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Review

Nonlinear Soil Response—A Reality?

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Abstract Geotechnical models consistently indicate that the stress-strain relationship of soils is nonlinear and hysteretic, especially at shear strains larger than $\sim 10^{-5}$ to 10^{-4}. Nonlinear effects, such as an increase in damping and reduction in shear-wave velocity as excitation strength increases, are commonly recognized in the dynamic loading of soils. On the other hand, these effects are usually ignored in seismological models of ground-motion prediction because of the lack of compelling corroborative evidence from strong-motion observations. The situation is being changed by recently obtained data. Explicit evidence of strong-motion deamplification, accompanied by changes in resonant frequencies, are found in the data from the 1985 Michoacan, Mexico, and the 1989 Loma Prieta, California, earthquakes, the events recorded by the vertical and surface accelerograph arrays in Taiwan, as well as a number of other events throughout the world. Evidence of nonlinear behavior becomes apparent beyond a threshold acceleration of ~ 100 to 200 gal. Nonlinearity is considerable in cohesionless soil but may be negligible in stiff soils. The findings of recent years indicate that nonlinear site effects are more common than previously recognized in strong-motion seismology.

Introduction

It has long been understood that the amplitude of seismic waves approaching the Earth's surface is magnified by passage through surficial layers of low impedance. Works by Kanai *et al.* (1956) and Gutenberg (1957) initiated a quantitative study of this phenomenon. Layer resonances and near-surface impedance gradients are the main factors that cause soil amplification in a simple horizontally layered structure (Haskell, 1960; Murphy *et al.*, 1971; Shearer and Orcutt, 1987).

The importance of soil amplification has been clearly demonstrated by the Michoacan (Mexico) earthquake of 19 September 1985, and the Loma Prieta (California) earthquake of 17 October 1989. Amplification of the ground motion by soft clays caused catastrophic damage in Mexico City in 1985 (Çelebi *et al.*, 1987; Seed *et al.*, 1988). Significant damage during the Loma Prieta earthquake occurred in areas of San Francisco and Oakland underlain by poor soil conditions (Borcherdt and Glassmoyer, 1992).

The significance of nonlinear soil behavior in seismically induced dynamic loading has been a controversial issue among seismologists and geotechnical engineers for decades. The central question of the discussion is whether soil amplification is amplitude dependent. The dependence of soil response on strain amplitude became a standard assumption in the geotechnical field (Finn, 1991); however, seismologists have rarely considered the importance of nonlinear site effects due to the lack of direct evidence from strong motion observations (Aki and Richards, 1980, p. 9). A comprehensive review of the site effects on earthquake ground motion compiled by Aki (1988) bypasses any detailed discussion of nonlinearity.

Evidence of nonlinear site response in seismological observations has appeared over the last ten years, due to an increase in the number of permanent strong-motion arrays and an improvement in data quality. For example, nonlinear site effects were reported during the Michoacan and the Loma Prieta earthquakes (see references below). These observations have increased seismological interest in the study of nonlinear seismic phenomena worldwide.

In this article, we describe how the presence of elastic nonlinearity affects soil amplification based on existing geotechnical models and then examine evidence of nonlinear soil response from the available seismological literature. Our goal is to infer the typical levels of ground acceleration or strain where significant nonlinear effects can develop. We hope to relate various observations scattered through geotechnical and seismological literature through a unitary approach, so that the conditions for nonlinear soil behavior could be outlined.

Seismic-Wave Amplification in the Nonlinear Case

Fundamental Concepts of Nonlinear Soil Behavior

It is well established in geotechnical engineering that soil response is nonlinear beyond a certain level of deformation (Erdik, 1987; Finn, 1991). Stress-strain relationships for the levels of shearing deformation produced by large earthquakes are nonlinear and hysteretic, as has been confirmed by numerous results of vibratory and cyclic loading tests on soil samples (Hardin and Drnevich, 1972a, 1972b).

A typical stress-strain relationship for cyclic shear is shown in Figure 1. This experimentally recorded behavior is composed of an initial loading (skeleton) curve and of hysteresis loops developed upon subsequent unloadings and reloadings.

The form of the skeleton curve and unloading and reloading branches is well defined in state-of-the-art soil engineering. Analytical approximations for the nonlinear skeleton curves can be found in the Ramberg–Osgood and the Martin–Davidenkov models (Erdik, 1987, pp. 516–517; Finn, 1988, p. 530). Iwan (1967) proposed a model composed of linear springs and Coulomb friction elements that can represent a broad range of nonlinear material behavior. The following simple hyperbolic form of the initial loading is widely accepted (broken line in Fig. 1):

$$\tau = f(\gamma) = \frac{G_{\max}\gamma}{1 + \dfrac{G_{\max}}{\tau_{\max}}|\gamma|}, \tag{1}$$

where τ is the shear stress, γ is the shear strain, $G_{\max}$ is the undisturbed shear modulus (taken at the origin), and $\tau_{\max}$ is the shear strength (the maximum stress that material can support in the initial state) (Hardin and Drnevich, 1972a, 1972b; Yu *et al.*, 1993).

Masing's rule (Masing, 1926) is often applied to extend the stress-strain path from the initial loading curve to the unloading and reloading phases, thereby introducing hysteresis. The rule states that unloading and reloading curves, starting from every reversal point (γ_r, τ_r) of a hysteresis loop (points A and B in Fig. 1), can be obtained from the skeleton curve by two operations: (1) translating the origin and (2) expanding the horizontal and vertical axis by a factor of 2. Thus,

$$\frac{\tau - \tau_r}{2} = f\left(\frac{\gamma - \gamma_r}{2}\right) \tag{2}$$

(Erdik, 1987, p. 517; Finn, 1988, p. 545). Two loops constructed in this way are shown in Figure 1.

Once hysteretic material behavior is adopted, two corollaries follow. First, as seen from Figure 1, the larger the maximum strain during the cycle, the lower the secant modulus $G_{\sec}$, obtained as the slope of the line between the origin and the turning point of the hysteresis loop. This means that the effective shear-wave velocity, defined by the shear modulus as $V = \sqrt{G/\rho}$, where ρ is density, decreases as the strain increases. Second, hysteresis involves a loss of energy in each deformation cycle; the energy loss is proportional to the area of the loop. As Figure 1 shows, an increase in maximum strain leads to an expansion of the loop that results in increased damping, $D/D_{\max} = 1 - G_{\sec}/G_{\max}$, where D is the damping ratio at strain corresponding to $G_{\sec}$, and $D_{\max}$ is its maximum asymptotic value (equation 4 of Hardin and Drnevich, 1972b). Strain-dependent damping tends to saturate at large strains (see Fig. 2). Thus, shear-wave velocity and damping in soils are amplitude dependent for the nonlinear hysteretic model.

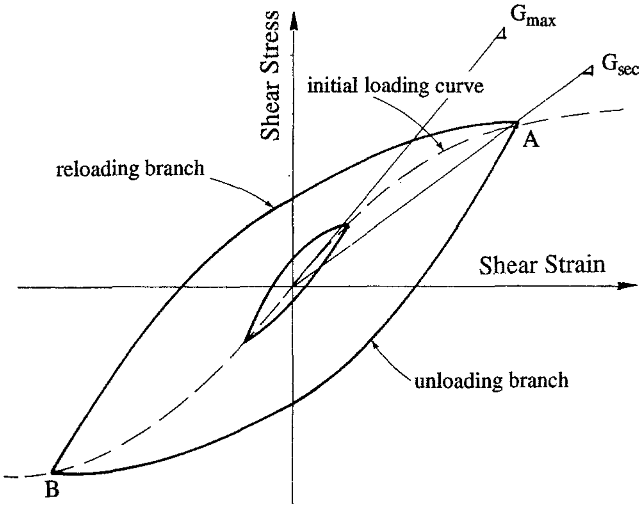

Figure 1. Typical stress-strain relationship of soil in cyclic shear deformation (adapted from Mohammadioun and Pecker, 1984). Initial loading curve has a hyperbolic form (broken line). Subsequent unloading and reloading phases track a hysteretic path. Two hysteresis loops constructed according to Masing's rules (Erdik, 1987, p. 517) are shown, where A and B mark the reversal points of the loop.

