Optical Doppler shift measurement using a rotating mirror

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Optical Doppler shift experiments are not simple because the light source cannot easily be moved in a sufficiently smooth and uniform manner to keep the level of noise well below that of the signal. Thus, most Doppler shift experiments are performed with sound or microwaves. Instead of using a moving mirror to produce a moving light source, we use a rotating mirror in which one beam is reflected from the advancing side and the other beam is reflected from the receding part of a rotating mirror. This arrangement can overcome many noise generating effects.

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I. INTRODUCTION

Although the Doppler shift is well understood, its demonstration using light is not simple. Using light is not simple because the moving mirror from which light is reflected cannot usually be moved in a sufficiently smooth and uniform manner to keep the level of noise below that of the signal. Small perturbations as small as the optical wavelength can produce a non-negligible noise. Instead, Doppler shift experiments for undergraduates are usually performed with sound or microwaves.

Previous undergraduate demonstrations of the optical Doppler shift have involved the use of Michelson interferometers with arms of approximately equal path lengths using an air-track glider[1] or a servo-mounted mirror on an isolated optics table;[2] light reflection from Scotchlite tape on a rotating turntable with a spectrum analyzer;[3] and direct frequency modulation produced by a model train engine.[4] Velocities up to 0.3 m/s were used, and thus Doppler beat frequencies were below 1 MHz.

One of the goals of the present work is to reach a room-size beat wavelength that corresponds to a beat frequency of about 30 MHz or more. Therefore, velocities of the order of 10 m/s are required. Using linear translation techniques at 10 m/s or a greater velocity in a standard classroom or laboratory is almost impossible. We have devised a rotational technique in which one beam is reflected from the advancing side and the other beam is reflected from the receding part of a rotating mirror. An apparatus was constructed that students can use to measure the Doppler shift as a function of the angular frequency of a rotating mirror.

II. DOPPLER SHIFT

Light frequencies are around 500 THz, well beyond the frequency resolution of light detectors. Therefore, the measured intensity fluctuation is the temporal mean of the optical perturbation over a time interval related to the detector time response, which is much larger than the period of the light wave. In contrast, the mixing or beat frequency corresponds to a much lower frequency and can be detected using a fast photodetector. Two effective sources with different velocities can be created using a rotating mirror. The beat frequency between the two sources is related to the first-order Doppler shift of the beam reflected by the rotating mirror.

A plane-polarized light wave of frequency f reflected from a uniformly moving mirror will undergo a frequency shift.[5] Let v_n be the velocity normal to its plane (positive toward the source) of a mirror in uniform motion, and consider a plane wavefront that makes an angle of incidence α with the mirror. Then, the frequency f' of the reflected beam to first order in v_n/c (c is the velocity of light) is[5]

$$f' = f\left(1 + 2\frac{v_n}{c}\cos\alpha\right). \tag{1}$$

Equation (1) follows from the fact that the light source moves with velocity $2v_n$ in the direction of the normal to the mirror, and, consequently, the component of this velocity in the direction of the reflected ray is $2v_n\cos\alpha$. (A simple geometrical explanation can be found, for example, in Ref. 6.) For nonuniform movement of the mirror Eq. (1) is valid using the instantaneous normal velocity of the reflecting surface.[7] For the geometry of our experiment (see Fig. 1) the normal velocity v_n is proportional to the distance R from the midpoint of the reflecting surface, according to

$$v_n = v\frac{R}{L}, \tag{2}$$

where v is the local velocity of the mirror and L is the distance from the axis of rotation. Because $v=\Omega L$ (Ω is the rotating angular frequency of the mirror), the frequency of a reflected beam at a distance R from the midpoint of the reflecting surface is (see Fig. 1)

$$f' = f\left(1 \pm 2\frac{R\Omega}{c}\cos\alpha\right), \tag{3}$$

where the sign is positive if the mirror advances to the source and negative if it recedes from the source.

If we take two beams reflecting from opposite points from the midpoint of the reflecting surface, we have

$$f_1 = f\left(1 + 2\frac{R\Omega}{c}\cos\alpha\right)$$

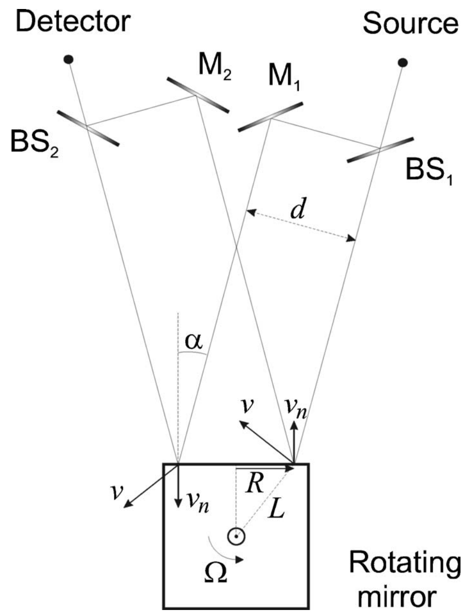

Fig. 1. Reflection of two beams separated by a distance d centered at the midpoint of the reflecting surface. The normal velocity at any point of the mirrored surface is proportional to the distance to its midpoint.

