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An integrated three-stage maintenance scheduling model for unrelated parallel machines with aging effect and multi-maintenance activities ☆

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ABSTRACT

We propose an integrated three-stage model for maintenance scheduling of unrelated parallel machines (UPMs) with aging effect and multi-maintenance activities (AEMMAs) using a variety of MODM techniques such as the fuzzy analytic hierarchy process (AHP), the technique for order of preference by similarity to ideal solution (TOPSIS), and goal programming (GP). We use fuzzy AHP in the first stage of the proposed model to account for the inherent ambiguity and vagueness in real-life maintenance scheduling problems. In the second stage, we use TOPSIS to reduce the multi-objective problem into an efficient bi-objective problem. Finally, we use GP to solve the resulting bi-objective problem and develop an optimal maintenance schedule in the third stage of the model. We use a numerical example to demonstrate the applicability of the proposed approach and exhibit the efficacy of the procedures and algorithms.

1. Introduction

Manufacturing firms are constantly under pressure to reduce their production costs. Maintenance costs, one of the main components of production costs, substantially add to this stress in the manufacturing environment (Bevilacqua & Braglia, 2000). Although manufacturing firms constantly seek to engage all levels and functions in an organization to maximize the overall effectiveness of production equipment, maintenance costs are unavoidable and play an important role in maintaining a machine's reliability and product quality. Although manufacturers have made great strides in controlling maintenance cost, maintenance remains an important topic for further study in production economics and research.

In deterministic scheduling problems, the job processing time is assumed constant and independent of its position or starting time in the scheduling process. However, there are many situations in which the actual processing times of the jobs may vary due to learning, aging or deterioration effects (readers should refer to Biskup (2008), Janiak & Rudek (2006) and Janiak & Rudek (2009) for the state of the current research). The motivation for this study stems from the metal or wood cutting process that cuts products to various sizes and shapes them in a parallel-machine setting. Due to wearing of the cutting tool, the actual processing time of the product increases with respect to the number of products already processed on the machine. The time required for processing a product depends on the quality of the cutting tool. Therefore, under normal circumstances, the cutting tool is replaced with a new one or is maintained after it has processed some products to improve its production efficiency.

Maintenance activities in the literature have been classified into two main categories: corrective and preventive (Li, Khoo, & Tor, 2006; Waeyenbergh & Pintelon, 2004). The corrective maintenance is the maintenance that occurs after systems failure (Swanson, 2001) while the preventive maintenance is the maintenance that is performed before systems failure in order to retain equipment in specified condition by providing systematic inspections, detection, and prevention of incipient failure (Moghaddam, 2013; Wang, 2002). The model proposed in this study falls in the preventive maintenance category.

One of the first steps of maintenance activities is to select the best repairmen from a pool of available repairmen. In this paper,

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the analytic hierarchy process (AHP) is used to select the most suitable repairmen based on a set of pre-specified evaluation criteria (Saaty, 1980). AHP can consider both quantitative and qualitative evaluation criteria. Most qualitative evaluation criteria in real-life are often accompanied by ambiguities and vagueness. Therefore, we consider fuzzy logic and fuzzy sets to represent ambiguous and vague information in the evaluation process. The integration of AHP and fuzzy set theory (Bellman & Zadeh, 1970) has resulted in the fuzzy AHP method.

Parallel machine scheduling (PMS) is concerned with the allocation of a set of jobs to a number of parallel machines. The studies on PMS in the literature have been generally categorized into three groups: identical, uniform and unrelated PMS problem (Cheng & Sin, 1990). Among these groups, unrelated PMS (UPMS) represents a generalization of the other two groups where different machines perform the same job but have different processing capacities or capabilities. However, solving real-life UPMS problems is a difficult task because they are mostly NP-hard (Torabi, Sahebjamnia, Mansouri, & Aramon Bajestani, 2013). This paper focuses on the UPMS problems, which have been addressed much less than the identical and uniform PMS problems in the literature (see Arnaout, Rabadi, & Musa, 2010; Chang & Chen, 2011).

In spite of huge advances in PMS research, multi-objective scheduling problems with simultaneous consideration of repairmen selection, aging effects and maintenance activities under unrelated parallel machine environment have not been thoroughly studied in the literature. Kuo and Yang (2008) studied single-machine scheduling problems with a cyclic process of aging effects and multi-maintenance activities. They investigated the problem with job-independent and position-dependent aging effects to minimize the makespan. Zhao and Tang (2010) extended the study of Kuo and Yang (2008) to the case with a job-dependent aging effect. Yang and Yang (2010a) studied single-machine scheduling with simultaneous consideration of job-dependent aging effects, multi-maintenance activities, and variable maintenance durations to minimize the makespan. Yang and Yang (2010b) further considered single machine scheduling with aging or deteriorating effects and deteriorating maintenance activities simultaneously to minimize the total completion time. Yang, Cheng, Yang, and Hsu (2012) studied UPMS problems considering aging effect and multi-maintenance activities (AEMMAs) to minimize total machine load. These studies have primarily formulated the problem with a single objective model. Very little work has focused on multi-objective scheduling problems (MOSPs).

Multi-objective decision making (MODM) techniques have attracted a great deal of interest due to their adaptability to real-life decision making problems. MODM problems often involve multiple conflicting objectives (Majazi Dalfard & Mohammadi, 2012; Zhang, Li, & Xiong, 2012) and decision makers (DMs) are required to search for a trade-off between the objectives. Generally, the MODM problem can be formulated as follows:

$$\text{MODM}: \begin{cases} Min \text{ or } Max: & \{f_1(x), f_2(x), \ldots, f_k(x)\} \\ s.t: & X \in S = \{X \in R^n | g(x) \leqslant b, X \geqslant 0\} \end{cases}$$

In this study we consider the following conflicting objectives: minimizing the makespan, minimizing the total maintenance cost, minimizing the maximum tardiness time of the jobs and minimizing the maximum earliness time of the jobs. In the proposed approach, a MODM problem is reduced to a bi-objective problem by using the technique for order preference by similarity to ideal solution (TOPSIS) (Khalili-Damghani, Sadi-Nezhad, & Tavana, 2013). Next, the resulting bi-objective problem is solved with goal programming (GP) to find solutions that simultaneously have a minimum distance from the positive ideal solution (PIS) and a maximum distance from the negative ideal solution (NIS).

