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Integrating noncyclical preventive maintenance scheduling and production planning for a single machine

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A B S T R A C T

This paper deals with the problem of integrating noncyclical preventive maintenance and tactical production planning for a single machine. We are given a set of products that must be produced in lots during a specified finite planning horizon. The maintenance policy suggests possible preventive replacements at the beginning of each production planning period, and minimal repair at machine failure. The proposed model determines simultaneously the optimal production plan and the instants of preventive maintenance actions. The objective is to minimize the sum of preventive and corrective maintenance costs, setup costs, holding costs, backorder costs and production costs, while satisfying the demand for all products over the entire horizon. The problem is solved by comparing the results of several multi-product capacitated lot-sizing problems. The value of the integration and that of using noncyclical preventive maintenance when the demand varies from one period to another are illustrated through a numerical example and validated by a design of experiment. The later has shown that the integration of maintenance and production planning can reduce the total maintenance and production cost and the removal of periodicity constraint is directly affected by the demand fluctuation and can also reduce the total maintenance and production cost.

1. Introduction

1.1. Motivation

Harmony between maintenance and production departments is necessary for the success of modern companies. These two activities are clearly linked and, together, contribute to the improvement of the profit margin and the company's effectiveness. However, in many cases, their relationship may become conflictual, since they share the same equipments. The production department has to satisfy customer demands within promised delays and service levels. If a production manager promises to a customer the satisfaction of his demand with a given service level, it is important to honor her/his promise in a timely manner. Thus, the production department pushes for the maximal use of the production equipments. However, the maintenance department should try to keep these equipments in good conditions through preventive actions. This antagonist environment promotes the lack of communication and internal conflict during the planning process. The synchronisation between the production planning and preventive maintenance (PM) activities may avoid failure, production delays and re-planning problems. The maintenance planning should be simultaneously planned with the production planning and scheduling in order to decrease the costs generated by the production interruptions.

1.2. Prior literature

There are a lot of papers in the literature dealing with tactical production planning issues. For example, in Argoneto et al. (2008), the authors cover the majority of advancement in this research area. The problem consists generally in minimizing inventory, production and set-up costs under machine capacities and demand satisfaction constraints. Solution methodologies for corresponding multi-product capacitated lot-sizing problems vary from traditional linear mixed integer programming, and associated branch and bound exact methods to heuristic methods.

Similarly, several maintenance planning models can be found in the literature. The advancement in this area is covered, for example, in Garg and Deshmukh (2006) where the authors present an interesting classification, based on the modeling approach used for the problem formulation, such as Bayesian approach, mixed integer linear programming, fuzzy approach, simulation, Markovian probabilistic models and analytic hierarchy process. These models are generally solved using optimization techniques

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to minimize equipment maintenance costs, or to maximize the equipment availability. Many preventive maintenance models are presented in a cyclic (or periodic) context. In Grigorieva et al. (2006), the authors present a literature review about periodic preventive maintenance problems. The periodic aspect of PM consists in a repetitive execution of the same optimal maintenance service (for the optimal preventive maintenance interval) in the time horizon. There is only a relatively limited literature on models presenting a general (*i.e.*, not necessarily periodic) preventive maintenance policy. The objective of these models is to determine either the best time for doing preventive replacements by new items, *i.e.*, perfect PM (Yao et al., 2004), or the optimal sequence for imperfect maintenance actions (Levitin and Lisnianski, 2000).

Budai et al. (2006) reviewed the majority of integrated maintenance and production models, and subdivided the research area into four categories: high level models, the economic manufacturing quantity models, models of production systems with buffer and finally production and maintenance rate optimization models. Cassady and Kutangolu (2005) proposed an integrated maintenance planning and production scheduling for a single machine, in order to find the optimal PM actions and job sequence minimizing the total weighted expected completion time. This model was solved by using genetic algorithms by Sortrakul et al. (2005). In Ashayeri et al. (1996), a mixed-integer linear programming model is developed to simultaneously plan preventive maintenance and production in a process industry environment. The model schedules production jobs and preventive maintenance jobs, while minimizing costs associated with production, backorders, corrective maintenance and preventive maintenance. The performance of the model is discussed and a branching solution procedure is suggested. Chelbi et al. (2008) proposed an integrated production and maintenance strategy for unreliable production system. The presented model focused on finding simultaneously the optimal value of the production lot size and the optimal preventive replacement interval, while considering the possibility of producing non-conform items. Song (2009) considered the problem of production and preventive maintenance control in a stochastic manufacturing system. The system is subject to multiple uncertainties such as random customer demands, machine failure and repair and stochastic processing times. A threshold-type policy is proposed to control the production rate and the preventive maintenance operation simultaneously. Jin et al. (2009) introduced a new methodology based on the financial stock options principles to maximize the average profit under uncertain demand, by generating the optimal number of PM works during the production plan. Chung et al. (2009a, 2009b) used the reliability acceptation function to minimize the production makespan in a multi-factory context. Berrichi et al. (2010) presented a mathematical model minimizing the production makespan and the system unavailability for parallel machine systems.