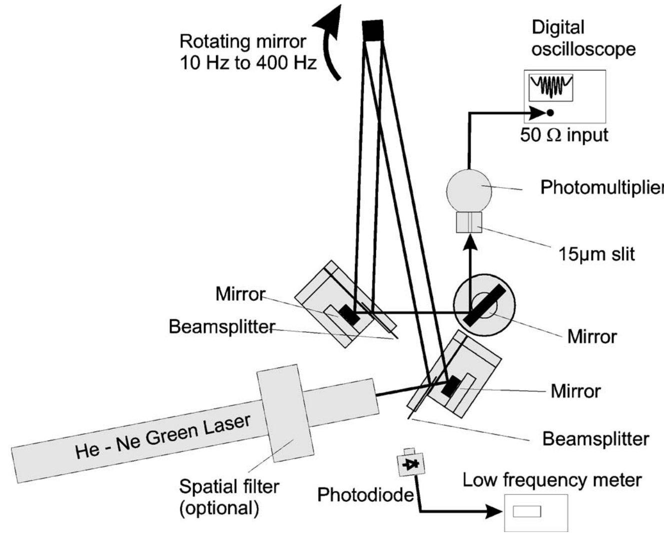

Fig. 2. Experimental arrangement. The arrows indicate the direction of the observed light. The curved arrow by the rotating mirror represents the direction of rotation.

$$f_2 = f\left(1 - 2\frac{R\Omega}{c}\cos\alpha\right). \tag{4}$$

The corresponding beat angular frequency of the beams is

$$f_b = f_1 - f_2 = \frac{4fR\Omega\cos\alpha}{c}. \tag{5}$$

Because the separation d between the two beams centered at the midpoint of the reflecting surface is (see Fig. 1)

$$d = 2R\cos\alpha, \tag{6}$$

we have

$$f_b = \frac{4\pi dF}{\lambda}, \tag{7}$$

where $F=2\pi\Omega$ is the frequency of rotation of the rotating mirror. Note that the beat frequency (7) depends on the beam separation rather than on the radial position where the beams hit the mirror. The same frequency will be obtained if both beams are at an equal but opposite distance R from the midpoint of the reflecting surface or one beam hits the mirror at its midpoint and the other at distance $2R$. That is, for this particular geometry, only the beam separation counts.

According to Eq. (7), the beat frequency also does not depend on the angle of incidence. If the angle of incidence of the parallel beam is larger, then the beam separation at the mirror surface will be larger by the same factor as the reduction of the component of the normal velocity in the direction of the reflected ray. Thus, the angle of incidence cancels, and the measurement is insensitive to perturbations of the mirror movement. This insensitivity is a clear advantage over translations for which the beams are not self-compensated. Any standard grade rotating tool is sufficient to obtain a good measurement of the optical Doppler shift.

The preceding considerations are valid to first order in v_n/c. At higher speed (not applicable to our experiment) second-order effects need to be considered,[5] including the modification of the angle of reflection (some examples of these effects can be found in Ref. 8).

Because both the source and detector are at rest in the laboratory frame of reference, an alternate way to obtain Eq. (7) is to calculate the difference in the path length from the source to the detector for the two arms of the interferometer.[9] By using an algebra software package, it is possible to obtain a more general derivation than this particular case. The interpretation as a path difference is easier to explain to undergraduate students. Also, using this approach it is easier to obtain the dependence of the intensity with time. Because the rotating beam is sweeping the photomultiplier slit, the intensity varies with time in a way similar to that from a lighthouse. For a given beam diameter it is possible to calculate a realistic time variation of the intensity as the geometrical overlap of the sweeping circular beam and the slit.

III. EXPERIMENTAL SETUP

A block diagram of the apparatus is shown in Fig. 2. The light source is a low-power green He–Ne laser (Melles Griot model 05-LGR-025). A beamsplitter separates the beam so that part is reflected from the advancing side of a rotating mirror and part is reflected from the receding part of a rotating mirror. They are recombined by a beam splitter and measured by a photomultiplier tube (PMT). Beating between the two beams produces fluctuations in the light intensity at the photocathode. The photocurrent is proportional to the light intensity and fluctuates due to the Doppler shift of the reflected beam from the rotating mirror.

