# Probabilistic Load Flow Computation Using the Method of Combined Cumulants and Gram-Charlier Expansion

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***Abstract*—Open access transmission has created a deregulated power market and brought new challenges to system planning. This paper proposes a new method to compute a probabilistic load flow in extensive power systems for the purpose of using it as a quick screening tool to determine the major investment on improving transmission system inadequacy. This innovative method combines the concept of Cumulants and Gram-Charlier expansion theory to obtain probabilistic distribution functions of transmission line flows. It has significantly reduced the computational time while maintaining a high degree of accuracy. This enables probabilistic analysis of power flow problems to be treated objectively and allows quantitative assessment of system reliability.**

***Index Terms*—Cumulants, Gram-Charlier expansion, open access transmission and deregulated power market, power system, probabilistic load flow, probability distribution function, transmission planning.**

## I. Introduction

RECENT changes in the U.S. electric power industry has caused dramatic increases in the use of the transmission system. Transmission planning has become an increasingly important topic due to the fact that federally mandated open access transmission has created a larger and more competitive bulk power market place. This competitive market place brings great challenges to the system planning field [1].

Computation of power flows in the electric power system is one of the major tasks facing power system planners. Deterministic load flow study requires specific values for loads, generation inputs and network conditions. In an open access environment, this information is not as certain as it used to be when the power system was a vertically integrated system. In system planning, it is desirable to assess bus voltages and line flows for a range of load and generation conditions. To carry out conventional load flow computations for every possible or probable combination of bus loads and generating unit outages is completely impractical because of the extremely large computational effort required. Performing probabilistic load flow studies gives system planning engineers a better feel of future system conditions and will provide more confidence in making judgments concerning investment.

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Application of probabilistic analysis to the power system load flow study was first proposed by Borkowa in 1974 [2]. Since then, there are two ways of adopting probabilistic approach to study load flow problems: Stochastic Load Flow (SLF) [3]–[6] and Probabilistic Load Flow (PLF) [7]–[12]. In SLF study, the load and generation at an instant time $t_i$ are treated as random variables. SLF investigates the impact of this uncertainty to the output of conventional power flow at each instant of time. Consequently, SLF deals with short time uncertainties and is useful for system operation. Since this paper investigates the effects of load and generation uncertainty over a long-term period on the adequacy of transmission network, we adopt Probabilistic Load Flow approach for planning study purpose.

In PLF study, many researchers have addressed the same question: what is the most efficient and sufficiently accurate method of obtaining the probabilistic density function (PDF) and/or statistical moments of the state vectors and line flows?

Monte Carlo simulation is one of the methods to obtain the PDF of the state vector and line flows. This technique involves a repeated selection of the value of input variables from their probability distribution and then for selected value of these input variables (active power and reactive power) obtaining the values of the state vector exactly in the same way as deterministic analysis. The final step is to obtain the probabilistic description of the state vector from the results of the repeated simulations. To obtain meaningful results, thousands of Monte Carlo simulations are usually required. The computation burden makes this process unattractive.

Previous researchers recognized that, although Monte Carlo Simulation method is able to provide accurate results, the computation is really time consuming, therefore is not suitable to handle practical systems. Most of researchers only use it for comparison purpose. The conventional convolution technique is another method to obtain the PDF of line flows and has been adopted by [7]–[13]. By applying linearization methods, the state vector and line power flows are represented as a linear combination of input variables. Therefore, assuming independence of all the variables, a convolution technique can be applied to obtain the PDF's of the desired variables. References [7], [8] extended probabilistic analysis techniques to handle ac load flow by modifying the linearization formulation. The convolution method was not changed.

The major problem in the conventional convolution method is to compute the equivalent discrete function since a function represented by $r$ impulses convolved with another represented by $s$ impulses will have $r$ times $s$ impulses. Reference [11] clearly stated that, even to obtain the PDF of a single line flow, the final number of discontinuous points could be extremely large when the number of discontinuous curve to be convoluted are large or each curve is represented by a large number of points. This process requires a large amount of storage and time especially when there are many functions involved due to large systems. Reference [11] proposed to calculate the expected value and standard deviation of the power flows initially in order to determine the optimum step size and appropriate number of points for injected powers at each node. Then, the complete density function of the power flow is computed. Reference [12] used a discrete frequency domain convolution techniques by applying Fast Fourier Transforms to reduce the computation time. Although, [11], [12] realized the problems of convolution method and tried to improve its efficiency. However, they are constrained by the convolution technique and can not essentially solve the problem. In addition, in order to obtain the Cumulative Distribution Function (CDF) of each line flow, integration of PDF over the range of line flow needs to be computed.

