{G1}, V_{G2}, \dots, V_{G_{N_G}} | Q_{DG1}, Q_{DG2}, \dots, Q_{DG_{N_{DG}}} | T_1, T_2, \dots, T_{N_T} | Q_{C1}, Q_{C2}, \dots, Q_{C_{N_C}}]^T$$

As mentioned above, the reactive power of  $Q_{DG}$  is continuous, while the capacity of RPC device installed  $Q_C$  is discrete. During the evolutionary process, treat  $Q_C$  as continuous variable and convert it into an integer when compute power flow. That's mean that the capacity of RPC device installed is calculated eventually according to Eq. (18).

$$Q_C = \text{round}\left(\frac{Q_C}{Q_{C\_unit}}\right) Q_{C\_unit} \quad (18)$$

where  $Q_{C\_unit}$  denotes the unit-capacity of RPC device.

### MORPO procedure

- step1** Enter the data related to the DG and distribution system, set the swarm size  $N$ , number of iterations  $M_t$ . Initialize the swarm  $\mathbf{P}$  according to the maximum and minimum reactive power capacities of DG and RPC devices. Initialize the individual best,  $\mathbf{p}_i$ .
- step2** Update  $\mathbf{p}_i$  and  $\mathbf{p}_g$  for each particle. Execute power flow computation and get the objective-function values corresponding to each particle, then find the non-dominated individuals in  $\mathbf{P}$  and select  $\mathbf{p}_g$  for each particle according to the strategy described in 'Selection of leader particle for each individual'. Meanwhile, Update  $\mathbf{p}_i$ .
- step3** Generate new swarm  $\mathbf{R}$  whose population size is  $2N$ . Create a swarm  $\mathbf{P}'$  according to Eqs. (15) and (13) based on velocity and position of each particle in  $\mathbf{P}$ . Combine  $\mathbf{P}$  and  $\mathbf{P}'$  to generate  $\mathbf{R}$ .
- step4** Identify non-dominated solutions from  $\mathbf{R}$ , and store the non-dominated ones in  $\mathbf{list}_{ND}$  and dominated ones in  $\mathbf{list}_D$ .
- step5** Select particles for next iteration according to the approach stated in 'Strategy for diversity promotion of Pareto solutions'.
- step6** Execute mutation operation. Determine whether mutation being needed, if yes, execute mutation operation, else go to step7.
- step7** Return to step2 until  $M_t$  is met.
- step8** Output the non-dominated solutions from the final iteration and regulate  $Q_C$  according to Eq. (18) and then store these solutions as Pareto solutions.

**Table 1**  
Mean value of the convergence measure GD and diversity  $\Delta$ .

Algorithm	ZDT1		ZDT2		ZDT3		ZDT4	
	GD	$\Delta$	GD	$\Delta$	GD	$\Delta$	GD	$\Delta$
NSGA-II	0.03348	0.3903	0.07239	0.4308	0.11450	0.7385	0.51305	0.7026
MOPSO	0.01148	0.6813	0.00089	0.6392	0.00418	0.8320	7.37429	0.9619
AEPSo	0.00016	0.6341	0.00008	0.6278	0.00061	0.6039	0.00016	0.6002
CDMOPSO	0.00690	0.2740	0.00682	0.2709	0.00720	0.3707	0.26300	0.2118
AIPSo	0.00448	0.5146	0.00329	0.4991	0.00612	0.5056	0.14462	0.5074
LH-MOPSO	0.00210	0.4088	0.00270	0.3803	0.00590	0.5607	0.48110	0.4089
DAMOPSO	0.00015	0.5753	0.00008	0.5564	0.00061	0.4877	0.00015	0.5657

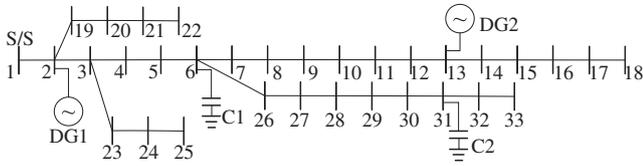


Fig. 1. Single line diagram of IEEE 33-bus system.

