Scheduling models for optimal aircraft traffic control at busy airports: Tardiness, priorities, equity and violations considerations

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A B S T R A C T

This work addresses the real-time optimization of take-off and landing operations at a busy terminal control area in case of traffic congestion. Terminal areas are becoming the bottleneck of the entire air traffic control system, in particular in the major European airports, where there is a limited possibility to build new infrastructure. The real-time problem of effectively managing aircraft operations is particularly challenging, since it is necessary to incorporate the safety regulations into the optimization model and to consider numerous performance indicators that are important to compute good quality solutions. However, in practice there is no well-recognized objective function and traffic controllers often use simple scheduling rules. In this paper, mixed integer linear programming formulations are proposed to investigate the trade-off between various performance indicators of practical interest, while taking into account the safety constraints with a high modeling precision. Experiments are performed for the two major Italian airports, Milano Malpensa and Roma Fiumicino, by simulating various sets of random landing and take-off aircraft disturbances. Practical-size instances are solved to (near)optimality via a commercial solver. The optimized solutions are also compared with a commonly used scheduling rule. A comprehensive computational analysis makes possible the selection of those solutions that are able to find a good compromise among the various indicators and, consequently, the investigation of the most representative formulation.

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1. Introduction

The ever growing demand of air transport is increasing the pressure on air traffic controllers, since air traffic in peak hours is getting closer to the capacity of the Terminal Control Area (TCA), at least in the major European airports where there is limited possibility of creating new infrastructure. Aviation authorities are thus seeking optimization methods to better use the available infrastructure [4,5,21,27,45]. However, the development and the implementation of effective optimization methods for such operational problems require the consideration of a number of aspects that are rarely taken into account simultaneously in the related scheduling theory:

- The optimization model should be able to incorporate all detailed information that is compliant with the safety regulations of the TCA, including information which is not relevant for the air traffic flow management in large networks with multiple airports and is therefore neglected in macroscopic models [17,22,48]. In most of the macroscopic models, the characteristics of the airport infrastructure are drastically simplified and the flight paths are aggregated, so that potential conflicts between single aircraft may not be visible, at least at the level of runways, ground and air segments of the TCA. A potential conflict occurs whenever aircraft traversing the same resource do not respect the minimum required safety distance.
- The time available for developing a new schedule of take-off and landing aircraft in the TCA can be very limited, since a computerized scheduler should be able to promptly react to any significant change occurring during operations.
- To a large extent, air traffic control operations and related issues are still scheduled by human controllers, who develop feasible aircraft schedules in the TCA based on their past experience, intuition and some scheduling rules without using any formally defined performance indicator. Recently, the push from SESAR and for CDM compliance [36] is making this less common though and airports have at least some automated support systems for some of the operations. For example, different commercial arrival manager systems are used at various airports [42,74]. However, the controllers usually have to fine tune the arrivals sequencing coming out of the systems themselves at...
the moment, since these systems do not usually (fully) take into account the fine details of the aircraft movement required to land in the correct order. Furthermore, we believe that further automated support is required in order to compute alternative (near)optimal ASP solutions and evaluate them in terms of a number of performance indicators in a short-term. In fact, the existing arrival manager systems incorporate various performance indicators that need to be fine-tuned across all airports. The lack of a generally recognized performance indicator to optimize places importance on the definition of acceptable objective functions. The quality of scheduling aircraft in the TCA typically involves several performance indices reflecting the interests of the different actors involved in air traffic management, such as the aircraft punctuality, the utilization level of airport resources, the costs incurred by different airline companies in terms of delays, broken flight connections and energy consumption, and so on. All these indices should be taken into account in the schedule development phase.

This paper addresses the first item by developing mixed integer linear programming (MILP) formulations, that take into account the relevant TCA safety aspects and various performance indicators with a high level of detail. As shown in the survey of Bennell et al. [18], the aircraft scheduling literature presents numerous models of the independent runway sequencing problem. This problem is modelled as a single machine scheduling problem. A natural way to model and solve a more accurate and extended aircraft scheduling problem with interdependent runways and air segments of the TCA is via job shop scheduling. The latter type of modelling approach permits to consider the airspace interactions between aircraft in order to compute better quality aircraft scheduling solutions in terms of delay management and traffic flow coordination in the TCA.

The MILP formulations proposed in this paper can be considered as a generalization of existing job shop scheduling models with blocking (no-store), no-wait and other additional constraints. These models are known under the name generalized disjunctive graph or alternative graph. Previous research on those job shop scheduling models has been successfully applied to model and solve complex benchmark instances on job shop scheduling [39, 55–57, 62], railway traffic management problems [25, 50, 52], and air traffic management problems [20, 27, 28, 65, 66].

The second item suggests that optimization models with a single objective function are more suitable than multi-objective approaches, since more efficient tools are available to solve these problems. This is also the most common choice in the literature (see, e.g., the reviews in [14, 18, 24, 46, 48, 61]).

The present paper investigates MILP formulations with single objective functions in order to find a good compromise among the different indices listed in the third item. Specifically, we observe that aircraft typically flies at constant speed in the TCA and that at constant speed the energy consumption is almost proportional to the flying time. We use the aircraft flow time as a surrogate of the energy consumption. Also, we adopt makespan-like objective functions in order to minimize the maximum completion time (i.e. the arrival time of the last aircraft), as a common surrogate for the throughput maximization, or the maximum tardiness (i.e. the largest aircraft delay). Moreover, we implicitly take into account the minimization of broken flight connections by minimizing the number of aircraft delayed more than a given threshold. All performance indicators can be measured in terms of aircraft arrival times at the entrance of the TCA and at the runways.

The aircraft scheduling problem (ASP), we deal with in this paper, can be summarized as follows. Given a set of landing/take-off aircraft and for each aircraft its path in the TCA, its current position, its scheduled runway occupancy time and the required time window to accomplish the arriving/departing procedures, the ASP is to assign the start time to each aircraft in all the resources it crosses in its path in such a way that all the potential conflict situations between aircraft are solved (at a microscopic level) and a suitable objective function is minimized.

This work follows the approach of Bianco et al. [20], based on the no-wait version of the job shop scheduling problem. However, this paper is based on the alternative graph model introduced by Mascis and Pacciarelli [55], that is able to model the ASP with an increased level of detail. The higher modeling precision includes further relevant TCA aspects such as holding circles, waiting in flight before landing, traveling in feasible time windows, hosting multiple aircraft simultaneously in air segments and individual aircraft simultaneously in runways. Previous works based on the alternative graph model of the ASP have been proposed recently [26–28, 65, 66]. D’Ariano et al. [26, 28] deal with the development of a branch and bound algorithm for the ASP. D’Ariano et al. [27] extend the ASP to a routing and scheduling problem and solve it with a tabu search algorithm. Samà et al. [65, 66] develop a rolling horizon approach for the original and extended ASP. However, all these works deal with the minimization of a makespan-like objective function.

The contribution of this work is to generalize the work done on the ASP modelled via alternative graphs. We investigate microscopic MILP formulations of the ASP with different objective functions and examine the differences between the ASP solutions in terms of various performance indicators. As far as we know, the proposed formulations increase the level of detail regarding the modeling of the constraints in the airspace nearby the TCA compared to the existing models, and permit to deal with any kind of objective function and constraint. We believe that the investigation of a suitable formulation of the ASP, that takes into account several performance indicators and models the constraints with high precision, is still an open problem in the related literature.

A computational study is presented for assessing the practical applicability of the proposed formulations. The ASP solutions are analyzed from the viewpoint of the above described performance indicators and trade-off between them, while previous research often focuses on a single performance indicator, with a myopic view in terms of other possible performance indicators. A procedure is proposed to develop a combined formulation with a good trade-off performance on several indicators.

