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Experimental study of granular stratification

Y. Grasselli, H.J. Herrmann

Abstract The flow of binary granular mixtures made of sand (rough) and glass spheres (smooth) into a vertical Hele Shaw cell give rises to heaps exhibiting different internal structures. We first give the phase diagram of heap morphologies classified as a function of the size ratio of rough to smooth particles. Granular stratification is one type of observable structure, which consists in the formation of alternating layers through the pile, and is found to occur only for a size ratio greater than 1.5. We present an experimental study of stratification and report that the wall separation and the mass flux can modify the layering process.

1
Introduction

Granular stratification is one type of size segregation induced by surface flows. Pouring a mixture of two granular media between two vertical plates forms a heap in which the grains can segregate in different ways, including stratification where different grain species spontaneously segregate in alternating layers (cf. fig. 1). This phenomenon, well known in geology [1], is also of interest in industrial processes and has recently experienced a growing interest as research subject. Granular stratification is found to result from the flow of a binary mixture composed of large rough and small smooth particles, the latter having a smaller angle of repose. The layered structures made of particles having different surface properties could be used to separate rough grains from smooth grains [2].

Makse et al. [3] have reported an experimental observation of stratification and have proposed a model taking into account the differences on the angles of repose between the two media which is able to reproduce stratification. Boutreux and de Gennes (BdG) [4] have then proposed a continuum approach of the flow of a granular mixture for the case of a size ratio of particles close to unity. In that case, a continuous segregation between the two media is predicted within the pile. Makse et al. [5] have extended this continuum theory and included their approach on the differences on angles of repose because an appearance of stratification was missing in the BdG model. Also several cellular automation models have been defined [6–9]. More recently, Koeppe et al. [10] have performed an experimental work on stratification using a mixture of sand and sugar. The authors have investigated the wall effects on stratification and clearly showed that the layer wavelength decreases with increasing wall separation.

In this paper, we present an experimental work on size segregation induced in two dimensional flows. We first report a phase diagram where heap morphologies have been classified as a function of the size of the particles. We then focus on stratification and present results obtained for size ratios, wall separation and filling rate effects and give a simple justification to our results.

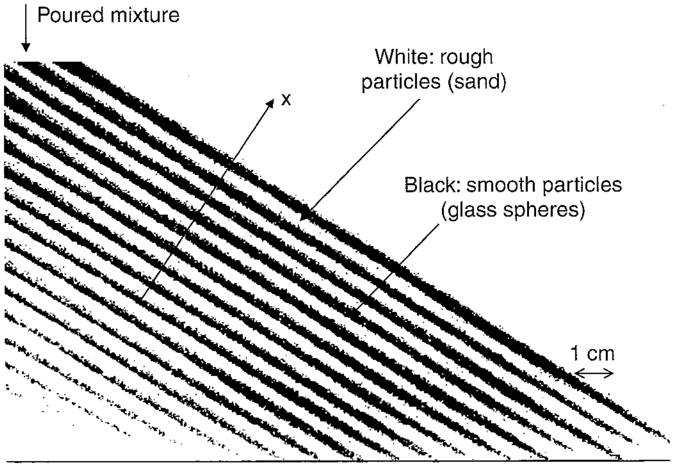

Fig. 1. Experimental binary picture of a stratified pile formed with a sand-glass spheres mixture (size ratio of rough to smooth particles: $\Phi = 3.6$, mass flux $W = 1.2\,\mathrm{g/s}$, wall separation $d = 3\,\mathrm{mm}$). No stratification exists in the initial part of the pile because no kinks can form due to the too small size of the pile. The size of this part strongly depends on W and d. The x-axis is perpendicular to the layers and is used to record intensity profiles (see experiments)