The rest of the paper is organized as follows: In Section 2 we introduce an integrated three-stage maintenance scheduling model for UPMs with AEMMAs. In Section 3, the problem is formulated as a multi-objective integer linear programming (MOILP) model. In Section 4 we use a numerical example to demonstrate the applicability of the proposed approach and exhibit the efficacy of the procedures and algorithms. Finally, conclusions and future directions are given in Section 5.

2. Proposed model

In this section we describe our three-stage approach for the repairmen selection and maintenance scheduling problem. Note that, before the first phase can be initiated, it is sometimes necessary for the DM to do an initial screening on the list of its potential repairmen. Once a suitable list of repairmen is constructed, the repairmen are evaluated and scored using a fuzzy multi-criteria decision making method in the first stage. In the second stage of the proposed approach, a multi-objective scheduling problem is reduced to a bi-objective problem with the TOPSIS method (Khalili-Damghani et al., 2013). Finally, in the third stage, the resulting bi-objective problem is modeled with goal programming to jointly determine the optimal maintenance frequencies and positions of the repairmen's maintenance activities and the optimal job sequences on the machines with respect to some supplementary constraints imposed on the mathematical model. The general framework for our proposed approach is illustrated in Fig. 1. In the following subsections, the three stages of the proposed model are discussed in more detail.

2.1. Stage 1: Evaluation of the alternative repairmen

Multi criteria decision making deals with the problem of choosing the best alternative, that is, the one providing the highest degree of satisfaction with respect to all the relevant criteria or goals. In order to obtain the best alternative a ranking process is required. AHP is one of the most popular and powerful multi-criteria decision making methods for decision making and has been used for years in service quality assessment. However, the AHP method, first developed by Saaty (1980), was inadequate and defective in handling the ambiguity of the concepts that are associated with a human being's subjective judgment. Therefore, the fuzzy AHP method, which combines traditional AHP with fuzzy set theory (Bellman & Zadeh, 1970), was developed for coping with uncertain judgments (Chen & Hung, 2010; Chiou, Tzeng, & Cheng, 2005; Naghadehi, Mikaeil, & Ataei, 2009) and to express preferences as fuzzy sets or fuzzy numbers which reflect the vagueness of human thinking (Cakir & Canbolat, 2008; Liou, Wang, Hsu, & Yin, 2011). In this study, triangular fuzzy numbers (TFNs) are used to represent the fuzzy relative importance of each alternative and criterion. The triangular fuzzy conversion scale used to convert such linguistic terms into fuzzy numbers in the evaluation model is given in Table 1. A TFN $N^{\sim}$ can be denoted by the triplet (a, b, c) with membership function $\mu_{N^{\sim}}(x)$, which is described as follows: the parameter "b" is the most likely value and the parameters "a" and "c" are the lower and upper bounds that limit the field of possible evaluation, as depicted in Eq. (1).

$$\mu_{N^{\sim}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leqslant x \leqslant b \\ \frac{c-x}{c-b}, & b \leqslant x \leqslant c \\ 0, & otherwise \end{cases} \tag{1}$$

The fuzzy arithmetic operations on two TFNs M_1 and M_2 derived by Dubois and Prade (1979) are as follows: if $M_1 = (a_1, b_1, c_1)$ and $M_2 = (a_2, b_2, c_2)$ then

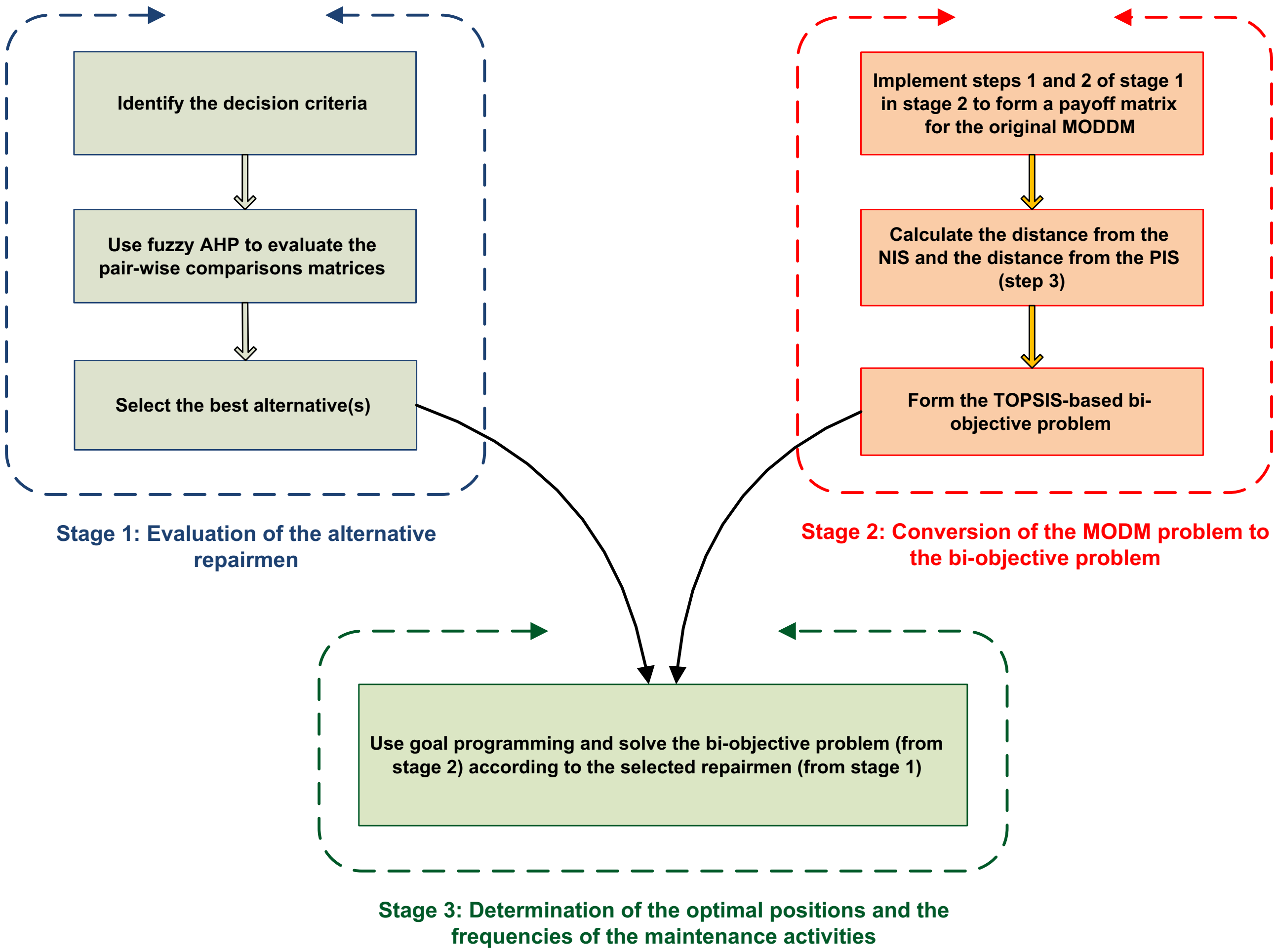

Fig. 1. Proposed three-stage framework.