This paper proposes a new method to compute the PDF and CDF of line flows. This method combines the concept of Cumulants and Gram-Charlier expansion theory to compute the PDF and CDF of line flows in a systematic way. Compared with other methods used by previous researchers [7]–[12], the method proposed in this paper avoids complex convolution calculation and replaces them with simple arithmetic process due to unique properties of Cumulants. Moreover, this new method is able to obtain the PDF and CDF of line flows with one run. This method significantly reduces the storage since low order Gram-Charlier expansion is able to achieve enough accuracy to approximate PDF and CDF of line flows. Therefore, the new method is able to handle large practical systems. Study results have shown that the new method can calculate the probability distribution accurately with much less computation effort. Consequently, system planning engineers are able to use it as a quick screening tool to determine the major investment on improving system adequacy.

## II. Probabilistic Load Flow Formulation

The load flow study computes the steady-state solution of the power system. In a deterministic load flow study, the known quantities are the injected active powers $(P_i)$ at all busbars (where P and Q or P and V are known) except the slack bus, the injected reactive powers $(Q_i)$ at all load busbars (where P and Q are known) and the voltage magnitude $(V_i)$ at all generator busbars (where P and V are known).

$$P_i = g_i(\delta_1, \delta_2 \ldots \delta_n, V_1, V_2, \ldots V_n) \tag{1}$$

$$Q_i = h_i(\delta_1, \delta_2, \ldots \delta_n, V_1, V_2 \ldots V_n) \tag{2}$$

where $\mathrm{i} = 1, 2, \ldots, \mathrm{n}$.

Since (1) and (2) are nonlinear in terms of the voltage magnitudes and angles (considered as state variables in this paper), the numerical solution must be based on an iterative method.

In probabilistic load flow (PLF) studies, the input variables $P_i$ and $Q_i$ are defined by probabilistic density functions. It is preferable to apply linear approximation to (1) and (2) so that the state variables could be solved as a linear combination of input variables. This, in turn, will not only allow us to solve load flow equations through fast direct methods but will also permit application of convolution techniques to arrive at the probabilistic description of the variables of interest.

In long term system planning, the main problem is to locate transmission and generation facilities in the appropriate places and in time to satisfy the customer's real power demand. Voltage problem is usually local problem when the system is adequate. Based on these considerations, this paper adopted dc load flow in the formulation of PLF problems. Since this program is used as a quick screening tool to determine the major investment on improving transmission system inadequacy, contingency analysis will be next step in system planning. Probabilistic contingency analysis will be our future research interest.

In addition, this paper mainly focuses on the methodology of computing probabilistic distribution functions of line flows. Although there are several other linearization methods applicable to PLF study, the methodology proposed in this paper is not restricted by the particular linearization formulation of PLF. It can be easily modified to study a.c. load flow.

Assume $V_i = V_k = 1$ p.u., $G_{ik} = 0$ and $\mathrm{Sin}\delta_{ik} = \delta_{ik}$

$$P_i = \sum \frac{\delta_{ik}}{X_{ik}} \tag{3}$$

where $X_{ik}$ is the reactance of the line joining bus i and k. The above equation can be formed in terms of a matrix expression

$$\mathbf{P} = \mathbf{Y}\boldsymbol{\delta} \tag{4}$$

where $Y_{ik} = (1/X_{ik}), Y_{ii} = \sum_{i \neq k}(1/X_{ik})$ in which the slack bus row and column are deleted. This equation is known as a dc form of the load flow problem. Therefore,

$$\boldsymbol{\delta} = \mathbf{Y}^{-1}\mathbf{P} = \mathbf{ZP} \tag{5}$$

where $\mathbf{Y}$ is the admittance matrix and $\mathbf{Z}$ is the impedance matrix.