### A series of simulations, analysis and discussion

The DAMOPSO algorithm is employed to determine the optimal reactive output of DGs and optimal capacities of RPC devices installed in a IEEE 33-bus system [27] as illustrated in Fig. 1 for reducing the active power loss, minimizing the voltage deviation and decreasing the total capacity of RPC. The parameters involved are listed in Table 2. The  $f_{loss}$  values under the two situations without installed DG and without RPO are 0.2015 MW and 0.1348 MW, respectively. And the  $f_{AV}$  values are 0.0916 and 0.0518, respectively. In order to demonstrate feasibility of the proposed model and effectiveness of the application of DAMOPSO, to obtain a deep insight into MORPO with different objectives and analyze the correlation between objectives, a series of simulations along with analysis and discussion are carried out.

**Case 1:** considering the three objectives:  $f_{loss}$ ,  $f_{AV}$  and  $f_Q$ ;

**Case 2:** considering the two objectives:  $f_{loss}$  and  $f_{AV}$ ;

**Case 3:** considering the two objectives:  $f_{loss}$  and  $f_Q$ .

First of all, consider Case 1, determine decision variables via optimize the three objectives  $f_{loss}$ ,  $f_{AV}$  and  $f_Q$ . Table 3 lists the Pareto solutions and the objective function values corresponding to each solution. Fig. 2 shows the well-distributed Pareto solutions obtained by DAMOPSO in the objective space. It is clear that Pareto solutions have satisfactory diversity characteristics. This is useful for decision maker to choose a reasonable choice. It also reveals that the objective functions are non-comparable and even conflict with each other. Besides, each solution results in the system active power loss and voltage deviation being reduced dramatically compared with those of without DG and without RPO. However, no

Table 2  
Parameter values.

Parameter	Value	Parameter	Value	Parameter	Value
$N$	100	$V_{rated}$	1 pu	$P_{DG1}/P_{DG2}$	1 MW
$w_0$	0.3	$Q_{Cunit}$	150 kvar	$Q_{C1min}/Q_{C2min}$	0 kvar
$c_0$	0.5	$Q_{C1max}$	4 * 150 kvar	$Q_{DG1min}/Q_{DG2min}$	-100 kvar
$M_t$	250	$Q_{C2max}$	7 * 150 kvar	$Q_{DG1max}/Q_{DG2max}$	500 kvar

Table 3  
Pareto solutions of Case 1.

Pareto solutions	$Q_{DG1}$ (Mvar)	$Q_{DG2}$ (Mvar)	$Q_{C1}$ (Mvar)	$Q_{C2}$ (Mvar)	$f_{loss}$ (MW)	$f_{AV}$	$f_Q$ (Mvar)
S1	0.157	0.421	0.600	0.900	0.0799	0.0175	1.50
S2	0.418	0.328	0.600	1.050	0.0799	0.0162	1.65
S3	0.500	0.398	0.450	0.900	0.0801	0.0188	1.35
S4	0.390	0.439	0.600	1.050	0.0806	0.0149	1.65
S5	0.461	0.489	0.450	0.750	0.0810	0.0200	1.20
S6	0.460	0.500	0.300	0.750	0.0822	0.0212	1.05
S7	0.464	0.481	0.150	0.750	0.0840	0.0229	0.90
S8	0.461	0.494	0.150	0.600	0.0865	0.0254	0.75
S9	0.365	0.500	0.150	0.450	0.0904	0.0282	0.60
S10	0.379	0.500	0.150	0.300	0.0953	0.0312	0.45
S11	0.301	0.500	0.150	0.150	0.1016	0.0345	0.30

solution can satisfy each objective function to be minimum at the same time.

Figs. 3 and 4 demonstrate the Pareto fronts of Cases 2 and 3, respectively. It can be seen that when the two objective functions are considered, the total capacity of RPC devices is conflict with the power loss. Similarly, voltage deviation is conflict with the power loss, unlike the relationship shown in Fig. 2(b). Tables 4 and 5 illustrate all the Pareto solution of Case 2 and selected ones of Case 3, respectively. Apparently, the Pareto solutions also bring about benefits to the system for the two cases. In addition, for Case 3, there are more Pareto solutions and the voltage deviation is reduced more markedly than the other two cases. Why? Analysis will be presented.

