Reliability Analysis of Steel Structures with Imperfections

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ABSTRACT: The verification of the design reliability of a steel element according to the concepts of standards EUROCODE 3 and EN1990 is presented in the article. Reliability is assessed by determining the failure probability, which is evaluated employing the Monte Carlo methodology. Reliability analysis enables the verification of reliability indices of the fore mentioned standards, generalization of obtained results and further development of design methods of structures according to the limit state theories. An overview of input random imperfections of steel structures whose histograms and statistical characteristics have been measured and monitored over a long time period is presented. The completeness of statistical information on input data for utilization in probabilistic studies is discussed. The quantification of uncertainty of statistical characteristics of hard to measure imperfections utilizing the fuzzy set theory is illustrated on a numerical example. The analysis of the influence of fuzzy uncertainty of input random variables on the fuzzy uncertainty of failure probability is presented. Uncertainty of computational models is discussed and modern instruments employable in the analysis of these uncertainties are listed. The analysis of the load carrying capacity of a steel plane frame with compression members is presented. Sobol's sensitivity analysis was applied for the identification of the dominant imperfections.

1 INTRODUCTION

At present, the best practical method of structural reliability verification of standards is according to the limit state methods. Standard procedures utilize characteristic and design values, which guarantee design reliability. Design values are obtained with the aid of partial safety factors, which are the basic indicators of reliability of structural design. EUROCODE 3 considers partial safety factors γ_M for resistance. A number of methods exist for the analysis of reliability. Probabilistic methods are frequently used. These methods are based on the assumption that input variables are of random character. This can be clearly demonstrated for e.g. on the strength characteristics of steel, whose histograms have been monitored over a long period and measured data are statistically evaluated and compared within the framework of individual EU countries (Melcher et al. 2004). The fundamental measured characteristic is the yield strength. Other random characteristics include the geometric characteristics for which relatively complete information is available in mass manufactured products. These information prove however insufficient in the elaboration of the probabilistic assessment of reliability of systems of structures comprising of more elements, where the influence of uncertainty of stiffness of external and internal bonds, eccentricity of force action and other imperfections must be taken into consideration. The sensitivity analysis and fuzzy analysis of sensitivity indices of system imperfections are presented in the article. The analysis of the influence of epistemic uncertainty of statistical characteristics of system imperfections is performed.

2 VERIFICATION OF STRUCTURAL STABILITY DESIGN PROCEDURES

2.1 *Reliability conditions of EUROCODES*

A simplified problem of a compression member loaded by permanent load action G combined with single variable load action Q is considered for the elaboration of a parametric study. The reliability condition of design according to EC3 and EN1990 can be written in the form:

$$\gamma_G \cdot G_k + \gamma_Q \cdot Q_k \leq R_{A\chi} \cdot f_{yk} / \gamma_M \tag{1}$$

where $R_{A\chi} = \chi A$ is the product of buckling coefficient χ and nominal cross-section area A, γ_M is the material partial safety factor, and values G_k, Q_k, f_{yk} are characteristic values of load actions and yield strength respectively. Design reliability is ensured by partial safety factors γ. The design reliability condition (6) can be rewritten as the inequality of design load action F_d and design load carrying capacity R_d. It is assumed in the numerical reliability study that the design load action is equal to the design load-carrying capacity,

$$F_d = R_d, \tag{2}$$

i.e., that the structure is designed for economic design with maximum load carrying capacity. Characteristic values G_k, Q_k are expressed by the ratio δ of load action Q_k to the total load action G_k+Q_k:

$$\delta = \frac{Q_k}{G_k + Q_k} \tag{3}$$

Characteristic values G_k and Q_k are calculated according to the relation:

$$1.35 \cdot G_k + 1.5 \cdot Q_k = 382.3\,\text{kN} \tag{4}$$

Equation (4) is derived from (1) for partial safety factors γ_G= 1.35; γ_Q= 1.5 (EN1990) and γ_M= 1.0 (EC3). The value R_d = 382.3 kN on the right side of the equation is the design load carrying capacity of IPE 200 strut of length 2.1m (non-dimensional slenderness $\bar{\lambda} = 1.0$) calculated acc. to EC3:

$$R_d = \frac{\chi_b \cdot f_y \cdot A_n}{\gamma_{M1}} = \frac{0.597 \cdot 235\text{MPa} \cdot 2.7248 \cdot 10^{-3}\,\text{m}^2}{1.0} = 382.3\,\text{kN} \tag{5}$$

where χ_b is buckling coefficient for the buckling strength curve b, A_n is the nominal cross-section area (evaluated from the nominal values of the cross section composed of rectangular segments), and f_{yk} is the characteristic value of yield strength.