At the tactical level, there are only a few papers discussing the issue of combining preventive maintenance and production planning. Weinstein and Chung (1999) examined the integration of maintenance and production decisions in hierarchical planning environment. In Aghezzaf et al. (2007), the authors present a production and maintenance planning model for a production system modeled as a single component, subject to cyclical PM with minimal repair at failure. An approximate algorithm based on Lagrangian decomposition is suggested in Aghezzaf and Najid (2008) to solve this problem for both cyclical, and noncyclical PM policies. In Nourelfath et al. (2010), the authors develop an integrated model for production and PM planning in multi-state systems.

1.3. Objective and outline

The present paper contributes to this small literature body on the integration of PM and production planning at the tactical level. The maintenance policy suggests possible preventive replacements at the beginning of each production planning period, and minimal repair at machine failure. This PM policy is said to be general, in the sense that it can be either cyclical or noncyclical. The production planning part corresponds to a multi-product capacitated lot-sizing problem. At this level, the decisions involve determination of quantities of items (lot sizes) to be produced in each period. Lot-sizing is one of the most important problems in production planning. Almost all manufacturing situations involving a product-line contain capacitated lot-sizing problems, especially in the context of batch production systems. The setting of lot sizes is in fact usually considered as a decision related to tactical planning, which is a medium-term activity. In aggregate planning, the lot sizing models are extended by including labor resource decisions. Tactical planning bridges the transition from the strategic planning level (long-term) to the operational planning level (short-term). Clearly, the time horizons may vary for each planning level depending on the industry. Typical values are one week (or less) for operational planning; one month (or more) for tactical planning; one year (or more) for strategic planning. In several modern production systems, the components are usually reliable and PM decisions should be integrated at the tactical level.

Unlike Weinstein and Chung (1999), we are not dealing with this problem in hierarchical planning environment. While the models in Aghezzaf et al. (2007) and Nourelfath et al. (2010) deal with cyclical PM, the present paper takes into account the possibility of noncyclical PM. To the best of our knowledge, the only existing model that deals with the same problem is the model in Aghezzaf and Najid (2008). The later assumes that maintenance actions carried out on the production system reduce its capacity without calculating this reduction. The model developed in this paper is different, and a method is proposed to evaluate the capacity reduction, the times and the costs of PM and minimal repair and the average production system capacity in each period.

The remainder of the paper is organized as follows. The next section describes the problem. Sections 3 and 4 develop, respectively, the mathematical model and the solution method. An illustrative example is presented in Section 5. A design of experiment is realized in Section 6, and conclusions are in Section 7.

2. Problem description

2.1. The preventive maintenance scheduling problem

Let us consider a single machine in a manufacturing system that is subject to random failures. Planned preventive maintenance and unplanned corrective maintenance can be performed on the machine. Whenever an unplanned machine failure occurs, a minimal repair (MR) is carried out, *i.e.*, the machine is restored to an operating condition without altering its age. In practice, MR happens when the machine operator does just enough maintenance to make the machine operable. Furthermore, we consider that the machine's hazard rate increases with the time, so that preventive maintenance is used to decrease the risk of failure. It is assumed that PM either restores the machine to "as-good-as-new" condition (perfect PM), or replaces the machine by a new one. We will sometimes refer to such perfect PM as preventive replacement (PR). We consider a general model, in the sense that it is possible to apply cyclical or noncyclical PR. The machine is considered as a binary-state system. It is characterized by its own nominal production rate, and its expected preventive and corrective maintenance times and costs. The expected maintenance cost during the planning horizon is the sum of preventive and corrective maintenance costs during each period. Given the failure, repair and PM characteristics of the machine, the objective of maintenance planning is to achieve an optimal compromise between PR and MR by scheduling PR actions to minimize the total maintenance cost.

2.2. The production planning problem

Suppose the machine is required to produce a set of products P, during a given planning horizon H including T periods. All periods have the same fixed length L. For each product $p \in P$, a demand d_{pt} is to be satisfied at the end of period t ($t=1, 2, \ldots, T$). The studied production planning problem consists of a multi-product capacitated lot-sizing problem. The decisions involve determination of quantities of items (lot sizes) to be produced in each period. The objective function minimizes the cost, while satisfying the demand for all products over the entire horizon. The constraints are related to the dynamics of the inventory and the backorder, the setup and the machine capacity.

