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## Two new multi-path routing algorithms for fault-tolerant communications in smart grid

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## ABSTRACT

Recently, smart grid, which is a newer generation of electricity supply network, is getting lots of attentions due to its huge benefits. One key component of the smart grid is an integrated communication network. To make the smart grid more dependable, it is extremely important to ensure that messages are exchanged over the communication network in a reliable and timely manner. A multiple path routing might be one way to achieve this goal. Unfortunately, the existing algorithms which compute multiple node-disjoint paths are not sufficient for this purpose since in a smart grid communication network, node failures can be co-related. Motivated by this observation, we introduce a new quality multiple routing path computation problem in a smart grid communication network, namely the min–max non-disrupting  $k$  path computation problem ( $M^2NkPCP$ ). We show this problem is NP-hard and propose two heuristic algorithms for it. In addition, we evaluate the average performance of the algorithms via simulation.

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### 1. Introduction

The recent advances in power network technologies have resulted in an automated modern power supply network called the *smart grid*. The smart grid collects and utilizes the real-time knowledge of its status as well as of the behaviors of electricity suppliers and consumers to improve the overall efficiency, sustainability, reliability, and the economics of the distribution and the production of electricity [1]. One crucial component of the smart grid which distinguishes itself from the conventional power supply network is the real-time communication network connecting the grid with electricity providers and consumers. It is known that this communication network is the

key enabler for the smart grid to provide a rich set of new services, such as the grids open-access market, distributed generation and storage devices, in-home networks, smart appliances, new software applications, which were previously not available [2,3].

The importance of the reliability of the communication network in the smart grid cannot be overemphasized. Many recent reports envision that in the near future, the smart grid will evolve into a highly complicated power network connecting various types of consumers from residential, industrial, and government sectors, and a wide variety of electricity sources such as traditional carbon fuel based power plants as well as emerging distributed renewable sources such as solar and wind [4]. Since the cost of the electricity generated by the renewable energy sources is much cheaper than the cost of that generated by the carbon fuel, the carbon fuel based power generation will be preferred only if the energy consumption of the consumers

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exceeds what the renewable energy sources can afford. In the power grid system, a power outage can occur if the power demand is greater than the power supply. Meanwhile, the smart grid uses the communication network to collect such demand and supply information in real time manner to cost-effectively facilitate carbon fuel based power plants (the amount of electricity supply exceeds the actual demand will be disposed). As a result, it is extremely important to ensure that the communication within the smart grid is secure, timely, and reliable [5]. This is one of the reasons why the smart grid is of great cyber security concern [6].

In a communication network, a path connecting a source node and a destination node is called a *routing path*. Currently, the routing algorithms employed by most communication networks compute a path with minimum cost, e.g. minimum number of hops or minimum total edge weight. A routing path fails if it fails to deliver a message from the source node to the destination node. In most cases, a routing path fails either by a link failure or by a node failure. Depending on the type of a communication network, one of the failures is much more frequent than the other. For instance, in a fiber optic network, link failure is highly unlikely and thus most routing failures happen at a node such as an intermediate router or repeater. On the other hand, in a wireless sensor network, link failure could be a main contributor of temporal failures and node failure can be a main contributor of permanent failures.

Briefly speaking, a multi-path routing is a routing strategy to concurrently transmit the copies of a message from a source node to a destination node throughout multiple paths. Intuitively, this is a good idea to improve the reliability of a communication since by sending multiple copies of the same message over separate paths, we have a much better chance to transfer data from a source to a destination on time despite the existence of faulty links and nodes. Therefore, multi-path routing algorithms have been introduced for reliable communications in various communication networks in the literature [7–15]. For those networks such as wireless sensor networks in which node failure is the predominating cause of permanent routing failure (link failures are more likely to be temporal) and the node failures are independent with each other, it is desirable for the multiple paths to deliver the copies of the same message to be node-disjoint with each other so that a node failure within a path would not affect the reliability of the other paths [16].

It is expected that the most common type of significant communication failure in the smart grid communication network is node failure [17–19]. However, unlike the most type of networks in which the failures are independent of each other, the node failures in the smart grid communication network can be co-related due to its unique architecture (see Fig. 1). In the smart grid, the components of the communication network such as routers need electricity to operate, and thus the power supply network is highly co-related with the communication network. As a result, a failure at the power network can result in an outage on the communication network and a failure at the communication network can result in an outage on the power network. The recent report by Nguyen et al. [20] observed

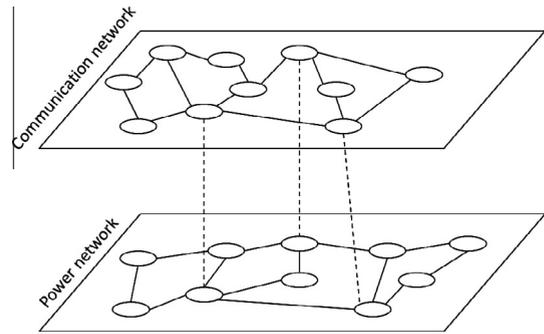


Fig. 1. This figure illustrates the interdependency between the power network and the communication network inside the smart grid.

that a node failure in a smart grid can cause the failures at some other nodes, and proposed a vulnerability assessment algorithm to evaluate the maximum possible effect of a single node failure within a smart grid. This means that the traditional node-disjoint path based multi-path routing is not proper to the smart grid communication network for reliable communication (see Fig. 2). Motivated by our observations that we discussed so far, in this paper, we introduce a new multi-path routing problem in smart grid communication networks. Largely, the contribution of this paper is twofold.