The experiments have been carried out on the main Italian airports in terms of passenger flows: Roma Fiumicino (FCO) and Milano Malpensa (MXP). Regarding the air traffic disturbances, 40 randomly delayed scenarios are considered for practical-size instances. The resulting problems are solved with a commercial solver to (near)optimality for each ASP formulation. The optimized solutions are also compared with the solutions computed by a practical scheduling rule.

Section 2 reviews the literature most relevant for this work. Section 3 formally describes the modelling of specific ASP constraints. Section 4 presents the mathematical formulations. Section 5 reports the experiments conducted on the FCO and MXP instances. Section 6 summarizes the paper results and outlines future research directions. Two appendices illustrate the alternative graph modeling and solving for a numerical ASP example.

2. Literature review

This section briefly reviews recent papers on some aspects of the air traffic flow management (ATFM) problem. We present various ATFM literature classifications and discuss our contribution. A more general discussion of the existing literature can be found e.g. in [1, 14, 18, 24, 33, 46, 48, 61].
A first classification of the ATFM literature is based on the following two basic categories: the traffic control between airports (see e.g. [17,21,22]) and the traffic control in the TCA of an individual airport (see e.g. [20,26,37,45,51]). For the former category, macroscopic models for large networks with multiple airports and aggregated flight paths are often adopted. For the latter category, microscopic models are proposed with identification and resolution of potential aircraft conflicts at the level of ground and air resources. This paper deals with the development of microscopic models for the resolution of potential conflicts in the TCA.

A second classification is based on the type of information. When dealing with static information (see e.g. [26,28,34,63]), the position and the speed of all aircraft are known in the traffic prediction. The case with dynamic information (see e.g. [15,43,65,75]) requires the computation of an aircraft schedule every time a new incoming aircraft is known. This paper studies an ASP based on static information, since all the data are known before the optimizer starts the computation of the ASP solution. However, this approach can be inserted in a dynamic system that iteratively solves an ASP problem with static information (see e.g. [65,66]).

A third classification is based on the algorithmic approaches. Among the heuristic approaches, fast heuristics are proposed in [27,28,34,60]. Exact procedures can be found in [26,28,34,35]. This paper does not focus on the investigation of new ASP algorithmic approaches or mathematical properties. We are interested in the comparison of (near)optimal ASP solutions in terms of several approaches or mathematical properties. We are interested in the paper does not focus on the investigation of new ASP algorithmic formulation and a general MILP solver.

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optimization approach to solve an aircraft recovery problem with minimization of the fuel consumption and the aircraft delay. Most of the literature is based on a simplified model of the TCA. We believe that this paper is a step forward in the definition of a flexible microscopic formulation for the ASP in the TCA, that is able to take into account different performance indicators, either in the objective function or in the set of constraints. Our work is complementary to the work done in the departure/take-off sequencing literature. We use a detailed modeling of inter-aircraft separation rules for arrival and departure aircraft. However, our work does not deal with the detailed management of ground movements. Our contribution is the coordinated management of the runways and the air segments of TCA. Specifically, our work is not the classical departure and take-off sequencing problem that usually schedules aircraft in independent runways, and thus can be viewed as a set of single machine scheduling problems. We deal with a problem with interdependent runways and air segments of the TCA. As reported in the existing literature (see e.g. the survey paper [18]), the problem studied in this paper can be viewed as a job shop scheduling problem, that is by far more complex than the independent runway scheduling problem.

We think that the traffic controllers should be informed of the existence of alternative ASP solutions in real-time and of the potential impact generated by implementing each ASP solution in terms of a number of relevant performance indicators. To this end, this paper studies some of the above discussed indices and their combinations. We evaluate them with special emphasis on extent of the impact of each specific indicator on the others. The evaluation is performed on MILP formulations based on the alternative graph model of [55], originally developed for a makespan-like objective function. In fact, previous works dealing with the alternative graph model of the ASP (see e.g. [26–28,65,66]) are focused on the minimization of the maximum consecutive delay in a busy TCA (i.e. the minimization of the largest delay caused by the resolution of potential conflicts between aircraft traveling in a busy TCA during a given time horizon of traffic optimization). In this work, we generalize the alternative graph model of the ASP, in order to deal with any kind of performance indicator. Practical-size instances are solved by an MILP solver to (near)optimality.

3. Problem description

In the TCA, landing (arriving) aircraft move from an air entry point of the TCA to a runway via landing air segments, following a standard descent profile, while maintaining a minimum safety distance between every pair of consecutive aircraft, depending on their type and position (at the same or different altitude). Final approach spacing tools can be of support for the computation of feasible sequencing, moving and spacing of landing aircraft in the TCA [31].

Similarly, take-off (departing) aircraft leave the runway flying toward the assigned exit point via take-off air segments along a standard ascent profile, still respecting the minimum safety distance (safety separation). The space distance can be translated into a time distance, setup time, by taking into account the different aircraft speeds. Setup times are considered sequence-dependent, since the minimum distance between different aircraft categories (heavy, medium, light and others) depends on the relative processing order of the common resources. For instance, the distance between heavy and light aircraft is much larger when light aircraft follows heavy aircraft than vice versa. Setup times do not only depend on the aircraft times but also on the route chosen for each aircraft.

Each aircraft has an assigned entry time into the TCA, which is the minimum time, release time, the landing/take-off procedure can start according to the current aircraft position and speed. Each aircraft has scheduled times, due date times, to start processing some TCA resources. Eventually, aircraft can also have a maximum time, deadline time, to start processing some TCA resources.

The runway is a blocking (no-store) resource [41] since it can only be occupied by one aircraft at a time, while the air segment is an uncapacitated resource since several aircraft can occupy it at the same time provided that the required safety separations are satisfied. Each aircraft has a processing time on each runway sand on the air segments before or after it, according to its landing/take-off profile. On the air segments, the processing time varies between a pre-defined time window, due to a limited possibility of aircraft speed changes.

Once an arriving aircraft enters the TCA, it should proceed to the runway. However, before entering the airport area, airborne holding time can be used to make aircraft waiting in flight until they can be guided through their landing procedure, that means flying in circles in specific areas named holding circles. On entry to a holding circle, the aircraft must fly at a fixed speed for a number of half circles, as prescribed by the air traffic controller. We assume that there are no aircraft sequencing decisions in the holding circle. Therefore, the aircraft can exit in a different order they enter in the holding circle and each holding circle resource is uncaptacitated.

Departing aircraft can be delayed in entering the TCA at ground level, i.e. before entering the runway. A departing aircraft is supposed to take-off within its assigned time window and is late whenever it is not able to accomplish the departing procedure within its assigned time window. Following the procedure commonly adopted by air traffic controllers, we consider a time window for take-off between 5 min before and 10 min after the Scheduled Take-off Time (STT). A departing aircraft is considered delayed in exiting the TCA if leaving the runway after 10 min from its STT. We assume that all take-off aircraft have a take-off time window with the same tolerance. Arriving aircraft are late if landing after their Scheduled Landing Time (SLT).

We use the following notation for the aircraft delays. Entrance delay is the delay of a landing/take-off aircraft on entry to the TCA. Total exit delay is the delay of a landing/take-off aircraft at the entrance in the runway/take-off air segment resource. The latter value is partly a consequence of a possible late entrance, which causes an unavoidable delay at the runway/take-off air segment resource. The latter value is partly a consequence of a possible late entrance, which causes an unavoidable delay at the runway/take-off air segment resource. The latter value is partly a consequence of a possible late entrance, which causes an unavoidable delay at the runway/take-off air segment resource. The latter value is partly a consequence of a possible late entrance, which causes an unavoidable delay at the runway/take-off air segment resource.