Table 1
Linguistic terms for the TFNs (Saaty, 1989).

Linguistic terms	Acronyms	TFN
Extremely more importance	EMI	(8, 9, 10)
Very strong importance	VSI	(6, 7, 8)
Strong importance	SI	(4, 5, 6)
Moderate importance	MI	(2, 3, 4)
Equal importance	EI	(1, 1, 2)

$$M_1 \oplus M_2 = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \quad (2)$$

$$M_1 \otimes M_2 = (a_1, b_1, c_1) \otimes (a_2, b_2, c_2) \cong (a_1a_2, b_1b_2, c_1c_2) \quad (3)$$

$$(M1)^{-1} = (a_1, b_1, c_1)^{-1} \cong (1/c_1, 1/b_1, 1/a_1) \quad (4)$$

The main steps of the fuzzy AHP are as follows:

(1) Structuring the decision hierarchy: Similar to conventional AHP, the first step is to break down the complex decision making problem into a hierarchical structure.

(2) Constructing the fuzzy pairwise comparison matrices: Buckley (1985) considered a fuzzy pairwise comparison matrix with n elements where pairwise comparison judgments are represented by TFN $a_{ij}^{\sim} = (l_{ij}, m_{ij}, u_{ij})$. In conventional AHP each set of comparisons for a level with n elements requires $n(n-1)/2$ judgments such as:

$$A^{\sim} = \left[a_{ij}^{\sim}\right] = \begin{bmatrix} a_{11}^{\sim} & \cdots & a_{1n}^{\sim} \\ \vdots & \ddots & \vdots \\ a_{n1}^{\sim} & \cdots & a_{nn}^{\sim} \end{bmatrix} \quad (5)$$

where $a_{ij}^{\sim}$ is the fuzzy comparison value of criterion (alternative) i with respect to criterion (alternative) j. To construct the fuzzy pairwise comparison matrix we use the linguistic terms of Table 1.

In this step a geometric mean technique defines the normalized vector of criterion (alternative) weights for each pairwise comparison matrix as follows:

$$e_j^{\sim} = \left(a_{j1}^{\sim} \otimes a_{j2}^{\sim} \otimes a_{j3}^{\sim} \otimes \ldots \otimes a_{jn}^{\sim}\right)^{\frac{1}{n}}, \quad j = 1, 2, 3, \ldots, n. \quad (6)$$

$$w_j^{\sim} = e_j^{\sim} \otimes \left(e_1^{\sim} \oplus e_2^{\sim} \oplus e_3^{\sim} \oplus \ldots \oplus e_n^{\sim}\right)^{-1}, \quad j = 1, 2, 3, \ldots, n. \quad (7)$$

(3) Aggregating the priorities and ranking the alternatives: The final step aggregates local priorities obtained at different levels of the decision hierarchy into composite global priorities for the alternatives based on the weighted sum method. If there are i alternatives and j criteria, then the final global priority of alternative i is given as:

$$A_i = \sum_{j=1}^{n} w_j^{\sim} \otimes w_{ij}^{\sim} \quad (8)$$

where $w_j^{\sim}$ is the weight of criterion j and $w_{ij}^{\sim}$ is the evaluation of alternative i with respect to criterion j.

The rationale of fuzzy methods is to defuzzify imprecise values at the end of the process, not in the beginning. Assuming $A_i = (a, b, c)$ is a TFN, the graded mean integration representation of A_i will be:

$$P(A_i) = \frac{a + 4b + c}{6} \quad (9)$$

The higher the value $P(A_i)$, the more preferred the alternative. Finally, assuming a threshold value on $P(A_i)$'s, say P^*, we select those alternatives A_i with $P(A_i)$ values greater than P^* as the ones qualified to be included in the next stage.

2.2. Stage 2: Conversion of the MODM problem to the bi-objective problem

The TOPSIS method, introduced by Hwang and Yoon (1981) for the first time, is a well-known multi-criteria decision making approach. A wide variety of TOPSIS applications have been reported in the MODM literature. Abo-Sinna and Amer (2005) and Abo-Sinna, Amer, and Ibrahim (2008) used TOPSIS to solve multi-objective large-scale non-linear programming problems. Abo-Sinna and Abou-El-Enien (2006) applied TOPSIS to large-scale multiple objective programming problems involving fuzzy parameters. Jadidi, Hong, and Firouzi (2009) used and extended the version of the TOPSIS method proposed by Abo-Sinna and Abou-El-Enien (2006) to solve the multi-objective supplier selection problem using mixed integer linear programming. Lai, Liu, and Hwang (1994) used the compromise properties of TOPSIS to generate solutions with the shortest distance from the PIS as well as the longest distance from the NIS while reducing a k-dimensional objective space to a two-dimensional objective space by a first-order compromise procedure. Recently, Khalili-Damghani, Tavana, and Sadi-Nezhad (2012, 2013) used a TOPSIS method to confine the objective dimension space of real-life large-scale multi-objective multi-period project selection problems. We briefly describe the TOPSIS method for the resulting bi-objective problem based on the original MODM problem with the following steps:

Step 1. Consider the original multiple-objective optimization problem with k conflicting objectives as follows:

$$\{Optimize\ f_i(x)\ i = 1,2,\ldots,k; g_j(x) \leqslant b\ j = 1,2,\ldots,m\} \tag{10}$$

Next we solve two sets of single objective optimization problems as follows:

$$\{Min\ f_i(x)\ i = 1,2,\ldots,k; g_j(x) \leqslant b\ j = 1,2,\ldots,m\} \tag{11}$$

$$\{Max\ f_i(x)\ i = 1,2,\ldots,k; g_j(x) \leqslant b\ j = 1,2,\ldots,m\} \tag{12}$$

Step 2. Using the pay-off table for the objective functions we obtain Z^+ and Z^- as follows:

$$Z^- = (Z_1^-, \quad Z_2^-,\ldots,Z_i^-,\ldots,Z_{k-1}^-,Z_k^-) \tag{13}$$

$$Z^+ = (Z_1^+, \quad Z_2^+,\ldots,Z_i^+,\ldots,Z_{k-1}^+,Z_k^+) \tag{14}$$

where Z^- is a vector of optimum values of the single objective problem which has been optimized in the contrary direction of the original MODM problem (i.e., NIS) and Z^+ is a vector of optimum values of the single objective problem which has been optimized in the same direction of the original MODM problem (i.e., PIS). The range of objective functions which are maximized in the original MODM problem can be estimated by $Z^+ - Z^-$. In contrast, the range of the objective functions which are minimized in the original MODM problem can be estimated by $Z^- - Z^+$.