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2
Experiments

Experiments have been performed using a vertical Hele Shaw cell (20 × 30 cm). The two glass walls of the cell are separated by exchangeable spacers allowing a range in wall separation from 1 to 6 mm. The filling system consists of a reservoir placed at the top left corner of the cell containing the mixture and having several exchangeable injection outlets which give filling rates of W equal to 0.32 g/s, 0.76 g/s, 1.2 g/s, 2.5 g/s and 5.25 g/s. The mixture is never completely homogeneous but we tried to reduce segregation effects by stirring carefully. We have checked that these fluxes remain constant for all the mixtures used. The mixtures were prepared with an equal volume of two media having nearly the same density but different surface properties (rough and smooth) like sand and glass spheres with a polydispersity of about 10%–15% and average sizes ranging respectively from 60 μm to 450 μm and from 70 μm to 630 μm. The angles of repose of sand were found to be about 10 degrees larger than the ones of glass spheres for all the wall separations used [11]. The first part of this study is dedicated to the determination of the conditions needed for stratification to occur. We then define Φ as being the size ratio of rough to smooth particles. Each mixture is classified as a function of Φ. We always suppose that sand grains have a larger surface roughness than glass spheres. This assumption was confirmed by direct optical microscope observations.

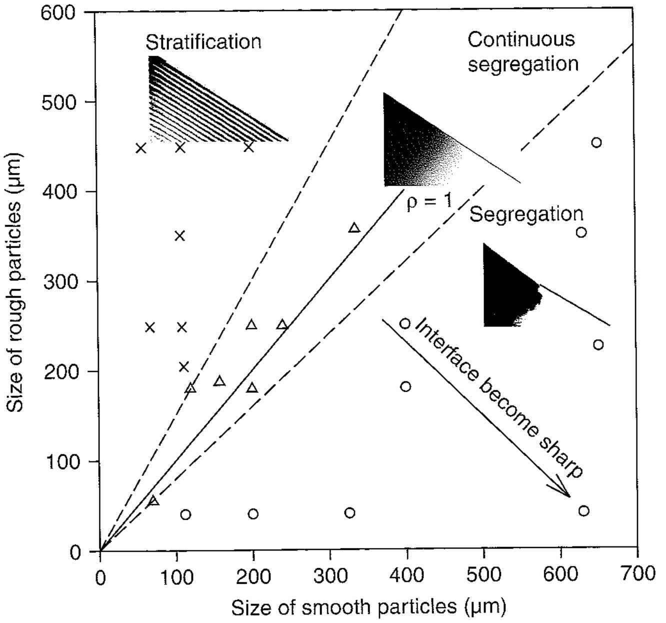

Fig. 2. Phase diagram of heap morphologies obtained by filling the cell with different granular mixtures (mass flux $W = 2.5$ g/s and wall separation $d = 3$ mm). Each structure has been classified by the size of the rough (sand) to the size of the smooth (glass spheres) particles. Granular stratification occurs for size ratios (rough/smooth) larger than approximately 1.5. Patterns are classified by: × stratification, △ continuous segregation, ○ segregation. The pictures are retrieved from typical experiments

2.1
Phase diagram

The internal structure of a heap formed with a binary granular mixture will depend on the nature and properties of the particles. In order to determine the resulting structure as a function of the grain properties, the cell thickness was set to 3 mm and mixtures were poured into with $W = 2.5$ g/s. For each mixture, several experiments were made to check reproducibility. We then classified each obtained heap morphology as a function of the size of rough and smooth particles. Three clearly distinct regions appear in the phase diagram (fig. 2). For values of Φ greater than 1.5, the pile exhibits a regular stratification morphology, as shown in figure 1, which depends on several parameters that we shall discuss later. The sand (200 μm)-sugar (800 μm) mixture used in reference [9], with $\Phi = 4$, fits in the stratification region of our phase diagram. For a value of Φ lower than 0.8, the heap shows demixing of the two components with the smaller particles on the top and the larger ones on the bottom of the pile. The two parts of the pile exhibit different angles which are equal to the angle of repose measured for each single species individually. The shape and width of the interface between the two media depends on Φ but also on W. This interface becomes sharper as Φ decreases for fixed W and also for decreasing W at fixed Φ. The regions of stratification and of total demixing have been reproduced by a simple model [12]. The intermediate region in the phase diagram is for values of Φ close to unity, actually between 0.8 and 1.5. In this region, heaps show a continuous segregation between the two species, i.e. the width of the transition zone exceeds the size of the pile. Therefore no interface can be properly defined between the two segregated media and we get a constant variation in the volume fraction of the two species from top to bottom and from left to right: increasing for larger particles and decreasing for smaller ones. The latter experimental observation confirms the theoretical results of Boutreux and de Gennes [4] which predicts a similar continuous segregation inside the pile for a size ratio close to unity. The limits given on the phase diagram (dashed lines in fig. 2) are not fixed and are only given to distinguish the three different regions. The type of transition from one region to another has to be clarified but we assume that the transition from stratification to continuous segregation is continuous. The transition from continuous segregation to pure segregation is even smoother.