2.3. The integrated problem

Suppose a machine possesses the failure, repair and PM characteristics defined in Section 2.1 and the production requirements described in Section 2.2. Furthermore, we assume that a replacement can be performed at the beginning of any planning period, except for the first period and at the end of the last period. Intuitively, separate production and maintenance plans are not always optimal with respect to the objective of minimizing the combined maintenance and production cost, and their combination may therefore reduce the total expected cost. Because of this, we propose an integrated model that solves the production planning and the PM scheduling simultaneously. This model allows us to determine jointly the optimal values of production and maintenance plans. The link between the production and maintenance is the machine capacity. The later is given by the machine production rate, which depends on the number of failures and on PM occurrences.

3. Mathematical model

Before presenting the integrated model, we evaluate the total maintenance cost and the production capacity of the machine as a function of failure, repair and PM characteristics.

3.1. Evaluating the maintenance cost and the production capacity reduction

Because we assume repair is minimal, we can model the occurrence of failures during $[0, x[$ using a non-homogeneous Poisson process. Then, the expected number of failures during $[0, x[$ is given by

$$M[0,x[= \int_0^x r(y)dy, \tag{1}$$

where $r(y)$ is the failure rate of the machine obtained from its probability density function $f(y)$ as follows:

$$r(y) = \frac{f(y)}{\int_y^\infty f(x)dx}. \tag{2}$$

Let us define the following binary variable:

$$z_t = \begin{cases} 1 & \text{if a PM action is performed at the beginning of period } t \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

We denote by a_t the effective age of the machine at the beginning of period t. Because we consider minimal repair and perfect PM, we have

$$a_t = (1-z_t)(a_{t-1}+L). \tag{4}$$

Eq. (4) expresses that if no PM is performed at the beginning of a period t ($z_t=0$), the effective age of the machine is the sum of its previous age with period length L. In contrast, when a PM is carried out ($z_t=1$), the machine's age becomes zero (because the machine is renewed). The vector $\mathbf{z}=\{z_t\}$ defines the machine PR decision binary variables ($t=1, 2, \ldots, T$).

If CMR and CPR are, respectively, the given expected MR cost and the PR cost of the machine, the expected maintenance cost during the planning horizon can be determined using the expected number of failure during each production planning period according to the following equation:

$$\begin{aligned} CM(\mathbf{z}) &= \sum_{t=1}^{T}(CPRz_t + CMRM[a_t,a_t+L[) \\ &= \sum_{t=1}^{T}\left(CPRz_t + CMR\int_{a_t}^{a_t+L} r(y)dy\right). \end{aligned} \tag{5}$$

To evaluate the average production rate of the machine during each period t, it is necessary to estimate the average availability of the machine per period. We assume that the length L is large enough, so that we can consider that a stationary regime is reached during each period. This assumption is realistic since a typical value for a tactical planning period in industry is one month or more. We denote by $A_t(\mathbf{z})$ the steady-state availability during a period t, which depends on the PM vector $\mathbf{z}$. More specifically, this availability depends on the expected number of failures during $[(t-1)L, tL[$, and on the occurrence or no. of preventive maintenance at the beginning of period t. In fact, within the time period $[(t-1)L, tL[$, the machine is expected to fail a number of times and be minimally repaired. Furthermore, if a preventive replacement is performed, the component will be unavailable. Let TPR and TMR, respectively, be the expected PR and the MR times of the machine. It follows that

$$A_t(\mathbf{z}) = \frac{L - TPRz_t - TMR\int_{a_t}^{a_t+L} r(y)dy}{L}. \tag{6}$$

Finally, if we denote by g the machine nominal production rate, the average production rate of the machine during a period t is

$$G_t(\mathbf{z}) = g(1-\alpha), \tag{7}$$

where $\alpha = TPR\ z_t + TMR\ M[a_t,a_t+L[/L$ is the capacity reduction factor.

3.2. The integrated model

The following additional notations are used:

- h_{pt} inventory holding cost per unit of product p by the end of period t
- b_{pt} backorder cost (lost opportunity and goodwill) per unit of product p by the end of period t
- Set_{pt} fixed set-up cost of producing product p in period t
- π_{pt} variable cost of producing one unit of product p in period t

The decision variables are as follows:

- z_t binary variables that are elements of the preventive maintenance vector $\mathbf{z}$
- x_{pt} quantity of product p to be produced in period t
- I_{pt} inventory level of product p at the end of period t

B_{pt} backorder level of product p at the end of period t
y_{pt} binary variable, which is equal to 1 if the setup of product p occurs at the end of period t, and 0 otherwise.