The power flow in the line joining the buses i and k becomes

$$P_{ik} = \frac{\delta_i - \delta_k}{X_{ik}} \tag{6}$$

The line flows can be represented as a function of voltage angles at busbars:

$$\mathbf{P_{Line}} = \mathbf{T}\boldsymbol{\delta} \tag{7}$$

Replacing $\boldsymbol{\delta}$ using (5), the line flows can be expressed as:

$$\mathbf{P_{Line}} = \mathbf{TZP} = \mathbf{HP} \tag{8}$$

The matrix H contains network distribution factors. The notation "(ik)j" represents the element of H in the row corresponding to line ik and in the column corresponding to bus j. An element $H_{(ik)j}$ of H represents the amount of real power flowing in line ik as a result of injection of 1 MW at bus j (with 1 MW absorbed by the slack bus). The distribution factors are obtained by

$$H_{(ik)j} = \frac{Z_{ij} - Z_{kj}}{X_{ik}} \tag{9}$$

in which, if node j is the slack bus, $H_{(ik)j} = 0$.

## III. Theoretical Background

### A. Definition of Moments

If, for a positive integer $v$, the function $X^v$ is integrable with respect to F(x) over $(-\infty, +\infty)$, the integral

$$\alpha_v = E(\xi^v) = \int_{-\infty}^{+\infty} x^v dF(x) \tag{10}$$

is called the moment of order $v$ or the $v$th moment of the distribution [14].

The moments about the mean, $m$, are often called the central moments

$$\beta_v = E[(\xi - m)^v] = \int_{-\infty}^{+\infty} (x-m)^v \, dF(x) \tag{11}$$

In the case of a linear function $\eta = a\alpha + b$, the $v$th moment of the variable $\eta$ is given by the expression

$$\alpha' = E[(a\xi+b)^v] = a^v \alpha_v + \binom{v}{1} a^{v-1} b \alpha_{v-1} + \cdots + b^v \tag{12}$$

### B. Definition of Cumulants

The mean value of the particular function $e^{it\xi}$ will be written

$$\varphi(t) = E(e^{it\xi}) = \int_{-\infty}^{+\infty} e^{itx} \, dF(x) \tag{13}$$

This is a function of the real variable t, and will be called the characteristic function of the variable $\xi$ [14].

If the $k$ th moment of the distribution exists, the characteristic function can be developed in MacLaurin's series for small values of t:

$$\varphi(t) = 1 + \sum_1^k \frac{\alpha_v}{v!}(it)^v + o(t^k) \tag{14}$$

$$\log \varphi(t) = \sum_1^k \frac{\gamma_v}{v!}(it)^v + o(t^k) \tag{15}$$

The coefficients $\gamma_v$ were introduced by Thiele and are called the semi-invariants or cumulants of the distribution.

The cumulants $\gamma'_v$ of a linear function $\eta = a\xi + b$ are obtained from the development:

$$\log[e^{bit}\varphi(at)] = \sum_1^k \frac{\gamma'_v}{v!}(it)^v + o(t^k) \tag{16}$$

Therefore,

$$\gamma'_1 = a\gamma_1 + b \quad \text{and} \quad \gamma'_v = a^v \gamma_v \quad \text{for } v > 1 \tag{17}$$

### C. Relationship Between Moments and Cumulants

The relationship between the moments and the cumulants can be deduced by substituting $\varphi(t)$ in (14) to (15),

$$\log\left(1 + \sum_1^k \frac{\alpha_v}{v!}(it)^v\right) = \sum_1^k \frac{\gamma_v}{v!}(it)^v + o(t^k) \tag{18}$$

It is seen that $\gamma_n$ is a polynomial in $\alpha_1, \ldots, \alpha_n$ and conversely $\alpha_n$ is a polynomial in $\gamma_1, \ldots, \gamma_n$. In particular,

$$\begin{aligned}
\gamma_1 &= \alpha_1 = m \\
\gamma_2 &= \alpha_2 - \alpha_1^2 \\
\gamma_3 &= \alpha_3 - 3\alpha_1\alpha_2 + 2\alpha_1^3 \\
\gamma_4 &= \alpha_4 - 3\alpha_2^2 - 4\alpha_1\alpha_3 + 12\alpha_1^2\alpha_2 - 6\alpha_1^4 \\
&\cdots\cdots
\end{aligned} \tag{19}$$

where $m$ denotes the mean value.