3
Stratification

We now focus on a situation where $\Phi > 1.5$, i.e. the stratification regime. The main description of the pile morphology can be achieved by looking at the wavelength λ of the layered structure. But before going on, let us first briefly describe the typical pile structure in this situation. In figure 1, we can notice in the lower left part of the pile a zone where no stratification occurs. The size of this region strongly increases with increasing W. For a given experimental condition (wall separation, filling rate), the size of the pile must exceed a critical size in order to allow for kink formation which is the main condition for getting stratification. But once the first kink can be formed, regular stratification follows. Moreover, in steady state the mass leaving the reservoir per unit time must be the same as the one flowing inside the cell. So, for a given wall separation, the thickness of the layer containing the rolling grains increases with W because of mass conservation. The size ratio needed to get stratification ($\Phi > 1.5$) is sufficiently high to consider that size segregation already exists in the moving layer. So, when an up-going kink appears on the surface of the pile, larger particles stop at a distance from the precedent layer which is almost the size of the moving layer. This process defines the layered structure observed in stratified pile.

The experimental study of stratified piles has been realized by image analysis. We first automatically determined the average slope of the layers and we record intensity profiles in a direction perpendicular to the layers as shown on figure 3. So, averaging the pixel positions weighted by the corresponding intensity gives with a good accuracy the distance between two extrema with an accuracy below the pixel precision. Repeating such measurements for different sections of the pile also allows us to check the regularity of the layers. We have observed that this regularity is altered for very low values of W (typically lower than 0.1–0.2 g/s). In that case, λ is not constant but increases with the size of the pile and then tends to a constant value. In such experimental conditions, the pile is formed by a succession of avalanches which is different from the experiments presented in the following where we have, during the pile formation, a continuous flow of grains on the surface. Moreover, stratification experiments with a mixture of particles having a large density difference (iron and sand) have shown that the layer thicknesses become thicker at the bottom for layers made of heavier particles. Inertia may play an important role in this case and can explain why heavy particles are preferentially situated at the bottom. Considering the two precedent observations, all experiments presented in this paper have been performed under conditions where the stratified structure is regular.

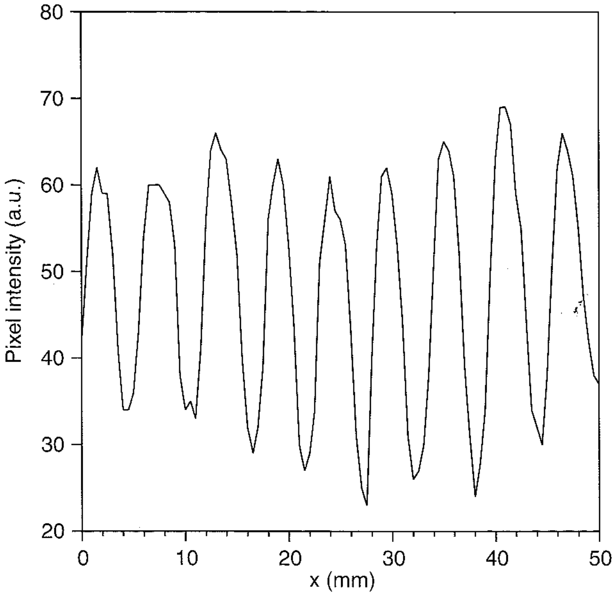

Fig. 3. Intensity profile recorded on a 256-grey leveled picture in a x-direction perpendicular to the stratification layers (see fig. 1). Averaging the positions allows us to obtain the average distance between two successive layers (i.e. the wavelength)