- (a) We introduce the min-max non-disrupting  $k$  path computation problem ( $M^2NkPCP$ ) whose goal is to compute  $k$  node-failure-disjoint paths (a node failure at one path does not lead to a node failure in another path) from a source to a destination such that the maximum cost (e.g. total Euclidean distance or hop distance) among the paths is minimized. Its formal definition is in Definition 5. To the best of our knowledge, this is the first effort to investigate

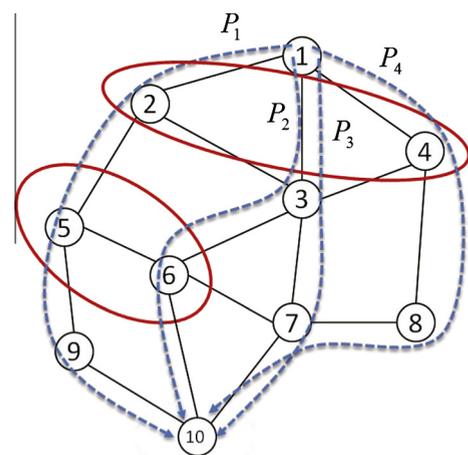


Fig. 2. In this figure, we are looking for two node-failure-independent paths from node 1 to node 10. In this example, node 5 and node 6 are interdependent, i.e. if one node fails, the other fails, and node 2 and node 4 are interdependent. Then, Path  $P_1$  and Path  $P_2$  may fail together if one of node 5 and node 6 fails. Also  $P_1$  and  $P_4$  can fail at them same time by a node failure. As a result,  $P_2$  and  $P_4$  would be better choice.

a multi-path routing problem in which node failures are co-related. Also, we believe this is the first work to study a multi-path routing problem in smart grid communication networks. We also show that this problem is NP-hard.

- (b) We propose two new heuristic algorithms for  $M^2NkPCP$ , each of which consists of three phases. Given a smart grid communication graph, the first phase of the algorithms computes an auxiliary graph. In the second phase,  $k$  node-disjoint paths in the auxiliary graph are computed. The two algorithms that we propose are differentiated by having a trade-off between running time and the quality of the output. In the final phase, the output of the second phase is used to recover a feasible solution of  $M^2NkPCP$ . We also conduct a simulation to evaluate the average performance of the proposed algorithms.

The rest of this paper is organized as follows. In Section 2, we introduce some related work. The network model and the formal definition of our problem is given in Section 3. The description of the two new heuristic algorithms for  $M^2NkPCP$  are given in Section 4. Our simulation results and corresponding analysis are in Section 5. Section 6 concludes this paper and presents some future works.

## 2. Related work

The smart grid is a newer generation of electricity distribution network and characterized by the existence of a tightly co-related communication network, which is used to collection information of the grid, electricity consumers and providers to improve the overall reliability, efficiency, sustainability, and the economics of the distribution and production of electricity [1]. While reliability of communications within the smart grid communication network is extremely crucial for the stability and reliability of the overall grid, there has been generally a lack of efforts made toward this issue.

One prominent approach to ensure to deliver a message reliably from a source node to a destination node is sending multiple copies of the message over independent paths concurrently [7–15]. Those works were targeting some of representative networks such as the Internet, mobile ad hoc networks, and wireless sensor networks. One typical assumption made by those works is that each node/link failure is independent of each other, which is largely correct in those networks. Therefore, most of those are not applicable to the smart grid communication network in which a failure of a node can result in a failure of another node [20].

The recent occurrences of natural disasters such as a massive earthquake occurred in Japan show that once such a natural disaster happens, more than one node can fail at the same time. This motivated various reliability-related researches in communication networks where some failures are co-related. In this set of researches, the physical shape of the network is a crucial factor to determine the co-relation among failures [21–23,25,24,26,27]. However, all of those works have focused on the survivability

analysis of a given network against a particular type of disaster.

Recently, Nguyen et al. observed that the node failures within a smart grid communication network is co-related and proposed a vulnerability assessment algorithm to evaluate the maximum possible effect of a single node failure within a smart grid [20]. To the best of our knowledge, Zhang and Perrig's work in [28] is the only work concerning multiple failure-independent path selection issue on the Internet. In this work, the authors recognized the failures at each path on the Internet can be co-related and studied on select  $k$  possibly failure-independent paths out of available paths based on the history information. Since our work focuses on the construction of routing paths, our work is significantly different from their work.

## 3. Network model and problem definition

In this paper, we consider a smart grid communication network graph  $G = (V, E, w)$ , where  $V = V(G) = \{v_1, \dots, v_n\}$  is the set of vertices,  $E = E(G)$  is the set of bidirectional edges, and  $w : E \rightarrow \mathbb{R}^+$  is the weight function over edges in  $E$ . For any vertex subset  $V' \subseteq V$ ,  $G[V']$  is the subgraph of  $G$  induced by  $V'$ . Similarly,  $G[E']$  is the subgraph of  $G$  induced by an edge subset  $E' \subseteq E$ . Now, we introduce some important definitions.

**Definition 1 (Interdependent nodes).** Given  $G$ , two nodes  $v_i$  and  $v_j$  are interdependent with each other if a failure of one node, e.g.  $v_i$ , results in a failure of another node, e.g.  $v_j$ , and vice versa. Otherwise, they are non-interdependent, or equivalently, are independent.

For any node  $v_i \in V$ , one can obtain the list of the interdependent nodes of  $v_i$  in  $V$  by using the vulnerability assessment algorithm by Nguyen et al. in [20]. It is easy to see that  $V$  can be partitioned into a collection of disjoint subsets of nodes  $\mathcal{S} = \{S_1, S_2, \dots, S_L\}$  such that each  $S_j \in \mathcal{S}$  is a subset of nodes which are interdependent with each other, i.e. a failure of a node in  $S_j$  means the failure of the rest of the nodes in  $S_j$ . Note that some  $S_j$  may include a single node.