In terms of the central moments $\beta_v$, the expressions of the $\gamma_v$ become

$$\begin{aligned}
\gamma_1 &= m \\
\gamma_2 &= \beta_2 = \sigma^2 \\
\gamma_3 &= \beta_3 \\
\gamma_4 &= \beta_4 - 3\alpha_2^2 \\
\gamma_5 &= \beta_5 - 10\beta_2\beta_3 \\
\gamma_6 &= \beta_6 - 15\beta_2\beta_4 - 10\beta_3^2 + 30\beta_2^3 \\
&\cdots\cdots
\end{aligned} \tag{20}$$

where $\sigma$ denotes standard deviation and conversely

$$\begin{aligned}
\beta_1 &= 0 \\
\beta_2 &= \gamma_2 = \sigma^2 \\
\beta_3 &= \gamma_3 \\
\beta_4 &= \gamma_4 + 3\gamma_2^2 \\
\beta_5 &= \gamma_5 + 10\gamma_2\gamma_3 \\
\beta_6 &= \gamma_6 + 15\gamma_2\gamma_4 + 10\gamma_3^2 + 15\gamma_2^3 \\
&\cdots\cdots
\end{aligned} \tag{21}$$

### D. Property of Cumulants

Let $\xi$ and $\eta$ be independent random variables with known cumulative function $F_1$ and $F_2$. The cumulative function $F(x)$ of the sum of two independent variables is given by

$$F(x) = \int_{-\infty}^{+\infty} F_1(x-z) \, dF_2(z) = \int_{-\infty}^{+\infty} F_2(x-z) \, dF_1(z) \tag{22}$$

$$F(x) = F_1(x) * F_2(x) \tag{23}$$

For the sum $\xi_1 + \xi_2 + \cdots + \xi_n$ of n independent variables, the cumulative function

$$F = F_1 * F_2 * \cdots * F_n \tag{24}$$

Let $\varphi_1(t), \varphi_2(t)$, and $\varphi(t)$ denote the characteristic function of $\xi, \eta$, and $\xi + \eta$ respectively.

$$\varphi(t) = E\left[e^{it(\xi+\eta)}\right] = E[e^{it\xi}] * E[e^{it\eta}] = \varphi_1(t) * \varphi_2(t) \tag{25}$$

If $\xi_1, \xi_2, \ldots, \xi_n$ are independent variables with the characteristic function $\varphi_1(t), \varphi_2(t) \ldots \varphi_n(t)$, the characteristic function $\varphi(t)$ of the sum $\xi_1 + \cdots + \xi_n$ is thus given by [9]:

$$\varphi(t) = \varphi_1(t) * \varphi_2(t) * \cdots * \varphi_n(t) \tag{26}$$

The multiplication theorem for characteristic functions gives

$$\log \varphi(t) = \log \varphi_1(t) + \log \varphi_2(t) + \cdots + \log \varphi_n(t) \tag{27}$$

Therefore,

$$\gamma_v = \gamma_v^{(1)} + \gamma_v^{(2)} + \cdots + \gamma_v^{(n)} \tag{28}$$

According to (20), it can be observed that

$$\begin{aligned} m &= m_1 + m_2 + \cdots + m_n \\ \sigma^2 &= \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2 \end{aligned} \tag{29}$$

### *E. Gram-Charlier Expansion*

Consider any random variable $\xi$ with a distribution of a continuous type and denote the mean value as $m$ and the standard deviation as $\sigma$. For the standardized variable $(\xi - m)/(\sigma)$, its cumulative function and density function are denoted as $F(x)$ and $f(x)$ respectively.