We have experimentally determined λ as a function of the wall separation d, the flux W and the size ratio of particles Φ. In order to get comparable results, we have prepared mixtures made of equal volume of sand and glass spheres of the same kind but with different Φ. In this way, we get similar mixtures with Φ of 3.6, 2.2, 1.8 and 1.6. The average sizes of rough and smooth particles for these mixtures were: 250 μm/70 μm ($\Phi = 3.6$), 250 μm/112 μm ($\Phi = 2.2$), 282 μm/157 μm ($\Phi = 1.8$) and 250 μm/157 μm ($\Phi = 1.6$). We have observed that for large W and small d, no stratification occurs. This is because it is difficult to form kinks during the filling process with a flux that is too strong. This does not mean that stratification does not exist at all but for such filling rates and wall separations, it could be that the size of the pile is not large enough, due to the size limitation of our cell, to ensure the presence of kinks.

3.1
Mass flux and wall effects

Makse et al. [5] have predicted a linear behavior of λ on the thickness of the rolling phase on the heap surface. Moreover, for a flux close to zero, λ is found to tend to a finite limit given by the size of the grains. Let us now express the layer wavelength λ considering simple mass conservation arguments. As layers are generated by up-going kinks on the surface of the pile, we suppose that the thickness of these kinks defines approximately the layer wavelength λ. For a constant filling rate W, the number of particles flowing out of the reservoir into the cell is the same as the number of particles stopping at the kink. We may then assume that the particle flux induced on the surface by the up-going kink is identical to W and we can express it as $W = \rho V d \lambda$ with d being the wall separation, V the average velocity of the kink (that might also depend on W) which is typically around 10 cm/s in our experiments and ρ the bulk density. We then get the following relation:

$$\lambda = W / \left(\rho V(W) d\right) \tag{1}$$

Equation (1) gives values of λ in good agreement with the values obtained experimentally taking an experimental determined velocity of the order of 10 cm/s but gives no stratification ($\lambda = 0$) for infinite wall separation although it has been observed experimentally [13]. In our experiments, the thickness of the rolling phase is given by the ratio W/d and like in ref. [5], we found that λ is linear in W/d. Experimental results obtained for λ have been plotted as a function of W/d and are presented for two different size ratios in figure 4 ($\Phi = 3.6$) and in figure 5 ($\Phi = 1.6$). The expected linear behavior of λ is not reproduced since one can notice a clear deviation for values of W/d tending to zero. The most striking deviation (most strongly pronounced in fig. 5) is that there is a finite

$\lambda = \lambda_0$ for zero flux (or infinite wall separation). λ_0 being more or less one order of magnitude greater than the typical size of the grains. In fig. 4, this non linear behavior is essentially given by the experimental points obtained at low flux ($W = 0.32\,\mathrm{g/s}$, black circles). One possible explanation could be that for a low flux, the heap is formed by intermitent avalanches. This could explain the differences observed between the experiments and the theory of ref. [5] (where one supposes that the heap is formed by kinks generated by a continuous flow). The use of mass conservation arguments or the theory of Makse et al. [4] are not able to explain the large values of λ_0 obtained experimentally, in comparison to the size of the particles. In order to reproduce the experimental results obtained for λ, one would have to take into account not only the ratio W/d but also the size of particles.

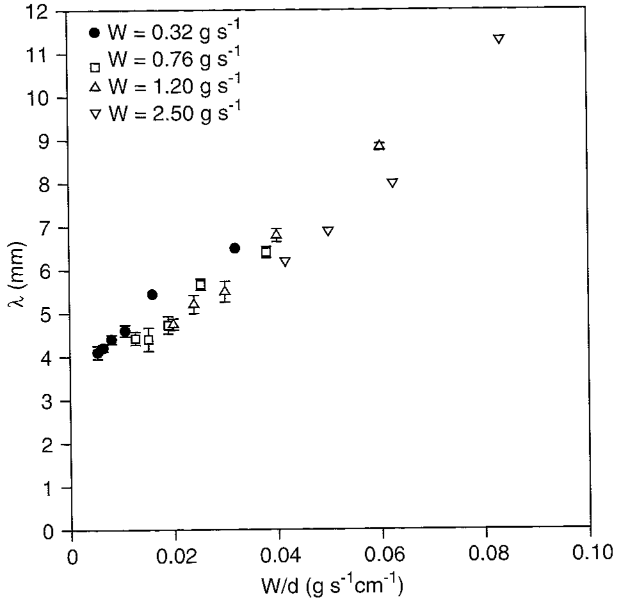

Fig. 5. Layer wavelength λ as function of W/d (Φ=1.6). Here the expected linear behavior is obtained. The λ-value for $W/d \to 0$ is about one order of magnitude greater than the typical size of the grains