The integrated model can be stated as follows:

$$\text{Minimize} \sum_{p \in P}\sum_{t=1}^{T}(h_{pt}I_{pt}+b_{pt}B_{pt}+\pi_{pt}x_{pt}+Set_{pt}y_{pt})+CM(\mathbf{z}), \quad (8)$$

$$\text{Subject to } I_{pt}-B_{pt}=I_{pt-1}-B_{pt-1}+x_{pt}-d_{pt}, \quad p \in P, \quad t=1,2,\ldots,T \quad (9)$$

$$x_{pt} \le \left(\sum_{q \ge t} d_{pq}\right) y_{pt}, \quad p \in P, \quad t=1,2,\ldots,T, \quad (10)$$

$$\sum_{p \in P} x_{pt} \le LGt(\mathbf{z}), \quad t=1,\ldots,T, \quad (11)$$

$$x_{pt}, I_{pt} \text{ and } B_{pt} \text{ integer}, \quad p \in P, \quad t=1,2,\ldots,T \quad (12)$$

$$y_{pt} \text{ binary}, \quad p \in P, \quad t=1,2,\ldots,T \quad (13)$$

$$z_t \text{ binary}, \quad t=1,2,\ldots,T. \quad (14)$$

The objective function (8) consists of a total holding cost of the inventory, a backorder cost (backlogs are allowed), a total production cost, a total setup cost and a total maintenance cost $CM(\mathbf{z})$ as given by Eq. (5). The first constraint (9) relates inventory or backorder at the start and end of period t to the production and demand in that period. Eq. (9) ensures simply that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. The second constraint (10) forces $x_{pt}=0$ if $y_{pt}=0$ and frees $x_{pt} \ge 0$ if $y_{pt}=1$. In Eq. (10), the quantity $(\sum_{q \ge t} d_{pq})$ is an upper bound of x_{pt}. Eq. (11) corresponds to the constraint of the available production capacity $G_t(\mathbf{z})$ given by Eq. (7).

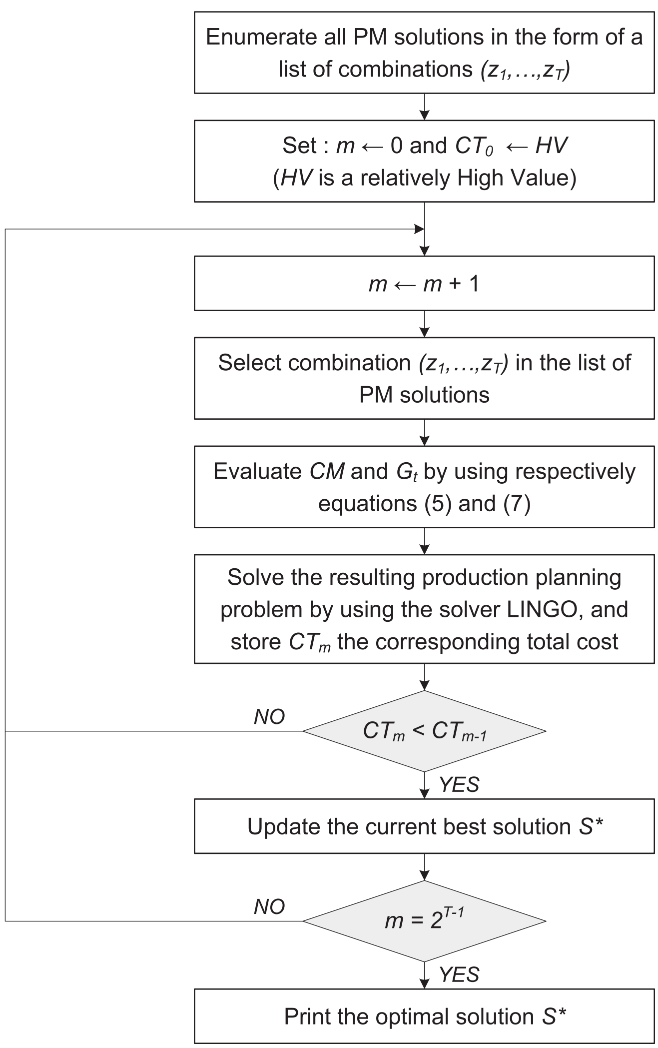

Fig. 1. Solution algorithm flowchart.