3.1. Performance indicators

The typical objectives of any real-time scheduling practitioner are (i) to find a good schedule in the short-term while (ii) trying to avoid negative long-term effects of the rescheduling decisions. In the air traffic environment under study, these needs correspond to schedule arrivals and departures with the aim of (i) reducing short-term delays and (ii) trying to recover the off-line plan of arrivals-departures as quickly as possible, in order to reduce long-term propagation of perturbations.

In this paper, the minimization of the maximum tardiness takes into account the first goal, while the minimization of the maximum completion time is a surrogate for the second goal. In
general, the minimization of a makespan-based objective function results in a more compact schedule compared to other practical objectives, which leaves more time available for accommodating future arrivals and departures and tries to reduce the long-term effects of past perturbations.

In real-time the ASP translates into an optimization problem for a limited time horizon of traffic optimization, with consequent myopic view on the overall traffic control horizon. In a companion paper [65], we deal with a rolling horizon approach in order to investigate the impact of traffic optimization in terms of the overall traffic control horizon. However, as far as we know, the rolling horizon approaches, or other problem decomposition approaches, cannot compute the optimal ASP solution for large-scale problems and thus are not useful in the context of this work.

There can be many ASP solutions with different view points (airline companies, local and global authorities). We choose a set of objective functions to be minimized in a given time horizon of traffic optimization, each one interested in looking at a particular aspect of the ASP.

Our interpretation of equity is different from the common equity consideration in the related literature that is based on some scheduling rule, such as the first scheduled, first served rule or the first come, first served rule. We agree that these rules can be considered useful measures of the deviation from an ideal (equitable) schedule. However, we observe that using such a scheduling rule may generate poor quality solutions in terms of the minimization of delay propagation or even infeasible schedules when applied to reschedule aircraft during operations. For these reasons, we use a new equity consideration that is viewed as the computation of an ASP solution that minimizes the consecutive delays, i.e. the delays due to the aircraft sequencing decisions. Specifically, we consider two equity measures: the minimization of the maximum consecutive delay (maximum tardiness), and the minimization of the largest difference of consecutive delays among the aircraft of each class (priority equity).

When minimizing the maximum tardiness, we focus on the minimization of the longest path in the graph, that often involves a few aircraft. The minimization of the average consecutive delays (average tardiness) is a more global vision since it requires the consideration of all delayed aircraft.

In our approach, aircraft priorities are also taken into account as follows: landing aircraft, due to safety measures, have greater priority than departing aircraft, which can wait at ground level with fewer risks. A landing, delayed aircraft has even greater priority, due to a lower level of fuel. In fact, a fairness concept in air traffic management is to give priority to late aircraft [14]. In this paper, the set of aircraft is divided into four classes which are ordered below in importance (from the highest to the lowest priority): (1) landing, delayed aircraft; (2) landing aircraft on time; (3) take-off, delayed aircraft; (4) take-off aircraft on time. The minimization of the weighted average consecutive delays (priority tardiness) requires a weighted consideration of the aircraft.

Mirroring the two different approaches used to minimize delays, in this priority scenario we compare the solutions that take into account equity, defined here as the minimization of the average difference between maximum and minimum tardiness for each class, with the ASP solutions that minimize a weighted average tardiness with weights assigned to each aircraft according to the corresponding level of priority.

Another aspect to be taken into account when solving the ASP is the use of existing, critical airport resources. This consideration translates into the maximization of the throughput, that can be viewed as the minimization of the maximum completion time (see e.g. [20]). In our view, it corresponds to the landing/take-off time of the last aircraft traveling in the TCA during the time horizon (time span) of the traffic optimization. This objective function is compared with the minimization of the average completion time. The latter objective looks at all aircraft in the studied time horizon of traffic optimization.

Finally, we evaluate the number of delayed aircraft exceeding a given tolerance thresholds and minimize the number of deadline violations, that imply additional operational costs for the airline companies, due e.g. to broken flight connections with other aircraft at the same TCA. This objective is focused on the aircraft that have a delay above the given threshold (tardy jobs).

3.2. Terminal control areas

Fig. 1 shows the TCA scheme of Milan Malpensa airport (MXP). There are two interdependent runways (RWY 35L, RWY 35R), used both for departing and arriving procedures. The MXP resources are three airborne holding circles (resources 1–3 in Fig. 1, named TOR, MBR, SRN), 11 air segments for arriving procedures (resources 4–14), a common glide path (resource 15), two runways (resources 16 and 17) and three air segments for departing procedures (resources 18–20, named SRN, TELVA, RMG). The common glide path resource includes two parallel air segments before the runways for which traffic regulations impose a minimum diagonal distance between landing aircraft added to a minimum longitudinal one.

Fig. 2 presents the scheme of another TCA, Roma Fiumicino airport (FCO). In this case, three interdependent runways (RWY 16L, RWY 16R, RWY25) can be used for departing and arriving procedures, but two of them (RWY 16R and RWY 25) cannot be used simultaneously and are thus considered as one. The FCO resources are three airborne holding circles (resources 1–3 in Fig. 2, named CIA, CMP, TAQ), seven landing air segments (resources 4–10), two runways (resources 12 and 13), a common glide path (resource 11) and three take-off air segments (resources 14–16, named BOL, RAVAL, ELIVIN).
4. Problem formulation

In the general job shop scheduling formulation of the ASP, an operation denotes the traversal of a resource (i.e. air segment, common glide path, runway, holding circle) by a job (i.e. aircraft). The sequence of operations related to an aircraft represents the (pre-defined) route associated with that aircraft. The variables of the ASP are the start time $t_i$ of each operation $i$ to be performed by an aircraft on a specific resource. A set of route timings is conflict-free if, for each pair of operations associated with the same resource, the minimum time separation constraints are satisfied.

The ASP is represented by the alternative graph model [55], since this approach permits an accurate and efficient representation of the ASP [26,27,65–67]. Let $G = (N,F,A)$ be the graph composed of the following sets: $N = \{0,1,...,n\}$ is the set of nodes, where nodes 0 and $n$ represent the start and the end operations of the schedule, while the other nodes are related to the start of the other operations; $F$ is the set of fixed directed arcs that model the sequence of operations to be executed by an aircraft; $A$ is the set of alternative pairs that model the sequencing decision and inter-aircraft safety separations. Each pair is composed of two alternative directed arcs.

Each node $i \in N$ of the graph is associated with the start time $t_i$ of operation $i$, and corresponds to the entrance of the associated aircraft to the associated resource. By definition, the start time of the schedule is a known value, e.g. $t_0 = 0$, and the end time of the schedule is a variable $t_n$.

Each fixed directed arc $(i,j) \in F$ has a length $w_{ij}$, which is uniquely determined by $i$ and $j$. The fixed arc length $w_{ij}$ models a minimum processing time between the start of $i$ and the start of $j$, such that $t_j \geq t_i + w_{ij}$. In particular, $\sigma(i)$ denotes the operation following $i$ on its route. It follows that $(i,\sigma(i)) \in F$ is the directed fixed arc connecting $i$ with $\sigma(i)$ and $t_{\sigma(i)} \geq t_i + w_{i\sigma(i)}$.