3.2
Dependence on the size ratio Φ

The dependence of λ as a function of the size ratio Φ for different wall separations is presented in figure 6 for a flux $W = 2.5\,\mathrm{g/s}$. The layer wavelength slowly increases with the size ratio, this increase being more as the wall separation decreases. For small d, the rigid walls of the cell play an important role and may greatly influence the heap formation by modifying the stratification pattern. The slow variation of λ as a function of Φ may be related to the fact that the absolute sizes of particles used in the mixtures are not very different. To clarify this point, we have prepared a mixture with $\Phi = 1.8$ but with different absolute sizes for rough and smooth particles : 450 μm/250 μm ($\Phi = 1.82$) which corresponds to an increase of the size of the particles by a factor 1.6. The results obtained for two different fluxes ($W = 1.2\,\mathrm{g/s}$ and $W = 2.5\,\mathrm{g/s}$) are presented in figure 7 where now a clear difference can be noticed for λ, the general behavior of the curves being similar (the slope obtained by leastsquare fit is almost the same). The vertical shift between the curves at fixed W is not directly proportional to the increase in the size of

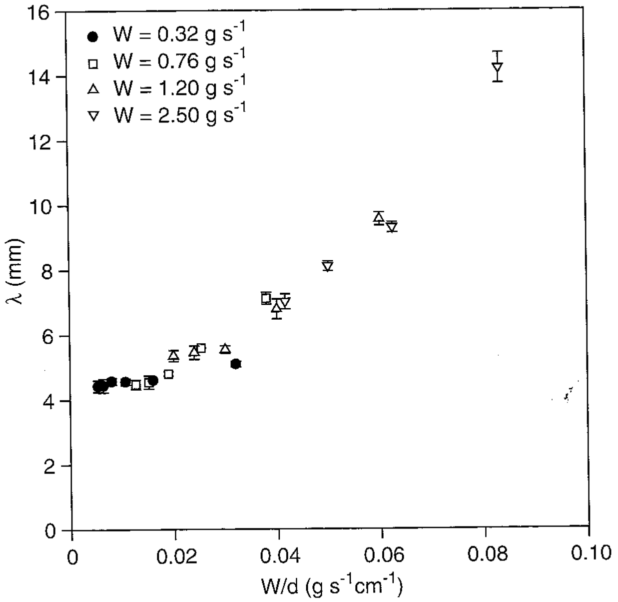

Fig. 4. Layer wavelength λ as a function of the ratio between the mass flux W and the wall separation d (size ratio of particles $\Phi = 3.6$). The experimental behavior differs from the one given by equation (1) (see text). This non linear behavior is due to the experiments performed at low flux (black circles) where the heap is mainly formed by avalanches rather than a continuous flow of particles over the surface

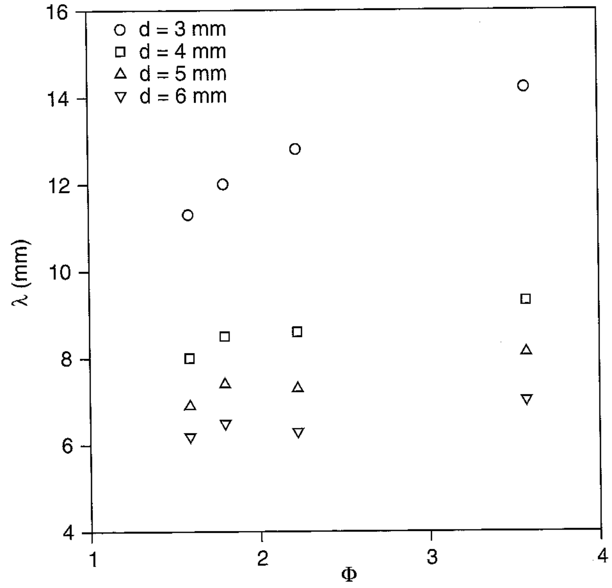

Fig. 6. Dependence of the layer wavelength λ as a function of the size ratio Φ for similar mixtures and different wall separations d (mass flux $W = 1.2\,\mathrm{g/s}$). λ slowly increases with Φ and depends also on the wall separation d. For small d, the influence of the walls is non negligible and modify the stratification process