4. Solution method

In the mixed integer problem formulated by (8)–(14), for each product p and for each period t, the decision variables are x_{pt}, I_{pt}, B_{pt}, y_{pt} and z_t. Each possible PM action is represented by a combination of the binary decision variables $(z_1,\ldots,z_T)$, which determines the number and the instants of the PM actions. For a given combination $(z_1,\ldots,z_T)$, the model (8)–(14) becomes a mixed integer linear production planning problem corresponding to the classical capacitated lot-sizing problem, which can be solved using any existing algorithm or mixed integer solver (LINGO, CPLEX or MATLAB for example). Our solution method consists of evaluating all the PM alternatives. This method can be represented by the flowchart of Fig. 1. To determine the integrated production and maintenance plans, the maximum number of mixed integer linear planning problems to be solved is given by the number of alternatives for PM solutions. Since each decision variable z_t $(t=1,\ldots,T)$ can have only two values, 0 or 1, (except for z_1, which equal to 1 because a new replacement is considered in the beginning of the planning horizon), the maximum number of combinations $(z_1,\ldots,z_T)$, as well as the number of LIP problems to solve is $N=2^{T-1}$. We remark also that for the N possible combinations representing the maintenance policy, only T combinations generate cyclical preventive maintenance plans.

5. Numerical example

5.1. Problem data

Let us consider a machine for which the characteristics are given in Table 1. The planning horizon H is 8 months composed of 8 periods ($L=1$ month). The machine has to produce two kinds of products in lots so that the demands are satisfied. For each product, the periodic demands are presented in Table 2. Table 3 gives the holding, backorder, set-up and production costs for each product. These costs are the same for all periods. Finally, we assume that the lifetime of the machine is distributed according to Weibull distribution with parameters (2, 2) in order to obtain the expected failure number per period (monthly) using the method in Section 3.1.

5.2. Results and discussion

In this part, we show that integrating PM and production planning reduces the total cost for both cyclical and noncyclical PM strategies. We focus on the PM periodicity constraint and its

Table 1
Characteristics of the machine.

g (items/month)	CPR ($)	CMR ($)	TPR (month)	TMR (month)
50	4000	1000	0.020	0.090

Table 2
Demands of products.

Period t	Demand of product 1 d_{1t} (items)	Demand of product 2 d_{2t} (items)
1	22	25
2	22	25
3	22	22
4	22	25
5	23	23
6	22	22
6	20	20
8	20	20

Table 3
Cost data of products.

Product p	Holding cost h_{pt} ($)	Backorder cost b_{pt} ($)	Set-up cost Set_{pt} ($)	Production cost π_{pt} ($)
1	40	240	1000	90
2	40	240	1000	90

Table 4
Evaluation of costs for each cyclical PM solution.

z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	Maintenance cost ($)	Production cost ($)	Total cost ($)
1	1	1	1	1	1	1	1	34000	**47950**	81950
1	0	1	0	1	0	1	0	20000	48230	68230
1	0	0	1	0	0	1	0	17500	49150	**66650**
1	0	0	0	1	0	0	0	**16000**	51790	67790
1	0	0	0	0	1	0	0	16500	56350	72850
1	0	0	0	0	0	1	0	18000	61520	79520
1	0	0	0	0	0	0	1	20500	64790	85290
1	0	0	0	0	0	0	0	20000	66150	86150

relationship with the demand. The value of using noncyclical preventive maintenance when the demand varies from a period to another will be then illustrated. Note that the numerical example solution has been realised with MATLAB and the execution time was 4.82 s using a 2.33 GHz processor.

5.2.1. Cost reduction by integrating cyclical PM and production planning

For 8 periods, we have only 8 possible periodic PM alternatives knowing that a preventive replacement is operated at the beginning of the planning horizon ($z_1=1$). Table 4 presents, for each periodic PM solution represented by a combination of ($z_1,\ldots,z_8$), the values of the total maintenance cost, the total production cost and the total cost (*i.e.*, the sum of total maintenance and production costs).

If we examine the pure production costs, we observe that the minimal production cost is obtained for $\mathbf{z}=(1,1,1,1,1,1,1,1)$. The minimal maintenance cost is obtained for $\mathbf{z}=(1,0,0,0,1,0,0,0)$. The total costs associated with each of these plans when both total production and maintenance costs are considered are, respectively, 81950$ and 67790$. However, the total cost of an optimal integrated production and cyclical maintenance plan is reduced to 66650$ and is obtained for $\mathbf{z}=(1,0,0,1,0,0,1,0)$. In other words, the optimal integrated production and periodic maintenance plan suggests that a preventive maintenance is performed for the machine every 3 months, and it is minimally repaired at failure. Table 5 shows the optimal production plan for the two products with integrating production and PM planning. From Table 4, we can calculate that this integration reduces the total cost by about 1.7%.