How will soil nonlinearity manifest itself in seismological observations? The fundamental resonance frequency of a low-impedance layer is proportional to the wave velocity:

$$f = \frac{V}{4H}, \tag{3}$$

where H is the layer thickness (Murphy *et al.*, 1971, p. 114). Increased strain will therefore decrease V and shift the resonance to lower frequencies; this shift may be observed in the spectra of recorded motions. In addition, increased dissipation will reduce soil amplification during strong motion relative to that in weak motion, causing a "strong-motion deamplification effect."

A number of nonlinear computer codes have been proposed to simulate realistic dynamic soil response; they implement various nonlinear constitutive laws and use different

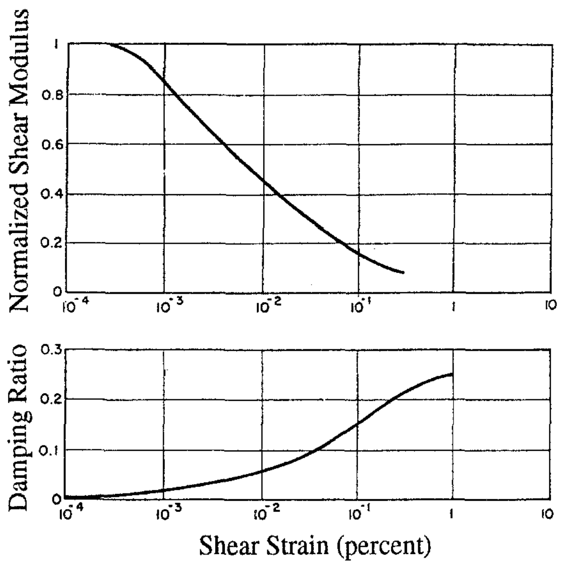

Figure 2. Typical curves showing the dependence of shear modulus and damping ratio of cohesive soil on strain amplitude (after Erdik, 1987). Shear modulus is normalized to its value at a strain of $3 \times 10^{-4}\%$.

numerical techniques. If the methods track the exact form of stress-strain relationship, they are usually referred to as the true nonlinear methods. Commonly used codes are NONLI3 (Joyner and Chen, 1975; Joyner, 1977), CHARSOIL (Streeter *et al.,* 1974; Priolo and Siro, 1989), MASH (Martin and Seed, 1978), and DESRA2 (Lee and Finn, 1978; Finn, 1988; Yu *et al.,* 1993). Details regarding the stress-strain models and the numerical methods used are reviewed elsewhere (e.g., Lam *et al.,* 1978; Erdik, 1987; Finn, 1988). Finn (1988, p. 531) concludes that true nonlinear methods provide essentially similar modeling accuracy. These rigorous simulation methods contrast with an "equivalent-linear" approach, implemented in a program SHAKE (Schnabel *et al.,* 1972), which is an approximate way of simulating nonlinear soil response based on empirical dependencies of shear modulus and damping on strain, as discussed in a later section.

Yu *et al.* (1993) used the DESRA2 code to outline the differences between linear and nonlinear response and marked off three separate frequency bands. In the lowest band of frequencies, the amplification is not affected by nonlinearity. In the central band, nonlinear deamplification takes place. In the high-frequency band, amplification is higher for nonlinear than for linear response.

The existence of specific frequency ranges where nonlinear and linear responses will differ can also be predicted from simple qualitative reasoning. In the low-frequency range, the wavelength is long, and so the waves are not greatly affected by the subsurface strata. In the intermediate frequency range, containing most of the radiated energy, the attenuation of strong motion by hysteretic damping reduces the strong-motion amplitudes relative to weak motion. In the high-frequency band, higher harmonics are generated, increasing strong-motion amplitudes. Harmonic generation is typical for wave propagation in nonlinear media (McCall, 1994). The position of these specific frequency bands is best quantified by relating them to a corner frequency (f_c) of a seismic source spectrum. As can be concluded from calculations by Yu *et al.* (1993), the transition from low-frequency to intermediate-frequency range occurs well below the spectral corner and depends on thickness of soil deposits. In this particular example, the crossover shifts from about one-third to about one-tenth of corner frequency as layer thickness increases from 20 to 6000 m (Fig. 22 of Yu *et al.,* 1993). The transition from intermediate-frequency deamplification to high-frequency amplification always takes place above the interval where most of the wave energy lies, that is, above the corner frequency. It decreases from about three times f_c to about one f_c for the same range of change in layer thickness (Yu *et al.,* 1993, p. 239). The spectral distortion of a broadband pulse propagating through a nonlinear medium, calculated by McCall (1994, Fig. 5), is in qualitative agreement with this pattern.

Using these behavioral traits as guidelines, observational data can be inspected for the presence of nonlinear effects.

Nonlinear Effects in Ground Motion Derived from Laboratory Tests and Dynamic Response Analysis

The nonlinear constitutive relations originate from empirical data. Laboratory tests consistently show the reduction in shear moduli and increase in damping with increasing shear strain. Typical test data were published by Seed and Idriss (1969, 1970) and have been extensively used in soil engineering since that time. Shear modulus and damping, as a function of strain, have been studied for different kinds of clays by Iwasaki *et al.* (1982), Sun *et al.* (1988), and Vucetic and Dobry (1988). An extensive review of nonlinear behavior of clays in the laboratory and under earthquake loading is presented by Dobry and Vucetic (1987). Hardin and Drnevich (1972a, 1972b) experimentally recorded hysteresis loops for a wide variety of soils; they found no effect of frequency on damping, suggesting that hysteretic damping dominates over viscous damping.

Figure 2 shows, as an example, the average Seed and Idriss curves, adapted from Erdik (1987). It can be seen that shear modulus can be reduced in half and damping can increase by a factor of 5 or so as strain increases from $\sim 10^{-4}\%$ to $10^{-2}\%$. Since typical strong-motion data are obtained in the form of acceleration time histories, it is useful to estimate acceleration values that correspond to these benchmark strains. Shear strain in a one-dimensional transverse wave is defined as $\gamma = \partial u/\partial z$, where u is the displacement and z is the propagation direction. For the sinusoidal input $u = u_0 \sin(2\pi ft - kz)$,

$$\max|\gamma| \equiv \max\left|\frac{\partial u}{\partial z}\right| = \frac{a_0}{2\pi fV}, \tag{4}$$

where u_0, f, and k are the displacement amplitude, frequency, and the wavenumber of the input wave, respectively, and a_0 is acceleration amplitude; $a_0 = \omega^2 u_0$. In the accelerograms of the large earthquakes (magnitudes more than 5), frequencies of less than approximately 5 Hz dominate (Fig. 5 of Beresnev *et al.,* 1994). The peak acceleration calculated from (4), given $f = 5$ Hz, $V = 250$ m/sec, and $\gamma = 10^{-4}$ (10^{-2}%), is ~80 gal (cm/sec^2). This value at 5 Hz corresponds to the particle velocity and displacement amplitudes of 2.5 cm/sec and 0.08 cm, respectively. These amplitudes are the upper-bound estimates, since typical frequencies in equation (4) may be lower than 5 Hz. We thus get a rather modest acceleration level at which significant nonlinear effects can be expected based on the characteristics of soil behavior in laboratory tests. A shift in the resonance frequency corresponding to a modulus reduction by a factor of 2 can be obtained from equation (3), which shows, for example, that resonance at 5 Hz would shift by ~1.5 Hz. This should be an easily observable quantity.