**Definition 2 (Interdependent subsets).** Given  $G$ , two subsets  $S_i$  and  $S_j$  in  $\mathcal{S}$  are interdependent if  $S_i \cap S_j \neq \emptyset$ . Otherwise, they are non-interdependent, or equivalently, are independent.

**Definition 3 (Interdependent paths).** Given  $G$  and a collection of independent subsets  $\mathcal{S} = \{S_1, \dots, S_L\}$  which partitions  $V$ , we call two paths  $P_1 = \{v_1^{(1)}, v_2^{(1)}, \dots, v_{|P_1|}^{(1)}\}$  and  $P_2 = \{v_1^{(2)}, v_2^{(2)}, \dots, v_{|P_2|}^{(2)}\}$  are interdependent with each other if there exists  $S_i \in \mathcal{S}$  such that  $S_i \cap P_1 \neq \emptyset$  and  $S_i \cap P_2 \neq \emptyset$  at the same time. Otherwise, they are non-interdependent (or equivalently, independent).

**Definition 4 (Non-disrupting paths).** Given  $G$ , a collection of independent subsets  $\mathcal{S} = \{S_1, \dots, S_L\}$  which partitions  $V$ , and a set of paths  $P_1, P_2, \dots, P_k$  from  $v$  to  $u$ , consider the sub-path  $P'_i$  in  $P_i$  such that  $P'_i = \{v_2^{(i)}, \dots, v_{|P_i|}^{(i)}\}$  for each

$1 \leq i \leq k$ . We call  $P_1, P_2, \dots, P_k$  are not disrupting with each other only if  $P'_1, P'_2, \dots, P'_k$  are independent with each other.

**Definition 5** ( $M^2NkPCP$ ). Given  $G = (V, E, w)$ ,  $S = \{S_1, \dots, S_L\}$ , and two distinct nodes  $s, t \in V$ , the min-max non-disrupting  $k$  path computation problem ( $M^2NkPCP$ ) is to find  $k$  non-disrupting paths  $\{P_1, \dots, P_k\}$  from  $s$  to  $t$  in  $G$  such that

$$\max_{1 \leq i \leq k} \text{Cost}(P_i) = \max_{1 \leq i \leq k} \sum_{e \in P_i} w(e)$$

is minimized.

**Theorem 1.**  $M^2NkPCP$  is NP-hard.

**Proof.** A special case of  $M^2NkPCP$  with  $|S_i| = 1$  for each  $1 \leq i \leq L$  is equivalent to the  $k$  min-max disjoint path problem, whose decision version is strongly NP-complete [29]. As a result, the decision version of  $M^2NkPCP$  is also NP-complete, and thus  $M^2NkPCP$  is NP-hard.  $\square$

In this paper, we assume the subgraph of  $G$  induced by  $S_i$  is connected for each  $i$ , which is highly likely in practice since the nodes fail together are geographically close enough to be powered by the same power source. We also assume that in the course of computing  $k$  non-disrupting paths between two nodes  $s$  and  $t$ , no node in  $S_s$  and  $S_t$  such that  $s \in S_s$  and  $t \in S_t$  will fail, otherwise at least one of  $s$  and  $t$  will fail and there is no feasible solution.

**4. Two heuristic algorithms for  $M^2NkPCP$**

This section introduces two different heuristic algorithms for  $M^2NkPCP$ . Given a smart grid communication graph  $G$ , a collection of independent subsets  $S$  such that  $V = \cup_{S_i \in S} S_i$ , and two distinct nodes  $s \in S_s$  and  $t \in S_t$  (where  $\{S_s, S_t\} \subset S$ ), each of the algorithms performs the following three distinct phases to compute  $k$  non-disrupting paths from  $s$  to  $t$  in  $G$ .

- Phase 1: transform  $G = (V, E, w)$  into a new edge-weighted graph  $G^* = (V^*, E^*, w^*)$  with directional edges.
- Phase 2: find  $k$  node-disjoint paths from  $s'$  to  $t'$  in  $G^*$  (each of which is associated with  $s$  and  $t$  in  $G$ , respectively) using a maximum flow algorithm such as Floyd–Warshall algorithm and a  $k$  minimum cost flow algorithm in [30].
- Phase 3: use the output of Phase 2 to recover  $k$  non-disrupting paths from  $s$  to  $t$  in  $G$ .

While the two algorithms share Phase 1 and Phase 3 in common, they differ in Phase 2. In the following subsections, we discuss about each phase.

**4.1. Phase 1: auxiliary graph  $G^*$  Induction**

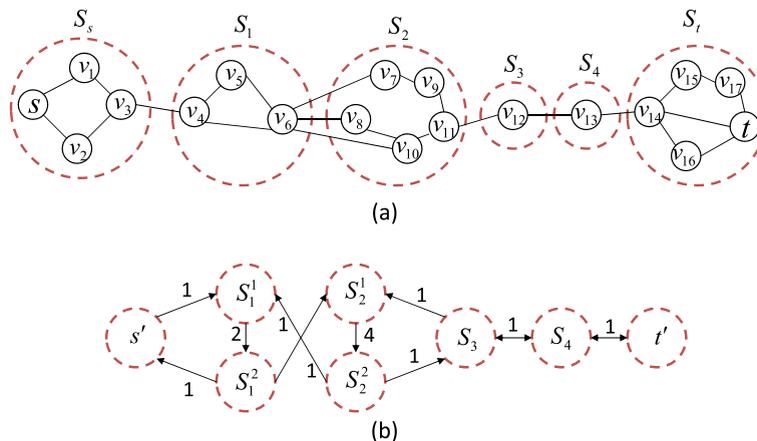
In this section, we explain how an auxiliary graph  $G^* = (V^*, E^*, w^*)$  is generated from  $G = (V, E, w)$  (refer Fig. 3).