The laser beam was used with no lenses. The unmodified laser beam is easy to position and to control directionally. Although not needed for demonstration purposes, an optional spatial filter was designed to be placed at the laser output.

The rotating mirror was handcrafted on a cube (20 mm side) of cobalt-steel. Only the front surface was mirrored, producing one pulse per rotation. A low-cost ($20), high-speed (up to 30 000 rpm) motor was used to drive the rotating mirror. A special support was constructed to hold the motor. The beamsplitters and the mirror were from standard optical Melles-Griot kits. Special separators were built to place the mirror and beamsplitter close enough to produce a 17 mm beam separation or recombination.

To detect the beat frequency a Hamamatsu R928 photomultiplier, powered by a 2 kV, 2 mA power supply, was used. The electrical signal was recorded by a 100 MHz, 500 MSa/s Tektronik TDS 320 digitizer or by a 500 MHz, 1 GSa/s Hewlett-Packard Infinium oscilloscope that has a built-in fast Fourier transform. A photomultiplier cage was made of aluminum. A photodiode connected to a low-frequency meter was used for measuring the rotating frequency. All the components were mounted on a 0.3×0.5 m iron base.

No special skills are needed for setting up the apparatus. With the mirror at rest the alignment is performed by using the first mirror-beamsplitter set and removing the motor to produce a parallel beam. Then, the motor is set in place and the mirror is manually rotated until one of the reflected beams hits the photomultiplier slit. Then, by means of the second mirror-beamsplitter set, the two reflected beams are recombined at the photomultiplier slit. The interference pattern between the two Airy disks can be easily produced on the slit. A final, minor adjustment can be performed while the mirror is rotating to obtain the best beating pattern.

At a given rotational frequency the duration of the beating signal can be changed by locating the detector at different distances from the rotating mirror. We obtained signal widths decreasing from 20 to 1 μs by moving the PMT up to 6 m from the mirror. Because the PMT is a very sensitive to light, some care should be taken to prevent room light from reaching the photomultiplier. Some screening may be needed depending on the configuration.

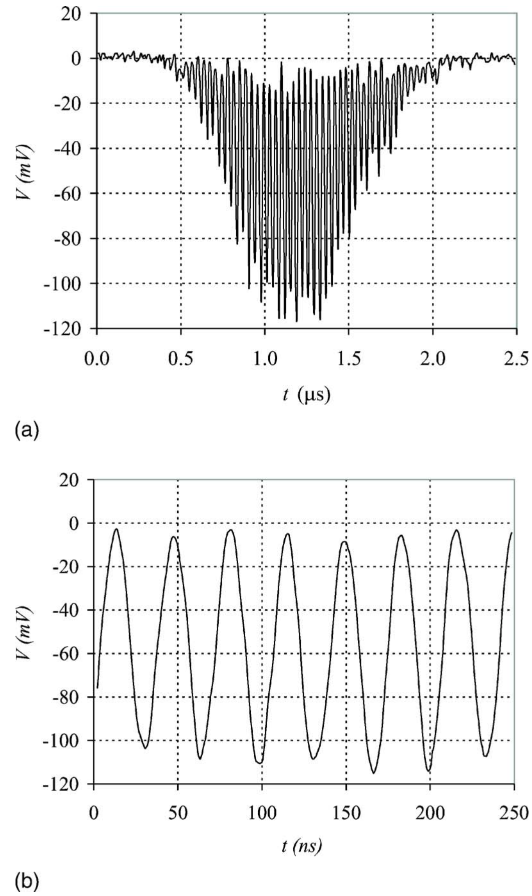

Fig. 3. Sample oscilloscope output of the Doppler beat frequency produced by a rotating mirror at 81.6 Hz measured at 4 m from the mirror: (a) extended plot, showing the time variation of the light intensity at 250 ns/div, 200 MSa/s; (b) expanded plot showing the beat pattern at 25 ns/div, 500 MSa/s.

IV. RESULTS

In our experiment $\lambda = 543.5$ nm (green He–Ne laser) and $d = 17$ mm. The frequency of the mirror was varied from 20 to 200 Hz. In Fig. 3 we show the temporal variation of the intensity for $F = 81.6$Hz. In this example the PMT was located 4 m from the rotating mirror to give an idea of the full pulse including its Doppler modulation inside. At shorter distance the pulse width increases and, because the Doppler beat frequency is the same, it is very difficult to see both the pulse and its details in a fixed scale. The measured frequency $f_b = 32.1$ MHz is in good agreement with the expected value according to Eq. (7).