Given the necessity of incorporating strain-dependent soil properties in dynamic response analysis, Idriss and Seed (1968b, 1970) introduced an equivalent-linear approach. It is based on the assumption that the nonlinear soil response can be simulated by a linear elastic model with damping, provided that its constants are assigned according to the average strain level achieved. The average strain is assumed to be constant throughout the excitation and is typically taken to be 0.65 times the maximum strain. The values of shear modulus and damping are adopted from experimental curves, such as those shown in Figure 2.

Idriss and Seed (1968a) used the equivalent-linear technique to calculate the amplification factor of a soil column, corresponding to a real site in San Francisco, for different base rock accelerations ranging from 5 to 128 gal. The soil was 42 m of clayey and silty sand. The record obtained on rock during the 1957 San Francisco earthquake was used as the input. The main theoretical features of nonlinear soil response were confirmed by this calculation: as base acceleration increased, the fundamental frequency decreased from 1.6 to 1.1 Hz, and the amplification factor decreased from 3.6 to 0.68. This illustrates that soil amplification may totally disappear for motions of sufficiently large amplitude. This is particularly noteworthy, since earthquake accelerations of the order of 128 gal are commonly observed. The authors concluded that "the ground response characteristics evaluated for one earthquake intensity cannot be extrapolated directly to obtain the ground response for motions of either a higher or a lower intensity. Thus the use of small earthquake records as the basis for evaluating site response during strong earthquakes may be misleading" (Idriss and Seed, 1968a, p. 2031).

The equivalent-linear method was implemented in the program SHAKE (Schnabel *et al.,* 1972), which has become a common tool of estimating dynamic ground response in geotechnical applications. As Finn (1991, p. 205) states, nonlinear site effects have been taken into account routinely in engineering practice since the 1970s. For instance, an equivalent-linear analysis based on the strain-dependent properties of the surficial deposits at Treasure Island (San Francisco Bay) gave an adequate explanation of the observed strong motions during the Loma Prieta earthquake with peak ground acceleration (PGA) of about 0.16 *g* (Idriss, 1990; Hryciw *et al.,* 1991) (see Tables 1 and 2 for more detailed event, site, and record description throughout the article). Chin (1992) calculated amplifications expected for the soft deposits at Treasure Island and the Gilroy#2 stiff alluvium site in central California (Table 2), using the equivalent-linear technique and a program analogous to NONLI3, respectively. For Treasure Island, he found that the constant amplification of about 2.5 given by the linear model would be expected for base accelerations of less than approximately 0.1 *g*. However, the amplification would decrease rapidly with increasing acceleration and become less than 1 at peak bedrock accelerations larger than 0.2 to 0.3 *g* (Fig. 5.8 of Chin, 1992). The Gilroy#2 site would experience even larger nonlinear response (perhaps because a true nonlinear method was applied). The linear amplification was again expected to be about 2.5, while the predicted amplification was less than 1.0 for accelerations exceeding 0.2 *g* (Fig. 5.20 of Chin, 1992). A remarkable result was that the amplification factors actually observed at Gilroy#2 during three major earthquakes in California were consistent with this calculation. Specifically, the soil amplification was only 0.75 for the Loma Prieta earthquake (PGA of ~0.35 *g*) and a factor of 2 for the 1984 Morgan Hill and the 1979 Coyote Lake earthquakes (PGAs of ~0.1 and 0.26 *g*, respectively) (see Table 1). In addition, computations for the Gilroy#2 site showed a leveling of surface acceleration at 0.4 *g* for input motions exceeding 0.5 *g*.

Note that we characterized the amplification factors here in terms of PGA, following the authors of the corresponding studies, although the effect may be frequency dependent, as pointed out by Yu *et al.* (1993).

Elton and Martin (1989) used SHAKE to calculate fundamental site periods at the city of Charleston, South Carolina, for input accelerations ranging from 0.2 to 0.45 *g*. In this range of input amplitudes, the calculated periods increased by 20%.

In the site effect model proposed by Sugito and Kameda (1990, Fig. 2), based on simulations, amplifications decrease from approximately a factor of 3 to less than 1 as bedrock accelerations increase from 0.02 to 0.20 *g*. Unfortunately, the authors do not provide the details of the simulation method, making it difficult to evaluate the theoretical basis of their model. The validation of the model by comparison with borehole acceleration data is also rather incomplete, since accelerations of more than ~0.02 *g* were not represented by any data (authors' Fig. 4).

Mohammadioun and Pecker (1984) examined peak horizontal accelerations from major earthquakes at alluvium sites worldwide and concluded that they tend to saturate (not exceed the level of) around 0.45 to 0.5 *g*. They carried out

Table 1

Significant Earthquakes Referred to in This Study

Event Name	Date	Location	Magnitude	Reference*
San Fernando	02/09/1971	Southern California	*M*† 6.7	Hudson (1972)
Coyote Lake	08/06/1979	Central California	M_L‡ 5.9	Joyner *et al.* (1981)
Coalinga	05/02/1983	Central California	*M* 6.7	Jarpe *et al.* (1988)
Chile	03/03/1985	Off Chilean coast	*M* 8.0	Çelebi (1987)
Michoacan	09/19/1985	Mexican Pacific coast	*M* 8.0	Singh *et al.* (1988a, 1988b)
Loma Prieta	10/17/1989	Central California	*M* 6.9	Darragh and Shakal (1991)
Northridge	01/17/1994	Southern California	*M* 6.7	Shakal *et al.* (1996)
Kobe	01/16/1995	Central-Western Japan	*M* 6.9	Kanamori (1995)

*See text for more complete reference list.

†Moment magnitude.

‡Local magnitude.

theoretical modeling of the saturation effect using CHARSOIL, for a test alluvial soil profile at Imperial Valley, California. In the calculations, deamplification occurred at bedrock accelerations higher than 0.25 *g* (Mohammadioun and Pecker, 1984, Fig. 24), and the computed surface accelerations saturated at a value of 0.35 *g*. This observational and theoretical evidence for the saturation effect seems to be contrary to the strong-motion data provided by the recent 1994 Northridge, California, and the 1995 Kobe, Japan, earthquakes (Table 1), where seemingly unrestricted accelerations of the order of 1 *g* have been recorded on soil (Kanamori, 1995; Shakal *et al.*, 1996). It remains an open question how this discrepancy can affect the existing soil deformation models.