5.2.2. Cost reduction by integrating noncyclical PM and production planning

If we remove the periodicity constraint in the PM plan, for 8 periods, there are $N=2^7=128$ possible PM solutions, represented by the combinations ($z_1,\ldots,z_8$), including the 8 periodic combinations presented in Table 4. Table 6 presents the combinations ($z_1,\ldots,z_8$) that optimize the total maintenance cost, the pure total production cost and the total cost.

As it shown in Table 6, the minimal production cost is obtained for the combination (1,1,0,1,0,0,0,0), and the minimal maintenance cost is obtained for (1,0,0,0,1,0,0,0). The total cost associated with each of these plans are, respectively, 69450$ and 67790$. However, the total cost of an optimal integrated production and maintenance planning is 65690$, and it is obtained for the combination (1,0,0,1,0,0,0,0). It follows that the integration of noncyclical preventive maintenance and production planning reduces the total cost by 3.2%. The corresponding optimal production plan is given in Table 7.

5.2.3. PM periodicity constraint and demand influence

In the previous example, we remark from Tables 4 and 6 that

- The optimal maintenance cost is the same for cyclical and noncyclical PM cases. It is equal to 16000$ and it is obtained for the combination (1,0,0,0,1,0,0,0), which means that the optimal replacement period is 4 months.
- For noncyclical PM policy, the optimal total cost is equal to 65690$ and it is obtained for the combination (1,0,0,1,0,0,0,0).
- For the cyclical case, the optimal total cost is equal to 66650$ and it is obtained for the combination.

The optimal total cost obtained for the general PM policy is then lower than for the cyclical policy. In fact, the removal of the PM periodicity constraint reduces the total cost by 1.5%. This cost reduction is explained by the dissimilarity between the demand tendency, and the nominal capacity generated by the optimal plan with periodicity constraint. As it is illustrated in Fig. 2, the optimal integrated production and cyclical maintenance plan (1,0,0,1,0,0,1,0) suggests a preventive replacement every 3 months. The nominal capacity associated with this plan is similar to the demand for the first 6 periods, same as the optimal integrated production and noncyclical PM plan (1,0,0,1,0,0,0,0). Starting from the 7th period, the preventive replacement imposed by the periodicity constraint makes the nominal capacity associated to the periodic plan dissimilar with the demand, while the nominal capacity associated to the general PM plan is still following the demand. As a result, the cost of the preventive replacement operating at the beginning of the 7th period is higher than the cost generated when removing the periodicity constraint, which leads to more inventory and more corrective maintenance.

As it is illustrated in Fig. 3, for the non integrated plan, the demand is affecting only the pure production optimization via the flow conservation constraint. The optimal total plan can be obtained by first optimizing the maintenance cost (thus no demand influence), then optimizing the pure production plan. The demand influence is higher in the case of integrated production and cyclical PM strategy. In this case, the integration of production and PM planning may lead to a better deal between preventive maintenance and production. Due to the demand tendency, the surplus of maintenance costs may be compensated by a lower production cost. Finally by removing the periodicity constraint, the integration of the preventive maintenance and

Table 5
Optimal production plan when integrating production and cyclical PM.

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	22	0	0	1	25	0	0	1
2	21	0	1	1	25	0	0	1
3	23	0	0	1	21	0	1	1
4	22	0	0	1	25	0	1	1
5	22	0	1	1	24	0	0	1
6	23	0	0	1	21	0	1	1
7	20	0	0	1	21	0	0	1
8	20	0	0	1	20	0	0	1

Table 6
Costs obtained for noncyclical PM.

Plan minimizing	Preventive maintenance vector								Maintenance cost ($)	Production cost ($)	Total cost ($)
	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8			
Maintenance cost	1	0	0	0	1	0	0	0	**16000**	51790	67790
Production cost	1	1	0	1	0	0	0	1	21500	**47950**	69450
Total cost	1	0	0	1	0	0	0	0	16500	49190	**65690**

Table 7
Optimal production plan where integrating production and general PM.

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	22	0	0	1	25	0	0	1
2	21	0	1	1	25	0	0	1
3	23	0	0	1	21	0	1	1
4	21	0	1	1	26	0	0	1
5	23	0	1	1	23	0	0	1
6	22	0	1	1	22	0	0	1
7	21	0	0	1	21	1	0	1
8	20	0	0	1	19	0	0	1