Step 3. Considering Z^+, Z^-, the range of the objective functions, the relative importance of the objective functions and dividing the objective functions into *two groups*, k_1 minimizing and $k - k_1$ maximizing, we calculate the distance function from the PIS and the distance function from the NIS as follows:

$$d_p^{PIS}(x) = \left[\sum_{i=1}^{k_1}\left[w_i \times \frac{f_i(x) - Z_i^+}{Z_i^- - Z_i^+}\right]^p + \sum_{i=k_1+1}^{k}\left[w_i \times \frac{Z_i^+ - f_i(x)}{Z_i^+ - Z_i^-}\right]^p\right]^{\frac{1}{p}} \tag{15}$$

$$d_p^{NIS}(x) = \left[\sum_{i=1}^{k_1}\left[w_i \times \frac{Z_i^- - f_i(x)}{Z_i^- - Z_i^+}\right]^p + \sum_{i=k_1+1}^{k}\left[w_i \times \frac{f_i(x) - Z_i^-}{Z_i^+ - Z_i^-}\right]^p\right]^{\frac{1}{p}} \tag{16}$$

Parameter p is a positive integer; specifically $p = 1$ refers to the rectangular distance, $p = 2$ refers to the Euclidean distance, etc. The parameter w_i is the relative importance of the objective functions in the original MODM problem whose sum is equal to unity. It is notable that $d_p^{PIS}(x)$ and $d_p^{NIS}(x)$ are scale independent since they are normalized to get values in the range $[0, 1]$ enabling us to compare the objective functions.

Step 4. The TOPSIS based bi-objective problem where S is the feasible solution space in the original MODM problem (i.e., $g_j(x) =< b, j = 1,2,\ldots,m$) is developed as follows:

$$Min\ d_p^{PIS}(x) \tag{17}$$

$$Max\ d_p^{NIS}(x) \tag{18}$$

$$s.t.\ x \in S \tag{19}$$

2.3. Stage 3: Determination of the optimal positions and the frequencies of the maintenance activities

Goal Programming (GP), one of the more powerful techniques for solving multi objective optimization problems, originated from the work of Charnes and Cooper (1961) and has been applied in a variety of situations. GP models aim to minimize deviations of the objective values from aspiration levels which are specified by the decision maker(s). The GP's solution depends on the metrics used for the deviations as well as the weighting method of the different goals such as the minimization of the weighted sum of goal deviations (Charnes & Cooper, 1977) or the minimization of the maximum deviation Flavell (1976). In this paper, we use the minimization of the weighted sum of goal deviations. A mathematical formulation of the GP model is given below:

GP

$$Min \quad \sum_{i=1}^{k} w_i(d_i^+ + d_i^-) \tag{20}$$

$$s.t. \quad f_i(x) + d_i^- - d_i^+ = G_i \quad i = 1,2,\ldots,k \tag{21}$$

$$x \in S \tag{22}$$

$$d_i^-, d_i^+ \geqslant 0, \quad i = 1,2,\ldots,k \tag{23}$$

where $f_i(x)$ is the function of the i-th goal, G_i is the aspiration level of the i-th goal, w_i is the relative importance of i-th goal, $d_i^+ = \max\{0, f_i(x) - G_i\}$ and $d_i^- = \max\{0, G_i - f_i(x)\}$ are the positive and negative deviations from the aspiration levels, respectively. Therefore, the GP model for solving the TOPSIS based bi-objective problem from stage 2 can be formulated as follows:

$$Min \quad [(w_{PIS} \times d_{PIS}^+) + (w_{NIS} \times d_{NIS}^-)] \tag{24}$$

$$s.t. \quad d_p^{PIS}(x) + d_{PIS}^- - d_{PIS}^+ = G_{PIS} \tag{25}$$

$$d_p^{NIS}(x) + d_{NIS}^- - d_{NIS}^+ = G_{NIS} \tag{26}$$

$$x \in S \tag{27}$$

$$d_{PIS}^-, \quad d_{PIS}^+, d_{NIS}^-, d_{NIS}^+ \geqslant \quad 0 \tag{28}$$

3. Application of the proposed model in the UPM problems with AEMMA

The proposed approach can be used in a variety of MODM applications. Here, we illustrate the applicability of the proposed approach to the UPM-AEMMA scheduling problem. Let us consider a factory that uses several repairmen to maintain its machines. Stage 1 consists of determining the best alternatives based on the relevant criteria, stage 2 converts MOILP to a bi-objective problem and finally stage 3 jointly determine job schedule, position and frequencies of the selected repairmen's maintenance activities by goal programming. The assumptions, indices, parameters, and decision variables of the problem are presented as follows:

Assumptions:

- There are n independent jobs $J = \{J_1, J_2, \dots, J_n\}$ to be processed on m unrelated parallel machines (M_h, $h = 1, 2, \dots, m$).
- Each machine is maintained by only one Repairman whereas each Repairman can take care of more than one machine.
- All the jobs are simultaneously available at time zero and job preemption is not allowed.
- A machine can process at most one job at a time and a machine cannot be idle when at least one non-assigned job exists.
- The actual processing time of a job increases if it is scheduled later due to the aging effect of the tools and the maintenance activities that may be performed on the machines to sustain their production efficiency.
- We assume that each machine may be subject to several maintenance activities over the scheduling horizon and the duration of each maintenance activity on machine h by Repairman s is a constant time t_{sh} and incurs a constant cost $\gamma_{sh}(h = 1, 2, \dots, m; s = 1, 2, \dots, S)$.
- We assume that each machine must process at least one job and $m < n$.
- We assume that a maintenance activity can be scheduled on each machine immediately after it has completed the processing of at least one job and the machine will revert to its initial conditions.