In summary, the geotechnical literature provides an abundance of examples of ground response modeling using an equivalent-linear code (SHAKE) or true nonlinear methods (Joyner and Chen, 1975; Lam *et al.*, 1978; Iwasaki *et al.*, 1982; Mohammadioun and Pecker, 1984; Erdik, 1987; Finn, 1988; Chang *et al.*, 1990; Chin, 1992; Finn *et al.*, 1993). Most of these studies provide a comparison of the performance of the equivalent-linear and the nonlinear methods. Both approaches can simulate observed strong-motion accelerograms and response spectra to a satisfactory extent for a wide range of peak accelerations. Comparisons consistently show that the equivalent-linear method exhibits significant discrepancies from true nonlinear techniques for the largest accelerations. This result arises from a limitation inherent to the equivalent-linear approach, which attempts to account for the response of a nonlinear system with a linear model whose parameters are chosen based on an average strain. For example, an equivalent-linear system apparently cannot account for harmonic generation effect. This point can be illustrated as follows. Durward *et al.* (1996) investigated a dependence of high-frequency decay of the spectra of ground accelerations on the peak velocity on ground surface. They modeled the decay-rate parameter κ with the equivalent-linear and true nonlinear methods and compared its values with those obtained from real earthquakes. The nonlinear methods correctly reproduced a decrease in spectral-decay rate at large input velocities, apparently due to harmonic-distortion effects, which was consistent with observations (Durward *et al.*, 1996, Fig. 9). However, an equivalent-linear method predicted an almost exponential increase in κ as strain increased, which was incompatible with data (Durward *et al.*, 1996, Fig. 7). An explanation of these results may lie in the fact that the equivalent-linear method failed to predict high-frequency energy creation at large strains because of the simplified treatment of nonlinearity inherent in it. True nonlinear methods are suggested for dynamic response analysis in the most general case.

Nonlinear dynamic response analysis, combined with the results of laboratory dynamic tests, suggests that the onset of appreciable nonlinear behavior of soils can be expected at accelerations above approximately 0.1 to 0.2 *g*. In the following section, we verify this conclusion based on field seismological observations.

Direct Seismological Evidence of Nonlinear Site Effects

In the previous section, we referred to a series of investigations where close agreement between calculated and observed records was achieved by applying nonlinear dynamic codes. A weakness of these approaches is that a linear model may also work. There are virtually no works that perform a concurrent-linear analysis and conclude that it is not appropriate. Also, most of the pronounced nonlinear effects in the high-acceleration range have been obtained by calculation rather than from field data. Alternative literature exists where the applicability of linear elastic models to strong ground motion evaluation is validated (e.g., Murphy *et al.* 1971; Joyner *et al.*, 1981; Seale and Archuleta, 1989). Jongmans and Campillo (1990) calculated ground amplifications during the 1983 Liege earthquake using the linear elastic transfer function of plane layers. Amplifications correlated well with the damage distribution. A similar conclusion was drawn by Elton and Martin (1989) for a historic Charleston, South Carolina, earthquake of 31 August 1886, who assumed a strain-dependent nonlinear soil behavior.

Table 2
Strong-Motion Sites and Records Significant for This Study*

Station Name and Location	Site Classification	Site Geology	Events Recorded	Peak Horizontal Acceleration	Reference
Yerba Buena Island, San Francisco Bay	rock	Franciscan sandstone and shale	Loma Prieta	0.07 *g*	Chin and Aki (1991) Darragh and Shakal (1991) Schneider *et al.* (1993)
Treasure Island, San Francisco Bay	soil	artificial fill over sand and bay mud; basement rock at 88 m	Loma Prieta	0.16 *g*	Idriss (1990) Chin and Aki (1991) Darragh and Shakal (1991) Hryciw *et al.* (1991) Schneider *et al.* (1993)
Gilroy#1, central California	rock	moderately weathered sandstone	Coyote Lake Loma Prieta	up to 0.44 *g*	Joyner *et al.* (1981) Chin and Aki (1991) Darragh and Shakal (1991) Schneider *et al.* (1993)
Gilroy#2, central California	stiff soil	alluvial-fan deposits; basement rock at 167 m	Coyote Lake 1984 Morgan Hill, California Loma Prieta	up to 0.35 *g*	Joyner *et al.* (1981) Chin and Aki (1991) Darragh and Shakal (1991) Chin (1992) Schneider *et al.* (1993)
Caltech Seismological Lab, southern California	rock	granite crystalline rock	San Fernando	0.18 *g*	Hudson (1972)
Caltech Athenaeum, southern California	soil	270 m alluvium	San Fernando	0.11 *g*	Hudson (1972) Rogers *et al.* (1984)
Hososhima, Japan	stiff soil	50 m of sand and clay on rock	earthquakes in Japan	up to 0.3 *g*	Tokimatsu and Midorikawa (1981) Takemura and Ikeura (1988) Kamiyama (1989)
Valparaíso, Chile	rock	Gneiss amphibolite	Chile	~0.2 *g*	Çelebi (1987)
Viña del Mar, Chile	soil	alluvium	Chile	~0.4 *g*	Çelebi (1987)
Garner Valley downhole array, southern California	soil	18 m of alluvial silty sands and silts; granite below	local earthquakes	up to 0.041 *g*	Archuleta *et al.* (1992)
UNAM and CU, Mexico City	rock	basalt	Michoacan	~0.03 *g*	Çelebi *et al.* (1987) Singh *et al.* (1988a, 1988b)
CDAO and similar sites, Mexico City	very soft soil	lake-bed clay	Michoacan	up to 0.17 *g*	Çelebi *et al.* (1987) Seed *et al.* (1988) Singh *et al.* (1988a, 1988b)
Lotung downhole array, Taiwan	deep cohesionless soil	silty sand	earthquakes in Taiwan	up to 0.26 *g*	Chang *et al.* (1989) Wen *et al.* (1994) Beresnev *et al.* (1995b)

*Stations in table appear in approximately the same order as in the text.

These conflicting results suggest that forward modeling to fit observations, as adopted in most works discussed above, cannot unambiguously prove or disprove nonlinear soil behavior. This partly explains the origin of the literature debate about the significance of nonlinear site effects (Finn, 1991, p. 205; Yu *et al.,* 1993, p. 218). As Aki and Irikura (1991, pp. 95–96) and Aki (1993, p. 108) state, most seismologists that deal with field observations and ground-motion prediction are reluctant to accept the nonlinearity of ground response. Their argument is that the linear elastic models of seismic energy generation, propagation, and near-surface transformation have worked reasonably well even at strong-motion levels (Aki, 1982; Yu *et al.,* 1993, p. 218). Is there any direct seismological evidence of nonlinear soil response? One proof would be to demonstrate with strong-motion records that the natural period and shear-wave velocity of sediments depend on the amplitude of the excitation. Another proof would be to confirm that empirical site amplification functions consistently diverge for weak and strong motions. In this section, we summarize evidence of this nature.

San Fernando Earthquake and Other Miscellaneous Observations

The San Fernando earthquake of 9 February 1971 (Table 1) provided an opportunity to compare weak- and strong-motion response at the same sites. Hudson (1972) analyzed the strong-motion records in the Pasadena area and compared accelerations recorded on thick alluvium (Caltech Athenaeum station) and granite crystalline rock (Caltech Seismological Laboratory) (Table 2) to earlier recordings of weak events at the same stations, as obtained by Gutenberg (1957). Hudson (1972, p. 1777) reports that, despite a general complexity of the peak acceleration distribution throughout the area, the maximum acceleration at the rock site was larger than that at the alluvium site. This result contrasted with Gutenberg's weak-motion observations, which indicated that the amplitudes at the alluvium site were 3 to 4 times greater than those at the rock site. The author admitted that the local distribution of ground shaking predicted on the basis of small earthquakes might not correspond to the distribution during a damaging earthquake (Hudson, 1972, p. 1786).

Peak accelerations recorded at the Seismological Lab (rock) and Athenaeum (alluvium) sites during the San Fernando earthquake were 179 and 109 gal, respectively. In this case, general conformity with geotechnical expectation (Idriss and Seed, 1968a) was observed. Hudson (1972), however, did not mention nonlinearity and was inclined to attribute the difference between the weak- and strong-motion amplifications to the effects of topography, source, and propagation path.

