Appropriate application of the Standardized Precipitation Index in arid locations and dry seasons

1. INTRODUCTION

The Standardized Precipitation Index (SPI) is widely accepted and used throughout the world in both research and operational modes because it is **normalize**d to a location and is normalized in time. This standardization allows the SPI to determine the rarity of a current drought event, as well as the probability of the precipitation necessary to end the current drought . It also allows the SPI to be computed at any location and at any number of time scales, depending upon the impacts of interest to the user. On the basis of an analysis of stations across Colorado, McKee et al. (1993) determined that the SPI is in mild drought 24% of the time, in moderate drought 9.2% of the time, in severe drought 4.4% of the time, and in extreme drought 2.3% of the time. These percentages are expected from a **normal distribution** of the SPI.

The first step in the SPI calculation is to determine the **probability density function** (PDF), which describes the long-term **observed precipitation**. Next, the **cumulative probability** of the observed precipitation is computed. The **inverse normal (Gaussian) function** is then applied to the cumulative probability, resulting in the SPI. This procedure is an **equiprobability transformation**. The essential feature of the equiprobability transformation is that the probability of being less than a given value of the obtained cumulative probability should be the same as the probability of being less than the corresponding value of the normal distribution.

Statistically, precipitation is not normally distributed. Since it is zero-bounded, and since non-precipitation days outnumber precipitation days in many cases, precipitation distributions are positively **skew**ed. Furthermore, a short-time scale will increase the precipitation **variability**, leading to a highly skewed distribution. When the probabilities of receiving given amounts of precipitation were estimated by **fitting** the **gamma distribution**, Barger et al. (1959(a)) noted 'These estimated probabilities are subject to error, of course. This error is greater for 1-week amounts than 2-week or 3-week totals and is greater in drier areas and seasons. Because the gamma distribution **fit**s individual storm rainfall better than the frequencies of rainfall totals in short periods of time, a high frequency of no-rain cases leads to poor fits of the observed frequencies.' Consequently, the parameters used in the gamma distribution to fit 1-, 2-, and 3-week precipitation totals were estimated by excluding zero or **trace** precipitations.

Guttman (1999) pointed out that different SPI values will be obtained if different probability distributions are used to describe the observed precipitation. He concluded that the **Pearson Type III distribution** (PE3) is the best universal model for the SPI calculation after comparing several models. In addition, **lognormal**, **extreme value**, and **exponential** models have been widely applied in the simulations of precipitation distribution.

In this study, the 2-parameter gamma PDF was chosen to fit the **frequency distribution** of precipitation for the SPI calculation because this distribution is currently used by the National Drought Mitigation Center (NDMC, drought.unl.edu), Western Regional Climate Center (WRCC, wrcc.dri.edu), and the National Agricultural Decision Support System (NADSS, nadss.unl.edu) and because the SPI computing software package based on the 2-parameter gamma model has been distributed to about 60 countries. In addition, the C++ code, developed by the Department of Computer Science and Engineering at the University of Nebraska-Lincoln to compute weekly SPI values, which will be used for further analyses in this study, is based on the same gamma model.

Theoretically, the SPI can be computed from as short as a 1-week time scale. Edwards and McKee (1997) indicated that, conceptually, although the SPI has a standard normal distribution with an expected value of zero and a **variance** of one, this is not always the case for the SPI at short time scales because of the skewed precipitation distribution. For dry climates or those with a distinct dry season where zero values are common, there will be too many zero precipitation values in particular seasons. As a result, the calculated SPI values at short time scales may not be normally distributed because of the highly skewed precipitation distribution and because of the limitation of the **gamma-fitted distribution** referred to by Bargeret al. (1959(a)). The biased SPI values were also mentioned by Lloyd-Hughes and Saunders (2002) and Sonmez et al. (2005). Under these circumstances, knowing how to apply and interpret the SPI appropriately is critical.

Thus, the objective of this study is to reveal the effects of arid climates and dry seasons on short-time-scale SPI values. To investigate whether the computed SPI values at short time scales from different precipitation regimes across the contiguous United States represent drought and flood events in a similar way, the **normality test** will be conducted on SPI values at various locations. Then, the reasons that lead to the biased SPI values will be explored. Finally, suggestions will be made with regard to the appropriate use and interpretation of the SPI on the basis of the **climate regimes**. We expect this study, along with a previous one, which shows the effect of the length of record on the SPI calculation, to provide guidance to the user in applying the SPI more appropriately, accurately, and effectively.