MOILP

$$Z_1 = Min \sum_{h=1}^{m} \sum_{s=1}^{S} \sum_{g=1}^{k_h} Q_{gsh} \times \gamma_{sh} \tag{29}$$

$$Z_2 = Min\ C_{max} \tag{30}$$

$$Z_3 = Min\ T_{max} \tag{31}$$

$$Z_4 = Min\ E_{max} \tag{32}$$

$$Z_{gh} \leqslant \sum_{s=1}^{S} Y_{sh} \quad \forall h, g = 1, 2, \dots \tag{33}$$

$$Y_{sh} + Z_{gh} \leqslant Q_{gsh} + 1 \& 2 * Q_{gsh} \leqslant Y_{sh} + Z_{gh} \quad \forall h, s, g \tag{34}$$

$$\sum_{h=1}^{m} \sum_{g=0}^{k_h} \sum_{r=1}^{u_{gh}} X_{jrgh} = 1 \quad \forall j \tag{35}$$

$$\sum_{j=1}^{n} \sum_{g=0}^{k_h} \sum_{r=1}^{u_{gh}} X_{jrgh} \geqslant 1 \quad \forall h \tag{36}$$

$$\sum_{s=1}^{S} Y_{sh} \leqslant 1 \quad \forall h \tag{37}$$

$$Z_{gh} \geqslant Z_{g+1h} \quad \forall h, g \tag{38}$$

$$\sum_{j=1}^{n} X_{jrgh} \geqslant \sum_{j=1}^{n} X_{jr+1gh} \quad \forall r, g, h \tag{39}$$

$$\sum_{j=1}^{n} X_{jrgh} \leqslant Z_{gh} \quad \forall r, g, h \tag{40}$$

$$\sum_{j=1}^{n} X_{j1gh} = Z_{gh} \quad \forall h, g \tag{41}$$

Indices

j Index for job	$(j = 1, 2, \dots, n)$
r Index for position of job	$(r = 1, 2, \dots, u_{gh})$
g Index for maintenance	$(g = 0, 1, 2, \dots, k_h)$
h Index for machine	$(h = 1, 2, \dots, m)$
s Index for repairmen	$(s = 1, 2, \dots, S)$

Parameters

n Number of jobs	
m Number of machines	
S	Number of repairmen (obtained from stage 1)
U_{gh}	Maximum position after gth maintenance on the hth machine
K_h	Maximum maintenance on machine hth
P_{jh}	Normal processing time of job j on machine h
α_{jh}	Aging factor of job j on machine h
d_j	Due date of job j
t_{sh}	Maintenance time of machine $h$by repairman s
γ_{sh}	Maintenance cost of machine h by repairman s
M	Large number

Decision variables

$$X_{jrgh} = \begin{cases} 1 & \text{if job } j \text{ is in the } r\text{th position after the } g\text{th maintenance on machine} h \\ 0 & \textit{otherwise} \end{cases}$$

$$Y_{sh} = \begin{cases} 1 & \text{if the } h\text{th machine is main tained by repair man} s \\ 0 & \textit{otherwise} \end{cases}$$

$$Z_{gh} = \begin{cases} 1 & \text{if maintenance } g \text{ is done on machine } h \\ 0 & \textit{otherwise} \end{cases}$$

$$Q_{gsh} = \begin{cases} 1 & \textit{if } Y_{sh} \times Z_{gh} \\ 0 & \textit{otherwise} \end{cases}$$

$$W_{jrgsh} = \begin{cases} 1 & \textit{if } Y_{sh} \times X_{jrgh} \\ 0 & \textit{otherwise} \end{cases}$$

C_j = completion time of job j

E_j = early time of job j

T_j = tardy time of job j

$$C_j - g \times t_{sh} - \sum_{j'}^{n}\sum_{g'}^{g-1}\sum_{r'}^{u_{g'h}} X_{j'r'g'h}(p_{j'h} + \alpha_{j'h}r') - \sum_{j''}^{n}\sum_{r''}^{r} X_{j''r''gh}(p_{j''h} + \alpha_{j''h}r'') \leqslant (1 - W_{jrgsh})M \quad \forall r,g,h,j,s \tag{42}$$

$$C_j - g \times t_{sh} - \sum_{j'}^{n}\sum_{g'}^{g-1}\sum_{r'}^{u_{g'h}} X_{j'r'g'h}(p_{j'h} + \alpha_{j'h}r') - \sum_{j''}^{n}\sum_{r''}^{r} X_{j''r''gh}(p_{j''h} + \alpha_{j''h}r'') + (1 - W_{jrgsh})M \geqslant 0 \quad \forall r,g,h,j,s \tag{43}$$

$$Y_{sh} + X_{jrgh} \leqslant W_{jrgsh} + 1 \& 2 * W_{jrgsh} \leqslant Y_{sh} + X_{jrgh} \quad \forall j,r,h,s,g \tag{44}$$

$$C_j + E_j - T_j = d_j \quad \forall j \tag{45}$$

$$C_{max} \geqslant C_j \quad \forall j \tag{46}$$

$$T_{max} \geqslant T_j \quad \forall j \tag{47}$$

$$E_{max} \geqslant E_j \quad \forall j \tag{48}$$

$$X_{jrgh}, Y_{sh} \& Z_{gh} = 0 \text{ or } 1 \quad \forall r,g,s,h,j \tag{49}$$

In the above MOILP model, Eqs. (29)–(32) are the objective functions to minimize the total maintenance cost, maximum completion time of all the jobs (Makespan), maximum earliness time and maximum tardiness time of the jobs, respectively. Eq. (33) shows that each maintenance activity is fulfilled by one Repairman. Eq. (34) is implemented to linearize Eq. (29). Eq. (35) dictates that each job should be assigned just once to one position of the machine. Eq. (36) guarantees that each machine should be assigned at least one of the jobs. Eq. (37) ensures that each machine is maintained by at most one Repairman. Eq. (38) shows that the $g+1$ th maintenance activity is not done on a machine, unless the *g-th* maintenance activity was done on the same machine. Eq. (39) ensures that the positions after each maintenance activity on each machine must be assigned to the jobs in order. Eq. (40) shows the relationship between the two binary variables, X and Z. Eq. (41) ensures that immediately after every maintenance activity on a machine, at least one job should be assigned to the machine.