2.2 *Probabilistic verification of structural stability design procedures*

The random characteristics of load action G and Q are calculated from characteristic values according to Tab. 1. It may be assumed for the dead load that the characteristic value G_k is also the mean value of the normal distribution. The variation coefficient of 0.1 was presumed according to (Holický et al. 2002, Kala 2007). Gumbel distribution with mean value m_Q = 0.6 Q_k and standard deviation S_Q = 0.21 Q_k was considered for the variable load in accordance with (Holický et al. 2002, Kala 2007). Failure probability of a strut designed according to (Kala 2007) occurs when the reliability condition (6), in which R is the random load-carrying capacity, and G, Q are random load action effects, is not satisfied.

$$G + Q < R \tag{6}$$

The variable quantifying reliability or unreliability is the probability that condition (6) isn't fulfilled during the life span of the structure with regard to structural, aesthetical, service, energetic, economic and ecological aspects. Attainment of the limit state (or in the more general sense, occurrence of failure) cannot be absolutely eliminated (due to technical and economical reasons) and so we try to design the structure so that the probability P_f that failure occurs is very small. The failure probability P_f is the most important and objective indicator of reliability and is commonly related to a certain reference time (usually 50 to 100 years), i.e. to a time interval within which the given degree of reliability should be maintained. The load carrying capacity R in (6) may be obtained from the response function. Load-carrying capacity R in (6) can be evaluated from the response function:

$$\sigma_x = \frac{R}{A} + \frac{R \cdot |e_0|}{(1 - R/F_{cr}) \cdot W_z} = f_y \quad \Rightarrow \quad R \tag{7}$$

where e_0 is the amplitude of initial strut curvature formatively identical to a half-wave of the sine function, A is cross-section area, F_{cr} is Euler's critical force of a bilaterally hinged steel strut, W_z is the sectional modulus to axis Z (axis perpendicular to the flange around which the section bends during buckling), and f_y is the yield strength.

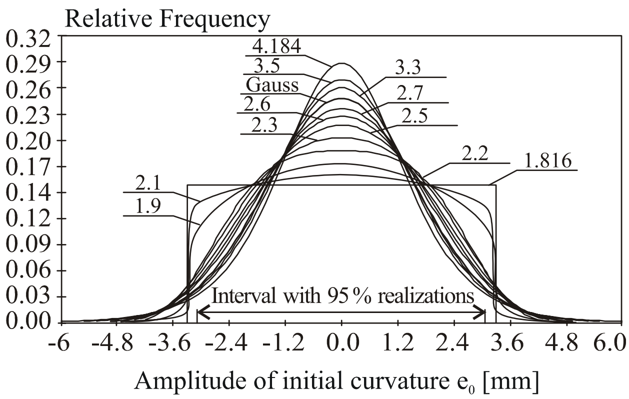
2.3 *Input random quantities*

Statistical characteristics h, b, t_1, t_2, f_y were considered as histograms of obtained results of experimental research (Melcher et al. 2004). Gaussian probability distribution with statistical characteristics given acc. to (Fukumoto et al. 1976, Soares 1988) was considered for Young's modulus.

Table 1. Input random quantities

	Symbol	Distribution	Mean value	St. deviation	Skewness	Kurtosis
1.	h	Gauss	200 mm	0.89 mm	0	3
2.	b	Gauss	100 mm	1 mm	0	3
3.	t_1	Gauss	5.6 mm	0.234 mm	0	3
4.	t_2	Gauss	8.5 mm	0.41 mm	0	3
5.	f_y	Gauss	297.3 MPa	16.8 MPa	0	3
6.	E	Gauss	210 GPa	10 GPa	0	3
7.	e_0	Hermite	0	Fuzzy	0	Fuzzy

