



## LETTER

# Estimation of GSM base station output power cumulative density function



Predrag Jovanović<sup>a,b,\*</sup>, Milan Bjelica<sup>a</sup>

<sup>a</sup> University of Belgrade, School of Electrical Engineering (ETF), Belgrade, Serbia

<sup>b</sup> Iritel a.d., Belgrade, Serbia

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## ABSTRACT

If the dependence of base station output power on system parameters in a mobile network would be known, it would be possible to achieve energy savings by adjusting the output power in each channel to the current traffic demands. Since power radiated in an active channel is a random variable, so is the total output power of a base station; its calculation can be a complicated task since it involves convolution of probability density functions associated to each active channel. In this letter, we propose a simulation approach to estimate the power of active GSM connections. We apply our model to an analytically tractable scenario and show that the results match.

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## 1. Introduction

Base station (BTS) power management enables not only the power savings in mobile networks, but also the reduction of electromagnetic radiation [1–5]. As the experimental results show, consumed power of both GSM and UMTS base stations depends on the traffic intensity that is served [6]. When discussing the output power of a base station, one can consider the instantaneous, mean, or peak power, probability density function (PDF) and cumulative density function (CDF). Amongst these quantities, the CDF is the most informative one as it provides data on both the consumed power and its temporal distribution. However, calculation of the CDF for BTS total output power is a nontrivial task, as it includes calculation of power consumed in each channel and also its dependence on traffic intensity.

In this letter, we consider the estimation of the CDF for GSM base station output power. We continue our work from [7] and propose a simulation model of low computational complexity which yields credible results.

## 2. Motivation

Although one might argue that GSM is surpassed by technologies like UMTS and LTE, we focus it in our research for the following reason: the GSM-based mobile networks are still widespread, with large user base. Moreover, in the case of multi-network mobile terminals, the pricing plans make GSM a very popular choice for voice calls and short text messaging. It is therefore unlikely that mobile operators will simply abandon the proven technology and switch to more advanced networks; instead, GSM will continue to coexist with UMTS and LTE, at least in the near future.

## 3. Model and assumptions

Power consumption of a BTS consists of a traffic-independent and traffic-dependent part. The first part corresponds to bias consumption of the electronic components, fan/cooler consumption, etc. and although being significant, currently is beyond the scope of our interest. In this paper, we focus on the second part, i.e. traffic-dependent consumption.

Let us consider a base station in GSM network. This base station is located in the centre of a circular cell of radius  $R$ . The mobile subscribers (MS), in general case, are arbitrarily distributed within the cell. The base station has  $N$  traffic channels and adjusts the output power in each of them,  $w$ , according to the distance between the MS and the BTS,  $d$ :

$$w = a \cdot d^\gamma, \quad (1)$$

\* Corresponding author at: University of Belgrade, School of Electrical Engineering (ETF), Belgrade, Serbia.

E-mail addresses: [pescchia@iritel.com](mailto:pescchia@iritel.com) (P. Jovanović), [milan@etf.rs](mailto:milan@etf.rs) (M. Bjelica).

where  $a$  describes path loss and  $\gamma \in [2, 5]$  [8].

A distinct channel should be allocated to each connection. If the traffic offered to the group of channels is  $A$ , then the served traffic is  $Y = (1 - B)A$ , with  $B$  being the call loss probability due to lack of idle channels. Let us note that the number of the MSs is typically much greater than  $N$ , so that the traffic process in the group of traffic channels can be modelled as the Erlang loss model [9]. We further assume random and independent generation of events, i.e. voice calls and data transmissions. Under these assumptions, the probability that  $i$  channels are busy,  $0 \leq i \leq N$ , is

$$Pr\{i, A, N\} \equiv ERL(i, A, N) = \frac{A^i / i!}{\sum_{j=0}^N A^j / j!} \quad (2)$$

It is not difficult to see that  $B = Pr\{N, A, N\} = ERL(N, A, N)$ , which is Erlang B formula.

Total base station power is the sum of powers in all active channels. Besides the distance of each MS from the BTS and the path loss, total BTS output power is also dependent on traffic. Total output power,  $W_{(n)}$ , when  $n$  channels are busy,  $n = 1, 2, \dots, N$  is equal to the sum of random values of powers of all active channels,  $W_{(n)} = \sum_{i=1}^n w_i$ . The PDF of  $W_{(n)}$  can now be calculated from the  $n$ -fold convolution:

$$f_{W_{(n)}}(x) = f_{w_1}(x) * f_{w_2}(x) * \dots * f_{w_n}(x), \quad (3)$$

where  $f_{w_i}(x)$  is the PDF of output power of  $i$ th busy traffic channel,  $i = 1, 2, \dots, n$ .

The CDF of base station output power when  $n$  channels are busy is now

$$F_{W_{(n)}}(x) = \int_0^x f_{W_{(n)}}(u) du. \quad (4)$$

The average CDF of total output power of base station with  $N$  traffic channels,  $F_W(N, x)$ , designates the probability that the total output power,  $W$ , is less or equal to  $x$ . By using Eq. (2), this can be expressed in the following form:

$$F_W(N, x) = \sum_{i=0}^N Pr\{i, A, N\} F_{W_{(i)}}(x) = ERL(0, A, N) + \sum_{i=1}^N ERL(i, A, N) F_{W_{(i)}}. \quad (5)$$

As each term in the sum involves calculation of an  $i$ -fold convolution, the calculation of base station output power CDF is computationally challenging; this is why we choose to obtain this result by simulation.