2. DATA SOURCES AND NORMALITY TEST

2.1 Data sources

The daily precipitation data used in the SPI calculation for this study were obtained from two sources. The first one was the High Plains Regional Climate Center (HPRCC, www.hprcc.unl.edu) of the United States. The HPRCC maintains quality-controlled daily precipitation records for 7 states including Colorado, North Dakota, Iowa, Kansas, Nebraska, South Dakota, and Wyoming. The lengths of records of the stations selected from the HPRCC database ranged from 1876–2004 to 1900–2004 and the selected stations did not have **missing data**. The second data source was the Applied Climate Information System (ACIS, www.rcc-acis.org), an internet-based system designed to provide direct access for user-specified queries to climate data archives. This system is operated by NOAA's Regional Climate Centers (RCCs) and the National Climatic Data Center (NCDC). In addition to the 146 stations obtained from the seven states within the HPRCC regional database, 72 more stations were selected from 33 other states using the ACIS database. Their station histories ranged from 1876–2004 to 1955–2004. Fewer stations were selected from the ACIS database than the HPRCC database because of the higher percentages of missing data and shorter lengths of the record period. We limited the missing data percentages of the selected stations to less than 3%. Furthermore, we replaced all the missing data with zero because no rain for a single day has a high probability. Figure 1 illustrates the distribution of the selected stations across the country for this study. Because of the limited data availability in some states, the selected stations are not evenly distributed. For example, Arizona, California, and Nevada are regions of interest because of their dry climate and unevenly distributed seasonal precipitation. However, the data limitations were such that only a few available stations were selected in California, and no stations were selected in Arizona and Nevada.

2.2. Normality test of frequency distribution of SPI values

SPI values at 1- to 24-week time scales were computed weekly for each year during the available periods using the daily precipitation data from the 218 selected stations. The distribution of these SPI values at each time scale for each week during the period of record was tested to see whether the SPI values were distributed normally through the equiprobability transformation. The normality test was conducted by graphically inspecting the **histogram** or probability plot of the data (Thode, 2002). In addition, three criteria to assess the normality using the PROC UNIVARIATE program within SAS (SAS Institute, Inc., Cary, NC, USA) were used. A distribution is considered non-normal when its variables related to the distribution meet three criteria simultaneously: (1) **Shapiro–Wilk statistic**, W, less than 0.96; (2) p-values less than 0.10; and (3) the absolute value of the **median** greater than 0.05. Otherwise, the distribution is normal. The **W statistic** is the ratio of the best estimator of the variance (based on the square of a **linear combination** of the **order statistics**) to the usual corrected **sum of squares** estimator of the variance. The p-value is the probability that is associated with the W statistic. The absolute value of the median less than 0.05 guarantees that the middle value when SPI values are sorted in order of increasing (or decreasing) magnitude is not greater than ±0.05. These criteria were checked for several more stations, which were found to behave in the same manner. The results of the test were analyzed for six representative stations (with long station histories) representing different climates regimes. Nationwide statistics on the test were also given. In addition, the causes of the non-normality were explored from both mathematical and statistical points of view.

3. RESULTS AND DISCUSSION

3.1. Frequency distribution **plot**s

Figure 2 shows the frequency distribution of seven **dry/wet categories** resulting from the 1-, 4-, 8-, and 12-week SPI values in the 1st, 9th, 17th, 25th, 33rd, 41st, and 49th week of the year for Columbus, New Mexico during 1910–2004. One needs to know that the time scale associated with an SPI value is the date for the end of the analysis period. For instance, the 12-week SPI in the 25th week of the year means ending the 12-week calculation on week 25. The x-axis is the seven categories suggested by McKee et al. (1993) including **extremely dry** (SPI<=−2.0); **severely dry** (−1.5>SPI>−1.99); moderately dry (−1.0>SPI>−1.49); near normal (−0.99<SPI<0.99); moderately wet (1.0<SPI<1.49); very wet (1.5<SPI<1.99); and extremely wet (SPI>=2.0). The y-axis shows the frequencies of the dry/wet events that occur within the seven categories. As suggested, some of the distributions, especially for shorter time scales, are lower bounded and do not have values less than−1.0, referred to the non-normal distribution in this study. Figure 3 illustrates a time series, also for Columbus, NM, of the 4-week SPI at the 25th week of each year (corresponds with late June) for the period 1910–2004. Obviously, the 4-week SPI values never are less than−0.5 during 1910–2004.