Eqs. (42) and (43) estimate the completion times of the jobs. These equations require some additional intuition. The ranges on which the new indices introduced are defined are given by $j', j'' = 1, 2, \ldots, n$, with $j', j'' \neq j$; $r' = 1, 2, \ldots, u_{g'h}$; $r'' = 1, 2, \ldots, r$ and $g' = 0, 1, 2, \ldots, g-1$. These equations define a limit interval for the completion time of job j based on the jobs processed before it. These jobs, denoted by j' and j'', are processed after performing either any of the g' previous maintenance activities (j' term on the left hand side of both equations) or the g current one (j'' term on the left hand side of both equations). That is, when accounting for all the other jobs located before job j, the decision maker has to consider the r' jobs processed after performing all the previous maintenance activities and the r'' jobs processed after performing the current maintenance activity g but before r, which indicates the position of job j. Eq. (44) is implemented to linearize Eqs. 42 and 43. Eq. (45) calculates the early/tardy time of the jobs. Eqs. (46)–(48) compute Makespan, maximum tardiness time and maximum earliness time of the jobs, respectively. Eq. (49) defines binary decision variables.

4. Numerical example and computational results

We consider a manufacturing company with unrelated parallel machines that wants to select the best alternatives of repairmen and assign them to the maintenance of the machines. The decision maker (Manager) considers four alternatives as repairmen (A_1–A_4) with the following criteria: Cost (C_1), Time (C_2) and Reliability (C_3). Stage 1 is applied as follows to evaluate these alternatives:

(1) Structuring decision hierarchy: Similar to conventional AHP, the first step is to break down the complex decision making problem into a hierarchical structure based upon AHP as shown in Fig. 2.
(2) Developing fuzzy pairwise comparison matrices: The decision maker uses the linguistic variables, and their corresponding triangular fuzzy numbers shown in Table 1, to form fuzzy comparison matrices for the objectives and for the alternatives with respect to the criteria which are presented in Tables 2 and 3. The resulting weights of the criteria and the alternatives for each fuzzy pairwise comparison matrix are calculated using Eqs. (6) and (7).

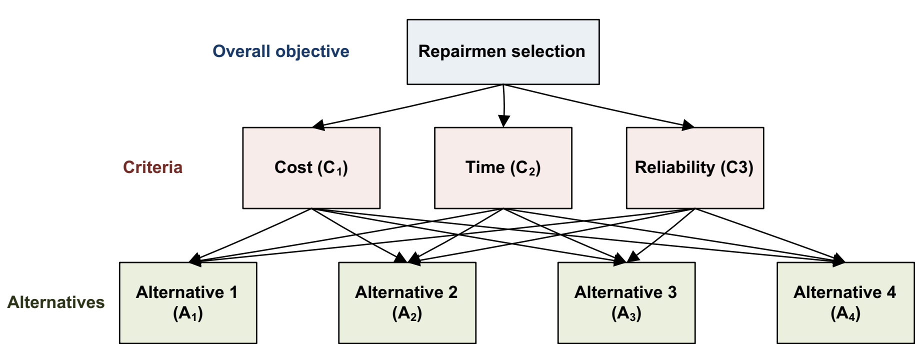

Fig. 2. Hierarchical structure of AHP.

Table 2
Evaluation of the criteria with respect to the goal.

Linguistic terms				Fuzzy terms			
Goal	C_1	C_2	C_3	C_1	C_2	C_3	Weight ($\tilde{w}_j$)
C_1	–			1	(1/6, 1/5, 1/4)	(1/8, 1/7, 1/6)	(0.05, 0.07, 0.09)
C_2	SI	–	MI	(4, 5, 6)	1	(2, 3, 4)	(0.40, 0.59, 0.83)
C_3	VSI		–	(6, 7, 8)	(1/4, 1/3, 1/2)	1	(0.23, 0.32, 0.46)

Table 3
Evaluation of the alternatives with respect to cost, time, and reliability.

Linguistic terms					Fuzzy terms				
Cost	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	Weight
Cost									
A_1	–	EI			1	(1, 1, 2)	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)	(0.07, 0.10, 0.17)
A_2		–			(1/2, 1, 1)	1	(1/6, 1/5, 1/4)	(1/4, 1/3, 1/2)	(0.06, 0.10, 0.14)
A_3	SI	SI	–	EI	(4, 5, 6)	(4, 5, 6)	1	(1, 1, 2)	(0.32, 0.44, 0.72)
A_4	MI	MI		–	(2, 3, 4)	(2, 3, 4)	(1/2, 1, 1)	1	(0.18, 0.34, 0.50)
Time									
Time	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	Weight
A_1	–		EI	EI	1	(1/4, 1/3, 1/2)	(1, 1, 2)	(1, 1, 2)	(0.12, 0.17, 0.34)
A_2	MI	–	MI	MI	(2, 3, 4)	1	(2, 3, 4)	(2, 3, 4)	(0.28, 0.50, 0.82)
A_3			–	EI	(1/2, 1, 1)	(1/4, 1/3, 1/2)	1	(1, 1, 2)	(0.10, 0.17, 0.29)
A_4				–	(1/2, 1, 1)	(1/4, 1/3, 1/2)	(1/2, 1, 1)	1	(0.08, 0.17, 0.24)
Reliability									
Reliability	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	Weight
A_1	–	MI	EI	SI	1	(2, 3, 4)	(1, 1, 2)	(4, 5, 6)	(0.27, 0.41, 0.71)
A_2		–		EI	(1/4, 1/3, 1/2)	1	(1/4, 1/3, 1/2)	(1, 1, 2)	(0.08, 0.12, 0.23)
A_3		MI	–	MI	(1/2, 1, 1)	(2, 3, 4)	1	(2, 3, 4)	(0.19, 0.36, 0.54)
A_4				–	(1/6, 1/5, 1/4)	(1/2, 1, 1)	(1/4, 1/3, 1/2)	1	(0.06, 0.11, 0.16)

(3) Aggregation of priorities and ranking the alternatives: this step aggregates local priorities obtained at different levels of the hierarchy into composite global priorities for the alternatives based on the weighted sum method and using Eqs. (8) and (9). The results are presented in Table 4.

Since $P(A_i)$, the range of the defuzzified final weight, belongs to [0, 1], a threshold value is set equal to $P^* = 0.3$, meaning that only alternatives with a score greater than or equal to 0.3 are selected. Therefore, alternatives A_2 and A_4 are selected for the next stage.

In the second stage, at first, we define the parameter values that are used in the MOILP.