Each alternative pair $(i,j),(h,k) \in A$ has two arcs with length $w_{ij}$ and $w_{hk}$. The alternative arc length $w_{ij}$ represents a minimum separation time between the start of $i$ and the start $j$. In particular, $w_{ij}(w_{hk})$ can be sequence-dependent, when nodes $i$ and $j (h$ and $k)$ are operations of different jobs. Also, there can be multiple alternative arcs between nodes $i$ and $j$.

In the tested ASP instances, the landing aircraft use both runways, while the take-off aircraft only used one (interdependent) runway resource for each TCA (i.e. runway resource 16 in Fig. 1 and runway resource 12 in Fig. 2). We do not perform rerouting measures for landing and take-off aircraft. D’Ariano et al. [27] and Samà et al. [66] focus on the combined aircraft reordering and rerouting problem at busy TCAs.

![Fig. 2. Fiumicino (FCO) terminal control area.](image)

![Fig. 3. Alternative graph of a holding circle (a), selected (b) and not selected (c).](image)

A selection $S$ is a set of alternative arcs, at most one from each pair. An ASP solution is a complete selection $S^*$, where an arc for each alternative pair of the set $A$ is selected, in which the connected graph $(N,F,S^*)$ has no positive length cycles. Note that a positive length cycle represents an operation preceding itself, which is an infeasibility. Given a feasible schedule $S^*$, a timing $t_i$ for operation $i$ is the length of a longest path from 0 to $i$ ($t^F(0,i)$). When minimizing a makespan-like objective function, an arc $(k,n)$ between the end node $k$ of each job and node $n$ is added to the alternative graph, and a selection $S^*$ is optimal if $t^F(0,n)$ is minimum over all the solutions.

The alternative graph can be viewed as a particular disjunctive program. We let $X$ be the set:

\[
X = \left\{ f \geq 0, x \in \{0,1\}^{\left| A \right|} : f_1 - f_2 + M(1-x_{ij}) \geq w_{ij} \right\}
\]

The variables of the ASP are the following: $|N|$ real variables $t_i$ associated with the start time of each operation $i \in N$ and $|A|$ binary variables $x_{ij}$ associated with each alternative pair $(i,j) (h,k) \in A$. The variable $x_{ij}$ is 1 if $(i,j) \in S$, and $x_{ij} = 0$ if $(h,k) \in S$. The constant $M$ is a sufficiently large number, e.g. the sum of all arc lengths.

The next subsections describe how the different types of TCA resources are modelled via alternative graphs, and show how each specific objective function can be formulated. A numerical ASP example of a traffic situation is illustrated for the FCO airport. For the proposed example, we give the trade-off between a set of non-dominated solutions, each one computed by solving a specific objective function to optimality. Graphs of the example are reported in Appendices A and B.
4.1. Resources in the alternative graph model

The TCA is composed of various types of resources. This section illustrates how each of them is modelled in the alternative graph. Fixed arcs are depicted with solid arrows and alternative arcs are depicted with dotted arrows.

Fig. 3(a) illustrates the formulation of a holding circle resource. We recall that holding circles are used by traffic controllers to let arriving aircraft wait before the start of their landing procedure when the TCA is congested. Let i be the entrance of aircraft A to the holding circle and let σ(i) be the start of the next operation, the holding circle resource is formulated by two fixed arcs (i, σ(i)) and (σ(i), i) (the two solid arrows), respectively of length $w^F_{fi} = 0$ and $w^F_{σ(i)i} = -δ$, where δ is the time required to perform a holding circle, plus a pair of alternative arcs ((i,σ(i)),(σ(i),i)) (the two dotted arrows) respectively of length $w^A_{fi} = δ$ and $w^A_{σ(i)i} = 0$. The formulation of multiple (half) circles can be easily done in a similar way.

Fig. 3 (b) shows the decision to perform a holding circle (the dotted arrow (i, σ(i)) is selected), while Fig. 3(c) the case with no holding circle (the dotted arrow (σ(i), i) is selected). The formulation of the holding circle constraints and variables is as follows:

$$
\begin{align*}
t_{σ(i)} - t_i & \geq w^F_{fi} = 0 \\
t_i - t_{σ(i)} & \geq w^F_{σ(i)i} = -δ \\
t_{σ(i)} - t_i + M(1 - x_{σ(i)i}) & \geq w^A_{fi} = δ \\
t_i - t_{σ(i)} + Mx_{σ(i)i} & \geq w^A_{σ(i)i} = 0
\end{align*}
$$

(2)

If $x_{σ(i)i} = 0$ then no holding circle is performed, as shown in Fig. 3(c). Fig. 4(a) presents the formulation of an air segment. We define operation i (σ(i)) as the entrance (exit) of aircraft A to the air segment. The processing time of aircraft A in the air segment must be included in the range $[θ_{min}, θ_{max}]$. To model this range of values $[θ_{min}, θ_{max}]$ are used with length $w^A_{σ(i)i} = θ_{min}$ and $w^A_{σ(i)i} = θ_{max}$, respectively. The following constraints are thus required for the processing time of aircraft A and B:

$$
\begin{align*}
t_{σ(i)} - t_i & \geq w^F_{fi} = θ_{min} \\
t_i - t_{σ(i)} & \geq w^F_{σ(i)i} = -θ_{max} \\
t_{σ(i)} - t_i & \geq w^A_{fi} = θ_{max} \\
t_i - t_{σ(i)} & \geq w^A_{σ(i)i} = -θ_{min}
\end{align*}
$$

(3)

In each landing/take-off air segment, air traffic regulations impose a minimal longitudinal and diagonal separation distance between consecutive aircraft, that varies according to the aircraft category. The minimum separation time between two aircraft (named A and B in Fig. 4) in the same air segment is thus formulated as a sequence-dependent setup time.

Since an overtake between two aircraft in the same air segment is not allowed for safety reasons, the entrance and exit orders between aircraft A and B must be the same. A sequence-dependent setup time between aircraft A and B is required at the entrance and exit of the air segment as follows: if A precedes B (B precedes A) the setup time at the entrance of the air segment is $µ_i^w(θ_{i1}) = Mx_{i1}$ and the setup time at the exit of the air segment is $µ_i^w(θ_{i2}) = Mx_{i2}$. The sequencing decision variables are the two pairs of alternative arcs in Fig. 4(a): $(i, σ(i), σ(i), i)$ with lengths $w^A_{fi} = µ_i^w$ and $w^A_{σ(i)i} = µ_{i2}^w$: $(j, i, σ(i), σ(j))$ with lengths $w^A_{ji} = µ_{i0}^w$ and $w^A_{ij} = µ_{i2}^w$.

In a feasible schedule (i.e. a complete selection with no positive length cycles), the selection of the two alternative arcs $(i, j)$ and $(σ(i), j)$ sequences aircraft A before B, i.e. the two constraints $t_j - t_i \geq w^A_{ji}$ and $t_{σ(i)} - t_i \geq w^A_{σ(i)i}$ are inserted in the graph. Otherwise, the two alternative arcs $(j, i)$ and $(σ(j), i)$ must be selected (aircraft B before A).

The formulation of the air segment sequencing decisions is next shown:

$$
\begin{align*}
t_j - t_i + M(1 - x_{ji}) & \geq w^A_{ji} = µ_{i1}^w \\
t_{σ(i)} - t_i + Mx_{σ(i)i} & \geq w^A_{σ(i)i} = µ_{i1}^w \\
t_{σ(i)} - t_i + M(1 - x_{σ(i)i}) & \geq w^A_{σ(i)i} = µ_{i2}^w \\
t_i - t_j + Mx_{σ(i)i} & \geq w^A_{σ(i)i} = µ_{i2}^w
\end{align*}
$$

(4)

If $x_{ji} = 0$ then the alternative arc of length $w^A_{jσ(i)i}$ is selected, i.e. aircraft B is scheduled first in the air segment (as shown in Fig. 4(b)). Consequently, the variable $x_{σ(i)i}$ must be set to 0 (i.e. the alternative arc of length $w^A_{jσ(i)i}$ is selected); otherwise the alternative arc of length $w^A_{jσ(i)i}$ is selected and a positive length cycle between nodes $σ(i)$ and $σ(j)$ is generated in the graph. In fact, there are only two possible sequencing decisions (A–B or B–A) between the two aircraft in the air segment, and therefore the variable $x_{σ(i)i}$ could be replaced with $x_{ji}$ in (4).