According to Gram-Charlier expansion, the cumulative and the density functions can be written as [14]:

$$F(x) = \Phi(x) + \frac{c_1}{1!}\Phi'(x) + \frac{c_2}{2!}\Phi''(x) + \frac{c_3}{3!}\Phi^{(3)}(x) + \cdots \tag{30}$$

$$f(x) = \varphi(x) + \frac{c_1}{1!}\varphi'(x) + \frac{c_2}{2!}\varphi''(x) + \frac{c_3}{3!}\varphi^{(3)}(x) + \cdots \tag{31}$$

where $\Phi(x)$ and $\varphi(x)$ represent the cumulative distribution function (CDF) and probabilistic density function (PDF) of normal distribution with $m = 0$ and $\sigma = 1; c_v$ are constant coefficients.

$$\begin{aligned} c_0 &= 1 \\ c_1 &= c_2 = 0 \\ c_3 &= -\frac{\beta_3}{\sigma^3} \\ c_4 &= \frac{\beta_4}{\sigma^4} - 3 \\ c_5 &= -\frac{\beta_5}{\sigma^5} + 10\frac{\beta_3}{\sigma_3} \\ c_6 &= \frac{\beta_6}{\sigma^6} - 15\frac{\beta_4}{\sigma^4} + 30 \\ &\cdots\cdots\cdots\cdot \end{aligned} \tag{32}$$

In this paper, we tested and compared Gram-Charlier series expansions from order 3 to 9 in order to identify which order(s) yield the highest accuracy for approximating the PDF of line flows using the combined Cumulants and Gram-Charlier expansion method.

## IV. Computation Procedure

Based on the above theories, the procedure of calculating PDF of line flows is summarized as follows,

1) Given the probabilistic description of generation and load, calculate the moments of injected active power according to (10).
2) Compute the cumulants of injected power according to the relationship between cumulants and moments expressed using (19).
3) Compute the cumulants of line flow according to the following equations:
   For the $i$th line flow,
   $$P_{\text{Line}\, i} = h_{i1}P_1 + h_{i2}P_2 + \cdots + h_{in}P_n.$$
   For the cumulants related with the $i$th line flow,
   $$\gamma_v = h_{i1}^v \gamma_v^{(1)} + h_{i2}^v \gamma_v^{(2)} + \cdots + h_{in}^v \gamma_v^{(n)}$$
   where $v = 1, 2, \ldots, 9$.
4) Compute central moments of each line based on (21).
5) Calculate the Gram-Charlier expansion coefficients using (32).
6) The cumulative distribution function and probabilistic density function of line flows can be obtained using (30) and (31) respectively.

## V. Case Study and Result Comparison

The method described in the preceding sections is applied to a WSCC (Western Systems Coordinating Council) test system, which consists of 179 buses and 263 lines. The network diagram is shown in Fig. 1, [15].

In order to demonstrate accuracy and efficiency of this method, it is compared with Monte Carlo simulation. Monte Carlo simulation repeats the process of deterministic load flow computation using, in each simulation, a particular set of values of the random variables generated in accordance with the corresponding probability distributions. Two simulations have been undertaken for different purposes. In order to compare calculation efficiency with the new method proposed in this paper, one simulation sets the termination criteria as the maximum difference of the mean values to all line flows less than 1 MW consecutively for three successive trials. Based on this criterion, Monte Carlo simulation converges at 753 trials. With consideration of accuracy comparison, another simulation sets the number of trials at 5000.

Table I lists the calculation time using each method. It can be seen that, depending on the order of Gram-Charlier expansion, the new method proposed in this paper is about 20–30 times faster than Monte Carlo Simulation with 753 iterations.

Fig. 2 shows the cumulative distribution curves of line flow LOS BANOS to MIDWAY, which is one of the two lines in Path 15 (the well-known bottleneck between southern and northern California). In order to display the graph clearly, we only show 3rd, 6th, and 9th orders of combined Cumulants and Gram-Charlier Expansion method. It can be seen that, with comparison of Monte Carlo Simulation, combined Cumulants and Gram-Charlier Expansion method can precisely calculate the CDF of line flows.