Our example consists of scheduling six jobs on the three machines using two selected repairmen (A_2 and A_4) from stage 1 where the aging factor matrix of each machine with respect to each job is presented as follows:

$$\alpha_{hj} = \begin{pmatrix} 6 & 12 & 5 & 1 & 10 & 3 \\ 7 & 11 & 6 & 2 & 10 & 4 \\ 6 & 14 & 5 & 3 & 11 & 3 \end{pmatrix}$$

Assume that the processing time matrix of each machine with respect to each job is as follows:

$$p_{hj} = \begin{pmatrix} 8 & 16 & 7 & 3 & 14 & 5 \\ 9 & 15 & 8 & 4 & 14 & 6 \\ 8 & 17 & 7 & 5 & 13 & 4 \end{pmatrix}$$

The maintenance cost of the machines with respect to the repairmen is given by the following matrix:

$$\gamma_{sh} = \begin{pmatrix} 100 & 200 & 200 \\ 100 & 100 & 50 \end{pmatrix}$$

The maintenance time of the machines with respect to the repairmen is as follows:

$$t_{sh} = \begin{pmatrix} 10 & 15 & 15 \\ 10 & 10 & 5 \end{pmatrix}$$

The due date of the jobs is presented as follows:

$$d_j = (28, 55, 12, 7, 70, 10)$$

The maximum maintenance activity is the same on all three machines. The maximum allowed job position is the same in each maintenance activity on the each machine as follows:

$$K_1 = K_2 = K_3 = 3; \quad U_{gh} = 4, \quad \forall g, h$$

Applying stage 2 of the proposed approach, Tables 5 and 6 present the decision variables for the single ideal objective optimization in the original MODM problem. Tables 7 and 8 present the decision variables for the single anti-ideal objective optimization in the original MODM problem. Table 9 presents both the payoff matrix of the single objective optimization and the range of the objective functions of the original MODM problem. The mathematical model has been coded in Lingo 9.0 and executed on a HP Laptop 4520s model with Core i3 due CPU, 2.4 GHz, and Windows 7 using 3 GB of RAM as follows:

According to Table 9 and Eqs. (15) and (16) the TOPSIS-based bi-objective problem will be achieved as follows. It is notable that the values of $w_i, i = 1, \ldots, 4$ and p are set to 0.25 and 1, respectively.

$$Min\ d_1^{PIS}(x) = \sum_{i=1}^{4} w_i \times \frac{f_i(x) - Z_i^+}{Z_i^- - Z_i^+} = 0.25 \times \left(\frac{Z_1 - 0}{600 - 0} + \frac{Z_2 - 34}{140 - 34} + \frac{Z_3 - 0}{118 - 0} + \frac{Z_4 - 0}{46 - 0} \right) \tag{50}$$

$$Max\ d_1^{NIS}(x) = \sum_{i=1}^{4} w_i \times \frac{Z_i^- - f_i(x)}{Z_i^- - Z_i^+} = 0.25 \times \left(\frac{600 - Z_1}{600 - 0} + \frac{140 - Z_2}{140 - 34} + \frac{118 - Z_3}{118 - 0} + \frac{46 - Z_4}{46 - 0} \right) \tag{51}$$

$$\text{s.t. } x \in S \tag{52}$$

Table 4
Final alternative rankings.

	Cost	Time	Reliability	Alternatives priorities	
$w_j^{\sim}$	(0.05, 0.07, 0.09)	(0.40, 0.59, 0.83)	(0.23, 0.32, 0.46)	Fuzzy final weight	$P(A_i)$ [Final ranking]
A_1	(0.07, 0.10, 0.17)	(0.12, 0.17, 0.34)	(0.27, 0.41, 0.71)	(0.08, 0.16, 0.45)	0.19 [3]
A_2	(0.06, 0.10, 0.14)	(0.28, 0.50, 0.82)	(0.08, 0.12, 0.23)	(0.17, 0.45, 1.07)	0.51 [1]
A_3	(0.32, 0.44, 0.72)	(0.10, 0.17, 0.29)	(0.19, 0.36, 0.54)	(0.08, 0.17, 0.41)	0.19 [3]
A_4	(0.18, 0.34, 0.50)	(0.08, 0.17, 0.24)	(0.06, 0.11, 0.16)	(0.11, 0.29, 0.74)	0.33 [2]

Table 5
Ideal position of the jobs and the maintenance activities on the machines with respect to Z_1 and Z_2.

Z_1	Maintenance activity 0				Maintenance activity 1				Maintenance activity 2				Maintenance activity 3			
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_1	J_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_4	J_6	J_5	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Z_2																
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_1	J_3	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_2	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_5	J_6	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

Table 6
Ideal position of the jobs and the maintenance activities on the machines with respect to Z_3 and Z_4.

Z_3	Maintenance activity 0				Maintenance activity 1				Maintenance activity 2				Maintenance activity 3			
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_4	J_1	J_5	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_6	–	–	–	J_2	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Z_4																
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_6	J_1	J_2	J_5	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

Table 7
Anti-ideal position of the jobs and the maintenance activities on the machines with respect to Z_1 and Z_2.

	Maintenance activity 0				Maintenance activity 1				Maintenance activity 2				Maintenance activity 3			
Z_1																
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_3	–	–	–	J_6	–	–	–	J_5	–	–	–	J_1	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Z_2																
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_3	J_1	J_5	J_2	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

Table 8
Anti-ideal position of the jobs and the maintenance activities on the machines with respect to Z_3 and Z_4.

	Maintenance activity 0				Maintenance activity 1				Maintenance activity 2				Maintenance activity 3			
Z_3																
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_1	J_5	J_2	J_3	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Z_4																
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_1	J_3	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_2	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_5	J_6	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

Table 9
Single-objective optimization payoff matrix for the original MODM problem.

	Z_1	Z_2	Z_3	Z_4
Ideal calculations				
Min Z_1	0	64	10	14
Min Z_2	0	34	27	46
Min Z_3	100	68	0	9
Min Z_4	0	135	65	0
Anti-ideal calculations				
Max Z_1	600	109	81	24
Max Z_2	0	140	85	3
Max Z_3	0	130	118	20
Max Z_4	0	34	27	46

$Z^+ = (0, 34, 0, 0)$, $Z^- = (600, 140, 118, 46)$.