The mass was not weighed. The mass variance can be obtained from measured geometrical characteristics. According to results of experimental research (Fukumoto et al. 1976), the dominant shape of initial curvature is given as a half-wave of the sine function. A Hermite four-parametric probability distribution, which makes provision for skewness and kurtosis was considered for the amplitude of initial imperfection e_0, see Figure 1. We know with certainty that the mean value and skewness for symmetrical elements comprised of IPE profiles is equal to zero. Standard deviation of the Hermite density function is designated based on the assumption that 95 % of the realizations of the amplitude of initial imperfection e_0 are found within the tolerance limits ⟨-3.15; 3.15⟩ mm of the standard EN 10034. Kurtosis is given as a fuzzy number, see Figure 1. The support of the membership function is ⟨1.816; 4.184⟩ and the kernel = 3. Input random variables are lucidly listed in Tab. 1.



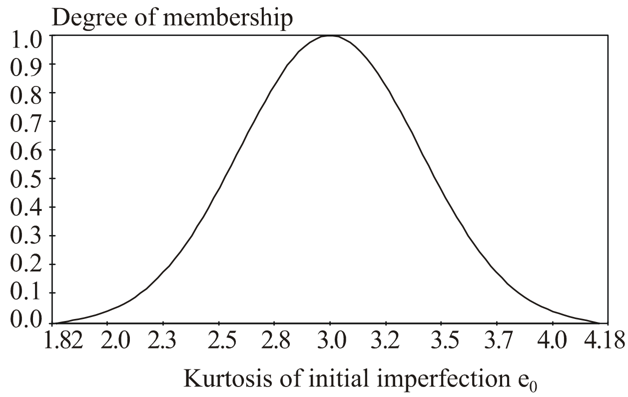


Figure 1. Hermite density distribution function and fuzzy number of kurtosis

The parameters of the four-parametric Hermite density function include: the mean value, standard deviation, skewness and kurtosis. The limit case arises for the kurtosis of 1.816, which corresponds to a rectangular density function. The kurtosis of the Gaussian density function has a value of 3.0. The maximum kurtosis was considered as 4.184, i.e. 3.0 + 3.0 - 1.816 leading to symmetrical minimal and maximal support values around the kernel. An example of a set of density functions is depicted in left part of Figure 1. The functions vary in values of kurtosis, which is listed for each function. The standard deviation of the amplitude of initial imperfection e_0 is also a fuzzy number, see Figure 2. The fuzzy number was evaluated utilizing the general extension principle (Zadeh 1965). Twenty cuts of the so-called α-cut method were utilized. The support of the fuzzy number of the standard deviation is given by the interval $(1.915; 1.565)$mm. The standard deviation of the Gaussian density function is given by the kernel value of 1.607. Even though the input fuzzy number of kurtosis is symmetrical, the fuzzy number of standard deviation is very asymmetrical.

2.4 *Fuzzy random analysis of load-carrying capacity*

The load-carrying capacity was evaluated utilizing equation (8) in which the axial stress was placed equal to the yield strength. The fuzzy analysis was performed utilizing the general extension principle (Dubois 1980). The statistical analysis of the load-carrying capacity was evaluated by means of the Monte Carlo method with 100 000 simulation runs. The histogram of load-carrying capacity was approximated by a Hermite polynomial, see Figure 2. The figure on the right represents the set of density functions of load carrying capacity corresponding to the set of density functions in Fig. 1.