In a later work by Rogers and Hays (1978) and Rogers *et al.* (1984), an extended data set of the 1971 San Fernando earthquake at 28 sites in the entire Los Angeles basin was reexamined. The site geology at these stations was broadly classified as an alluvium, sedimentary rock, and crystalline rock. Earthquake motions were statistically compared with weak motions produced by nuclear tests at the Nevada test site. The spectral ratio technique was applied to reduce source and path effects. Rogers *et al.* (1984, Figs. 11 and 12) calculated 10 alluvium-to-crystalline-rock spectral ratios in weak and strong motions, including a pair analyzed by Gutenberg (1957) and Hudson (1972). The magnitude of the weak-motion amplification at the Athenaeum site was in quantitative agreement with Gutenberg's results, while strong-motion deamplification between approximately 0.5 and 5 Hz paralleled Hudson's report. However, deamplification was not detected at all of the alluvium sites. Following an unbiased statistical approach, Rogers *et al.* (1984) calculated a coefficient R', which was the ratio of strong- to weak-motion amplifications averaged over all frequencies, components, and stations pairs available. R' was expected to be unity. However, the overall R' was around 0.9, which suggests nonlinear soil behavior (Rogers *et al.,* 1984, Table 2). Nevertheless, the authors again explained the differences in weak- and strong-motion amplifications by differences in source and path effects. One can note that the values of R' calculated in the article were not the best indicator of a possible nonlinear site response, since the calculation included even the ratios between stations with similar geology, for example, crystalline-rock-to-crystalline-rock ratios. As a result, the obtained values of R' could overestimate those involving a true soil response. Consequently, both Hudson (1972) and Rogers *et al.* (1984) could not completely rule out nonlinear response at the sedimentary sites during the San Fernando earthquake.

Seed *et al.* (1976) summarized data from eight earthquakes with magnitudes of about 6.5, occurring in the western part of the United States, in the form of amplitude-dependent amplifications for different soil types. Their result is reproduced in Figure 3. The mean curves obtained from 40 to 60 individual observations are shown for rock, stiff soil, and deep cohesionless soil. The lower curve in Figure 3, as authors point out, is rather speculative as it is not deduced from real data. Instead, this curve is based on the authors' experience and theoretical considerations only. Similarly, the values of all curves above 0.3 g are extrapolations. The range of scatter relative to the mean curves is not shown in the original article. This indicates that some caution should be exercised in interpreting site-response characteristics presented in Figure 3; however, any speculations involved in their derivation are based on reasonable assumptions. The plots show that amplification factors smaller than 1 apply to all characteristic soil types at base accelerations above 0.2 g.

Kamiyama (1989) performed regression analysis of the maximum surface particle velocity for 228 strong-motion records observed at 26 sites in Japan. After calculating the coefficients of the regression equation using all the data, the author applied the regression to the determination of frequency-dependent amplification functions for five representative soft sites, relative to a slate outcrop. At two sites (Shiogama and Hososhima) (see Table 2 for Hososhima site), which consisted of hard rock overlain by soft deposits, the predominant period lengthened and amplifications between 1 and 5 Hz decreased as the maximum velocity increased. Three other sites did not exhibit this behavior; however, their near-surface geology was more complex. In addition, only one reference rock station was used, while the soil stations were scattered throughout Japan. One may note that grouping a variety of site effects in one generalized model might smear possible signs of nonlinear response. Earlier site-specific analysis at Hososhima by Tokimatsu and Midorikawa (1981) demonstrated that this site experienced a shear modulus decrease at strains of 10^{-5} to 10^{-3}. Finally, the Shiogama and Hososhima sites were also addressed in Takemura and Ikeura's (1988) work, in which a method of synthesizing strong motions using a superposition of small earthquake was tested. Although the method performed reasonably well, it failed to fit the observed strong-motion accelerograms at these two particular stations, presumably because of inherent nonlinear effects.

A comparison of weak- and strong-motion amplifications was carried out by Çelebi (1987) for the mainshock and the aftershocks of the 3 March 1985 earthquake off the Chilean coast (Table 1). Ratios of Fourier amplitude spectra between the Viña del Mar alluvium site and Valparaíso amphibolite gneiss reference site (Table 2) were calculated. Acceleration at the alluvium station attained almost 400 gal. Çelebi (1987, p. 1161) pointed out that the weak-motion amplifications were greater than those for strong motion over the entire frequency range of 1 to 10 Hz and that nonlinear soil response was one of the plausible mechanisms. On the other hand, the spacing between the two stations is comparable to their distance to the mainshock epicenter, so that the dissimilarity in the spectral ratios might be attributed to the azimuthal variation in source radiation and differences in wave propagation paths.

The question of nonlinear site amplification was addressed by Jarpe *et al.* (1988) for accelerations up to 0.7 *g*, using data from several aftershocks of the 1983 Coalinga earthquake in California (Table 1). Ratios of the spectra in the alluvial valley to those of a sandstone site were calculated. Average ratios for 23 weak- and 7 strong-motion events showed that the strong-motion amplification was significantly lower for frequencies between 10 and 12 Hz at the very edge of the frequency band analyzed. Nonlinear soil behavior was mentioned as a most likely cause for this difference, though the frequency range of nonlinear response was clearly limited. In our opinion, to prove that this result was not fortuitous in a more convincing way, the authors could verify if the reduced strong-motion ratios extended beyond 12 Hz, the frequency that limited plots in their work.

A unique opportunity to detect nonlinear amplification effects is provided by the use of borehole vertical arrays. If a downhole instrument is installed in the bedrock, the surface-to-bedrock spectral ratio gives a pure transfer function of the soil, nearly isolated from source and path effects. An intriguing result has been published by Archuleta *et al.* (1992). At the Garner Valley downhole array (Table 2), one of the accelerometers is located at a depth of 220 m in granite, and one is installed at the surface. Two earthquakes with nearly coincident hypocenters but different local magnitudes (4.2 and 2.5) were recorded, and spectral ratios at 0 to 220 m were calculated (Fig. 13 of Archuleta *et al.,* 1992). The authors noted that weak-motion amplification was significantly larger than strong-motion amplification in a wide frequency band, from 3 to 40 Hz (authors' Fig. 4). The most pronounced resonance is slightly shifted to the left. The horizontal acceleration amplitude in the M_L 4.2 event was about

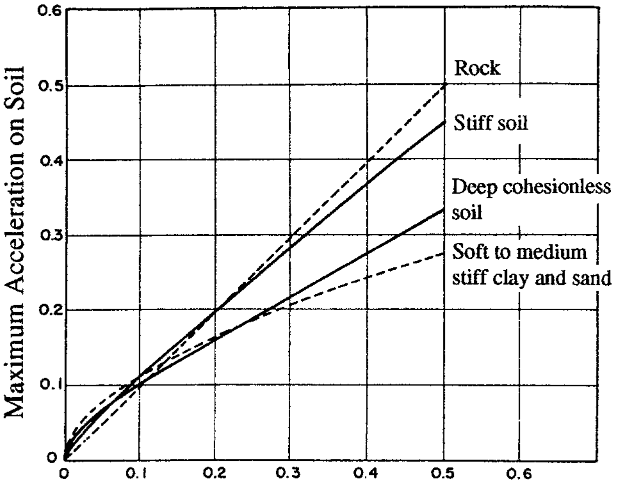

Figure 3. Curves showing amplification of bedrock accelerations by different classes of soil (after Seed *et al.,* 1976). Accelerations are in fractions of *g*.