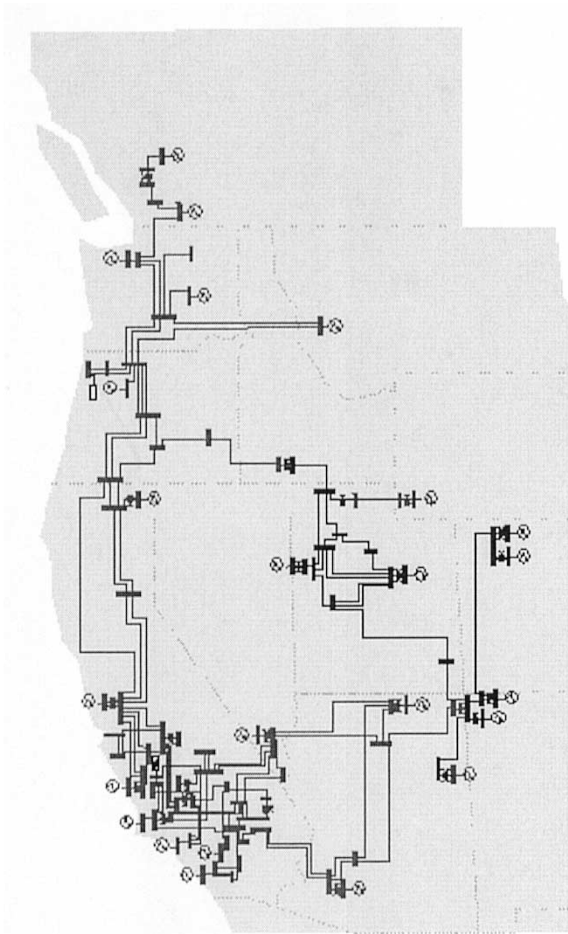

Fig. 1. WSCC Test System Diagram.

TABLE I
COMPUTATION TIME COMPARISON

| Methods | Computation Time (seconds) |
|---|---|
| Monte Carlo (753 iterations) | 203.44 |
| Monte Carlo (5000 iterations) | 941.59 |
| Cumulants & Gram-Charlier (3rd) | 8.84 |
| Cumulants & Gram-Charlier (4th) | 9.77 |
| Cumulants & Gram-Charlier (5th) | 10.44 |
| Cumulants & Gram-Charlier (6th) | 11.48 |
| Cumulants & Gram-Charlier (7th) | 12.08 |
| Cumulants & Gram-Charlier (8th) | 12.96 |
| Cumulants & Gram-Charlier (9th) | 13.84 |

In order to demonstrate the accuracy of this method, Average Root Mean Square (ARMS) error is computed using the Monte Carlo 5000 iteration results as reference. ARMS is defined as:

$$\mathrm{ARMS} = \frac{\sqrt{\sum_{i=1}^{N} (\mathrm{CG}_i - \mathrm{MC}_i)^2}}{N} \tag{33}$$

where

$\mathrm{CG}_i$ is the ith point's value on the cumulative distribution curve calculated using the method of combined Cumulants and Gram-Charlier expansion;

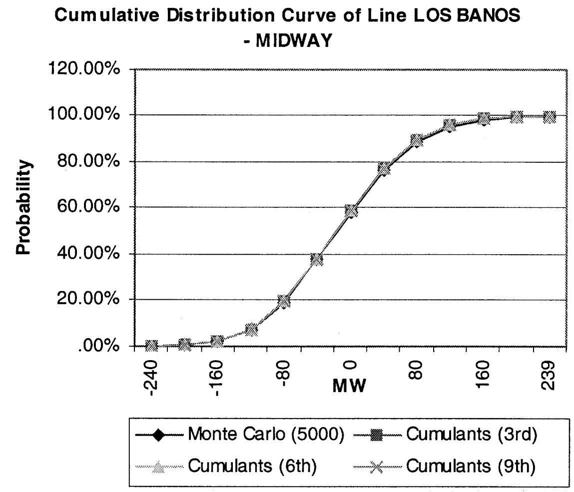


Fig. 2. Cumulative Distribution Curves of Line LOS BANOS-MIDWAY.