Briefly speaking, we construct  $G^*$  based on our assumption that the induced graph of  $G$  by each  $S_i \in S$  is connected. By relying on this assumption, we contract each  $S_i$  in  $G$  into one or two nodes in  $G^*$  and setup directional edges accordingly. The details of this construct are as follows.

- (a) for each subset  $S_i \in S \setminus \{S_s, S_t\}$  such that  $|S_i| \geq 2$ , add a pair of nodes  $\{S_i^1, S_i^2\}$  into  $G^*$ . For each node pair  $\{S_i^1, S_i^2\}$  added in this way, we add a directed edge  $S_i^1 \rightarrow S_i^2$  from  $S_i^1$  to  $S_i^2$  with an edge weight of

$$w^*(S_i^1, S_i^2) = \text{diameter}(G[S_i]) = \max_{\forall v_i, v_j \in S_i} \text{len}(SP(v_i, v_j)),$$

where  $SP(v_i, v_j)$  is the shortest path between  $v_i$  and  $v_j$  in  $G[S_i]$  (the subgraph of  $G$  induced by  $S_i$ , which is a connected subgraph based on our assumption), and  $\text{len}(p)$  denotes the length of the path  $p$ . In (b) of Fig. 3,  $\langle S_1^1, S_1^2 \rangle$  and  $\langle S_2^1, S_2^2 \rangle$  are the corresponding



**Fig. 3.** A path in  $G^*$  shown in figure (b) can be used to construct a path in  $G$  shown in figure (a). In this example, hop distance is used as an edge weight.

transferred directed edges of subset  $S_1$  and  $S_2$  in (a) of Fig. 3 respectively.

Also, for each pair of edge pairs  $\{S_{i_1}^1, S_{i_1}^2\}$  and  $\{S_{i_2}^1, S_{i_2}^2\}$  added to  $V^*$  in this way, we add two directed edges  $S_{i_1}^2 \rightarrow S_{i_1}^1$  and  $S_{i_2}^2 \rightarrow S_{i_1}^1$  with an edge weight of

$$\min_{\forall v_i \in S_{i_1}^1, \forall v_j \in S_{i_2}^2} w(v_i, v_j)$$

if there exists an edge  $(u, v) \in E$  for some  $u \in S_{i_1}$  and  $v \in S_{i_2}$ . For example,  $\langle S_1^2, S_1^1 \rangle$  and  $\langle S_2^2, S_1^1 \rangle$  in (b) of Fig. 3 are such kind of directed edges for  $S_1$  and  $S_2$  in (a) of Fig. 3 respectively.

(b) for each subset  $S_i \in \mathcal{S} \setminus \{S_s, S_t\}$  such that  $|S_i| = 1$ , e.g.  $S_i = \{u\}$ , we add  $u'$  to  $V^*$ . We also add two directional edges to connect  $u'$  with the other nodes in  $V^*$ . In detail,

(i) if there exists two different subsets  $S_i = \{u\}$  and  $S_j = \{v\}$  in  $\mathcal{S}$  such that there exists an edge between  $u$  and  $v$  in  $G$ , then add two directional edges  $u' \rightarrow v'$  and  $v' \rightarrow u'$  to  $E^*$  and set  $w^*(u', v') \leftarrow w(u, v)$  and  $w^*(v', u') \leftarrow w(u, v)$ . The bidirectional edge between  $S_3$  and  $S_4$  in Fig. 3(b) is the corresponding edge for  $S_3$  and  $S_4$  in Fig. 3(a).

(ii) for each pair  $u$  and  $S_i$  such that  $|S_i| \geq 2$  in  $G$ , if there exists at least one node in  $S_i$  which is connected to  $u$ , we add two directional edges  $u \rightarrow S_i^1$  and  $S_i^2 \rightarrow u$  to  $E^*$  and set the weight of each edge to be  $\min_{v \in S_i} w(u, v)$ . We can find an example of this case: edges  $\langle S_2^2, S_3 \rangle$  and  $\langle S_3, S_2^1 \rangle$  in Fig. 3(b) are the transferred edges for  $S_3$  and subset  $S_2$  in Fig. 3(a).

(c) for  $S_s$  including  $s$ , we add  $s'$  to  $V^*$ . For each  $S_i \in \mathcal{S} \setminus \{S_s\}$  such that  $|S_i| \geq 2$ , if there exists  $u \in S_i$  such that  $u$  is adjacent to a node  $v \in S_s$  in  $G$  ( $v$  can be  $s$ ), we add a direct edge from  $s'$  to  $S_i^1$  to  $E^*$  with an edge weight of  $len(SP(s, u))$  in  $G[S_s \cup S_i]$ , which can be shown by edges  $\langle s', S_i^1 \rangle$  and  $\langle S_i^2, s' \rangle$  in Fig. 3(b). For each  $S_i \in \mathcal{S} \setminus \{S_s\}$  such that  $S_i = \{u\}$ , if  $u$  is adjacent to a node  $v \in S_s$  in  $G$  ( $v$  can be  $s$ ), we add a direct edge from  $s'$  to  $u'$  to  $E^*$  with an edge weight of  $len(SP(s, u))$  in  $G[S_s \cup S_i]$ .

(d) for  $S_t$  including  $t$ , we add  $t'$  to  $V^*$ . For each  $S_i \in \mathcal{S} \setminus \{S_s\}$  such that  $|S_i| \geq 2$ , if there exists  $u \in S_i$  such that  $u$  is adjacent to a node  $v \in S_t$  in  $G$  ( $v$  can be  $t$ ), we add a direct edge from  $S_i^2$  to  $t'$  to  $E^*$  with an edge weight of  $len(SP(u, t))$  in  $G[S_t \cup S_i]$ . For each  $S_i \in \mathcal{S} \setminus \{S_t\}$  such that  $S_i = \{u\}$ , if  $u$  is adjacent to a node  $v \in S_t$  in  $G$  ( $v$  can be  $t$ ), we add a direct edge from  $u'$  to  $t'$  to  $E^*$  with an edge weight of  $len(SP(u, t))$  in  $G[S_t \cup S_i]$ , which can be shown by bidirectional edge between  $S_4$  and  $t'$  in Fig. 3(b).