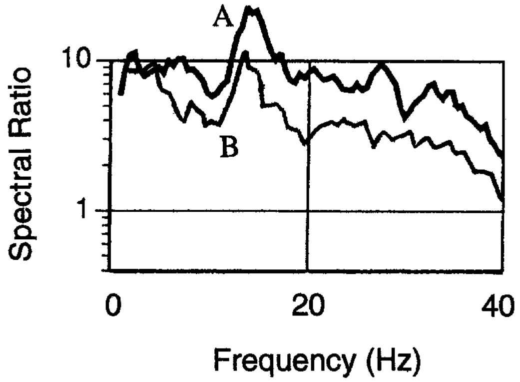

Figure 4. Ratios of Fourier amplitude spectra between the surface and 220-m-deep instrument in granitic rock, obtained for two local earthquakes with different magnitudes. The curves A and B correspond to the *M*2.5 and *M*4.2 events, respectively. The difference in amplification level is clearly seen. We combined these curves on a single plot from the original data of Archuleta *et al.* (1992, Fig. 13).

0.04 *g* at the surface. Despite this modest PGA, this event may have caused a detectable nonlinear soil behavior, considering that the upper 18 m is granular soil, the reference instrument is in unweathered granite, and the data are supposed to be free of distorting source and path effects. Interestingly, the geologic structure used in a numerical simulation of nonlinear site effects by Yu *et al.* (1993) is remarkably similar to the Garner Valley site. The model consists of a 20-m soil stratum overlying hard rock. As shown in Figures 5 and 6 of Yu *et al.* (1993), noticeable differences in amplification given by linear and nonlinear responses arise at base peak accelerations as low as 0.01 *g*. "Nonlinear" interpretation of Archuleta *et al.*'s (1992) result is therefore naturally expected from a realistic soil rheology, as incorporated in DESRA2. Other investigations similarly showed that nonlinear soil response modeled by DESRA2 is close to observations (Wen *et al.,* 1994). Note that the authors in Archuleta *et al.* (1992) have not given an explanation for the observed difference in weak- and strong-motion amplification, nor have they mentioned nonlinearity, perhaps due to the preliminary character of their analysis.

Seismological Evidence of Nonlinear Site Response during the 1985 Michoacan Earthquake

Çelebi *et al.* (1987) compared spectral ratios between lake-bed stations (very soft clay) and a rock (basalt) station UNAM (Autonamous National University of Mexico) for the Michoacan earthquake with the weak-motion ratios generated by traffic noise (Tables 1 and 2). Weak-motion ratios were significantly larger than strong-motion ratios over the frequency range of 0.1 to 4 Hz, for which the results were presented. Nevertheless, this evidence was not interpreted as an indication of nonlinear site response, because the weak motions recorded by separate stations were supposedly not generated by the same source.

In another work by Singh *et al.* (1988b), spectral ratios between one of the same soil stations CDAO (Central de Abastos Office) and the hill zone rocky station CU (Ciudad University) were again computed for the Michoacan and three smaller earthquakes all having epicenters more than 300 km from the sites, almost precluding the influence of source or path dissimilarities on the ratios. Singh *et al.* (1988b, Fig. 2) pointed to the clear evidence of nonlinear clay behavior during the Michoacan earthquake: in strong motion, the spectral ratio was noticeably lower and peaks were shifted toward longer periods compared to the weak motions. The effect was distinct between 0.2 and 4 Hz. Higher frequencies were not present in the spectra because of the large epicentral distance.

Contrary to Singh *et al.* (1988b), Finn (1991, p. 208) and Aki (1993) share the viewpoint that the Michoacan earthquake exhibited a linear response in Mexico City. Finn (1991, pp. 208–211) mostly bases this conclusion on the laboratory-measured elastic properties of clays that show little dependence on strain up to the order of 0.1% while paying little attention to the results of field observations. Aki (1993) calls the case of Mexico City a "most spectacular demonstration of the applicability of linear theory," referring to another study by Singh *et al.* (1988a), and concludes that "the ground motion at the lake bed is amplified 8 to 50 times . . . with very little evidence for nonlinearity" (Aki, 1993, p. 97). However, Singh *et al.* (1988b, p. 658) revised their result, mentioned by Aki, and explained the apparent linear behavior reported previously by the poor data quality. A nonlinear response was revealed after a more careful examination.

A point of view contrary to that of Finn and Aki has also been developed by Lomnitz (1990, 1995). The author argues that significant shear-modulus reduction in Mexico City during the Michoacan earthquake led to a widespread loss of rigidity in clays that cover a large part of the city. As a result, shear waves propagated virtually as gravity waves on water. Lomnitz (1990, 1995) derives his conclusions from clay properties, theoretical estimates, and limited supporting data. This opinion is not shared by all investigators, though, as a lively debate indicates (Chávez-García and Bard, 1995; Lomnitz, 1995).

Our viewpoint is that the most direct method of site-response estimate applied by Singh *et al.* (1988b), who calculated spectral ratios between neighboring stations, gives a clear evidence of nonlinear response of clays during the Michoacan earthquake.

Seismological Evidence of Nonlinear Site Response during the 1989 Loma Prieta Earthquake

The Loma Prieta earthquake (Table 1) provides established cases of ground nonlinearity.

The first field reports showed that there was a relative independence of peak ground acceleration on the recording site conditions (soil or rock) in the epicentral area (Boore *et al.,* 1989).

Darragh and Shakal (1991) applied the spectral ratio technique to study the response at Treasure Island and Gilroy#2 soil sites with respect to reference rock stations at Yerba Buena Island and Gilroy#1, respectively (Table 2). Treasure Island gave a much lower amplification in the Loma Prieta mainshock than in the aftershocks in the frequency range of 0.5 to 7 Hz (Fig. 5), despite the relatively low PGA of 0.07 *g* at the reference station. The first two resonances in the strong motion almost disappear, and the third resonance is clearly shifted. This can be regarded as a seismological proof of the nonlinear response at this location revealed by geotechnical modeling using the equivalent-linear approach (Idriss, 1990; Hryciw *et al.,* 1991; Chin, 1992). Gilroy#2, a stiff alluvial site, displayed a less pronounced but still evident nonlinear effect. When compared to the 13 aftershock ratios, the Loma Prieta mainshock deamplification clearly emerged around 1 Hz and between 2 and 3 Hz. The more ambiguous nonlinear response at Gilroy#2, relative to that at Treasure Island, could be expected geotechnically considering the different soil types. The Treasure Island station is underlain by fill, whereas Gilroy#2 is on stiff

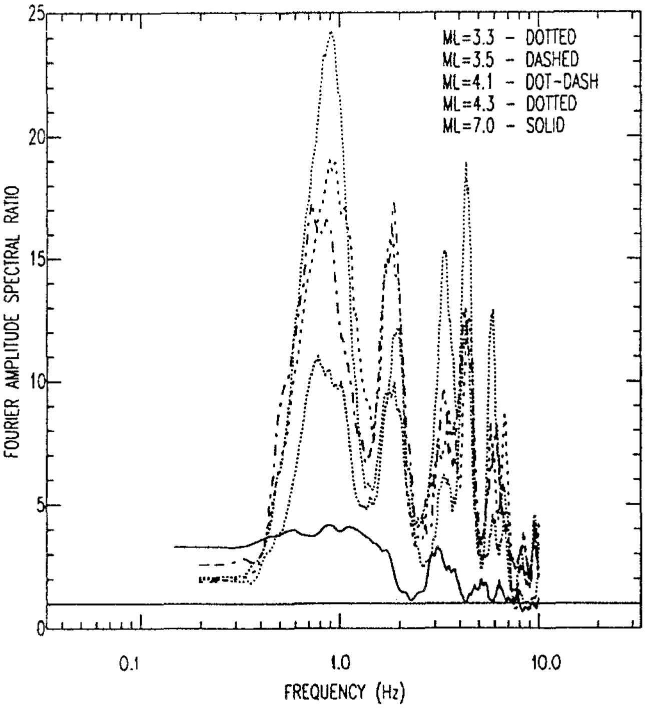

Figure 5. Spectral ratios between the soft-soil Treasure Island relative to the rock site at Yerba Buena Island for the Loma Prieta mainshock and aftershocks. Solid line corresponds to the mainshock. Ratios were calculated for the direct *S* wave (after Darragh and Shakal, 1991).

alluvial deposits. They may be classified as soft and stiff soils, respectively. Figure 3 shows that the latter type has amplification characteristics closer to the rock.