In order to solve the above TOPSIS-based bi-objective problem, we use stage 3 of the proposed approach (i.e,. GP) as follows:

$$Min \quad [(w_{PIS} \times d^+_{PIS}) + (w_{NIS} \times d^-_{NIS})] \tag{53}$$

$$\text{s.t.} \quad 0.25 \times \left(\frac{Z_1 - 0}{600 - 0} + \frac{Z_2 - 34}{140 - 34} + \frac{Z_3 - 0}{118 - 0} + \frac{Z_4 - 0}{46 - 0}\right) + d^-_{PIS} - d^+_{PIS} = G_{PIS} \tag{54}$$

$$0.25 \times \left(\frac{600 - Z_1}{600 - 0} + \frac{140 - Z_2}{140 - 34} + \frac{118 - Z_3}{118 - 0} + \frac{46 - Z_4}{46 - 0}\right) + d^-_{NIS} - d^+_{NIS} = G_{NIS} \tag{55}$$

$$x \in S \tag{56}$$

$$d^-_{PIS}, \quad d^+_{PIS}, d^-_{NIS}, d^+_{NIS} \geqslant \quad 0 \tag{57}$$

Weights w_{PIS} and w_{NIS} of each goal in the objective function are set as $(w_{PIS}, w_{NIS}) = (0.5, 0.5)$. The aspiration levels of the goals are equal to $(G_{PIS}, G_{NIS}) = (0, 1)$. Note that since goals 1 and 2 have associated weights in the final objective function, the total fractional deviation is less than the sum of the individual fractional deviations from goals 1 and 2. The results of solving the GP are summarized in Table 10 as follows:

Some final remarks regarding the scale of the problems considered are due. Note that, for obvious reasons of manuscript length, the numerical example provided is a small scale one. In this regard, any problem arising from the dimensionality of the model relates directly to those of the standard MOILP models. However, the current setting provides an intuitive direction in which to proceed if the dimensionality of the model becomes an issue. The current proposal is mainly intuitive and can be further developed in future extensions of the paper.

Assume that the number of machines, the number of jobs that must be processed, or both, lead to a MOILP model that is not solvable due to its dimension. In this case, a potential solution would consist of partitioning the sets of machines, jobs and repairmen in two or as many subsets as necessary to obtain a numerical solution. The partition of the sets of jobs and machines in two (or more) subsets must be done randomly. This is due to the lack of a ranking mechanism based on the heterogeneity of both variables in terms of aging and processing time, as can be observed from our previous numerical example.

However, the partition of the set of repairmen can be based on their relative efficiency, $P(A_i)$. We can exploit this measure of performance to obtain efficient (from a repairmen viewpoint) computable solutions for each one of the (sub)models generated. Assume, for expositional simplicity, that we partition the sets of jobs and machines in two subsets of $j/2$ and $h/2$ elements, respectively. The number of elements can be rounded by the decision maker if the total number of jobs or machines is odd. Similarly, a new (less efficient) repairman may be added from the available pool if required to equate the size of the resulting subsets.

Assume that the set of $s = 1, 2 \ldots, S$ repairmen is ordered based on their defuzzified efficiency score $P(A_i)$ from the highest to the lowest one. The suggested procedure would consist of the following steps

1. Partition the set of repairmen in two ordered subsets based on their efficiency scores and assign them to the groups of machines and jobs defined previously.
2. Assign the more efficient half of repairmen to a given subset of machines and jobs and the less efficient half of repairmen to the complement subset of machines and jobs.
3. Solve the resulting model to obtain the goal score achieved relative to the corresponding aspiration levels.
4. Switch the subset of repairmen between groups.
5. Solve the resulting model to obtain the goal score achieved relative to the corresponding aspiration levels.
6. Compare the goal scores achieved in both cases.
7. Consider the case with the higher score and start shifting either one or a subset of repairmen form the less efficient to the more efficient group. The shift must be done orderly, starting with the initial members of the less efficient set and exchanging them with the final members of the more efficient one. Perform the resulting calculations and solve the model again.
8. If the goal score increases, proceed orderly with further shifts. Otherwise, stop. If subsets of repairmen are shifted, then individual shifts can be performed within each subset until the highest goal score is achieved. Note that, in the current example, a complete shift of repairmen from the less to the more efficient group would lead to the complement subsets considered initially and providing a lower goal score. In this regard, note that additional shifting combinations must be considered when accounting for more than two subsets.

This process can be repeated until the highest goal score is obtained. The intuition behind this process follows from the

Table 10
Optimal position of the jobs and the maintenance activities on the machines.

	Maintenance activity 0				Maintenance activity 1				Maintenance activity 2				Maintenance activity 3			
Machine 1																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_6	J_1	J_5	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 2																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	J_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Machine 3																
Position	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4	r_1	r_2	r_3	r_4
Repairman A_2	J_3	J_2	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Repairman A_4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

completely asymmetric distribution of repairmen and the assumption that more efficient repairmen will perform better than the less efficient ones. Note that the stochastic component of machine and job group selection would remain when implementing this approach, which constitutes the price to pay for numerical tractability. However, despite this stochastic component, we can still aim at partitioning efficiently the set of repairmen among the corresponding subsets of jobs and machines.

5. Conclusions and future research directions

In this paper we proposed a scheduling model for solving maintenance scheduling problems with UPMs and AEMMAs. In the first stage of the evaluation process we use a fuzzy AHP approach for repairmen selection. In the second stage we present a procedure based on the TOPSIS approach to reduce the MODM problem to an efficient bi-objective problem. Finally, in the third stage, we use GP and solve the resulting TOPSIS problem based on a bi-objective integer linear programming model in which the two goals of total distance from the PIS and the NIS are taken into consideration. A numerical example was presented to illustrate the applicability of our proposed approach.

The contribution of the proposed performance measurement system is fourfold: (1) In spite of tremendous advances in PMS research, multi-objective scheduling problems with simultaneous consideration of repairmen selection, aging effects and maintenance activities in a parallel machine environment have not been thoroughly studied in the literature; (2) We proposed a comprehensive repairmen selection and scheduling problem that combines fuzzy AHP with TOPSIS and GP in a structured and simple to use framework; (3) We considered fuzzy logic and fuzzy sets to represent ambiguous, uncertain and imprecise information in a manufacturing environment; and (4) The proposed method was capable of synthesizing a representative outcome based on qualitative judgments and quantitative data.

A very practical extension of the model proposed in this study is to consider other MODM methods in the proposed framework. Applications of the proposed method to other problems in the manufacturing or service environments constitute another potential extension of the proposed method.

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