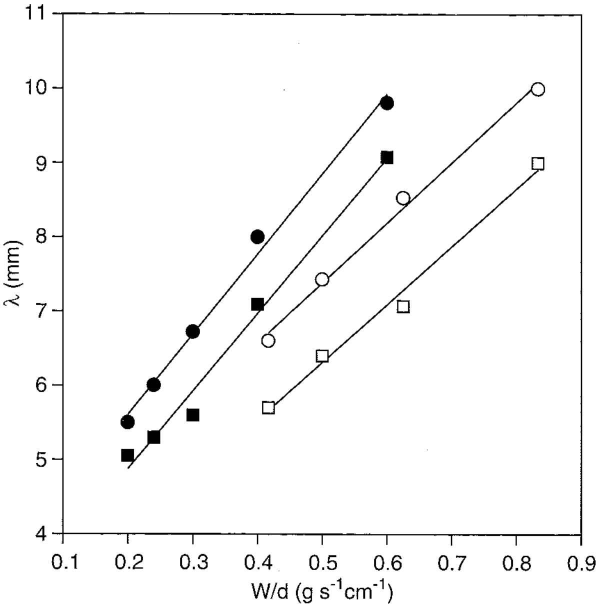

Fig. 7. Layer wavelength, λ, as a function of W/d (d being the wall separation and W the mass flux) for mixtures having nearly the same size ratio Φ (rough/smooth) and for mass flux $W = 1.2\,\text{g/s}$ (black points) and $W = 2.5\,\text{g/s}$ (white points). The squares represent a mixture of $\Phi = 1.8$ with rough particles of 282 μm and smooth ones of 157 μm. The circles correspond to a similar mixture with $\Phi = 1.82$ but with rough particles of average size 450 μm and smooth ones of 250 μm

the particles and depends on the particle flux. This means that for identical experimental conditions, the number of particles per layer of each species of grains depends on Φ and on W. Since it is difficult to consider very different sizes experimentally, we cannot determine the dependence of λ on the absolute size more quantitatively. The effect of the size of the particles is difficult to predict because for a fixed W and different Φ, the number of particles flowing into the cell per unit time decreases and we have actually no way to access it experimentally. Considering equation (1), the slope of the curve $\lambda = f(W/d)$ is related to the kink velocity V. Using the experimental data of figure 5 we get a velocity of about 8 cm/s which is in good agreement with the kink velocity determined experimentally. But we need to investigate in more detail the velocity of the kinks and understand how it depends on the experimental parameters.

4
Conclusion

We have performed an experimental study of the flow of several granular mixtures between two vertical plates. These mixtures were composed of rough (sand) and smooth (glass spheres) particles. The latter having a smaller angle of repose. We have reported a phase diagram for the heap morphologies as a function of the size ratio (rough/smooth). It clearly appears that stratification only occurs for a size ratio greater than 1.5. For values below 0.8, we get bulk segregation between the two media: the width of the interface formed between the two media strongly depends on the size ratio and on the filling rate of the mixture and tends to give a continuous segregation for size ratios close to unity (actually between 1.5 and 0.8). Granular stratification was found to depend on the wall separation d of the cell and on the mass flux W of the mixtures. Experimentally, we report that λ is determined by W/d for a given mixture. For values of W/d close to zero, λ tends to a finite value which is approximately one order of magnitude greater than the size of the grains. We have observed that λ depends on the size of the particles. We also proposed a simple determination of λ by writing an equation of mass conservation. The predicted behavior of λ is qualitatively in good agreement with the experimental observations but does not reproduce the non zero values obtained for vanishing W/d.

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