#### 4. Our proposal

As shown in Fig. 1, simulation framework that we propose combines traffic generation and random user placement within the area of the observed BTS.

Two independent random numbers  $X$  and  $Y$  are generated by Mersenne Twister generator [10]. The GSM traffic is simulated according to the value of  $X$ ; let us remember that our model accounts for total traffic, as regarding the BTS energy consumption it is irrelevant if this traffic was generated by voice calls or data services. A new call/data service is generated if  $0 < X < A$ , while the previously busy channel  $k$  is released when  $k - 1 < X < k$ ; between these two events, channel  $k$  is occupied and consumes the power  $w_k$ . Random distance between a MS and the BTS (and, subsequently, channel power for a new connection) is either read from

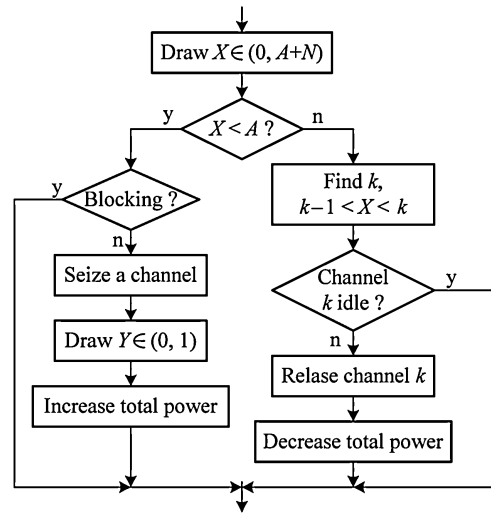


Fig. 1. Simulation main routine.

a preloaded file, or determined by the value of  $Y$ , using probability transformation; in either case, the corresponding output power is calculated by Eq. (1). The value of this new connection power is added to the total BTS power, while the power of a terminated connection is subtracted from the total BTS power.

Each run of the routine from Fig. 1 gives one BTS power value; thus, a series of simulation runs produces a sample from the unknown power distribution, which is then used to estimate the empirical CDF.

Simulation source code is available at the website <http://telekomunikacije.etf.rs/research/een>.

#### 5. Numerical example

We illustrate the performance of our framework by considering the case where mobile stations are uniformly distributed in a cell. The user distance  $d$  now has the PDF of a form  $f_d(x) = 2x/R^2$ .

Let us adopt the following numerical values:  $N = 5$ ,  $R = 8$  km,  $a = 0.625$  watt/(km<sup>2</sup>),  $\gamma = 2$ ,  $W_{max} = 40$  watt (class 4 BTS),  $A = 3$  E. It can be shown by probability transformation that  $f_{w_i}(x) = 1/(aR^2)$ , i.e. the powers of active channels are uniformly distributed. The total BTS power CDF now can be expressed in a closed form, by Irwin–Hall distribution [11,12]; this analytical solution is drawn in Fig. 2 with dashed line. It is worth noting that the offset for

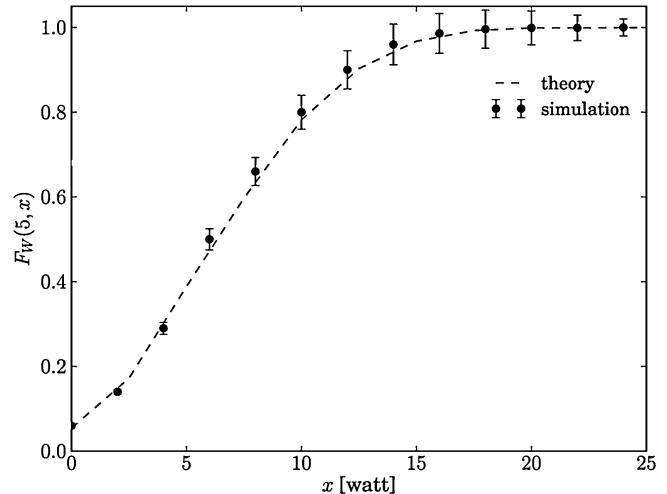


Fig. 2. Theoretical and simulation results, with 95% confidence interval borders.

$x=0$  is due to the first term in Eq. (5), which reflects the non-zero probability of all channels being idle even during the busy hour.

Simulation results are depicted with circles in Fig. 2. The transformation used to generate random values of user distance from uniformly distributed random variable  $Y \in (0, 1)$  was  $d = R\sqrt{Y}$ . About 2000 events per channel were simulated for the CDF relative error (estimated by batch means) not to be greater than 5% with confidence level of 95% [13]. The vertical bars in Fig. 2 mark the borders of these intervals.

As it can be seen, the difference between the theoretical and simulation results is negligible. Based on this observation, we expect that the proposed simulation method would produce credible results also in those cases where mobile stations are arbitrarily distributed in a cell, when analytical solution might not exist.

## 6. Conclusion

In this letter, we presented simulation method for fast and accurate estimation of the CDF of BTS output power. We illustrated its application in a rare case with analytical solution and showed that the results matched. This could help develop efficient strategies for energy savings in mobile networks, which would consider both traffic conditions and user distribution within the cells.

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