3.2. Non-normality plots

We selected six stations with long periods of record from various climate regimes to illustrate when non-normal SPI distributions occur by season, and at what time scales and in which locations this occurs. The six stations (Figure 1) and their periods of record are: Alliance 1 WNW, Nebraska (1894–2004), Bozeman Montana St Univ., Montana (1893–2004), Clayton 1 SSW, Georgia (1894–2004), Columbus, New Mexico (1910–2004), Elkins, West Virginia (1926–2004), and Sacramento WSO City, California (1878–2004).

Figure 4 presents the SPI non-normality distributions for the six stations based on the three predetermined test criteria. As indicated, the SPI with time scales up to 4 weeks (about 1 month) is distributed non-normally in the winter season for Alliance 1 WNW. For stations Bozeman Montana St Univ. and Clayton 1 SSW, most of the 1-week SPIs are not normally distributed. By the end of the year, the SPI with non-normal distribution increases at longer time scales for Bozeman at Montana St University. It appears that the SPI is distributed normally at almost all time scales throughout the year for Elkins. In contrast, Columbus and Sacramento WSO City have obvious seasonal spikes in their SPI non-normality distributions. In Columbus, SPI values with time scales up to 12 weeks are distributed non-normally at most times of the year except during the 28th through 36th weeks of the year (early July to early September). In Sacramento, the time scales of SPI values with non-normal distributions begin to increase from the 20th week (in late May) and reach their highest point at the 38th week of the year (in September), and then gradually drop back by the end of the year.

What causes the differences in the SPI distribution among the six stations? Figure 5 displays the corresponding average 4-week precipitation total distribution over a year for the six stations. The precipitation distributions suggest that the SPI non-normality distribution is closely related to the precipitation distribution. Most of the precipitation for Alliance falls in the middle of a year, with only little amounts occurring during the winter season. The precipitation distribution pattern of Bozeman is similar to that of Alliance. However, one should note that the ratio of the maximum precipitation total to the minimum precipitation total for Bozeman (R=4.21) is less than that for Alliance (R=8.69), resulting in fewer zero precipitation values for Bozeman. The precipitation distributions for Clayton (R=1.55) and Elkins (R=1.84) are even throughout the year. The SPI distributions for these two stations, thus, are normal at almost all time scales during the year. On the other hand, precipitation distributions for Columbus (R=11.11) and Sacramento (R=956.83) exhibit strong seasonality. According to the distributions, it is easy to see why the SPI values distribute normally during the 28th–36th weeks of the year for Columbus and why SPI non-normal distributions for Sacramento occur during the 20th–48th weeks.

3.3. Nationwide statistics on non-normality rates

Table I lists the average non-normality rates (percentages) for the SPI values at 1- to 24-week time scales for each state. The states are clustered by their climate regions. The non-normality rates of the states within the same climate region are not **homogeneous** because of **topographic** contrasts among the stations, leading to a variety of precipitation seasonalities. Generally, the non-normality rates within the Southwest, Northwest and High Plains regions range from a 1-week up to an 8-week SPI or longer. Most of the states in the remaining regions have high non-normality rates for 1- and 2-week SPI values only. The 4-week SPI non-normality rates of each state are also labeled in Figure 1.

In order to reveal the relationship between the non-normality rates and seasons, Figure 6 depicts the changes of the normal versus non-normal percentages of the 4-week SPI over the year for some states that have a relatively high non-normality percentage and have different precipitation seasonalities. These states include four states from the Northern Plains (North Dakota, South Dakota, Nebraska and Kansas), New Mexico from the Southwest and three from the West Coast (California, Oregon and Washington). Obviously, the non-normality rate changes with the seasons, which, in turn, is related to the precipitation distributions. About 30% of the 4-week SPI values are found to be non-normal in January, November, and December for the Northern Plains states. For New Mexico, the non-normally distributed 4-week SPI values are spread over the whole year except from late July to early September. In the West Coast states, over 40% of the non-normal 4-week SPI values appear between June and October.