Fig. 5(a) shows the runway formulation. Let i, j be the entrance of aircraft A, B to the runway and $σ(i), σ(j)$ their exit. The processing times of aircraft A and B in the runway are $θ_A$ and $θ_B$. These processing times are modelled by the two fixed arcs (i, $σ(i)$) and (j, $σ(j)$) with lengths $w^F_{fi} = θ_A$ and $w^F_{ji} = θ_B$:

$$
\begin{align*}
t_{σ(i)} - t_i & \geq w^F_{σ(i)i} = θ_A \\
t_{σ(j)} - t_j & \geq w^F_{σ(j)j} = θ_B
\end{align*}
$$

(5)

In a feasible schedule, a sequence-dependent setup time is required at the runway. If A precedes B (B precedes A) the setup time at the runway is $θ_{i2}^w$ ($θ_{j2}^w$). Furthermore, the runway is a no-store resource, since only one aircraft at a time can be scheduled.

![Fig. 4. Alternative graph of an air segment (a), and a feasible arc selection (b).](image)
The no-store formulation is:

\[ t_j - t_{\sigma(j)} + M(1 - x_{\sigma(i),j}) \geq w_{\sigma(i),j} = \eta_{AB} \]  
\[ t_j - t_{\sigma(i),j} + Mx_{\sigma(i),j} \geq w_{\sigma(i),j} = \eta_{BA}. \]  

(6)

The sequencing decision is modelled via the alternative pair \((\sigma(i), j), (\sigma(j), i))\), in which the alternative arc \((\sigma(i), j)\) has length \(w_{\sigma(i),j} = \eta_{AB}\) and the alternative arc \((\sigma(j), i)\) has length \(w_{\sigma(j),i} = \eta_{BA}\). In Fig. 5(b), aircraft A is scheduled before aircraft B and the alternative arc \((\sigma(i), j)\) is selected. In this case the binary variable \(x_{\sigma(i),j} = 1\).

The delay of each aircraft can be measured by means of suitable due date times. In this paper, we consider two due date times for each landing aircraft, measured at the entrance of the TCA and at the entrance of the runway. Take-off aircraft have a single due date time at the runway. Both landing and take-off aircraft have a release time at the entrance of the TCA.

Fig. 6 presents the alternative graph modeling of the release and due date times for a landing aircraft A. Node f models the first operation of aircraft A in the TCA, while node g models its runway operation. The runway due date time \(d_j\) is the scheduled entrance time of aircraft A in the runway, modelled by the runway due date arc \((j, n)\) between nodes j and n of length \(w_{j,n} = -d_j\). The entrance due date time \(d_i\) is the scheduled entrance time of aircraft A in the TCA, modelled by the entrance due date arc \((i,j)\) between nodes i and n of length \(w_{i,j} = -d_i\). The entrance time of aircraft A in the TCA is constrained by a release time \(r_i\) that is the minimum time at which this aircraft can enter the TCA, modelled by the release arc \((0,i)\) between nodes 0 and i of length \(w_{0,i} = r_i\). The formulation of release and due date arcs for aircraft A is as follows:

\[ t_n - t_j \geq w_{j,n} = -d_j \]  
\[ t_n - t_i \geq w_{i,n} = -d_i \]  
\[ t_j - t_0 \geq w_{0,i} = r_i. \]  

(7)

Additional scheduling requirements can be modelled in the alternative graph when defining the weight of runway due date arcs. For instance, one can take into account integrated recovery strategies between the runway and ground scheduling solutions as follows. The runway due date arc \((j,n)\) can have length \(w_{j,n} = -d_j - s_j\), in which \(s_j\) is a slack time that can have a positive (negative) value if one want to have a more (less) flexible due date time at the runway. Alternatively, one can use the deadline arc \((j,0)\) between nodes j and 0 of length \(w_{j,0} = -D_j\) in order to fix a deadline time \(D_j\) to start processing the runway: \(t_0 - t_j \geq -D_j\). For instance, the latter type of arc can be used to model a rigid coordination time requirement between the runway and ground scheduling solutions.

### 4.2. Formulations with different objective functions

This section presents the MILP formulations of the ASP used in this paper. For each formulation, we consider all problem constraints introduced in the previous section and we describe how the specific objective functions have been modelled.

The first group of formulations is related to aircraft delay minimization. Here, we use the following types of due date arcs: the entrance due date arcs are associated with the first operation of landing aircraft and measure their entrance delay in the TCA; the runway due date arcs are associated with the operation modelling the entrance/exit to/from a runway resource of landing/take-off aircraft and measure the delay caused by the resolution of potential aircraft conflict in the TCA. Both types of due date arcs measure the delay generated by the scheduling decisions, i.e. by the selection of one alternative arc from each alternative pair of the graph.

The length of entrance and runway due date arcs are defined in the alternative graph as follows. We let \(i\) be the first operation of an aircraft in the TCA, \(\gamma_i\) be its scheduled entrance time and \(q_i\) be the entrance delay. We assume that the latter information is an input data for the traffic controller. For the landing aircraft, the entrance due date arc has length \(-d_i = -(\gamma_i + q_i)\), and the consecutive delay at the entrance of the TCA is \(\max(0, \gamma_i - d_i)\).

The total exit delay is the sum of the unavoidable delay (which cannot be recovered by aircraft rescheduling, even if the aircraft travel in the TCA with their minimum processing time) plus the consecutive delay (which is required to solve potential conflicts).

We let \(j\) be the arriving/departing operation at/from a runway \(r\) of a landing/take-off aircraft \(A, d_j\) be its scheduled arrival/departure time and \(r_j\) be the earliest possible entrance/exit time to/from the runway \(r\). For each node \(j\) of a landing aircraft, \(r_j\) is computed as the sum of the release time plus the minimum processing time into the landing air segments. In case of a departing aircraft, \(r_j\) is the sum of the release time plus the scheduled processing time in the runway. The total exit delay of aircraft \(A\) at \(r = t_j - d_j\). Since we want to minimize the consecutive delay at \(r\), the runway due date arc has length \(-d_j = -\max(r_j, \delta_j)\). The unavoidable delay at the runway is \(\max(0, t_j - d_j)\), while the consecutive delay at the runway is \(\max(0, t_j - \delta_j)\).