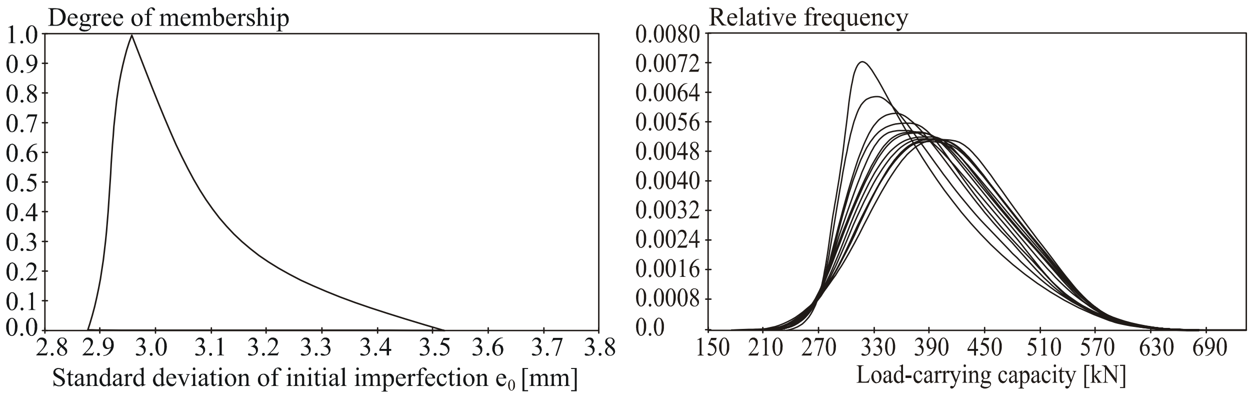


Figure 2. Fuzzy numbers of standard deviation and load-carrying capacity

2.5 *Fuzzy random analysis of failure probability*

The fuzzy analysis of the misalignment of failure probability is performed. Fuzzy analysis is employed for the analysis of fuzzy uncertainty of the shape of the density distribution function of imperfection e_0. Input density functions and degrees of membership are depicted in Figure 1. The failure probability was evaluated for $\delta \in \{0; 0.1; 0.2; ...; 1.0\}$. Sufficient runs of the Monte Carlo method were employed in the probabilistic analysis to ensure that equation (6) was not fulfilled at least 200 times. This guarantees a balanced probability estimation error of 7 %. The procedure is as follows:

- A value of δ (e.g. δ=0) was selected and 11 values of failure probability for the kurtosis value with input degree of membership from Figure 1 were evaluated.
- The functional dependence between kurtosis and failure probability was approximated by a linear spline, i.e., the so-called response surface method.
- The membership function of the failure probability was evaluated according to the general extension principle.

The membership function of failure probability was evaluated for each values of δ in this manner. The discrete values were approximated by the Hermite approximation polynomial.

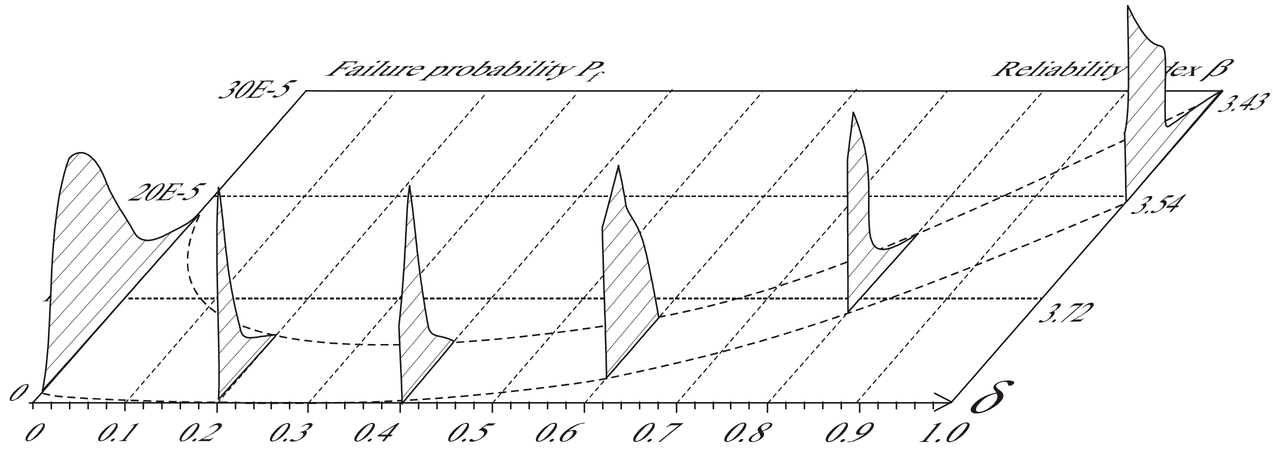


Figure 3. Fuzzy random analysis of misalignment of failure probability

The support and kernel of the failure probability distribution is depicted in Figure 4. The support is indicated by the dot-and-dash curves, and the kernel is depicted by a red solid curve. The main output of the study is the dashed curve, which was obtained utilizing the COG (centre of gravity) defuzzification method.