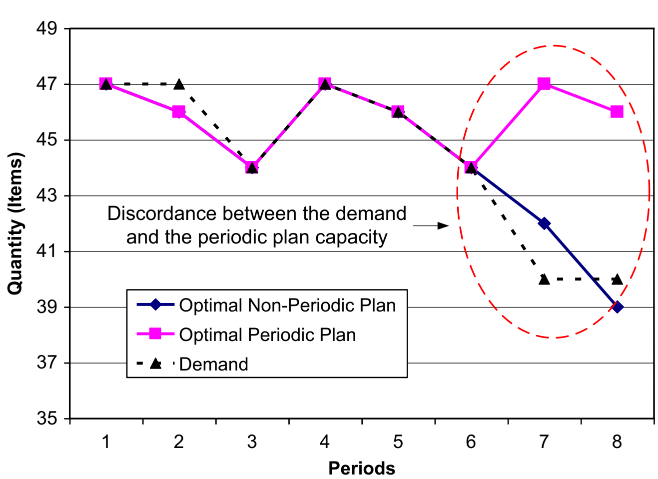

Fig. 2. Dissimilarity between the demand and the optimal cyclical PM plan capacity.

production planning can generate a better matching between the nominal system capacity and the demand, which may reduce the production costs (less inventory and backorder). In that case, the demand tendency is affecting the production planning, the maintenance planning and also the nominal system capacity (see Fig. 3).

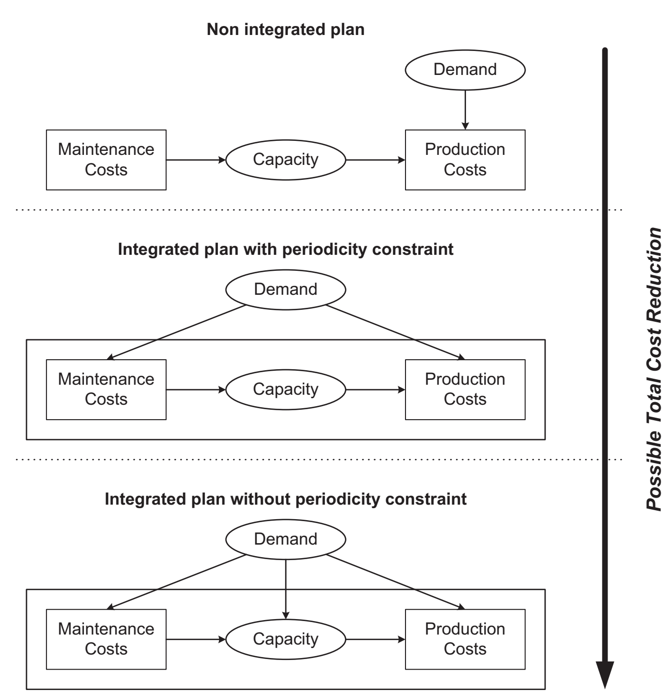

Fig. 3. Demand influence for the different production and preventive maintenance plans.

6. Design of experiment

The numerical example presented in the previous section has illustrated that

- The integration of maintenance and production planning can reduce the total maintenance and production cost;
- The removal of periodicity constraint is directly affected by the demand fluctuation and can also reduce the total maintenance and production cost.

In order to validate these conclusions, a design of experiment based on the variation of some important parameters is performed. We consider the same single machine problem presented in Section 5 with 8 planning periods ($L=1$ month). The system has to satisfy a monthly two product demand ($P=2$) with a total capacity of 50 units per month. Production costs π_{pt} and setup costs Set_{pt} are the same for all periods and for both products and are ,respectively, equivalent to 90$ and 1000$. Maintenance times and costs in addition to holding and backorder costs will be considered as the experiment design parameters and will take low, medium and high values as presented in Table 8. The component lifetime distribution can be a Weibull distribution with parameters (2,2) or (3,3). The demand for each product p is assumed to have a stationary mean demand $\overline{d}_p$. The period by period demand for each product d_{pt} is determined from the stationary mean demand with a random fluctuation generated according to a discrete uniform distribution within a fixed interval $[-\overline{d}_p w, \overline{d}_p w]$, where $w \in [0,1]$ is called the fluctuation parameter. Consequently, the period-by-period demand for each product d_{pt} will be generated randomly from the interval $[(1-w)\overline{d}_p, (1+w)\overline{d}_p]$.

The experiment design realization is divided into two parts. For the first part, 6 trials are performed where each trial is featured by the fluctuation parameter w, chosen from the set {0.05, 0.10, 0.15, 0.20, 0.25, 0.30}. Problems with all combinations of parameters in Table 8 are solved for both Weibull (2,2) and Weibull (3,3) system lifetime distributions, which make a total of 1458 maintenance and production planning problems to solve for each trial. In all these problems, the demand is generated randomly according to the methodology described previously with a stationary demand $\overline{d}_p = 23$ and a fluctuation parameter w.