Chin and Aki (1991, 1996), Chin (1992), and Schneider *et al.* (1993) applied a stochastic modeling technique to simulate observed Loma Prieta accelerograms. Chin and Aki (1991) show good agreement between the observed and predicted records in their duration and spectral content; however, observed peak accelerations at soil sites at close epicentral distances are systematically lower than predictions. The authors conclude that weak-motion amplification factors assigned to soil sites in the simulations did not correctly reproduce the actual amplifications that occurred in strong motion, which were reduced by nonlinear effects. The threshold acceleration inferred for the onset of nonlinear behavior was about 0.1 *g*, which does not contradict accepted geotechnical models.

Interestingly, Chin and Aki (1991) found a large nonlinear response at Gilroy#2 site, while Treasure Island almost matched the linear prediction (Fig. 15 of Chin and Aki, 1991). Essentially, the same result is reported by Schneider *et al.* (1993). In the latter work, the authors show that the equivalent-linear calculation with strain-dependent soil properties provides a good fit to the observed data at Gilroy#2 site, while purely linear calculation overestimates data significantly (Fig. 6 of Schneider *et al.,* 1993). In contrast, nonlinear and linear predictions at Treasure Island are almost indistinguishable. These results clearly disagree with a strong deamplification effect at Treasure Island reported by Darragh and Shakal (1991), and possible explanations for this disagreement should be addressed.

Treasure Island, San Francisco Bay, is 98 km from epicenter and 2.5 km from the reference Yerba Buena Island. In these conditions, spectral ratios can provide a high accuracy of site-response estimate (Beresnev and Wen, 1996). This accuracy is corroborated in Figure 5, where all weak-motion ratios resolve the same fine-resonance structure of local response. Chin and Aki (1991) and Schneider *et al.* (1993) developed a technique that was not reference-site dependent and thus required specification of source and path models that are often ambiguous. The method approximates the source spectrum by the ω^2 shape and introduces path attenuation through the frequency-dependent Q factor and geometric spreading of 1/distance. Though these assumptions can lead to a reasonable accuracy in simulating bedrock motion at near-epicenter distances, this accuracy generally decreases with distance (Beresnev and Atkinson, 1996). The use of this technique for site-effect assessment is warranted when the effect cannot be isolated by the more direct methods; otherwise its accuracy may be generally lower than that of a spectral ratio technique, especially at large distances from the source. In addition, Chin and Aki (1991) use amplification values that are not site specific. Wennerberg (1996) raised concerns about the adequacy of path-effect representation in Chin and Aki's (1991) work and indicated that their reported deviations between the predicted and observed peak accelerations could be removed in many cases by using site-specific amplifications. These may be the reasons for inconsistency between the soil response estimates at Treasure Island obtained by Darragh and Shakal (1991) and by the other authors. There is little doubt that response at Treasure Island during the Loma Prieta mainshock was strongly nonlinear, which is further corroborated by clear signs of liquefaction (Darragh and Shakal, 1991).

The generic behavior of fill and mud deposits within the San Francisco Bay area for weak and strong motion was summarized by Borcherdt and Glassmoyer (1992). Average spectral ratios (0.5 to 2.5 Hz) of horizontal accelerations between fill/mud sites and the Franciscan firm complex demonstrate a decrease in the amplification from 5.7 (low strain) to 3.8 (large strain) (Table 5 of Borcherdt and Glassmoyer, 1992). Stiffer sites do not show such a reduction in strong-motion amplification. Although the authors consider this result to be biased by distance effects and do not connect it to nonlinear ground behavior, the nonlinearity hypothesis cannot be rejected.

The contradicting results regarding the extent of nonlinear soil behavior during the Loma Prieta earthquake aroused a lively debate. The inconsistency between the different observations is yet to be resolved, possibly through accumulation and analysis of new data.

Evidence of Nonlinear Site Response at Strong-Motion Arrays in Taiwan

Chang *et al.* (1989), Wen (1994), and Wen *et al.* (1994, 1995) analyzed spectral ratios of surface to downhole accelerations at the Lotung vertical accelerograph array in northeastern Taiwan. The instrumented borehole was drilled to a depth of 47 m in the alluvial deposits within the area of the SMART1 surface array (Table 2). The site is composed of silty sands and is geotechnically classified as "deep cohesionless soil." A total of about 30 local events were recorded. The maximum acceleration obtained from the surface instrument of the vertical array is approximately 0.26 *g* in the two *M*6.5 earthquakes that occurred at epicentral distances of 24 and 66 km. Wen (1994) observed a gradual reduction in the effective shear-wave velocity measured from borehole data, as acceleration amplitude increased from 0.05 *g* to 0.20 to 0.26 *g*. Velocities decreased to as little as 50% of the low-strain values.

Wen *et al.* (1994, 1995) found that soil amplifications closely followed the pattern expected from a hysteretic soil behavior. Figure 6 shows the uphole/downhole *S*-wave spectral ratios calculated for a strong local earthquake (surface PGA of 0.19 *g*) (thick line B) and the weak-motion ratios for the foreshocks, aftershocks, and coda of this earthquake (thin lines labeled A). All weak-motion ratios are close to each other. By comparison, the strong motion is deamplified between approximately 2.6 and 11 Hz, and the resonances are shifted downward (line B). Note a close qualitative resemblance of the amplification functions depicted in Figures 5 and 6, despite the overall differences in the conditions under which they were obtained.

Wen *et al.* (1994) pointed to a frequency-dependent character of nonlinear response and noted that their observations were generally consistent with the theoretical predictions made by Yu *et al.* (1993) using DESRA2. Wen *et al.* (1995) also demonstrated that, while nonlinearity was significant in the cohesionless alluvial deposits, it was not detectable at comparable acceleration levels from borehole data at stiffer site.

Beresnev *et al.* (1995a, 1995b) examined weak- and strong-motion amplifications at the SMART1 array using traditional soil-to-rock spectral ratios. Spectral ratios were calculated between the alluvium site and a bedrock site spaced at 4.8 km. Weak motions selected for the analysis had a peak horizontal acceleration of less than 30 gal, while strong motions were in the range of 100 to 267 Gal. A clear reduction in strong-motion amplification was found in the frequency range between ~2 and 9 Hz, exhibited by both the average responses calculated for weak and strong motions and the individual responses obtained from mainshocks, foreshocks, aftershocks, and coda.

At the SMART2 array in eastern Taiwan, where there are no rock sites available, spectral ratios were calculated between the older terrace deposits and the more recent alluvium. The SMART2 data provided accelerations as large as 300 gal. Strong-motion ratios consistently deviated from those in weak motion in the frequency band between ~1 and 10 Hz, showing a difference in nonlinear response of soils having different age and composition.