3.4. Reasons for the non-normal SPI distributions

The formulas used to calculate the SPI values were investigated to explore the reasons for the non-normal SPI distribution. The SPI values used in this study were calculated on the basis of the following theory.

As discussed before, the gamma distribution is used to fit precipitation time series. It is defined by (Thom, 1966):

Where β is a **scale parameter**, α is a **shape parameter**, and $\Gamma(\alpha)$ is the **ordinary gamma function** of α. The estimations of β and α can be found in Thom (1966), and Edwards and McKee (1997).

The distribution function, from which probabilities can be obtained, is:

Since a precipitation distribution may contain zeros, the **mixed** distribution function of zeros and continuous precipitation amounts needs to be employed, given by (Thom, 1951)

Where q is the probability of a zero, and is estimated by $\frac{m}{n}$, in which *m* is the number of zeros in a precipitation time series *n*.

Finally, the SPI is estimated by the **rational** approximation approach (Hastings, 1955; Abramowitz and Stegun, 1965):

Equation (4) computes the negative SPI values, while Equation (5) computes the positive values. In order to have balanced negative and positive values, *t* must be the same under the two situations: $0<H(x)\leq 0.5$and $0.5<H(x)\leq 1.0$. The parameter *t* is determined by Equation (6) when $0<H(x)\leq 0.5$ and by Equation (7) when $0.5<H(x)\leq 1.0$. Thus, *H(x)* is critical in determining whether negative and positive SPI values are symmetric, which leads to a normal distribution.

Figure 7(a) and (b) illustrate the changes of the 3- and 6-month (about 12- and 24-week) SPI values with *H(x)*, *G(x)* and *t* in August for Sacramento, CA, respectively. The previous tests showed that 3-month SPI values in August are non-normally distributed (the intersection of x=32 and y=12 in Figure 4), and 6-month SPI values are normally distributed (the intersection of x=32 and y=24 in Figure 4). At the 3-month time scale, the lowest SPI value is −0.40, while the highest value is 2.58. At the 6-month time scale, the lowest value reaches −2.27 and the highest reaches 2.84. It is found that, at the 3-month scale, the *H(x)* curve separates from the *G(x)* curve significantly for low-precipitation amounts, and the two converge slowly as the precipitation amount increases, indicating *q*, the probability of a zero, is large (see Equation (3)). In other words, there is a high probability of zero values within the August 3-month precipitation total. In fact, there are 43 zeros in the 125 precipitation time series used to compute the 3-month SPI values for August. The unusually high *H(x)* when $0<H(x)\leq 0.5$ leads to the *t* values that are not symmetric when $0.5<H(x)\leq 1.0$. As a result, SPI values are non-normally distributed. On the contrary, because all the 6-month precipitation totals are greater than zero, the *H(x)* curve is completely overlaid with the *G(x)* curve. Therefore, *t* is symmetric, resulting in the SPI being normally distributed.

To further demonstrate the effects of low-precipitation seasons on the SPI calculation, Figure 8 depicts the gamma PDF derived from the 3- and 6-month precipitation observations in August for Sacramento, CA. The 3-month curve (often referred to as a ***hyper-exponential distribution***) shows a typical characteristic of precipitation distribution in low-precipitation seasons or dry climates, suggesting that the probability of having a low precipitation total is very high. In contrast, the 6-month curve is a ***unimodal distribution*** with a slightly positive skew. These two different shapes of PDFs will define two different **cumulative probability distribution functions** (CDFs), which will be used to estimate the SPI through the equi-probability transformation. Figure 9 illustrates the equi-probability transformations from the fitted 3- and 6-month gamma CDFs to the standard normal distribution, resulting in the 3- and 6-month SPI values, respectively. As can be seen, because the 3-month precipitation observations contain too many zero and trace precipitation amounts, the cumulative probability of a very small amount of precipitation is very high. As a result, a small precipitation amount will correspond to a high SPI value. Referring back to Equation (3), the high probability of zero frequencies will produce a high *q*, and the high *q* will be the lower untransformed bound. Consequently, the SPI values will always be greater than a certain value. For instance, a 0.5 mm precipitation total for the 3-month time scale will result in an SPI value of about 1.8, while for the 6-month time scale, the same amount of precipitation will result in an SPI value of−1.5.