In order to evaluate the obtained results, for each trial we define N_1 as the percentage of problems where the integration of maintenance and production planning reduces the total maintenance and production cost. N_2 is the total percentage of problems where the optimal solution suggests a non-periodic planning (regardless of a gain we obtained with integration or not). N_3 is defined as the percentage of the number of problems where the optimal solution proposes a non-periodic maintenance planning and the integration of maintenance and production planning generates a total cost reduction.

The experimental results presented in Table 9 shows that, when the demand is subject to more fluctuations, the number of problems where the integration proposes a total cost reduction (N_1) increases, as well as the number of problems where the optimal solution suggests a non-periodic maintenance plan (N_2 and N_3).

For the second part of the experiment design, 3 trials are realized where, for each trial, the mean stationary demand $\overline{d}_p$ is chosen from the set {23, 24, 25} with a constant fluctuation parameter $w=0.10$. The period-by-period demand is generated randomly according the same methodology described previously. The experimental results, presented in Table 10, indicate that N_1, N_2 and N_3 increase when the mean stationary demand is getting closer to the system nominal capacity $g=50$ (25 unit per month for each product). For $\overline{d}_p = 25$, more than 80% of problems generate a total cost reduction through the maintenance and production planning integration. Almost all the integrated problems (97%) suggest a non-periodic maintenance plan.

Note also that, for all the 9 trials presented in Tables 9 and 10 with over 11,000 problems executed, the mean solution time per problem is 3.83 s using MATLAB and a 2.33 GHz processor. For problems where the integration of maintenance and production planning generates a total cost reduction, the average cost reduction is around 5.87%.

The experiment design presented in this section confirms the observations made from the illustrative example in Section 5. The demand variation affects both production and maintenance planning and the integration of maintenance and production planning activities can obtain a better deal in order to reduce the total maintenance and production costs. The removal of periodicity constraint for the maintenance plan affects the system capacity, which can fit more with the demand fluctuation.

Table 8
Experiment design parameters.

Parameters	Values		
	Low	Medium	High
CPR ($)	4000	6000	8000
CMR ($)	500	1000	2000
TPR (month)	0.02	0.05	0.07
TMR (month)	0.05	0.07	0.09
h_{pt}, $p=1,2$ ($)	40	60	80
b_{pt}, $p=1,2$ ($)	150	200	250

Table 9
Experiment design results for a fixed stationary mean demand and different values of the demand fluctuation parameter.

Trials	Total solved problems	w	Mean stationary demand		N_1 (%)	N_2 (%)	N_3 (%)
			$\overline{d}_1$	$\overline{d}_2$			
1	1458	0.05	23	23	31.82	19.82	35.99
2	1458	0.10	23	23	35.39	24.07	48.26
3	1458	0.15	23	23	38.82	28.81	58.83
4	1458	0.20	23	23	41.70	31.39	62.83
5	1458	0.25	23	23	43.07	34.91	67.20
6	1458	0.30	23	23	44.31	37.11	70.43

Table 10
Experiment design results for different stationary mean demand with a fixed fluctuation parameter.

Trials	Total solved problems	w	Mean stationary demand		N_1 (%)	N_2 (%)	N_3 (%)
			$\overline{d}_1$	$\overline{d}_2$			
7	1458	0.10	23	23	35.39	24.07	48.26
8	1458	0.10	24	24	61.80	61.59	89.46
9	1458	0.10	25	25	80.04	85.53	97.34

7. Conclusion

In this paper, we developed a model for planning production and noncyclical preventive maintenance simultaneously for a single machine, subjected to random failures and minimal repairs. A non-linear mixed programming model was developed in order to minimize the production and the maintenance costs. The integrated problem was solved by comparing the results of several multi-product capacitated lot-sizing problems. The present contribution extends our previous work (Nourelfath et al., 2010) by taking into account the possibility of noncyclical PM. The present model is then more general, in the sense that it relaxes the cyclic restriction. This relaxation is important at least for two reasons. First, in many practical situations additional constraints, such as limited size and number of maintenance crews combined with the productivity requirement, can make it difficult to implement a cyclical preventive maintenance strategy. Second, noncyclical preventive maintenance can be advantageously used when the demand to be satisfied varies considerably from one period to another. The value of using noncyclical preventive maintenance was illustrated through a numerical example. It was shown that the removal of the periodicity constraint of the preventive maintenance policy allowed for more cost reduction. This is due to the similarity between the capacity generated by the noncyclical PM plan and the demand tendency. Our design of experiments has shown that the integration of maintenance and production planning can reduce the total maintenance and production cost and the removal of periodicity constraint is directly affected by the demand fluctuation and can also reduce the total maintenance and production cost. The production system was modeled as a single machine. An extension of the proposed model to the case of multiple machines, while considering noncyclical PM and dependencies, is currently under investigation.

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