The MAX TARDINESS is the formulation that minimizes the maximum consecutive delay both for the entrance and runway due date times, that is the largest deviation from the entrance and due date times due to the resolution of potential conflicts in the TCA during the time horizon considered. This objective function can be represented as the minimization on the longest path from node 0 to node \(n\) in the alternative graph with entrance and runway due date arcs [25,26]. We observe that all aircraft have the same relevance with this objective function, meaning that this is the most equitable approach. The formulation is next shown:

\[
\min t_n \quad \text{s.t.} \quad t_n - t_k \geq -d_k \quad \forall (k, n) \in E \\
\{x, t\} \in X
\]

(8)

The AVG TARDINESS is the minimization of the average consecutive delay both for the entrance and runway due dates. The
We call this formulation value aircraft that have a consecutive delay above a given threshold to maximize the satisfaction of airline company requirements, all airlines and the traffic costs, according to service contracts between the airline companies. In our approach, in order to satisfy airline company requirements, we attempt to minimize the number of deadline violations that often translate into penalty costs, according to service contracts between the airline companies and the traffic control authorities. The second formulation takes into account aircraft priorities and focuses on maximizing the equity between the aircraft of each priority class. Here, the equity is defined as the difference between the largest ($D_{\text{max}}$) and the smallest ($D_{\text{min}}$) consecutive delays between all aircraft of the same class $c \in C$. This formulation is named PRIORITY EQUITY:

$$\min \frac{1}{|C|} \sum_{c=1}^{C} |D_{\text{max}} - D_{\text{min}}|$$

s.t.

$$z_k - t_k \geq -d_k \quad \forall (k, n) \in F$$

$$z_k - D_{\text{max}} \leq 0 \quad \forall (k, n) \in F \land k \in J_c \quad \forall C$$

$$z_k - D_{\text{min}} \geq 0 \quad \forall (k, n) \in F \land k \in J_c \quad \forall C$$

where $|C|$ is the number of classes and $J_c$ is the set of aircraft belonging to class $c \in C$.

The last group of formulations is related to the maximization of throughput. To deal with throughput, we fix $d_k=0 \ \forall k \in K$. The formulation MAX COMPLETION minimizes the exit time of the last aircraft from the runway. This formulation corresponds to the following makespan minimization problem:

$$\min t_n$$

s.t.

$$t_n - t_k \geq 0 \quad \forall (k, n) \in F$$

$$|x, t| \in X$$

4.3. A numerical example

This section describes a simple traffic situation at the Roma Fiumicino (FCO) TCA, highlighting the difference between the optimal solutions computed for each model of the previous section. For each solution, we provide the value of all the other performance indicators.
Fig. 7 presents a schematic view of the TCA and provides the route of each aircraft: A and C are landing aircraft, while B and D are take-off aircraft. A and D are delayed aircraft, i.e., they have an entrance delay that changes their release time. The entrance delay of A is 170 and the one of D is 489. Furthermore, all aircraft have to use the same runway resource (12), since the other one (13) is not available. The presence of disturbances causes potential conflict in the TCA and, therefore, the ASP must be solved. The alternative graph of the traffic situation is shown in Appendix A.

Table 1 gives the optimal solution value for each objective function (one per row in bold), and the corresponding value for all other performance indicators. The optimal solution is obtained by solving the ASP formulations of Section 4.2. Regarding the formulations with aircraft priorities, we consider a different class for each aircraft as described in Section 3. According to the aircraft types and the delayed aircraft, the adopted weights are: Priority Tardiness (second formulation with aircraft priorities), Avg Completion (third formulation), and Priority Equity share the same solution. The different ASP solutions in Table 1 are also illustrated in Appendix B.

Table 2 describes the 20 ASP instances that we generated with different weights can be used in the objective functions). The assumptions made in the data sets have been inspired by the current practice at the studied airports. In general, the proposed MILP formulations can deal with any routing combination at runways (i.e., all runways can be eventually modelled in mixed-mode), and any prioritization of take-off and landing aircraft (i.e., different weights can be used in the objective functions).

For each terminal control area (MXP or FCO), we deal with practical-size instances of 30-min traffic optimization. All traffic optimizations start at time t0 = 0. The tests have been performed in a laboratory environment by using real-world data from MXP and FCO TCAs. The studied ASP instances are characterized by two types of aircraft (named medium and heavy aircraft). Consequently, sequence-dependent setup times are modelled in each ASP instance. The processing and setup times are computed for each aircraft category according to standard descent and ascent profiles, disregarding the actual aircraft passenger and freight load. The release and due date times are computed from a reference timetable.

Only one runway is used in a mixed mode for each TCA (resource 16 at MXP and resource 12 at FCO). There are up to 13 (12) aircraft scheduled in each runway during half hour traffic optimization at MXP (FCO). This can be considered a dense traffic while comparing the traffic flows at MXP and FCO with the other Italian airports. The impact of studying a limited time horizon of traffic optimization is not investigated in this paper.

Table 2 is organized as follows. Column 1 presents the TCA (FCO and MXP). Columns 2 and 3 the number of landing and take-off aircraft, Columns 4 and 5 the maximum and average entrance delays (in seconds), Columns 6 and 7 the maximum and average unavoidable delays (in seconds). The latter delays are significantly

5. Experimental results

This section presents the computational results for the ASP formulations of Section 4.2. The tests have been performed in a laboratory environment. We consider real-world ASP instances for FCO and MXP. The ASP solutions are computed via the solver IBM ILOG CPLEX 12.0, with a given time limit of computation. The experiments are executed on a processor Intel Dual Core E6550 (2.33 GHz), 2 GB of RAM and Windows XP.

5.1. Description of the ASP instances

The assumptions made in the data sets have been inspired by the current practice at the studied airports. In general, the proposed MILP formulations can deal with any routing combination at runways (i.e., all runways can be eventually modelled in mixed-mode), and any prioritization of take-off and landing aircraft (i.e., different weights can be used in the objective functions).

For each terminal control area (MXP or FCO), we deal with practical-size instances of 30-min traffic optimization. All traffic optimizations start at time t0 = 0. The tests have been performed in a laboratory environment by using real-world data from MXP and FCO TCAs. The studied ASP instances are characterized by two types of aircraft (named medium and heavy aircraft). Consequently, sequence-dependent setup times are modelled in each ASP instance. The processing and setup times are computed for each aircraft category according to standard descent and ascent profiles, disregarding the actual aircraft passenger and freight load. The release and due date times are computed from a reference timetable.

Table 2 describes the 20 ASP instances that we generated with random entrance delays (10 with negative exponential distribution and 10 with Gaussian distribution). The entrance delays are randomly generated for the first half aircraft entering the TCA according to a given distribution. The exit delays are measured as a positive deviation from the scheduled take-off/landing time, that is derived from the reference timetable. Each row reports average information on a terminal control area. In total, the computational study is based on 40 ASP instances.

Table 2 is organized as follows. Column 1 presents the TCA (FCO and MXP). Columns 2 and 3 the number of landing and take-off aircraft, Columns 4 and 5 the maximum and average entrance delays (in seconds), Columns 6 and 7 the maximum and average unavoidable delays (in seconds). The latter delays are significantly

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smaller than the entrance delays, since we compute the free-net
Solutions for the ASP formulations.
Table 4
Variables and constraints for each ASP formulation.

Table 3
Variables and constraints for each ASP formulation.

Table 4
Solutions for the ASP formulations.

The next subsections will show the computational results obtained for the 40 ASP instances. We tested the 8 ASP formulations: Max and Avg Tardiness dealing with pure delay minimization, Priority Tardiness and Priority Equity dealing with aircraft classes, Max and Avg Completion dealing with throughput minimization, Tardy Job \( P=0 \) and Tardy Job \( P=300 \) dealing with deadline violations. In addition, we tested the 40 ASP instances with a practical scheduling rule and a combined formulation. As described in Section 3, we used four classes of aircraft: (1) landing, delayed aircraft; (2) landing aircraft on time; (3) take-off, delayed aircraft; (4) take-off aircraft on time. Their weights are: \( f_1 = 20 \), \( f_2 = 10 \), \( f_3 = 2 \) and \( f_4 = 1 \).