### Analysis and discussion

#### Rethink and reanalyze about objectives

Is weighted aggregation reasonable? Reinvestigate Figs. 2 and 3. When the three objectives are selected  $f_Q$  is conflict with  $f_{loss}$  and  $f_{AV}$ . As Eq. (2) shows, appropriate voltage promotion decrease voltage deviation, and it's known that voltage promotion also results in the reduction of power loss along the lines. As Fig. 4(a–c) demonstrate, the voltage magnitude of each bus is prompted with RPO. While part of contribution is caused by the output of RPC devices. To some extent, the more output of RPC devices, the less voltage deviation and system active power loss. Appropriate voltage promotion is good both for reducing power loss and decreasing voltage deviation, but the relationship between  $f_{loss}$  and  $f_{AV}$  is complex, as shown in Figs. 2 and 3(b). Especially, when only  $f_{loss}$  and  $f_{AV}$  are considered, they are conflict with each other. Thus, the relationship between the three objectives is complex, non-comparable and even conflict with each other. In other word, is it reasonable to convert three of them or two of them into a single objective [4,5,10]? It deserves a further study.

The Pareto solutions of Case 3 shown in Fig. 3(b) are much more than Cases 1 and 2. It seems that lack of  $f_Q$  in the objective functions accounts for the results. In Cases 1 and 2, minimization of  $f_Q$  is one of the objectives, while in Case 3, it's out of consideration. As stated above, to some extent, more output of RPC devices do goods for power loss and voltage deviation, most of  $f_Q$  corresponding to each Pareto solution in Case 3 is equal to 1.65 MW, the value of which is the maximum allowable total capacity of RPC devices. The reactive power output of DG is regarded as continuous and a small change in  $Q_{DG}$  will result in different  $f_{loss}$  and  $f_{AV}$ . Hence, in this case, there are many combination of  $Q_{DG1}$  and  $Q_{DG2}$  to generate different combination of  $f_{loss}$  and  $f_{AV}$ .

#### Effects on voltage magnitude and objective functions caused by allocation of DG and RPC devices

As Fig. 4(a–c) indicate, the installations of DG units and RPC devices have a positive effect on the promotion of voltage

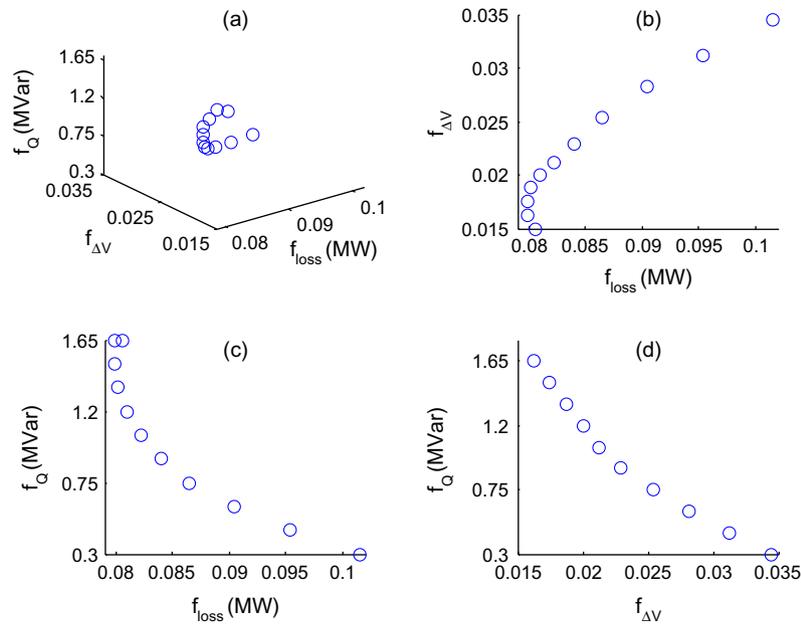


Fig. 2. Pareto solutions of Case 1 in the objective space.