The measurements proceeded as follows. Using a variable transformer the speed of the rotating mirror was varied from 20 to 200 Hz. At approximately 10 Hz intervals the photomultiplier signal was saved to the computer together with the photodiode measure of the mirror frequency (depending on the digitizer, up to 32 768 data points were recorded). A fast Fourier transform (FFT) was applied to each signal and the peak frequency was recorded. In Fig. 4 the FFT corresponding to Fig. 3 is plotted. Some digitizers have a built-in FFT function. For 32 768 data points the typical full width at half maximum (FWHM) of the FFT was smaller than 1% of the beat frequency, giving enough precision for demonstration purposes. In Fig. 5 the measured beat frequency as a function of the rotating mirror frequency is shown. The line is the theoretical prediction from Eq. (7). All experimental values agree with the theoretical values within the error limits.

V. DISCUSSION

The beat frequency difference between the Doppler shifted light from a rotating mirror was observed for different rotational frequencies. Very good results were obtained using low-precision motors. Neither sophisticated optics nor electronics is needed. The main items required are a low-powered laser, mirrors, optical elements, and a detector.

The techniques described in this paper provide a quantitative demonstration of the Doppler effect with light and overcome the usual problems caused by motional instabilities associated with the moving mirror, because the frequency shift depends on beam separation rather than radial position or axial displacement.

The velocities of the moving mirror were up to 10 m/s, which is more than an order of magnitude larger than in previous work.[1–4] This fact lets us use an optical beat frequency up to 80 MHz in a 1 to 20 μs pulse length. This pulse can be used to perform other measurements including the speed of light without needing extra special equipment because the wavelength is expected to be from 4 to 15 m.

Another advantage is that the experiment is not limited to a restricted temporal window as in methods that employ pure translation of the source (the mirror must start and stop at some point). We can continuously acquire data using a rotating mirror, which is especially useful for demonstrations.

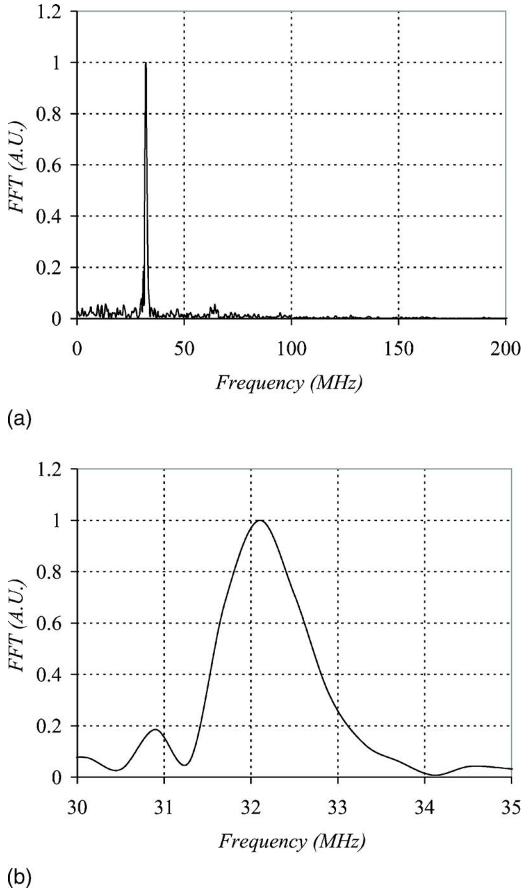

Fig. 4. Fast Fourier transform of 1024 data points from the signal shown in Fig. 3(a). (a) The full spectrum; (b) an enlargement around the main peak showing the frequency of the peak and its FWHM (depending on the digitizer up to 32 768 data points were recorded, thus reducing the FWHM).

The present method can in principle be used at a lower rotational speed. We have not investigated this low speed limit because our low-cost motor was unable to smoothly rotate below 10 Hz. As suggested by a reviewer, we note that the rotational angular speed of the Earth $(15°/\text{hour})$ corresponds to a Doppler beat of about 8 Hz. Thus, the rotating mirror Doppler technique described here could be a very interesting way to show students how to detect very low rotational speeds.

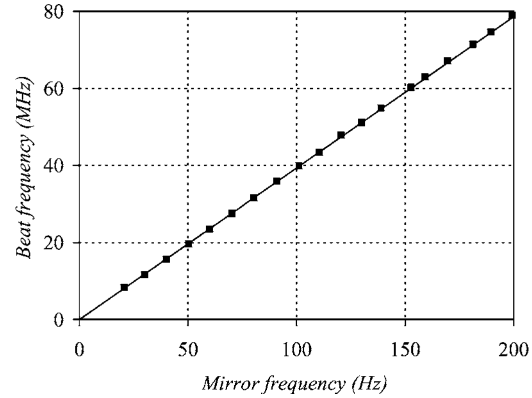

Fig. 5. Doppler beat frequency as a function of the rotational frequency of the mirror. Full line, theoretical values; squares, experimental values (the error bars are within the dimensions of the squares).

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