5.2. ASP formulations

Table 4 presents the average computational results obtained for each ASP formulation. Each column reports the average results on the 20 ASP instances for a traffic control area. Table 4 is organized in blocks of eight rows per formulation: Row 1 gives the objective function, Row 2 the average computation time (in seconds), Row 3 the number of problems that were solved to optimality by CPLEX, Row 4 the average upper bound value (in seconds, named UB-1) at up to 60 s, Row 5 the average upper bound value (in seconds, named UB-2) at up to 10,800 s. The experiments with a large computation time allowed us to get further information on the optimal solutions. The best known value of upper bounds is reported in bold. Row 6 gives the average best known value of lower bound (in seconds, named LB) obtained with the largest computation time, Row 7 the average optimality gap (in percentage, named GAP-1) computed as follows: \( (UB-2 - LB)/LB \). Row 8 the average optimality gap (in percentage, named GAP-2) computed as follows: \( (UB-2 - LB)/LB \).

From Table 4, we have the following observations. All ASP instances are solved to (near)optimality (the optimality gap is always below 1%, except for the average tardiness at MXP airport showing around 10% optimality gap). The results obtained at 60 s of computation are similar to the ones obtained with larger computation times (comparing UB-1 versus UB-2). We conclude that the ASP instances can be efficiently solved in a short computation time (up to 60 s on a standard processor), compatible with real-time application.

Regarding the specific performance of the various objective functions, the Tardy Jobs formulations present the larger number of optimal solutions, while Avg Tardiness and Avg Completion the lowest number. More optimal solutions are generally obtained for the objective functions based on a maximum delay minimization compared to the ones based on an average delay minimization. Also, Priority Equity presents a larger number of optimal solutions than Priority Tardiness. The problem of minimizing the maximum consecutive delay, even if this is done per priority class, is thus easier to solve to optimality by CPLEX than the weighted average consecutive delay minimization.

5.3. Optimizing an objective and looking at the other objectives

This subsection studies how the optimal ASP solutions computed for a specific indicator infer the quality of the other indicators. The proposed analysis permits to assess the quality of non-dominated solutions for one objective function in terms of the other performance indicators.

Fig. 8 presents average results on the 35/40 ASP instances solved to optimality by all ASP formulations. Each plot in this figure reports the average optimality gap in terms of all performance indicators.
follows: (value obtained for the secondary indicator / optimal value for that indicator). For instance, the top-left plot reports the average results obtained for the Max Tardiness formulation, in which Avg Tardiness has 105% optimality gap, Priority Equity 41% and so on.

From Fig. 8, Avg Tardiness presents the best compromise solutions, since the optimality gap is, on average, up to 70% for all secondary indicators. In general, the objective functions based on a maximum delay minimization present larger gaps compared to the ones based on an average delay minimization (see left versus right plots). Specifically, Avg Tardiness presents a gap of 28% for Max Tardiness and smaller gaps than Max Tardiness for all secondary indicators but Priority Equity and Tardy Jobs. The combination of Avg and Max Tardiness is thus worthy of investigation.

Regarding the objective functions based on classes and weights, optimizing with priorities does not cause a serious drop of the related performance indicators. In fact, Priority Equity presents good values of Max Tardiness, that is the most equitable performance indicator. Similarly, Priority Tardiness gives good values in terms of Avg Tardiness.

When looking at the throughput as a secondary indicator, most of the objective functions have performance very close to the optimal values of Max Completion and Avg Completion. We conclude that these indicators can easily be taken into account by the studied ASP formulations.

![Fig. 8. Optimal solutions for an indicator viewed in terms of the other indicators.](image-url)
Tardy Jobs are among the less equitable objective functions, since they prefer to significantly penalize the behavior of a few aircraft in order to have the set of tardy aircraft as small as possible. Comparing the two corresponding formulations in terms of the other performance indicators, Tardy Jobs $P=300$s outperforms Tardy Jobs $P=0s$. In general, the latter formulation presents the worst values in terms of the various performance indicators.

It is interesting to note that, even if the considered objective functions are based on apparently similar performance indicators, the solutions optimized for a single indicator have the drawback to deteriorate the performance related to some of the other performance indicators.

5.4. A practical scheduling rule

This subsection studies the quality of the solutions computed by a commonly used scheduling rule: First Come First Served (FCFS). We observe that more elaborated approaches than FCFS, such as algorithms based on max position shifting \cite{10,29,30}, can be a more fairer comparison against real world behavior, since controllers usually modify the FCFS solution by switching groups of two or more aircraft around in order to improve throughput on some bottleneck resources.

Table 5 shows the average value (named UB, in seconds) on the 40 ASP instances of Section 5.1 obtained for the FCFS rule, and the number of optimal solutions for each performance indicator. The average computation time is less than 1 s.

Fig. 9 presents a plot with average results on 35/40 instances of Section 5.1 obtained for the FCFS rule, and the number of optimal solutions for each performance indicator. The average optimality gap is less than 1 s.

Table 6 shows the average value (named UB, in seconds) on the 35/40 instances of Section 5.1 obtained for the FCFS rule, and the number of optimal solutions for each performance indicator. The average optimality gap is less than 1 s.

<table>
<thead>
<tr>
<th>Objective function</th>
<th>MXP (UB (s))</th>
<th>N. of optimal sol.</th>
<th>FCO (UB (s))</th>
<th>N. of optimal sol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Tardiness</td>
<td>529.4</td>
<td>4</td>
<td>493.6</td>
<td>3</td>
</tr>
<tr>
<td>Avg Tardiness</td>
<td>128.2</td>
<td>0</td>
<td>115.1</td>
<td>0</td>
</tr>
<tr>
<td>Priority Equity</td>
<td>212.5</td>
<td>0</td>
<td>148.8</td>
<td>1</td>
</tr>
<tr>
<td>Priority Tardiness</td>
<td>2302.0</td>
<td>0</td>
<td>2230.2</td>
<td>0</td>
</tr>
<tr>
<td>Max Completion</td>
<td>3458.1</td>
<td>7</td>
<td>3657.6</td>
<td>9</td>
</tr>
<tr>
<td>Avg Completion</td>
<td>2990.4</td>
<td>0</td>
<td>2755.3</td>
<td>0</td>
</tr>
<tr>
<td>Tardy Jobs $P=0$</td>
<td>13.2</td>
<td>0</td>
<td>13.4</td>
<td>0</td>
</tr>
<tr>
<td>Tardy Jobs $P=300$</td>
<td>4.9</td>
<td>6</td>
<td>5.3</td>
<td>10</td>
</tr>
</tbody>
</table>

5.5. Combined formulation

When dealing with several performance indicators, we believe that a weighted objectives approach is not reasonable, since it would be very difficult to identify the right value of the weights. For this reason, we present the following combined formulation. First of all, we identify the ASP formulation of Section 4 that has the best trade-off performance. Then, we extend that formulation by introducing a set of additional constraints related to the non-optimized performance indicators. The additional constraints are generated via the following constraint generation procedure:

Phase I: A constraint $\alpha_i \leq \beta_i$ is added regarding the value $\alpha_i$ of each non-optimized indicator $i$ in order to limit the deterioration of its performance up to a given threshold value $\beta_i$.

Phase II: The resulting ASP formulation is solved to (near) optimality, provided that any ASP solution exists for the given additional constraints.

Phase III: If at least a feasible ASP schedule exists, the procedure returns the best solution computed by the solver (we use CPLEX with a time limit of 10,800 s). Otherwise, the new constraints introduced in phase I are revised by increasing the threshold value $\beta_i$ related to the value $\alpha_i$ of each performance indicator $i$.

The procedure iterates the three phases till a feasible ASP schedule is found.