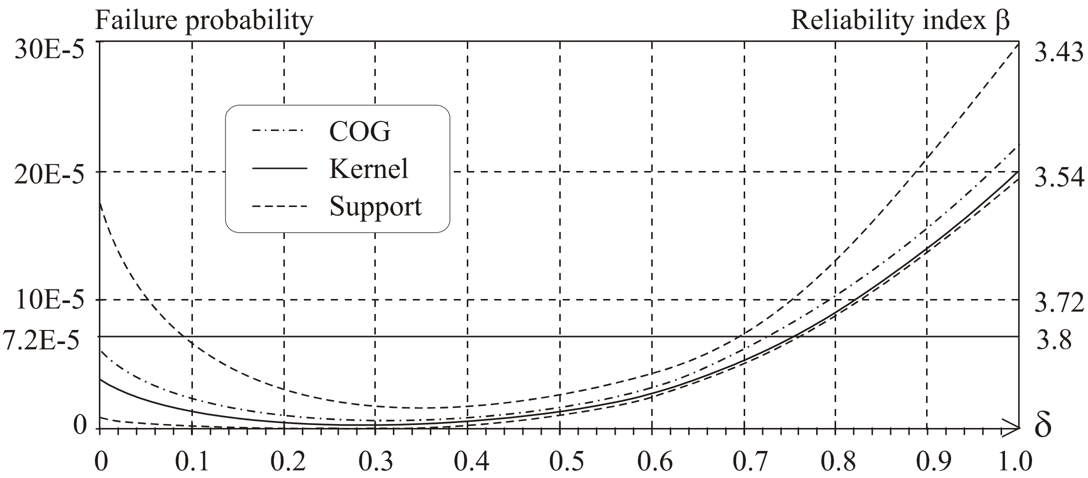


Figure 4. Support, kernel and greatest crisp control output of the fuzzy analysis of failure probability

2.6 *Conclusion remarks*

The presented study illustrates the fuzzy uncertainty of failure probability resulting from the vague (fuzzy) uncertainty of kurtosis of the random amplitude of initial member curvature e_0. Discrepancies between results are considerable and advert to the necessity of fuzzy analysis whenever the input random variables are assigned subjectively. It is evident that the defuzzified values (centroids) represented graphically by the dashed curves were higher than the values evaluated by means of purely stochastic analysis (kernels) represented by the full curves in all cases. The non-linear membership functions despite the symmetric input membership function of kurtosis are apparent from Figure 3. Results of the application of probabilistic analysis point out the significant discrepancies of design reliability of steel structures acc. to the EUROCODE concept, from which the need for further calibration of reliability indices arises.

3 ULTIMATE LIMIT STATE AND SENSITIVITY ANALYSIS OF A STEEL PLANE FRAME

3.1 *Computational model*

The second study is aimed at the analysis of the ultimate limit state of a steel plane frame, see Figure 5. The analysis of the limit state requires utilization of the geometrical non-linear solution. The frame geometry was modelled using beam elements with initial curvature in the form of a parabola of the 3rd degree. The geometrical non-linear Euler incremental method combined with the Newton-Raphson method was employed (Kala 2005).