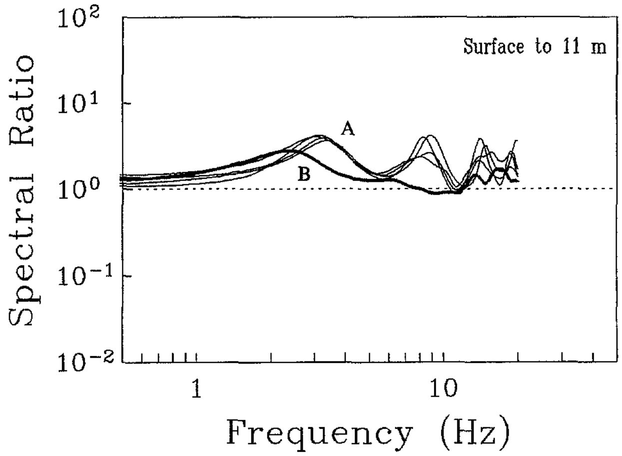

Figure 6. Smoothed spectral ratios of accelerations between the surface and the depth of 11 m at the Lotung downhole array in Taiwan. Ratio for an M_L 6.2 earthquake (thick line) is labeled by the letter B; a group of curves labeled A (thin lines) corresponds to the foreshocks, aftershocks, and shear-wave coda of this earthquake.

Thus, various Taiwan data provide evidence of nonlinear response for peak accelerations between ~100 and 300 gal.

Known Negative Results (No Nonlinear Effects at High-Acceleration Levels)

Several investigators explored the significance of nonlinear ground behavior at high strains and reported no substantial nonlinear phenomena.

Murphy *et al.* (1971) compared observed soil-to-rock spectral ratios to those predicted by a linear model, for two nuclear explosions. Two stations underlain by fill were compared to a common station on tuff. Pseudo-relative velocity spectra (PSRV) were used. The distance to the explosions was 16.6 km, and the maximum acceleration was 0.75 *g*. The authors reported no significant deviations of the observations from the linear theory in the entire frequency interval from 0.4 to 12 Hz. Transverse and radial components of the observed records found in the article have similar amplitudes, and the analysis was applied to both of these components. This shows that the waves from the explosions were not dominantly compressional, at least at the receiver sites, and that conclusions apply to shear waves as well.

Murphy *et al.* (1971) speculate that the analyses of PSRV and Fourier spectral ratios are equivalent. In reality, ratios of response spectra have specific features that need to be carefully interpreted. Response spectra characterize the response of a single-degree-of-freedom linear oscillator to a recorded time history as a function of its natural period. This is not equivalent to the Fourier frequency spectrum. Yu *et al.* (1993, p. 234) examined these differences numerically and showed that the use of response spectral ratios for recognizing nonlinearity is precarious. For instance, the authors generated seismograms for magnitude 4, 5, 6, 7, and 8 earthquakes at a distance of 15 km from the source and demonstrated that differences between corresponding linear and nonlinear response spectra in a frequency range from 0.1 to 10 Hz are negligible. The results using Fourier spectra were clearly different (Fig. 14b of Yu *et al.*, 1993). Note that the frequencies in Murphy *et al.*'s (1971) investigation were almost entirely within this band, which is unfavorable for the detection of nonlinear effects using the PSRV spectral ratios.

Rogers and Hays (1978, Fig. 4) present similar arguments in favor of linearity. They compared the PSRV ratios between alluvium and welded tuff stations for two nuclear explosions, one of which caused a strong and the other one a weak acceleration (PGAs at alluvium of 0.54 and 0.037 *g*, respectively). The frequency range was 0.2 to 20 Hz. The authors showed that the weak- and strong-motion ratios were similar. Nevertheless, numerical modeling predicts that significant differences in PSRV spectra in linear and nonlinear cases should occur above 10 Hz, where the linear response spectrum is overamplified (Fig. 14a of Yu *et al.*, 1993). This pattern can actually be seen in Figure 4 of Rogers and Hays (1978), between 10 and 20 Hz.

These remarks indicate that the reinterpreted data of Murphy *et al.* (1971) and Rogers and Hays (1978) may actually be supportive of a nonlinearity assumption. One important inference from the work by Yu *et al.* (1993) is that response and Fourier spectral ratios behave as antipodes in terms of differences between weak (linear) and strong (nonlinear) motion.

Spectral ratios between the alluvial Gilroy#2 and sandstone Gilroy#1 sites in the 1979 Coyote Lake earthquake in California (Tables 1 and 2) were examined by Joyner *et al.* (1981). They found that both the linear viscoelastic model and Iwan's nonlinear model (Iwan, 1967) gave predictions that are fairly compatible with observations. Peak accelerations on alluvium and bedrock were 0.26 and 0.11 *g*, respectively. The authors point out that the approximately linear behavior at the Gilroy#2 site in this acceleration range is expected even from Iwan's nonlinear model. As stated earlier, Chin (1992, Fig. 5.20) used the same nonlinear method to calculate that the nonlinear behavior at Gilroy#2 could occur above accelerations of 0.1 to 0.2 *g*. Thus, the Coyote Lake earthquake occurred at the very edge of the transition to the nonlinear regime. This hypothesis is strengthened by more evident nonlinear response at this site during the Loma Prieta earthquake, also discussed earlier.

A widely cited demonstration of the linearity in the soil response to strong motion is Tucker and King's (1984) result. The authors studied spectral ratios between stations located in a sediment-filled valley; they observed no tangible differences between weak- and strong-motion ratios, for accelerations reaching 0.2 *g*. Unfortunately, the description of the sediment properties is not available from this study, so that it is not possible to link them to any generic soil type. Also, it is possible that the ratios were calculated between two sedimentary sites that have the same soil conditions. One of them is located in the middle and the other at the edge of the valley. It may have happened that these sites both responded to the strong shaking nonlinearly, but in a similar manner, so that taking the ratio has eliminated the signs of nonlinear response.

Conclusions

The significance of nonlinear-elastic soil response to earthquakes has long been a contentious matter. On the one hand, soil samples behave nonlinearly in laboratory tests at strains larger than 10^{-5} to 10^{-4}, a result that is standard in geotechnical research. On the other hand, it is also routine in seismology to assume that soil amplification factors measured from weak motions apply to strong motions, i.e., effects of nonlinearity are neglected.

The identification of nonlinearity in site response is challenging because the site effect itself is poorly isolated. The source and path effects are individual for every earthquake and tend to dominate the spectra of ground motions. The spectral ratio technique only partly relieves this problem. It is also true that the bulk of good-quality digital strong-motion data has been meager until very recently. Perhaps, the reality of nonlinear soil response could be considered completely hypothetical a decade ago.

In recent years, several large earthquakes have been recorded by modern digital surface and vertical arrays. Downhole data ensure a reliable evaluation of soil transfer functions. Observations in different parts of the world already provide established direct evidence of the significance of nonlinear site effects.

Various seismological observations referred to in the previous section indicate that the threshold acceleration beyond which nonlinearity becomes perceptible is ~100 to 200 gal. This range is slightly higher than could be estimated from the laboratory-derived material behavior but should be considered a realistically established value. In this acceleration range, reduced amplification occurs due to amplitude-dependent damping. It appears at this time that material behavior under laboratory conditions is slightly more nonlinear than in nature. Otherwise, geotechnical and seismological data are apparently being reconciled.

Nonlinearity under strong ground shaking depends on the physical properties of soils. Nonlinearity may be considerable for soft clays and sands and negligible for stiffer materials. One example is the study of two downhole array data by Wen *et al.* (1995), where soil behaved in accordance with nonlinear hysteretic law in one case, but nonlinearity was not revealed in the second case at comparable acceleration levels. This may explain some of the negative results of the searches for nonlinear seismic effects and should be taken into account in the practical work leading to microzonation or ground-motion forecast.

Acknowledgments

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