TABLE II
ARMS OF LINE FLOW FROM LOS BANOS TO MIDWAY

| Methods | ARMS |
|---|---|
| Cumulants & Gram-Charlier (3rd) | 0.099% |
| Cumulants & Gram-Charlier (4th) | 0.104% |
| Cumulants & Gram-Charlier (5th) | 0.104% |
| Cumulants & Gram-Charlier (6th) | 0.102% |
| Cumulants & Gram-Charlier (7th) | 0.102% |
| Cumulants & Gram-Charlier (8th) | 0.102% |
| Cumulants & Gram-Charlier (9th) | 0.102% |

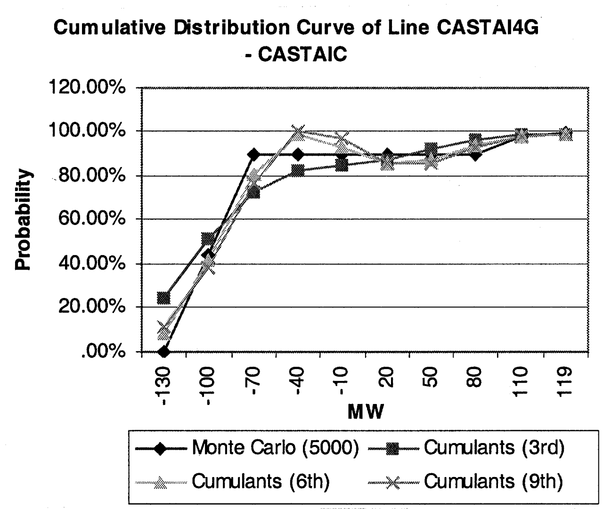


Fig. 3. Cumulative Distribution Curves of Line CASTAI4G-CASTAIC.

$\mathrm{MC}_i$ is the ith point's value on the cumulative distribution curve calculated using the Monte Carlo method;

$N$ represents the number of points.

The relevant ARMS results of power flow on the line LOS BANOS to MIDWAY are shown in Table II.

Fig. 3 shows the cumulative distribution curves of line flow from CASTAI4G to CASTAIC calculated using different methods. In our simulation, we assume that there is only one generator located at CASTI4G with 200 MW output capability

TABLE III
ARMS OF LINE FLOW FROM CASTI4G TO CASTAIC

| Methods | ARMS |
|---|---|
| Cumulants & Gram-Charlier (3rd) | 3.259% |
| Cumulants & Gram-Charlier (4th) | 1.964% |
| Cumulants & Gram-Charlier (5th) | 1.824% |
| Cumulants & Gram-Charlier (6th) | 1.693% |
| Cumulants & Gram-Charlier (7th) | 1.809% |
| Cumulants & Gram-Charlier (8th) | 1.812% |
| Cumulants & Gram-Charlier (9th) | 2.334% |

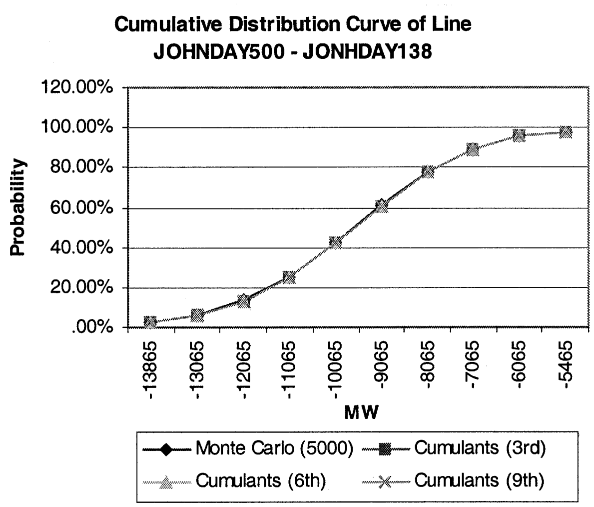


Fig. 4. Cumulative Distribution Curves of Line JOHNDAY500-JOHNDAY138.

in total. Line CASTAI4G-CASTAIC is the sole line connecting that generator with the whole system.

Table III shows the ARMS of power flow on line CASTI4G-CASTAIC. We can observe that the method proposed in this paper can approximate Cumulative Distribution Function (CDF) of line flow from LOS BANOS to MIDWAY more accurately than the ones of line CASTAI4G-CASTAIC. This is due to the fact that line CASTAI4G-CASTAIC is located near the generator at CASTI4G and therefore its CDF is dominated by that generator's probabilistic distribution pattern. Since we usually use binomial distribution modeling the PDF's of generators, the PDF of the generator at CASTI4G is a discrete function. Because the Gram-Charlier expansion method is derived from Central Limit theory, it provides better approximation when the number of independent variables tends to infinity and the probability distribution function of each variable is continuous rather than discrete. This leads to the better approximation of the CDF of line LOS BANOS-MIDWAY.