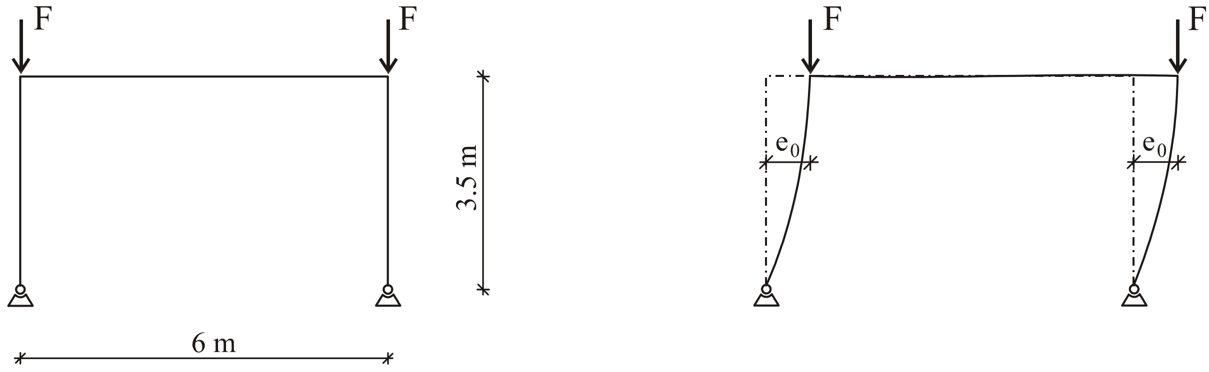


Figure 5. Frame geometry and shape of the first eigen mode of buckling

3.2 *Input random quantities*

Experimentally obtained material and geometrical characteristics of steel products of a dominant Czech producer, see (Melcher et al. 2004) were utilized for the analysis of the problem. For non-measured quantities (e.g., Young's modulus), the study was based on data obtained from technical literature; for e.g. statistical characteristics of Young's modulus are listed in (Fukumoto et al. 1976, Soares 1988). The statistical characteristics of input quantities are listed in Tab. 1. All input random quantities are considered statistically independent.

Table 1. Input random quantities

Symbol		Meaning	Probability distribution	Mean Value	Standard deviation
h_1	Left Column	Cross-sectional height	Histogram	270.27 mm	1.196 mm
b_1	Left Column	Cross-sectional width	Histogram	136.81 mm	1.341 mm
t_{w1}	Left Column	Web thickness	Histogram	6.963 mm	0.277 mm
t_{f1}	Left Column	Flange thickness	Histogram	10.126 mm	0.466 mm
f_{y1}	Left Column	Yield strength	Histogram	297.3 MPa	16.8 MPa
E_1	Left Column	Young's modulus	Gauss	210 GPa	12.6 GPa
h_0	Cross beam	Cross-sectional height	Histogram	360.36 mm	1.595 mm
b_0	Cross beam	Cross-sectional width	Histogram	172.3 mm	1.689 mm
t_{w0}	Cross beam	Web thickness	Histogram	8.44 mm	0.335 mm
t_{f0}	Cross beam	Flange thickness	Histogram	12.611mm	0.582 mm
f_{y0}	Cross beam	Yield strength	Histogram	297.3 MPa	16.8 MPa
E_0	Cross beam	Young's modulus	Gauss	210 GPa	12.6 GPa
h_2	Right Column	Cross-sectional height	Histogram	270.27 mm	1.196 mm
b_2	Right Column	Cross-sectional width	Histogram	136.81 mm	1.341 mm
t_{w2}	Right Column	Web thickness	Histogram	6.963 mm	0.277 mm
t_{f2}	Right Column	Flange thickness	Histogram	10.126 mm	0.466 mm
f_{y2}	Right Column	Yield strength	Histogram	297.3 MPa	16.8 MPa
E_2	Right Column	Young's modulus	Gauss	210 GPa	12.6 GPa
e_0		System imperfection	Gauss	0	3.5 mm

3.3 *Sensitivity analysis results*

The Sobol's sensitivity analysis belongs among the variance based methods, which provide more complex information on the sensitivity of an output quantity to an input quantity than, for e.g., the correlation analysis between the input and output (Sobol 1990). The Monte Carlo method was employed for the calculation of sensitivity indices (Saltelli 2004). The model output Y is the load carrying capacity calculated in each simulation run of the Monte Carlo method. 5000 simulation runs were applied in our study. Results of sensitivity analysis depicted in Figure 6 illustrate that the interaction of higher orders are very small and that the dominant variable is that of the system imperfection e_0.