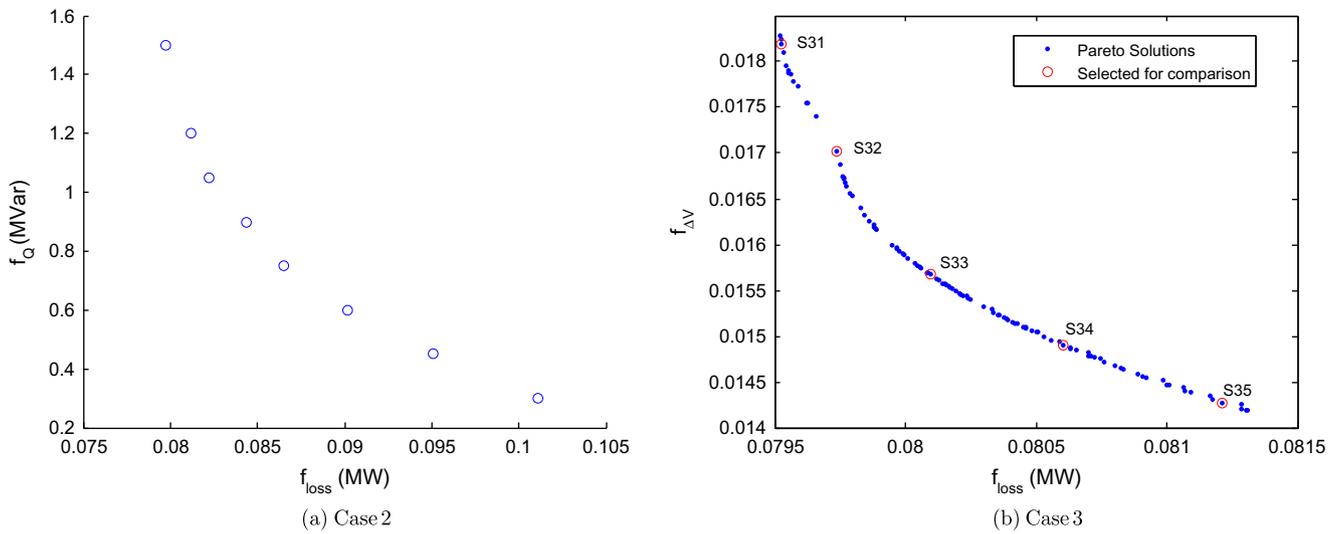


Fig. 3. Pareto solutions of Cases 2 and 3 in the objective space, respectively.

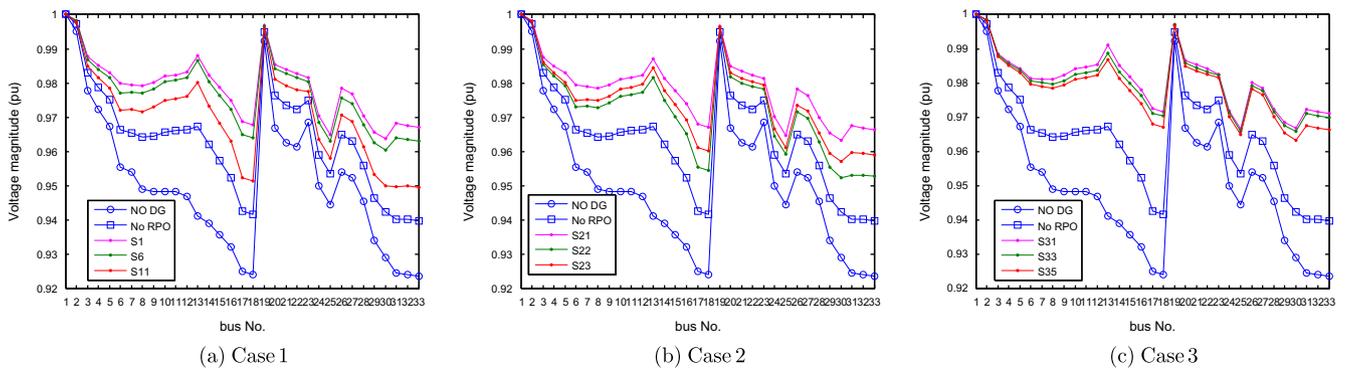


Fig. 4. Voltage magnitude of each bus corresponding to the selected Pareto solutions in Cases 1, 2 and 3, respectively.