Fig. 4 shows the cumulative distribution curves of line JOHNDAY500-JOHNDAY138 where 10 generators are located at bus JOHNDAY138. The ARMS of this line flow is shown in Table IV. Results shown in this figure and this table have proved that the accuracy of Gram-Charlier expansion method improves as the number of independent variables increases.

TABLE IV
ARMS OF LINE FLOW FROM JOHNDAY500 TO JOHNDAY138

| Methods | ARMS |
|---|---|
| Cumulants & Gram-Charlier (3rd) | 0.079% |
| Cumulants & Gram-Charlier (4th) | 0.078% |
| Cumulants & Gram-Charlier (5th) | 0.078% |
| Cumulants & Gram-Charlier (6th) | 0.081% |
| Cumulants & Gram-Charlier (7th) | 0.082% |
| Cumulants & Gram-Charlier (8th) | 0.082% |
| Cumulants & Gram-Charlier (9th) | 0.081% |

TABLE V
MW VALUE OF 10% AND 90% CONFIDENCE LEVEL OF LINE FLOW FROM LOS BANOS TO MIDWAY

| Methods | MW (10%) | MW (90%) |
|---|---|---|
| Monte Carlo (753 iterations) | -106 | 77 |
| Monte Carlo (5000 iterations) | -109 | 87 |
| Cumulants & Gram-Charlier (3rd) | -109 | 84 |
| Cumulants & Gram-Charlier (4th) | -109 | 84 |
| Cumulants & Gram-Charlier (5th) | -109 | 84 |
| Cumulants & Gram-Charlier (6th) | -109 | 84 |
| Cumulants & Gram-Charlier (7th) | -109 | 84 |
| Cumulants & Gram-Charlier (8th) | -109 | 84 |
| Cumulants & Gram-Charlier (9th) | -109 | 84 |

In planning, we are interested in the 10% and 90% confidence levels that the line flow will not exceed because this would indicate roughly the desired capacity of the path. Consequently, accurate estimation of MW value at 10% and 90% confidence levels has important meaning to system planning engineers. For example, the MW value at 10% and 90% confidence levels of line LOS BANOS-MIDWAY is calculated using different methods and displayed in Table V.

In conclusion, computation results have proved that the combined Cumulants and Gram-Charlier expansion method enables system planners to estimate 10% and 90% confidence levels accurately enough to determine the security level and the maximum power rating of the line with confidence.

## VI. CONCLUSION

Open access of transmission systems brings many uncertainties into system planning. Compared with deterministic load flow studies, probabilistic load flow studies give system planning engineers the ability to appraise the system in a much wider sense and enables quantitative assessment to be made in reliability studies.

The combined Cumulants and Gram-Charlier expansion method proposed in this paper provides a new way of computing probabilistic distribution functions of line flows for reliability evaluations in system expansion planning. With the comparison of Monte Carlo 5000 iteration results, the new method is able to accurately approximate the cumulative distribution function of transmission line flows. More importantly, this method is a significant improvement in reducing storage, therefore is able to handle extensive system. Results have shown that it is 20–30 times faster than Monte Carlo simulation. Theoretically, the method will not increase computation burden dramatically with increase of system size. Future research will consider applying this method to larger size systems.

Based on our observation, order 6 of combined Cumulants and Gram-Charlier method gives better approximation to the entire CDF curve. However, order 7 provides better estimation results at the tail ends (e.g., 10% and 90% confidence levels) of the distribution. Since it is more important for planning engineers to estimate the reasonable upper range of line flows, we recommend that the combined Cumulants and Gram-Charlier expansion method be applied with 7th order.

In summary, this method enables system planners to obtain the possible ranges of power flow and the probability of occurrence quickly enough to meet planning requirements in the deregulated power markets. With this information, the probability of any line being overloaded can be easily computed. Therefore, objective decisions can be made with regard to reinforcement plans.

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