#### 4.2. Phase 2: two different strategies to compute $k$ disjoint paths in $G^*$

In this section, we introduce two different strategies to compute  $k$  disjoint paths in  $G^*$ . The two strategies are

trade-off algorithms in terms of their running time and quality of outputs.

**Algorithm 1.** Min-max  $k$  node-disjoint path computation algorithm ( $G^* = (V^*, E^*, w^*), s', t'$ )

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1: Set  $E' \leftarrow \emptyset, \mathcal{P}^* \leftarrow \emptyset, MM \leftarrow \infty$ .
2: Sort the edges in  $E^*$  according to their weights in the non-decreasing order and store the order in  $E' = \{e_1, e_2, \dots, e_q\}$ .
3: for  $i = 1$  to  $q$  do
4:    $\mathcal{P}_i \leftarrow \emptyset, MM_i \leftarrow 0$ 
5:    $E' \leftarrow E' \setminus \{e | e \in E^* \text{ and } w(e_i) \leq w(e) < weight(e_{i-1})\}$ 
   (if  $i = 1$ , replace this line with
    $E' \leftarrow E' \setminus \{e | e \in E^* \text{ and } w(e_i) \leq w(e)\}$ )
6:   if there is  $k$  node-disjoint paths from  $s'$  to  $t'$  in  $G^*[E']$  then
7:     Apply the  $k$  minimum cost flow algorithm on  $G^*[E']$  to find  $k$  node-disjoint paths and store them in  $\mathcal{P}_i$ . Also, calculate the length of each path in  $\mathcal{P}_i$  and put the length of the longest one into  $MM_i$ .
8:   else
9:      $E' \leftarrow E' \cup \{e | e \in E^* \text{ and } w(e_i) \leq w(e) < weight(e_{i-1})\}$  (if  $i = 1$ , replace this line with
      $E' \leftarrow E' \cup \{e | e \in E^* \text{ and } w(e_i) \leq w(e)\}$ ).
10:  end if
11:  if  $MM_i < MM$  then
12:    Set  $\mathcal{P}^* \leftarrow \mathcal{P}_i$  and  $MM \leftarrow MM_i$ .
13:  end if
14: end for
15: Return  $\mathcal{P}^*$ .

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**Strategy 1.** The main idea of our first strategy is simple: replace the weight of all edges in  $G^*$  to 1 and apply the maximum flow algorithm from  $s'$  to  $t'$ , and obtain  $m$  flows. Then, from the  $m$  flows, we restore  $m$  node-disjoint paths in  $G^*$  (for the details, please refer [31]). Finally, among the  $m$  paths, pick the first  $k$  shortest paths as the output of Phase 2.

**Strategy 2.** The second strategy consists of two steps. In the first step, we sort the edges in  $E^*$  in the non-increasing order of their weights and obtain an ordered edge set  $E' = \{e_1, e_2, \dots, e_q\}$ . In the second step, we remove some edges in  $E'$  and obtain  $\bar{E}^*$  so that the subgraph of  $G^*$  induced by the edges in  $\bar{E}^*$  includes  $k$  node-disjoint paths from  $s'$  to  $t'$ . In detail, for each  $e_i \in E'$ , starting from  $i = 1$  to  $q$ , we remove all the edges  $e \in E$  whose weight satisfies  $w^*(e_i) \leq w^*(e) < w^*(e_{i-1})$ , and check if the subgraph of  $G^*$  induced by  $E'$  without those edges is still  $k$ -connected, which can be verified using a maximum flow algorithm as stated in Strategy 1.

If  $k$ -connected, we permanently remove those edges from  $E'$  (update  $E'$ ), obtain  $k$  node-disjoint paths using the  $k$  minimum cost flow algorithm on the subgraph of  $G^*$  induced by the updated  $E'$ , compute the length of longest path,  $MM_i$ , among them, and proceed with  $i + 1$ . Otherwise, we keep those edges in  $E'$  and proceed. Here, we can set  $w^*(e_0) \leftarrow \infty$ . After we process the case  $i = q$ , then we pick

the  $k$  node-disjoint paths which correspond with the minimum  $MM_i$  as an output. The details of this strategy is described in Algorithm 1.

4.3. Phase 3: Getting  $k$  non-disrupting paths in  $G$  from  $k$  node-disjoint paths in  $G^*$

In Phase 2, we have obtained a set  $\mathcal{P}^*$  of the  $k$  node-disjoint paths from  $s'$  to  $t'$  in  $G^*$ . In this phase, we explain how to use  $\mathcal{P}^*$  to obtain  $k$  non-disrupting paths from  $s$  to  $t$  in  $G$ . Our main idea for this phase is that by our construction of  $G^*$ , for any path  $P' \in \mathcal{P}^*$  from  $s'$  to  $t'$  in  $G^*$ , there exists a corresponding path  $P$  from  $s$  to  $t$  in  $G$  (refer Fig. 4). In detail, suppose  $P' = s' \rightarrow v_1 \rightarrow v_2 \rightarrow \dots \rightarrow v_p \rightarrow t'$ . Then, we construct  $P$  from  $P'$  by replacing each edge in  $P' \subseteq E^*$  into one or more edges in  $E$  (along with their end points). Initially, we start from the first node in  $\mathcal{P}^*$ , which is  $s'$  with  $P = \{s\}$ . Each edge in  $P' \in \mathcal{P}^*$  is one of the following four types.