**Table 4**  
Pareto solutions of Case 2.

Pareto solutions	$Q_{DG1}$ (Mvar)	$Q_{DG2}$ (Mvar)	$Q_{C1}$ (Mvar)	$Q_{C2}$ (Mvar)	$f_{loss}$	$f_{AV}$ (MW)	$f_Q$ (Mvar)
S21	0.15	0.36	0.60	0.90	0.080	0.018	1.50
S22	0.50	0.50	0.15	0.45	0.090	0.028	0.60
S23	0.50	0.50	0.15	0.30	0.095	0.031	0.45
S24	0.50	0.50	0.15	0.15	0.101	0.034	0.30
S25	0.47	0.50	0.30	0.75	0.082	0.021	1.05
S26	0.50	0.50	0.30	0.60	0.084	0.024	0.90
S27	0.27	0.48	0.45	0.75	0.081	0.020	1.20
S28	0.45	0.50	0.15	0.60	0.086	0.025	0.75

**Table 5**  
Pareto solutions of Case 3.

Pareto solutions	$Q_{DG1}$ (Mvar)	$Q_{DG2}$ (Mvar)	$Q_{C1}$ (Mvar)	$Q_{C2}$ (Mvar)	$f_{loss}$ (MW)	$f_{AV}$	$f_Q$ (Mvar)
S31	0.50	0.49	0.60	1.05	0.0812	0.0143	1.65
S32	0.50	0.44	0.60	1.05	0.0806	0.0149	1.65
S33	0.50	0.37	0.60	1.05	0.0801	0.0157	1.65
S34	0.50	0.26	0.60	1.05	0.0797	0.0170	1.65
S35	0.50	0.35	0.60	0.90	0.0795	0.0182	1.50

magnitude in the system. In addition, the voltage magnitude of the bus, where DG units (or RPC devices) are installed, and those of its nearby buses are prompted significantly, i.e., bus 13 where a DG is located and bus 31 where a RPC device is installed. Especially in Case 3, the output of RPC devices is larger than Cases 1 and 2, and results in more voltage promotion. But, the investment on RPC devices is dramatically more than those of Cases 1 and 2.

Since the system active power loss and voltage deviation are related to voltage magnitude of each bus in the system, and the fact that the voltage magnitude of the bus, where DG units and (or) RPC devices are installed, is prompted significantly as well as those of its nearby buses, diverse optimally located DGs and RPC devices with appropriate capacity would bring about more benefits to the system active power loss and voltage deviation than those brought about by fixed location of DGs and RPC devices. In other words, both location and sizes of DGs and RPC devices should be decision variables, and techniques should be employed to determine the optimal allocation of DGs and RPC devices along with the other conventional RPO decision variables.

## Conclusion

This study attempted to investigate the MORPO of distribution system integrated with DG. To effectively replicate different perspectives of the RPO problem and provide the designer with diverse alternative options, a MOO model with technical and operational constraints has been constructed to reduce the total active power loss, minimize the voltage deviation and decrease the total capacity of RPC devices simultaneously. A DAMOPSO has been successfully applied to the MORPO problem, suggesting that the proposed approach is capable of providing higher quality and a wider range of Pareto solutions so that the decision makers can have a more flexible and reasonable choice. Analysis and discussion have been conducted to obtain a deep insight into MORPO with different objectives. It can be concluded that optimization results depend on the selected objective functions, and that the relationship between the objective functions is complex, non-comparable and even conflict with each other, which make us to rethink whether it's reasonable to convert the conflict objective functions into a single one.

It's known that the intermittence nature of some DG (e.g. wind and photovoltaic) and the variability of the load influence the operation of electric power system. And as stated in 'Effects on voltage

magnitude and objective functions caused by allocation of DG and RPC devices', both locations and capacities of DGs and RPC devices affect the voltage of each bus in the system, and eventually affect the optimization results. Hence, further study considering intermittence nature of some DG and the variability of the load into MORPO and taking both locations and capacities of DGs and RPC devices as decision variables together with the conventional RPO decision variables will be investigated.

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