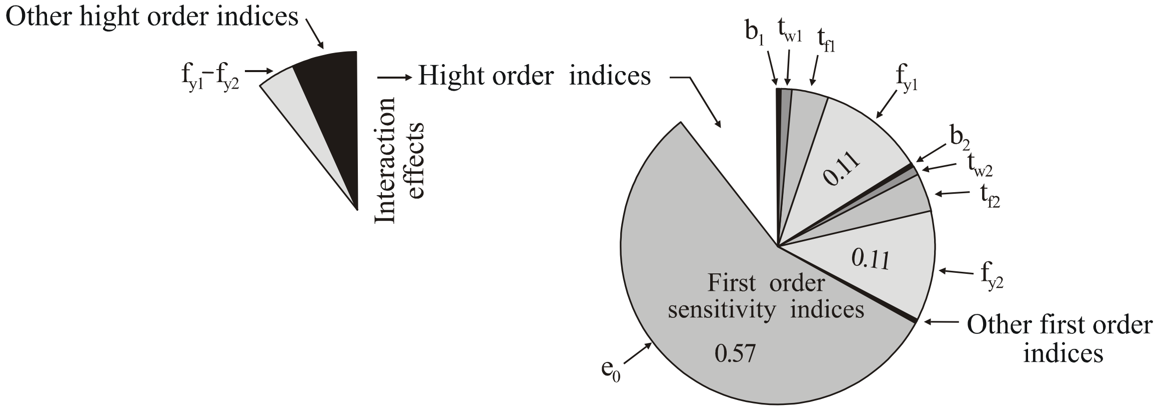


Figure 6. Sobol sensitivity analysis results

Acquired results of the sensitivity analysis illustrate that the statistical characteristic of system imperfections should be determined with increased accuracy, which is however difficult or practically impossible in heavy service conditions. The uncertainty occurring in this case is not of stochastic character. Detailed statistical information on system imperfection would require information on the verticality of each column and also information on the initial axial curvature of each column.

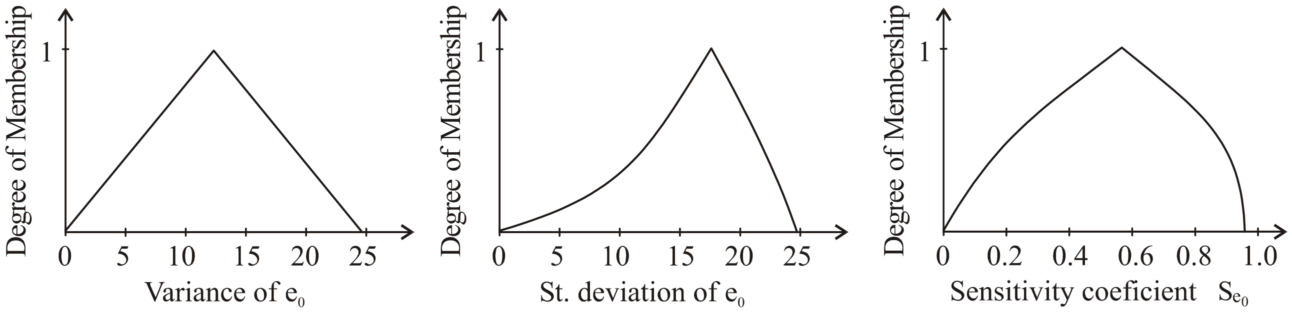


Figure 7. Sobol sensitivity analysis results with used fuzzy logic

The fuzzy logic was applied (Zadeh 1965). The fuzzy uncertainty of initial system imperfection of the frame modelled using the first eigen mode buckling shape is in the variance of e_0. The mean value is with certainty equal to zero for a symmetrical frame, i.e. it can be considered as a singleton of value zero. Due to the fact that the frame is symmetrical, the skewness is also equal to zero. For the fuzzy analysis, we shall consider that the initial system imperfection e_0 has a Gaussian density probability function and that the variance of e_0 is a fuzzy number with symmetrical membership function, See Figure 7. The fuzzy numbers of sensitivity indices were determined using the general extension principle (Dubois 1980). The fuzzy number of the first order sensitivity coefficient of imperfections e_0 is shown in Figure 7.

3.4 *Conclusion remarks*

It is clear from obtained sensitivity analysis results that system imperfections have a dominant influence on the variability of the load carrying capacity of the analysed frame. Since this variable is burdened with relatively high epistemic uncertainty, the variance of imperfection e_0 was considered as a fuzzy number with triangular membership function. The result of the fuzzy analysis is a nonlinear and asymmetrical membership function of the sensitivity index of system imperfection despite the linear and symmetric membership function of the variance of the imperfection e_0.

4 GENERAL CONCLUSION

Results of the application of probabilistic analysis point out the significant discrepancies of design reliability of steel structures acc. to the EUROCODE concept, from which the need for further calibration of reliability indices arises. Fuzzy analysis of failure probability illustrates that the discrepancies of failure probability may be significantly covered by the fuzzy uncertainty of input data and computational procedures, which may significantly prevail over stochastic uncertainty in complex structures. It is necessary to continue monitoring material and geometrical characteristics of industrially produced structures and aim at r a wider cooperation and collaboration of specialists of individual fields. In the future it is necessary to continue in theoretical studies of reliability of commonly produced bearing structures and in perfecting methods of reliability analysis.

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