- (a)  $s' \rightarrow v_1$  (Type 1): In this case,  $v_1$  is either
  - Case (i):  $t'$  in  $G^*$ , which represents the subset  $S_t$  including  $t'$  in  $G$ . In this case,  $P'$  consists of exactly one edge from  $s'$  to  $t'$  in  $G^*$ . Thus,  $P$  is the minimum weight path from  $s$  to  $t$  in  $G[S_s \cup S_t]$ . The last node of  $P$  becomes  $t$  and we are done.
  - Case (ii): a node  $u'$  in  $G^*$ , which represents a subset  $S_i$  in  $G$  such that  $|S_i| = 1$ , e.g.  $S_i = \{u\}$ . In this case, we add the minimum weight path from  $s$  to  $u$  in  $G[S_s \cup \{u\}]$ . The last node of  $P$  becomes  $u$ .
  - Case (iii): a node  $S_i^1$  in  $G^*$ , which represents a subset  $S_i$  such that  $|S_i| \geq 2$ . In this case, we add the minimum weight path from  $s$  to a node  $u$  in  $S_i$  in  $G$  to  $P$ . The last node of  $P$  becomes  $u$ . Edge  $\langle s', S_i^1 \rangle$  in (a) of Fig. 4 is an example for this case, which is restored to edge  $\langle v_3, v_4 \rangle$  in (b) of Fig. 4.
- (b)  $S_i^1 \rightarrow S_i^2$  (Type 2): In this case, we first look ahead the node  $u'$  coming after  $S_i^2$  in the path  $P'$ . There are following three cases.

- Case (i):  $u'$  is  $S_i^1$  such that  $|S_i^2| \geq 2$ . Then, we add a minimum weight path within  $G[S_i^1]$ , which, in  $G$ , connects the last node of  $P$  to a node  $v$  in  $S_i^1$ , which is adjacent to a node  $u$  in  $S_i^2$  such that  $w(u, v)$  is minimum, to  $P$ . The last node of  $P$  becomes  $u$ . For example,  $\langle S_1^1, S_1^2 \rangle$  and  $\langle S_2^1, S_2^2 \rangle$  in (a) of Fig. 4 are such kind of edges, which are restored to  $\langle v_4, v_6 \rangle$  and  $\langle v_{10}, v_{11} \rangle$  in (a) of Fig. 4 respectively.
- Case (ii):  $u' \neq t'$  and it represents  $S_i^2 = \{u\}$ . Then, we add a minimum weight path within  $G[S_i^1]$ , which, in  $G$ , connects the last node of  $P$  to a node  $v$  in  $S_i^1$ , which is adjacent to the only node  $u$  in  $S_i^2$  such that  $w(u, v)$  is minimum, to  $P$ . The last node of  $P$  becomes  $u$ .
- Case (iii):  $u' = t'$ . Then, we add a minimum weight path within  $G[S_i^1 \cup S_t]$ , which, in  $G$ , connects the last node of  $P$  to  $t$  to  $P$ . The last node of  $P$  becomes  $t$  and we are done. Edge  $\langle S_4, t' \rangle$  in (a) of Fig. 4 is an example for this case, which is restored to edge  $\langle v_{13}, v_{14} \rangle$  in (b) of Fig. 4.
- (c)  $S_i^2 \rightarrow u'$  such that  $|S_i^2| \geq 2$  (Type 3): In this case,
  - Case (i): if  $u' = S_i^2$  such that  $|S_i^2| \geq 2$ , we add an edge, which, in  $G$ , connects the last node  $u$  of  $P$  to another node  $v$  in  $S_i^2$  such that  $w(u, v)$  is minimum, to  $P$ . The last node of  $P$  becomes  $v$ . We can find an example of this case: edges  $\langle S_1^2, S_2^1 \rangle$  in Fig. 4(a) is transferred to edge  $\langle v_6, v_{10} \rangle$  in Fig. 4(b).
  - Case (ii): if  $u' = S_i^2$  such that  $S_i^2 = \{u\}$ , we add an edge, which, in  $G$ , connects the last node of  $P$  to  $u$ , to  $P$ . The last node of  $P$  becomes  $u$ . For this case, there is an example in Fig. 4: edges  $\langle S_2^2, S_3 \rangle$  in (a) is transferred to edge  $\langle v_{11}, v_{12} \rangle$  in (b).
  - Case (iii):  $u' = t'$  is handled by Case (iii) of Type 2.
- (d)  $u' (\neq s') \rightarrow v'$  such that  $u'$  represents  $S_i = \{u\}$  (Type 4): In this case,
  - Case (i): if  $v'$  represents  $S_i^2$  such that  $|S_i^2| \geq 2$ , we add an edge, which, in  $G$ , connects  $u$  which is the last node of  $P$  to another node  $v$  in  $S_i^2$  such that  $w(u, v)$  is minimum, to  $P$ . The last node of  $P$  becomes  $v$ .

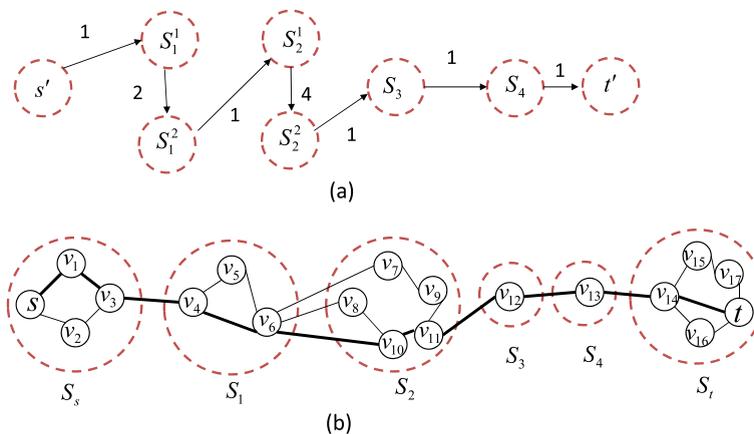


Fig. 4. A path in  $G^*$  shown in figure (a) can be restored to a path in  $G$  shown in figure (b), which is composed of bold edges. In this example, hop distance is used as an edge weight.