Therefore, from a mathematical point of view, the SPI values are lower bounded because of a high probability of zero precipitation events. In addition, we need to question the confidence of the computed SPI values by the 2-parameter gamma model. As mentioned before, the model selected to simulate the precipitation distribution could affect the confidence in the SPI results, because the gamma model used in this study only has two free parameters, which would not give the best goodness-of-fit for the given data. Wilks (1990) proposed a method to more precisely estimate gamma distribution parameters using data containing zeros. It is not clear whether this solution could improve the accuracy of the SPI estimation. Also, other alternatives models (e.g. PE3) are worthy to be studied. Thus, further study will focus on the methods for the estimation of the precipitation distribution parameters for arid climates or for those with a distinct dry season.

The other factor that affects the confidence of the computed SPI values is the limited sample size that can be used in the gamma-distribution fitting of precipitation data because of the high chances for zero precipitation values at the shorter time scales. Guttman (1994) concluded that more records are needed for a stable estimation in the tail of the SPI distribution, which is related to extreme events (especially drought events in this study). In the given example for Sacramento, there are 43 zeros out of the 125 precipitation time series used to compute the 3-month SPI values for August. The sample size that can be used in the estimation of the precipitation distribution is reduced by 1/3. In this case, the accuracy of the estimation of the tails is, therefore, suspect.

Although the short-time-scale SPI values accurately portray the mixture of a lot of dry days and a few wet days in dry areas and times, the lower bounded SPI values (or non-normally distributed SPI values) fail to indicate a drought occurrence. In fact, the SPI represents a cumulative probability of precipitation associated with a specific location or time scale. It does not have to indicate a drought or flood. The appropriate use and interpretation of the SPI values under these circumstances should be done with caution. The discussion of short term drought in dry climates, or low precipitation, is meaningless since no rain is a normal part of the local climate. In such climate regimes, drought occurrence should be related to the time scale. For regions with dry climates or low-precipitation seasons, periods without precipitation are very common. Short periods without rain would not make a drought. The critical feature is how long the drought lasts, rather than how dry it is (Clark, 1993).

It is also worth noting that the SPI is a statistical product of the available data set at a given location since the SPI calculation is based on the input data set. The computed SPI values will be slightly different, as the data set changes either temporally or spatially. This character makes it difficult to rationalize, for instance, when the drought officially started and when it ended according to the varying SPI values. Therefore, the SPI is a research tool rather than an attempt to link the input data to the physical functioning of the Earth system.

4. SUMMARY

In this study, the effects of low-precipitation seasons and dry climates on the SPI calculation were demonstrated on the SPI values from 1 to 24 weeks for each week of the year for 218 selected weather stations from 40 states in the contiguous United States. From a mathematical point of view, the SPI values are lower bounded when a high frequency of zero values (no precipitation cases(occurs, leading to a non-normally distributed SPI. Under these circumstances, the SPI fails to adequately indicate a drought occurrence. Nationwide statistics suggest that the non-normality rates are closely related to local precipitation regimes. In the eastern United States, SPI values at short time scales can be used in drought/flood monitoring and research for any season, while in the western United States (in those areas having distinct seasonal precipitation distributions), the appropriate explanation of this index becomes complicated. This would be the case in other arid regions as well.

Although the SPI approach is reasonable, the 2-parameter gamma distribution we used to simulate the precipitation data would reduce the reliability of the computed SPI values because of its limitation in short time-scale simulations. In addition, because of the limited sample size used to simulate precipitation distributions in dry climates, the estimations of the model parameters from the small data samples are prone to large errors, especially in the tails of the distributions, which is what we are most interested in. Therefore, we remind the SPI user to be cautious when applying and interpreting SPI values in and between various regions having variable climate regimes. In dry climates, the analyst should focus on the duration of the drought rather than on just its severity. Furthermore, the SPI is only a statistical product of the input data, limiting its role in revealing the complexity of the drought events.