The reason for studying the combined formulation is to investigate whether ASP solutions exist with a limited deterioration of the non-optimized indicators. Furthermore, the combined formulation does not require to set parameters in the objective support systems for solving the ASP based on the consideration and optimization of a number of performance indicators.
function, while it would be difficult to identify the right value when using e.g. parameters in the objective function in order to weight the different performance indicators.

From the results in Section 5.3, Avg Tardiness presents the best trade-off performance, since it is the only ASP formulation that is, on average, up to 70% from the optimal solution of each performance indicator. We therefore applied the constraint generation procedure to this ASP formulation and each of the 40 ASP instances of Section 5.1. Initially, the threshold value $\beta_i$ has been fixed equal to the best known value $\alpha_i$ of each secondary indicator $i$. At each iteration of the constraint generation procedure, the current threshold value $\beta_i$ is increased of $0.1 \alpha_i$ for each secondary indicator $i$ until a feasible schedule is found. Specifically, the following number of feasible schedules is found: 7 for 30% increase of the initial threshold value, 10 for 40% increase, 13 for 50% increase, 3 for 60% increase, 4 for 70% increase, and 3 for 80% increase. The computation of a trade-off schedule is thus a complex problem and a significant relaxation to the best known value of some secondary indicator is required to compute a feasible schedule for the studied ASP instances.

Table 6 presents average results on the 40 ASP instances for the combined formulation generated by the constraint generation procedure, in terms of the same information reported in Table 5.

Fig. 10 presents the average optimality gap for each performance indicator as in Section 5.3. The plot presents the average results on 35/40 instances of Section 5.1 for which an optimal ASP solution has been computed for all performance indicators. The computational results in Fig. 10 show that the best trade-off solutions are now computed by the solver for the combined formulation. The average distance from the optimal solution is reduced in the three worst cases (55% for Tardy Jobs $P=0$s, 43% for Tardy Jobs $P=300$s and 39% for Priority Equity) compared to Avg Tardiness. However, this improvement is obtained at the cost of a light deterioration the performance of other performance indicators, including the one minimized in the objective function (that has the largest worsening, on average, up to 5%).

### 6. Conclusions and further research

This paper presents microscopic formulations of the ASP with high precision modeling and the evaluation of alternative objective functions. We examine the trade-off between some classical performance indicators in the take-off and landing aircraft scheduling literature, since we believe that there is still not a generally recognized objective function for the ASP. We observe that optimal solutions are very often computed by a commercial solver within one minute of computation on a standard processor. Furthermore, the solver computes different ASP solutions for the ASP formulations with different objective functions. Finally, a pool of (near) optimal ASP solutions is provided to the traffic controller along with quantitative information on numerous performance indicators.

An extensive set of computational experiments shows the existence of relevant gaps between the ASP solutions computed focusing on the studied aspects of the ASP. A trade-off between the quality of various performance indicators is found for the ASP solutions computed via the different formulations. In particular, the Avg Tardiness results to be a good trade-off formulation, and its solutions outperform the FCFS solutions in terms of all performance indicators. However, better trade-off solutions exist if one introduces additional constraints in Avg Tardiness.

In general, the development of an ASP formulation taking into account multiple performance indicators is a challenging problem. The solutions computed for a combined formulation may improve some indicator, while they may have the drawback to deteriorate the performance related to some other indicator. However, we believe that this work moves the interest of researchers and practitioners in paying more attention to the various modeling aspects and performance indicators related to the ASP, and thus on the inherent multi-objective nature of this problem.

Further research will be concentrated on developing real-time efficient (eventually heuristic) scheduling algorithms for specific ASP formulations that would (1) reduce the optimality gap found by CPLEX, (2) reduce the time to compute the best solution, (3) solve large-size ASP instances to (near)optimality. Furthermore, we intend to further improve the quality gaps in terms of the best known values of each indicator. The latter result can be achieved by investigating ASP formulations with multiple objectives, or by introducing additional ASP constraints while optimizing a single indicator (as shown in this paper for the combined formulation).

Other promising research directions should focus on the coordination of the ASP solutions with related problems, such as the en-route, ground and gate scheduling problems [32,59]. Additional factors should be considered, such as evaluating the impact of a dynamic system setting, integrating the ASP solutions with the ground scheduling solutions and the en-route scheduling solutions, dealing with other objectives, constraints and variables (e.g. aircraft routing and speed control).

### Acknowledgment

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### Appendix A

The traffic situation of Section 4.3 is modelled by the alternative graph of Fig. 11. Each node of the alternative graph represents an operation, e.g. A12 is aircraft A entering runway 12. Black solid arrows represent fixed directed arcs, while coloured dashed

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arrows represent alternative directed arcs. The length of each arc is depicted in the graph for the routes of Fig. 7.

In this example there are 16 alternative pairs. For the two landing aircraft A and C, we assume that the capacity in the holding circle is unbounded, so there are no potential conflicts between them at the entrance of the TCA. However, we use four alternative pairs (two between nodes A1 and A4 and two between nodes C1 and C4) in order to model the possibility to perform one or two circles in the airborne holding. The shortest circle takes 180 (see the red alternative pairs ((A1,A4), (A4,A1)) and ((C1,C4), (C4, C1)), while the longest 240 (see the green alternative pairs ((A1, A4), (A4,A1)) and ((C1,C4), (C4,C1))). Since aircraft A and C have the same route in the TCA, four alternative pairs model the air segments: the (blue) pair ((A4,C4), (C9,A7)) and the (brown) pair ((C4, A4), (A7,C9)) for air segment 4; the (blue) pair ((A11,C11), (C12, A12)) and the (brown) pair ((C11,A11), (A12,C12)) for air segment 11. Another alternative pair between A and C is required on the runway resource: the (orange) pair ((Aout,C12), (Cout, A12)).

The two take-off aircraft B and D have a potential conflict on the runway resource with the landing aircraft. We thus have to use another five alternative pairs for resource 12: ((Aout,B12), (B14, A12)) (depicted in red), ((Aout,D12), (D14,A12)) (depicted in green), ((B14,C12), (Cout,B12)) (depicted in violet), ((B14,D12), (D14,B12)) (depicted in blue turquoise), ((Cout,D12), (D14,C12)) (depicted in grey). Aircraft B and D also have a potential conflict on air segment 14, that is modelled by the blue pair ((B14,D14), (Dout, Bout)) and the brown pair ((D14,B14), (Bout,Dout)).

Appendix B

Given the example of Section 4.3, Fig. 12 shows the optimal solution for the formulation “Tardy Jobs P = 0”. The alternative arcs selected in the solution are shown with coloured dashed arrows. The corresponding (complete) selection of the alternative arcs is the following: ((A4,C4) and (A7,C9) (aircraft A is scheduled first on air segment 4); ((A11,C11) and (A12, C12) (aircraft A is scheduled first on air segment 11); (C1,C4) of length 180 and (C4,C1) of length 180 (aircraft C must perform circles in the holding of length 180); ((A4,A1) of length –180 and (A4,A1) of length 0 (aircraft A does not perform circles in the holding); (D14,A12), (D14,B12), (D14,C12), (Aout,B12), (Aout,C12) and (B14,D12) (the runway sequence is D – A – B – C); (D14,B14) and (Dout,Bout) (aircraft D is scheduled first on air segment 14).
Fig. 13. Gantt charts for the optimal ASP solutions for the numerical example.
Fig. 13 illustrates the Gantt chart of the 7 optimal ASP solutions for the numerical example (we note that the optimal solution for Avg Tardiness and Priority Equity is the same ASP schedule).

References


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