- Case (ii): if  $v'$  represents  $S_i$  such that  $S_i = \{v\}$ , then, we add an edge, which, in  $G$ , connects  $u$  which is the last node of  $P$  to  $v$ , to  $P$ . The last node of  $P$  becomes  $v$ . Edge  $(S_3, S_4)$  in (a) of Fig. 4 is an example for this case which is restored to edge  $(v_{12}, v_{13})$  in (b) of Fig. 4.
- Case (iii): if  $v' = t'$ , then we add a minimum weight path within  $G[S_t \cup \{u\}]$ , which connects  $u$  which is the last node of  $P$  to  $t$ , to  $P$ . The last node of  $P$  becomes  $t$  and we are done.

Clearly, the running time of our strategies is polynomial. Also, it produces a feasible solution of  $M^2NkPCP$  if a given problem instance has a solution.

## 5. Simulation results and analysis

In this section, we conduct a simulation to compare the average performance of our algorithms for  $M^2NkPCP$  which differ in the second phase. It is easy to expect that the length of the shortest path is significantly shorter than the maximum cost path among the  $k$  paths computed by each of the algorithms. Still, we also compute the length of the shortest path as a reference lowerbound (LB).

**Network construction.** For fair comparison, we need to run the algorithms over random network graphs. One easy and widely accepted way to generate such random networks is computing unit disk graphs. In detail, we consider a  $100 \times 100$  unit distance virtual two-dimensional space, set the transmission range  $R$  of each node as 20 unit distance, and deploy  $n$  nodes over it. Then, we assume two nodes are connected with each other if their distance is no greater than 20 unit distance. We add a source node  $s$  and a destination node  $t$ . We connect  $s$  to any node which is at most 20 unit distance far from the left border of the virtual space. Similarly, we connect  $t$  to any node which is at most 20 unit distance far from the right border of the virtual space. As a result, we obtain a network graph  $G = (V, E)$ .

In  $G$ , the weight of an edge connecting a node to  $s$  or  $t$  is  $R = 20$ . For the other edges, their length become their weight. The collection of the independent subsets whose size is no less than 1 is denoted as  $L^*$ , i.e.  $L^* = \{S_i \mid |S_i| \geq 2, \forall S_i \in S_L\}$ . In the initial stage, if a network instance is not  $k$ -connected, we discard it and produce a new one until a  $k$ -connected network can be obtained. We adopt two schemes (uniform one and randomized one) to construct a collection of independent subsets. Each of the scheme consists of the following two steps:

- Set the size  $p$  of each  $S_i$  to be 10 (uniform) or to be a random integer from  $[1, \frac{n}{10}]$  (randomized).
- For each  $S_i \in L^*$ , we pick a random integer  $i_0$  from  $[1, n]$  and add  $v_{i_0} \in V$  into  $S_i$ . Then, for each  $S_i = \{v_{i_0}\}$ , we randomly select  $p - 1$  nodes from  $V$  and add them to the subset such that  $G[S_i]$  is connected. Any used  $i_0$  for some  $S_i$  cannot be reused for another  $S_i$ .

For each parameter setting, we produce 50 graph instances. For each graph, we apply each algorithm and produce  $k$  non-disrupting paths. The quality of an output ( $k$  paths) is evaluated based on the cost of maximum cost path among the  $k$  paths. For the ease of the discussion, we will notate a graph instance with a collection of uniform size independent subsets of interdependent nodes by  $G_U$ , and the other (random) by  $G_R$ .

**Simulation results.** In this simulation, we will study how each algorithm is affected by following three important parameters: the number of sensor  $n$ , the number  $|L^*|$  of the independent subsets with more than one node, and the number of disrupting paths  $k$ . In particular, we consider the following three different settings:

- $|L^*| = 8, k = 4$ , and  $n$  varies from 100 to 400 with the increment of 50.
- $n = 200, k = 4$ , and  $|L^*|$  varies from 4 to 10 with the increment of 1.
- $n = 250, |L^*| = 8$ , and  $k$  varies from 1 to 5 with the increment of 1.

We first study the impact of the number of nodes on the performance of the strategies. From Fig. 5(a), we can observe that in  $G_R$ , the cost of both of strategies decreases smoothly as the size of the network grows. This is natural because by the way that we construct  $G_R$ , as we have more nodes, we will have more number of edges and therefore more paths between  $s$  and  $t$ . At the same time, we will have more number of shorter paths. From Fig. 5(b), we can observe that in  $G_U$ , Strategy 1 does not barely show any improvement even though the size of the network grows. Meanwhile, Strategy 2 shows some improvement. Overall, our result shows that the way to construct the collection  $L^*$  of the independent subsets of interdependent nodes does not impact the performance of the algorithm significantly.

Next, we exam the impact of the size of  $L^*$  on the performance of the strategies. Fig. 6(a) and (b) show that as the size of  $L^*$  grows, the cost of the outputs of both strategies increases. Especially, in  $G_R$ , the performance gap between the strategies grows. This is significant since as the interdependency of the nodes behind the communication network caused by the power network becomes more complicated, our second strategy's extra effort is getting more effective.

Fig. 7 shows that in both  $G_U$  and  $G_R$ ,  $k$ , the required number of non-disrupting paths to compute has an impact on the cost of the output of both strategies. Clearly, in both  $G_R$  and  $G_U$ , as  $k$  increases, the performance gap between the strategies increases. In conclusion, our simulation shows that the extra running time in Strategy 2 makes it to outperform Strategy 1 and this becomes more significant as  $k$  and  $|L^*|$  grows.

## 6. Concluding remarks and future works

In this paper, we investigate a new multiple routing path computation problem in a smart grid communication network. We observe that the node failures in the smart grid network can be co-related and the conventional  $k$  node-disjoint routing path computation algorithms do

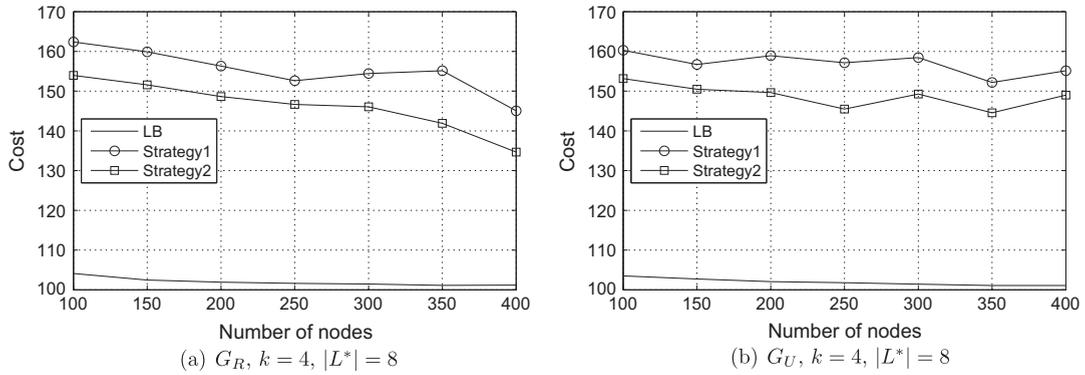


Fig. 5. The maximum length vs. the number of nodes.

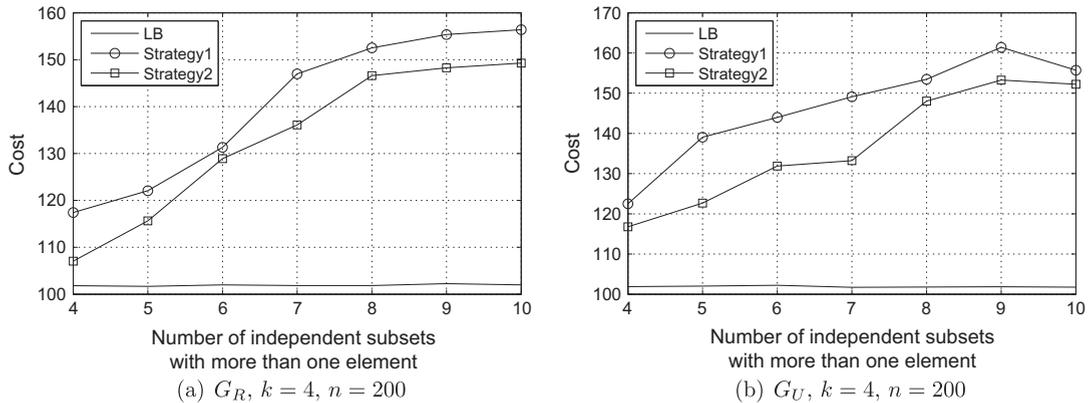


Fig. 6. The maximum length vs. the number of independent subsets with more than one element.

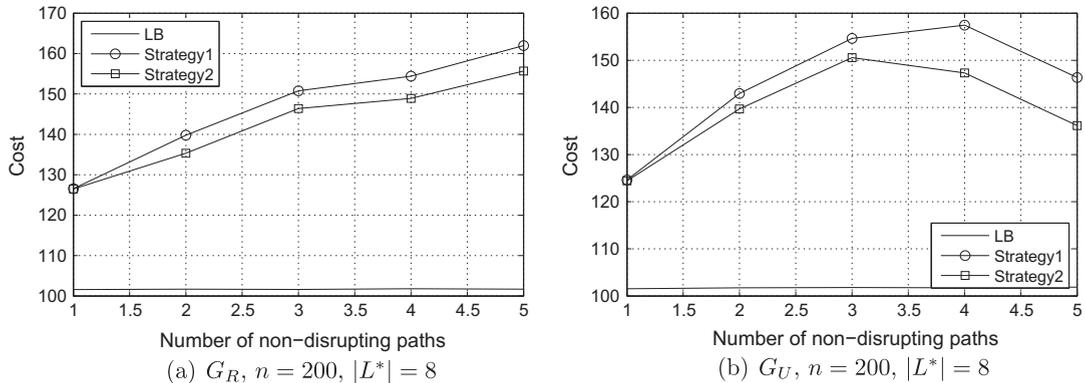


Fig. 7. The maximum length vs. the number of non-disrupting paths.

not provide promised level of reliability under such co-related node failures. We formally define this problem and show it is NP-hard. We also provide heuristic algorithms for the problem and evaluate the average performance of the algorithms via simulation.

In this paper, our algorithms are designed under the assumption that a set of nodes, which are

failure-interdependent, forms a connected subgraph of the original smart grid communication network. While this is highly likely scenario, it is not guaranteed. Therefore, we plan to further investigate this general case. Since the proposed problem of interest of this paper is NP-hard, it is of great theoretical interest to design and analysis approximation algorithm for the problem.

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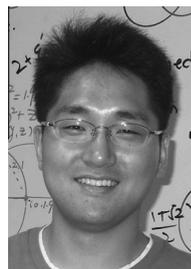
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